2022 ABET ASSESSMENT MECH 322 FLUID MECHANICS/ MECH 420 HEAT TRANSFER

Dr. K. J. Berry, PE ASME FELLOW

| FLIUD MECHANICS/HEAT TRANSFER PROJECT |  |  |  |
| :---: | :---: | :---: | :---: |
| SPRING 2022 Fluids/Heat Transfer |  |  |  |
| PERFORMANCE INDICATOR |  |  |  |
| GROUP | Identify | Facts | Convert |
| 1 | 4 | 4 | Demonstrate |
| 2 | 4 | 4 | 4 |
| 3 | 4 | 4 | 4 |
| 4 | 1 | 0 | 0 |
| 5 | 4 | 4 | 3 |
| 6 | 4 | 4 | 4 |
| 7 | 4 | 4 | 4 |
| 8 | 4 | 4 | 3 |

# MECH-420 HEAT TRANSFER GROUP (2) PROJECT TYPED, COMPUTER PLOTS, EXCEL FILE, PDF FILE SUBMISSION DUE FRIDAY WEEK \#10 <br> <br> 10:00 AM BB 

 <br> <br> 10:00 AM BB}

As engineers for Parametric Design, Inc you are requested to evaluate a facility cooling concept on your clients remote Pacific Ocean resort complex 500 miles west of California. Air at 26C is routed through thick wall cooper tubing, $\mathrm{Di}=0.20 \mathrm{~m}, \mathrm{Do}=0.35 \mathrm{~m}$ that flows through the cool pacific waters at a depth of 50 m at a volume flow rate that is to vary from $0.02 \mathrm{~m} 3 / \mathrm{s}$ to $1 \mathrm{~m} 3 / \mathrm{s}$. The velocity and temperature profiles can be assumed as shown below.



Your engineering staff is requested to study the parametric relationships between the submerged pipe length (L), volume flow rate, pressure drop, fan power to overcome friction within the smooth tube, and heat transfer rates, and annual operating cost at $\$ 0.15 / \mathrm{kWh}$, to ensure a discharge air temperature of 19C.

You are to use your engineering judgement to determine what data is needed, what calculations are needed, what plots are needed, and what assumptions are needed to complete the engineering analysis report and the ROAD MAP, and to meet Berry's (CEO) expectations as you know his proclivity for details and logic.

Provide a 2-4-page, 1.5 line spacing, 11 pitch font, research essay regarding the threat and impact of global warming and climate change, and your perspective on the engineering challenges your generation must face to save mankind.

All reports ae to be typed professionally as they will be posted online for display and review for future students and professionals.

Jacob Feenstra and Cody Dolby
MECH-420-01
Dr. K. J. Berry
6/10/2022


- Pipe Length

Heat Transfer Design Project

## Variables:

- Volume Flow Rate
- Pressure Drop
- Fan Power
- Heat Transfer Rates
- Annual Operating Cost


## Constraints:

- Inlet Air Temperature: $26^{\circ} \mathrm{C}$
- Outlet Air Temperature: $19^{\circ} \mathrm{C}$
- Tubing

$$
\begin{array}{ll}
\circ & D_{-} \mathrm{i}=0.20 \mathrm{~m} \\
\circ & \mathrm{D}_{\mathrm{B}} \mathrm{o}=0.35 \mathrm{~m}
\end{array}
$$

- Depth of Tubing: 50 m
- Volume Flow Rate: $0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}$ to $1 \mathrm{~m}^{\wedge} 3 / \mathrm{s}$
- Cost of Power: $\$ 0.15 \mathrm{~kW} / \mathrm{hr}$

Provided Graphs:


Note: Temperature is in ${ }^{\circ} \mathrm{C}$


Note: Velocity is in $\mathrm{cm} / \mathrm{s}$, not $\mathrm{m} / \mathrm{s}$

## Analysis

The geometry of the problem is a cylinder with internal flow and external convection. This geometry will dictate how the heat transfer rate, q , is solved for. In the process, L is solved for, which can be used to calculate pressure drops, power needed for internal flow, and cost of power.

## Heat Transfer Rate Calculations

To start, property values will be found and the gemoetry defined. For the internal flow, a mean temperature is estimated with
$\frac{T_{i}+T_{o}}{2}=\frac{299.15 K-292.15 K}{2}=295.65 K$
and the external flow's temperature is fixed (as read from the depth graph) at 280.9 K .

The assumption was made that the properties of the air were similar to that at atmospheric pressures (from Table A.4) and that the properties of the water were similar to that of a saturated fluid (Table A.6).

Thus, the properties for the internal flow are as follows:

$$
\begin{aligned}
& \rho_{i}=1.1817 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} ; \mathrm{Pr}_{i}=0.7081 ; \mu_{i}=0.00001824 \frac{\mathrm{Ns}}{\mathrm{~m}^{2}} ; c_{p, i}=1007 \frac{\mathrm{~J}}{\mathrm{kgK}} ; k_{i}=0.02595 \frac{\mathrm{~W}}{\mathrm{mK}} ; \\
& v_{i}=f\left(\dot{V}_{i}\right)
\end{aligned}
$$

The properties for the external flow are as follows:

$$
\begin{aligned}
\rho_{\infty} & =1000 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} ; \operatorname{Pr}_{\infty}=9.999 ; \mu_{\infty}=0.001387 \frac{\mathrm{Ns}}{\mathrm{~m}^{2}} ; c_{p, \infty}=4196.38 \frac{\mathrm{~J}}{\mathrm{kgK}} ; k_{\infty}=0.5834 \frac{\mathrm{~W}}{\mathrm{mK}} ; \\
v_{\infty} & =14.675 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

The geometries and properties of the pipe are as follows:
$D_{i}=0.2 m ; D_{o}=0.35 m$
$k_{\text {pipe }}=46.7825 \frac{\mathrm{~W}}{m^{2} \mathrm{~K}}\left(\right.$ Calculated from Table A.1, commercial bronze @ $\left.T_{\text {mean }}\right)$

Ultimately, $U A\left[\frac{W}{K}\right]=\frac{1}{\Sigma R_{\text {th }}\left[\frac{K}{W}\right]}$ will drive the solution for different internal flow rates.
$U A=\frac{1}{\Sigma R_{t h}}$ can be expanded to better fit the specified problem. Assuming no fouling factors, and

$$
U A=U_{o} A_{o}=U_{i} A_{i}=\frac{1}{\frac{1}{h_{i} A_{i}}+\frac{\ln \left(\frac{r_{o}}{r_{i}}\right)}{2 \pi k_{p i p e} L}+\frac{1}{h_{o} A_{o}}}
$$

$U_{o}$ can be evaluated direclty, via
$U_{o}\left[\frac{W}{m^{2} K}\right]=\frac{1}{\frac{A_{o}}{h_{i} A_{i}}+\frac{A_{o} \ln \left(\frac{r_{o}}{r_{i}}\right)}{2 \pi k_{p p p e} L}+\frac{A_{o}}{h_{o} A_{o}}}$
Given $A_{i}=\pi D_{i} L\left[m^{2}\right]$ and $A_{o}=\pi D_{o} L\left[m^{2}\right]$,

$$
U_{o}\left[\frac{W}{m^{2} K}\right]=\frac{1}{\frac{\pi D_{0} L}{h_{i} \pi D_{i} L}+\frac{\pi D_{o} L * \ln \left(\frac{r_{o}}{r_{i}}\right)}{2 \pi k_{p i p L} L}+\frac{\pi D_{o} L}{h_{o} \pi D_{o} L}}=\frac{1}{D_{o}\left(\frac{1}{h_{i} D_{i}}+\frac{\ln \left(\frac{r_{o}}{r_{i}}\right)}{2 k_{p i p e}}+\frac{1}{h_{o} D_{o}}\right)}
$$

Another equation that the definied gemoetry gives validity to is the following equation:
$\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}[$ unitless $]=e^{\left(\frac{-U A}{\dot{m} * c_{p, i}}\right)}$
This will provide another constraint that L can be solved for using differing internal flow rates.

To solve for $U_{o}, h_{i}$ and $h_{o}$ must be solved for using the Reynolds number and Nusselt's number. To solve for $R e_{D, i}$, the mean velocity must be solved for inside the pipe. This will be done as a function of volumetric flow rate, $\dot{V}_{i}$.
$\dot{V}_{i}\left[\frac{m^{3}}{\mathrm{~s}}\right]=V_{\text {mean }, i}\left[\frac{m}{\mathrm{~s}}\right] * A_{\text {cross-section, }, i}\left[m^{2}\right]$, therefore, $V_{\text {mean }}=\frac{\dot{V}_{i}}{A_{\text {cross-section, } i}}$, and thus,
$R e_{D, i}[$ unitless $]=\frac{\rho_{i} V_{\text {mean, } i} D_{i}}{\mu_{i}}$
Since the Reynolds number for internal flow is always greater than 2300, there is always turbulent flow. The following equation solves for all internal flow Nusselt numbers ( $\mathrm{n}=0.3$ due to cooling):
$N U_{i, \text { turbulent }}[$ unitless $]=0.023 * R e_{D, i}^{4 / 5} \operatorname{Pr}^{0.3}$
Knowing $N U_{i, \text { turbulent }}, h_{i}$ can be found:
$h_{i}\left[\frac{W}{m^{2} K}\right]=\frac{N U_{i, \text { turbulent } *} * k_{i}\left[\frac{W}{m K}\right]}{D_{i}[m]}$
Since $V_{\infty}$ is known, $R e_{D, \infty}$ can be solved directly.
$R e_{D, \infty}[$ unitless $]=\frac{\rho_{\infty} V_{\infty} D_{o}}{\mu_{\infty}}$

Since this Reynolds nubmer is greater than $10^{6}$, the following equation will be used to calculate Nusselt's number for the external flow:

$$
\begin{equation*}
\overline{N u}_{D}=0.3+\frac{0.62 R e_{D}^{1 / 2} \operatorname{Pr}^{1 / 3}}{\left[1+(0.4 / P r)^{2 / 3}\right]^{1 / 4}}\left[1+\left(\frac{R e_{D}}{282,000}\right)^{5 / 8}\right]^{4 / 5} \tag{7.46}
\end{equation*}
$$

Knowing $N U_{\infty}$,

$$
h_{o}\left[\frac{W}{m^{2} K}\right]=\frac{N U_{\infty} * k_{\infty}\left[\frac{W}{m K}\right]}{D_{o}[m]}
$$

Now knowing $h_{i}$ and $h_{o}, U_{o}$ can be solved for:

For each value of $U_{o}$ (unique to a specific internal flow rate), the lenght can be calculated to achieve the desired values for $T_{m, i}$ and $T_{m, o}$. If $U A=U_{o} A_{o}$ and $A_{o}=\pi D_{o} L$,

$$
\begin{aligned}
& \frac{T_{\infty}-T_{m, 0}}{T_{\infty}-T_{m, i}}[\text { unitless }]=e^{\left(\frac{-u A}{\dot{m} * c_{p, i}}\right)} \text { can be rewritten as } \frac{T_{\infty}-T_{m, 0}}{T_{\infty}-T_{m, i}}[\text { unitless }]=e^{\left(\frac{-u_{o} \pi D_{0} L}{\dot{m} * c_{p, i}}\right)} \\
& \frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}[\text { unitless }]=e^{\left(\frac{-U_{o} \pi D_{o} L}{\dot{m} * c_{p, i}}\right)} \text { can be rewritten as }
\end{aligned}
$$

$$
\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right)[\text { unitless }]=\frac{-U_{o}\left[\frac{\mathrm{~W}}{m^{2} K}\right] \pi D_{o}[m] L[m]}{\dot{m}\left[\frac{\mathrm{~kg}}{\mathrm{~s}}\right] * C_{p, i}\left[\frac{\mathrm{~J}}{\mathrm{kgK}}\right]} \text { which in turn can be rewritten as }
$$

Finally, a value for q can be calculated using the unique values determined for each flow rate. Recognizing that,
$U_{o}\left[\frac{W}{m^{2} K}\right] \pi D_{o}[m] L[m]=U_{o}\left[\frac{W}{m^{2} K}\right] A_{o}\left[m^{2}\right]=U A\left[\frac{W}{K}\right]$ and $\frac{\left(\Delta T_{o}-\Delta T_{i}\right)[K]}{\ln \left(\frac{\Delta T_{o}}{\Delta T_{i}}\right)[\text { unitless }]}=\Delta T_{L M}[K]$ (with $\Delta T_{o}=T_{\infty}-T_{o}$ and $\Delta T_{i}=T_{\infty}-T_{i}$ ),
each heat transfer rate, q , can be solved for:
$q[W]=U A\left[\frac{W}{K}\right] * \Delta T_{L M}[K]$

## Pressure Drop Calculations

The pressure drop for each volume flow rate can be calculated. Since every volume flow rate creates turbulent flow, the following equation can solve for the friciton factor:

$$
\frac{1}{\sqrt{f}}[\text { unitless }]=-1.8 \log _{10}\left(\left(\frac{\left(\frac{\epsilon[m]}{D_{i}[m]}\right)}{3.7}\right)^{1.11}+\frac{6.9}{\text { Re }_{D, i}[\text { unitless }]}\right)
$$

Assuming a smooth pipe condition, (with $\epsilon=0$ )

$$
\frac{\epsilon[m]}{D[m]}=0
$$

Therefore, friction factor, f , can be solved for:
$f[$ unitless $]=\left(-1.8 \log _{10}\left(\left(\frac{\left(\frac{\epsilon}{D_{i}}\right)}{3.7}\right)^{1.11}+\frac{6.9}{R e_{D, i}}\right)\right)^{-2}$

This friction factor can be used to solve directly fro the pressure drop. Using each unique value for $v_{\text {mean }}$ and $f$, and considering that $\Delta x=L$, the pressure drop can be solved for.
$\Delta p[\mathrm{~Pa}]=f[$ unitless $]\left(\frac{\left.\rho_{i}\left[\frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right] v_{\text {mean }\left[\frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}\right]}^{2}\right)\left(\frac{\mathrm{L}[\mathrm{m}]}{D_{i}[m]}\right), ~\left(\frac{1}{2}\right)}{2}\right)$

## Flow Power

To calculate the power that must be available for a specific volume flow rate, the mass flow rate must be calculated first.
$\dot{m}\left[\frac{k g}{s}\right]=\dot{V}_{i}\left[\frac{m^{3}}{s}\right] * \rho_{i}\left[\frac{k g}{m^{3}}\right]$

Then, the following equation can be used to solve for the power necessary for the volumetric flow rate:
$P_{i}[W]=\frac{\dot{m}\left[\frac{k g}{s}\right] \Delta p[P a]}{\rho_{i}\left[\frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right]}$

## Cost of Flow Power

To calculate the cost of the flow power required for each specific volume flow rate, multiply the power required by the cost of power $\left(0.15\left[\frac{\$}{k W h r}\right]\right)=\left(0.00015\left[\frac{\$}{W h r}\right]\right)$.

$$
\frac{\operatorname{Cost}}{\text { hour }}\left[\frac{\$}{h r}\right]=P_{i}[W] * 0.00015\left[\frac{\$}{W h r}\right]
$$

To generate an annual cost, convert the hours to years (1 year $=8760$ hours).
Thus, the annaul cost of power can be calculated:
$\operatorname{Cost}[\$]=P_{i}[W] * 0.00015\left[\frac{\$}{W h r}\right] * 8760 h r$

## Graphs:




Pressure Drop vs Volumetric Flow Rate


Internal Flow Power vs Volumetric Flow Rate


Annual Cost vs Volumetric Flow Rate



Below is a diagram of the engineering challenge at hand. The challenge is to evaluate various parameters of the system if the outlet temperature of the pipe remains at a constant temperature of 19C while the volumetric flow rate is varied.


ROADMAP
Our engineering staff have determined this problem is a "Special Case: Internal Flow/External Convection" problem type. An equation provided for this scenario can be rearranged to study what pipe length is required to keep the outlet temperature at the desired temperature of 19C while the volumetric flow rate through the pipe is varied.

1. Solve for $T_{\text {mean }}$, then find the properties of air flowing inside the pipe using Table A. 4
2. Use $T \infty$ to find the properties of water flowing around the pipe using Table A. 6
3. Solve for $h_{i}$ : internal pipe flow ( $h_{i}$ will vary with $R e$, which varies with internal mass flow)
a. Solve the Reynold's number for flow inside the pipe, determine Laminar or Turbulent, and chose appropriate formula for Nusselt \#
b. Calculate Nusselt \#, and plug into Dittus-Boelter to solve for $h_{i}$
4. Solve for $h_{0}$ : cross flow over a cylinder
a. Solve for the Reynolds number and find values for $m$ and $C$
b. Plug values for $m$ and $C$ into Nusselt equation and solve for $h_{\circ}$
5. Using the equation for UA, solve for $U$ and cancel L's.
a. Plug in values for $h_{o}$ and $h_{i}$
i. U will change as the internal volumetric rate changes.
6. With values for $U$, mass flow, and $c_{p}$ the special case equation can be rearranged to solve for $L$ while $T_{m, o}$ is held constant.
7. With values obtained for $h_{o}$ and $h_{i}$, and the given Temperature gradient the heat transfer rate of the pipe, pressure drop, fan power, and cost can be solved for - all will vary based on internal mass flow of the pipe.

Special Case: Internal Flow/External Convection

$$
\begin{aligned}
\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}} & =\exp \left(\frac{-\bar{U} A}{\dot{m} c_{p_{\text {air }}}}\right) \\
A & =\pi D L
\end{aligned}
$$

Rearrange to solve for L as a function of $\bar{U}$

$$
L=-\frac{\dot{m} c_{p_{a i r}} \ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right)}{\bar{U} \pi D}=\frac{\left(\frac{\mathrm{kg}}{\mathrm{~s}}\right)\left(\frac{\mathrm{J}}{\mathrm{~kg} \cdot \mathrm{~K})}\right)}{\frac{\mathrm{W}}{m^{2} \cdot K}(m)}=\frac{\frac{\mathrm{J}}{\mathrm{~s} \cdot \mathrm{~K}}}{\frac{\bar{s}}{m \cdot K}}=m
$$

To solve for L , must first find $\dot{m}, c_{p_{\text {air }}}, \bar{U}$

## Solving for $\dot{m}$

$$
\begin{gathered}
\dot{m}=\rho * v_{m} * A_{c} \\
v_{m}=\frac{\dot{Q}}{A_{c}}
\end{gathered}
$$

Where $v_{m}$ is the average velocity of the internal fluid, $\dot{Q}$ is the volumetric flow rate, and $A_{c}$ is cross sectional area of the pipe.

Substitute $v_{m}$, cancel $A_{c}$

$$
\dot{m}\left[\frac{k g}{s}\right]=\rho\left[\frac{\mathrm{kg}}{\mathrm{~m}^{3}}\right] * \dot{Q}\left[\frac{\mathrm{~m}^{3}}{\mathrm{~s}}\right]
$$

Mass flow rate will change as the volumetric flow rate is varied.

## Solving for $c_{p_{\text {air }}}$

Solving for $\mathrm{T}_{\mathrm{m}}$ to use Table values for air

$$
T_{m_{\text {air }[K]}}=\frac{T_{m, i}-T_{m, o}}{2}
$$

Using Table A.4, values for various properties of air can be interpolated using $\mathrm{T}_{\mathrm{m}}$

## Solving for $\bar{U}$

Solving for $T_{m}$ to use Table values for air

$$
\bar{U} A=\frac{1}{\sum R_{t h}}
$$

Substitute for $R_{t h}$ : Note THICK WALL $\rightarrow$ Conductive Term, using $\mathrm{k}_{\text {copper }}$

$$
\overline{U_{o}}=\frac{1}{A_{o}\left(\frac{1}{h_{i} A_{i}}+\frac{1}{h_{o} A_{o}}+\frac{\ln \left(\frac{r_{o}}{r_{i}}\right)}{2 \pi k_{c u} L}\right)}
$$

Cancelling $\pi$ and L's by plugging in $A=\pi D L$

$$
\overline{U_{o}}=\frac{1}{D_{o}\left(\frac{1}{h_{i} D_{i}}+\frac{1}{h_{o} D_{o}}+\frac{\ln \left(\frac{r_{o}}{r_{i}}\right)}{2 k_{c u}}\right)}=\frac{1}{m\left(\frac{1}{\left(\frac{W}{m^{2} \cdot K}\right)(m)}\right)}=\frac{W}{m^{2} \cdot K}
$$

Assuming that the copper pipe's temperature is approximately that of $T_{\infty}$, a value for $k_{c u}$ can be found using Table A. 1 - Note that $k$ does not change drastically with temperature so our team believes this assumption will not affect end results.
To solve for $\overline{U_{o}}$, values of $h_{i}$ and $h_{o}$ must be found.

Solving for $h_{o}$ : Cross Flow over a cylinder
Solve for Re

$$
R e=\frac{\rho * v_{m} * D_{o}}{\mu_{h 20}}=\frac{\left(\frac{\mathrm{kg}}{\mathrm{~m}^{3}}\right)\left(\frac{\mathrm{m}}{\mathrm{~s}}\right)(\mathrm{m})}{\frac{N s}{\mathrm{~m}^{2}}}=\frac{\frac{\mathrm{kg} \cdot \mathrm{~m}}{\mathrm{~s}}}{\mathrm{~N} . \mathrm{s}}
$$

Using Re, Table 7.2 gives values for m and C .

$$
\begin{gathered}
\frac{\overline{h_{o}} * D_{o}}{k_{h 20}}=C * \operatorname{Re}^{m} * \operatorname{Pr}^{1 / 3} \\
\text { Rearrange to find } h_{o} \\
\overline{h_{o}}=\frac{k_{h 20} * C * R e^{m} * \operatorname{Pr}^{1 / 3}}{D_{o}}=\frac{\frac{W}{m \cdot K}}{m}=\frac{W}{m^{2} \cdot K}
\end{gathered}
$$

$\mu_{h 20}, k_{h 20}$, and Pr found using properties of water at T $\infty$ (Table A.6)

## Solving for $h_{i}$ : Internal Cylinder Flow

Must first solve for Re to determine Turbulent or Laminar flow $\rightarrow$ determines which formula for $\overline{N U}$ will be used.

$$
\begin{gathered}
\operatorname{Re}=\frac{\rho * v_{m} * D_{i}}{\mu_{\text {air }}} \\
v_{m}=\frac{\dot{V}}{A_{c}}, A_{c}=\frac{\pi * D_{i}^{2}}{4}
\end{gathered}
$$

Rearrange and substitute:

$$
R e=\frac{4 * \rho * \dot{V}}{\pi * D_{i} * \mu_{\text {air }}}
$$

Reynold's number will change as the volumetric flow rate is varied.
All Re's above $2300 \rightarrow$ Turbulent Flow

$$
\begin{gathered}
\bar{h}_{l}=k_{\text {air }} \frac{\overline{N U}}{D_{i}} \\
\overline{N U}=0.023 * R e^{4 / 5} * \operatorname{Pr}^{n} \\
\text { Where } \mathrm{n}=0.3, \text { cooling: } \\
\bar{h}_{l}=k_{\text {air }} \frac{0.023 * R e^{4 / 5} * \operatorname{Pr}^{n}}{D_{i}}=\frac{\frac{W}{m \cdot K}}{m}=\frac{W}{m^{2} \cdot K}
\end{gathered}
$$

Pr of Air already found using Table A. 4 and $\mathrm{T}_{\mathrm{m}}$
Since Re varies with volumetric flow rate, $\bar{h}_{l}$ will also vary with volumetric flow rate.
L can now be solved for using $\dot{m}, c_{p_{\text {air }}} \bar{U}$.

## Solving for Pressure Drop

$$
\begin{gathered}
\Delta P=f\left(\frac{\rho * v_{m}^{2}}{2}\right)\left(\frac{\Delta x}{D_{i}}\right)=\frac{\left(\frac{k g}{m^{3}}\right)\left(\frac{m^{2}}{s^{2}}\right)(m)}{m}=\frac{k g}{m \cdot s^{2}}=\frac{N}{m^{2}}=P a \\
\Delta x=L
\end{gathered}
$$

Solving for $f$
Re inside pipe already solved $\rightarrow$ Turbulent Flow gives the equation:

$$
\frac{1}{\sqrt{f}}=-1.8 \log _{10}\left(\left(\frac{\frac{\varepsilon}{D}}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)
$$

Given a smooth tube $\rightarrow \frac{\varepsilon}{D}=0$
Solve for $f$ as a function of Re

$$
f=\left(\frac{1}{\left.-1.8 \log _{10}\left(\frac{\frac{\varepsilon}{D}}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)}\right)^{2}
$$

Pressure drop is a function of pipe length, $L$ and the friction factor, $f$.

## Solving for Fan Power

$$
\begin{gathered}
W_{\text {flow }}^{\cdot}=\frac{\dot{m} * \Delta P}{\rho}=\dot{V} * \Delta P=\frac{m^{3}}{s} \cdot \frac{N}{m^{2}}=\frac{N m}{s}=W \\
\text { Temperature Rise } \\
\Delta T=\frac{W_{\text {flow }}}{\dot{m} c_{p}}=\frac{W_{\text {flow }}}{\rho \dot{V}\left(c_{p} * 1000\right)}=\frac{\frac{J}{S}}{\left(\frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)\left(\frac{\mathrm{m}^{3}}{s}\right)\left(\frac{\mathrm{J}}{\mathrm{~kg} \cdot \mathrm{~K})}\right.}=K
\end{gathered}
$$

## Solving for Heat Transfer

$$
\begin{aligned}
& \text { Special Case: Constant } T_{\infty} \\
& \qquad q_{c o n v}=\bar{U} A \frac{\Delta T_{o}-\Delta T_{i}}{\ln \left(\frac{\Delta T_{o}}{\Delta T_{i}}\right)}
\end{aligned}
$$

OR
Newton's Law of Cooling:

$$
q_{c o n v}=\dot{m} c_{p}\left(T_{m, o}-T_{m, i}\right)=\left(\frac{\mathrm{kg}}{\mathrm{~s}}\right)\left(\frac{\mathrm{J}}{\mathrm{~kg} \cdot \mathrm{~K}}\right)(K)=\frac{\mathrm{J}}{\mathrm{~s}}=W
$$

Using either method to solve for q yields the same heat transfer rate relative to mass flow.

## Results



Graph 1: Volumetric Flow versus required Pipe Length


Graph 2: Pressure Drop versus required Pipe Length


Graph 3: Fan work versus required Pipe Length


Graph 4: Heat Transfer versus required Pipe Length


Graph 5: Cost versus required Pipe Length

## Conclusions

For the required outlet air temperature of 19C, the length of pipe will be dependent on the selected volumetric flow rate. Volumetric flow and Pipe Length will have a logarithm relationship, as shown in Graph 1. Conceptually, this is due to the relationship between heat transfer and flow rate. With an increased flow rate, the only way to maintain the desired temperature is to increase the surface area of the system, by doing this a higher heat transfer rate can be achieved (shown in Graph 4) and the desired outlet air temperature can still be maintained. Consequently, pressure drop will also increase due to the increase in pipe length if a higher flow rate is required, as shown in Graph 2. This also correlates with an increase in Fan Work and cost per hour, shown in Graphs 3 and 5.


Cory Mazure \& Arthur Jillson

## Problem Description

As engineers for Parametric Design, Inc you are requested to evaluate a facility cooling concept on your clients remote Pacific Ocean resort complex 500 miles west of California. Air at 26C is routed through thick wall cooper tubing, $\mathrm{Di}=0.20 \mathrm{~m}, \mathrm{Do}=0.35 \mathrm{~m}$ that flows through the cool pacific waters at a depth of 50 m at a volume flow rate that is to vary from $0.02 \mathrm{~m} 3 / \mathrm{s}$ to $1 \mathrm{~m} 3 / \mathrm{s}$. The velocity and temperature profiles can be assumed as shown below.



Your engineering staff is requested to study the parametric relationships between the submerged pipe length (L), volume flow rate, pressure drop, fan power to overcome friction within the smooth tube, and heat transfer rates, and annual operating cost at $\$ 0.15 / \mathrm{kWh}$, to ensure a discharge air temperature of 19 C .

You are to use your engineering judgement to determine what data is needed, what calculations are needed, what plots are needed, and what assumptions are needed to complete the engineering analysis report and the ROAD MAP, and to meet Berry's (CEO) expectations as you know his proclivity for details and logic.

## Given + Property Information

Much of this information was retrieved from the book

- Resort 500 miles west of california
- Initial Air Temp $26^{\circ} \mathrm{C}$, Final Air Temp $19{ }^{\circ} \mathrm{C}$, Mean $=22.5^{\circ} \mathrm{C}$

$$
\begin{array}{ll}
\circ & \rho @ 22.5^{\circ} \mathrm{C}=1.227\left(\mathrm{~kg} / \mathrm{m}^{\wedge} 3\right) \\
\circ & \mathrm{cp} @ 22.5^{\circ} \mathrm{C}=1006.72(\mathrm{~J} / \mathrm{kg}-\mathrm{K}) \\
\circ & \mu @ 22.5^{\circ} \mathrm{C}=0.0000182425\left(\mathrm{~N}-\mathrm{s} / \mathrm{m}^{\wedge} 2\right) \\
\circ & \mathrm{k} @ 22.5^{\circ} \mathrm{C}=0.02594(\mathrm{~W} / \mathrm{m}-\mathrm{k}) \\
\circ & \operatorname{pr} @ 22.5^{\circ} \mathrm{C}=0.71064
\end{array}
$$

- Copper tubing carrying air
- $\mathrm{Di}=0.2 \mathrm{~m}$
- $\mathrm{Do}=0.35 \mathrm{~m}$
- Depth $=50 \mathrm{~m}$
- Volumetric flow rate
- From .02-1 $\mathrm{m}^{3} / \mathrm{s}$
- Copper properties
- Kwall=401[W/m-k]
- Water Properties
- Temperature @ $50 \mathrm{~m}=\mathrm{T} \infty=13^{\circ} \mathrm{C}=286.15^{\circ} \mathrm{K}$
- Velocity @ $50 \mathrm{~m}=1465[\mathrm{~cm} / \mathrm{s}]=14.65[\mathrm{~m} / \mathrm{s}]$
- $\rho @ 13^{\circ} \mathrm{C}=1000\left(\mathrm{~kg} / \mathrm{m}^{\wedge} 3\right)$
- $\mu @ 13^{\circ} \mathrm{C}=0.00119165\left(\mathrm{~N}-\mathrm{s} / \mathrm{m}^{\wedge} 2\right)$
- $\mathrm{k} @ 13^{\circ} \mathrm{C}=0.59184$ (W/m-k)
- Pr@ $13^{\circ} \mathrm{C}=8.56$
- Specific Volume @ $13^{\circ} \mathrm{C}=0.00100023\left(\mathrm{~m}^{\wedge} 3 / \mathrm{kg}\right)$


## Requested to Study

- Submerged pipe length
- Volumetric flow rate
- Pressure drop
- Fan power to overcome friction
- Heat transfer rates
- Annual operating cost $\$ 0.15 / \mathrm{kWh}$ to ensure discharge air temp is $19^{\circ} \mathrm{C}$


## Equations + Roadmaps

Here you will find the equations and worked out roadmaps that allowed for us to power the data seen in the attached spreadsheet.

## Tmean for Air Properties

We started with analyzing the mean temperature for the air and reporting it in Kelvin

Tmean $=\frac{T_{m, i}+T_{m, o}}{2}=\frac{26+19}{2}=22.5 C=295.65[\mathrm{~K}]$

## Finding Pr

The Pr is relevant in the Nusselt number calculation. It must be calculated for the internal and external conditions. It is found within table A.6.

Water: Fluid (13 C, 286K)

Table A. 6 Thermophysical Properties of Saturated Water ${ }^{a}$

| Temperature, $T$ (K) | Pressure,$p \text { (bars) }^{b}$ | Specific Volume ( $\mathrm{m}^{3} / \mathrm{kg}$ ) |  | Heat of Vaporization, $\boldsymbol{h}_{f g}$ (kJ/kg) | $\begin{gathered} \text { Specific } \\ \text { Heat } \\ (\mathrm{kJ} / \mathrm{kg} \cdot \mathrm{~K}) \end{gathered}$ |  | Viscosity$\left(\mathbf{N} \cdot \mathrm{s} / \mathrm{m}^{2}\right)$ |  | Thermal Conductivity (W/m $\cdot \mathbf{K}$ ) |  | Prandtl Number |  | Surface <br> Tension, $\begin{gathered} \sigma_{f} \cdot 10^{3} \\ (\mathrm{~N} / \mathrm{m}) \end{gathered}$ | $\begin{gathered} \text { Expansion } \\ \text { Coeffi- } \\ \text { cient, } \\ \boldsymbol{\beta}_{f} \cdot 10^{6} \\ \left(\mathbf{K}^{-1}\right) \end{gathered}$ | Temperature, $T(\mathrm{~K})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $v_{f} \cdot 10^{3}$ | $v_{g}$ |  | $c_{p, f}$ | $c_{p, g}$ | $\mu_{f} \cdot 10^{6}$ | $\mu_{g} \cdot 10^{6}$ | $k_{f} \cdot 10^{3}$ | $k_{g} \cdot 10^{3}$ | $\mathrm{Pr}_{f}$ | $\boldsymbol{P r}_{g}$ |  |  |  |
| 273.15 | 0.00611 | 1.000 | 206.3 | 2502 | 4.217 | 1.854 | 1750 | 8.02 | 569 | 18.2 | 12.99 | 0.815 | 75.5 | -68.05 | 273.15 |
| 275 | 0.00697 | 1.000 | 181.7 | 2497 | 4.211 | 1.855 | 1652 | 8.09 | 574 | 18.3 | 12.22 | 0.817 | 75.3 | -32.74 | 275 |
| 280 | 0.00990 | 1.000 | 130.4 | 2485 | 4.198 | 1.858 | 1422 | 8.29 | 582 | 18.6 | 10.26 | 0.825 | 74.8 | 46.04 | 280 |
| 285 | 0.01387 | 1.000 | 99.4 | 2473 | 4.189 | 1.861 | 1225 | 8.49 | 590 | 18.9 | 8.81 | 0.833 | 74.3 | 114.1 | 285 |
| 290 | 0.01917 | 1.001 | 69.7 | 2461 | 4.184 | 1.864 | 1080 | 8.69 | 598 | 19.3 | 7.56 | 0.841 | 73.7 | 174.0 | 290 |
| 295 | 0.02617 | 1.002 | 51.94 | 2449 | 4.181 | 1.868 | 959 | 8.89 | 606 | 19.5 | 6.62 | 0.849 | 72.7 | 227.5 | 295 |
| 300 | 0.03531 | 1.003 | 39.13 | 2438 | 4.179 | 1.872 | 855 | 9.09 | 613 | 19.6 | 5.83 | 0.857 | 71.7 | 276.1 | 300 |


| Temp | Prf |  | Temp | Prf |
| :---: | :---: | :---: | :---: | :---: |
| 285 | 8.81 |  | 286 K | 8.56 |
| 290 | 7.56 |  |  |  |

$\operatorname{Pr}$ water $=8.56$ through interpolation on excel sheet
Air: Fluid (26 C, 299K)
Table A. 4 Thermophysical Properties of Gases at Atmospheric Pressure ${ }^{a}$

| $T$ <br> (K) | $\underset{\left(\mathrm{kg} / \mathrm{m}^{3}\right)}{\rho}$ | $\begin{gathered} c_{p} \\ (\mathbf{k J} / \mathrm{kg} \cdot \mathrm{~K}) \end{gathered}$ | $\begin{gathered} \mu \cdot 10^{7} \\ \left(\mathbf{N} \cdot \mathbf{s} / \mathbf{m}^{2}\right) \end{gathered}$ | $\begin{aligned} & \nu \cdot 10^{6} \\ & \left(\mathrm{~m}^{2} / \mathrm{s}\right) \end{aligned}$ | $\begin{gathered} k \cdot 10^{3} \\ (\mathrm{~W} / \mathrm{m} \cdot \mathrm{~K}) \end{gathered}$ | $\begin{aligned} & \alpha \cdot 10^{6} \\ & \left(\mathrm{~m}^{2} / \mathbf{s}\right) \end{aligned}$ | Pr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

$$
\mathrm{Air}, \mathcal{M}=28.97 \mathrm{~kg} / \mathrm{kmol}
$$

| 100 | 3.5562 | 1.032 | 71.1 | 2.00 | 9.34 | 2.54 | 0.786 |
| ---: | ---: | ---: | ---: | :---: | :---: | ---: | ---: |
| 150 | 2.3364 | 1.012 | 103.4 | 4.426 | 13.8 | 5.84 | 0.758 |
| 200 | 1.7458 | 1.007 | 132.5 | 7.590 | 18.1 | 10.3 | 0.737 |
| 250 | 1.3947 | 1.006 | 159.6 | 11.44 | 22.3 | 15.9 | 0.720 |
| 300 | 1.1614 | 1.007 | 184.6 | 15.89 | 26.3 | 22.5 | 0.707 |

Pr air $=.707$

## Reynolds Number

We had to find the internal and external Reynolds number to determine the flow condition.

Internal Re
Internal Re $=\frac{\rho u_{m} D}{\mu_{\text {fluid }}}$
See Excel Sheet: All flow rates are turbulent (greater than 2300 Re )

## External Re

External $R e=\frac{V D}{v}=\frac{14.65^{*} 0.35}{0.00100023}=5126$, Turbulent (greater than 2300 Re )
See Excel Sheet: All flow rates are turbulent (greater than 2300 Re )

## Nusselt Number and "h"

In this section of the roadmap, the nusselt number will be calculated, which allows for h to be found. This is done for both the internal and external flow, after solving for the Reynolds number.

Internal Nusselt Number
In the internal condition, it is cooling, so n will equal .3.
$\overline{N U}=\frac{\bar{h} D}{k_{\text {fluid }}}=0.023 R e^{4 / 5} \operatorname{Pr}^{n}$
$\mathrm{n}=0.4$ Heating
$\mathrm{n}=0.3$ Cooling

Finding Internal "h"
$\bar{h} i\left[\frac{W}{m^{2}-K}\right]=\frac{\left(0.023 R e^{4 / 5} P r^{0.3}\right)^{*} k_{f l u i d}}{D}$

## Nusselt Number (External)

$\overline{N U}=\frac{\bar{h} D}{k_{f l u i d}}=C \operatorname{Re}^{m} \operatorname{Pr}^{1 / 3}$

In order to solve for Nusselt and eventually get h, C and m must be found.

Finding "C" and "m"
For both the air and the water, $\operatorname{Pr}>.7$. The ReD varies from 8 k to 440 k .

## $\operatorname{Pr} \geq 0.7$

Table 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

| $\boldsymbol{\boldsymbol { R } \boldsymbol { e } _ { \boldsymbol { D } }}$ | $\boldsymbol{C}$ | $\boldsymbol{m}$ |
| :---: | :---: | :---: |
| $0.4-4$ | 0.989 | 0.330 |
| $4-40$ | 0.911 | 0.385 |
| $40-4000$ | 0.683 | 0.466 |
| $4000-40,000$ | 0.193 | 0.618 |
| $40,000-400,000$ | 0.027 | 0.805 |

Table 7.2: used to find C and m after determining Renolds above

$$
\mathrm{C}=0.193
$$

$$
\mathrm{m}=0.618
$$

## Finding K fluid

## Exterior: Water

The k of the exterior fluid can be determined through interpolation, as seen in the excel file.

| K of fluid (water) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Temp (K) | $\mathrm{k}\left(\mathrm{W} / \mathrm{m}^{\star} \mathrm{K}\right)$ |  | Temp (K) | $\mathrm{k}\left(\mathrm{W} / \mathrm{m}^{\star} \mathrm{K}\right)$ |
| 285 | 0.59 |  | 286.15 | 0.59184 |
| 290 | 0.598 |  |  |  |

$K$ of fluid water $=.592 \mathrm{~W} / \mathrm{m} * \mathrm{~K}$

## Interior: Air

The table A. 4 in the book can be used in order to determine the k value
Table A. 4 Thermophysical Properties of Gases at Atmospheric Pressure ${ }^{a}$

| $\boldsymbol{T}$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | $c_{p}$ <br> $(\mathrm{kJJ} / \mathrm{kg} \cdot \mathrm{K})$ | $\boldsymbol{\mu} \cdot 10^{7}$ <br> $\left(\mathrm{~N} \cdot \mathbf{s} / \mathrm{m}^{2}\right)$ | $\boldsymbol{\nu} \cdot 10^{6}$ <br> $\left(\mathrm{~m}^{2} / \mathrm{s}\right)$ | $\boldsymbol{k} \cdot 10^{3}$ <br> $(\mathbf{W} / \mathrm{m} \cdot \mathrm{K})$ | $\alpha \cdot 10^{6}$ <br> $\left(\mathrm{~m}^{2} / \mathrm{s}\right)$ | $\operatorname{Pr}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

$$
\mathrm{Air}, \mathcal{M}=28.97 \mathrm{~kg} / \mathrm{kmol}
$$

| 100 | 3.5562 | 1.032 | 71.1 | 2.00 | 9.34 | 2.54 | 0.786 |
| ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: |
| 150 | 2.3364 | 1.012 | 103.4 | 4.426 | 13.8 | 5.84 | 0.758 |
| 200 | 1.7458 | 1.007 | 132.5 | 7.590 | 18.1 | 10.3 | 0.737 |
| 250 | 1.3947 | 1.006 | 159.6 | 11.44 | 22.3 | 15.9 | 0.720 |
| 300 | 1.1614 | 1.007 | 184.6 | 15.89 | 26.3 | 22.5 | 0.707 |

$$
\mathbf{K} \text { of fluid (air) }=26.3 \times 10^{-3} \mathrm{~W} / \mathrm{m}^{*} \mathrm{~K}
$$

## Finding External "h"

With the previously calculated information, $h$ can now be found
$\bar{h} o\left[\frac{W}{m^{2}-K}\right]=\frac{\left(C R e^{m} P^{1 / 3}\right)^{*} k_{\text {fluid }}}{D}=\frac{\left(0.193 * 5126^{0.618} 8.56^{1 / 3}\right)^{*} 0.59184}{0.35}=130.966$

## Submerged Pipe Length

## Finding thermal resistance

$U A=\frac{1}{\frac{1}{h_{i} A_{i}}+\frac{\ln \left(\frac{r_{o}}{r_{i}}\right.}{2 \pi k_{w_{\text {wal }}}}+\frac{1}{h_{o} A_{o}}}=L\left(h_{i}\left(\pi D_{i}\right)+\frac{2 \pi k_{\text {wall }}}{\ln \left(\frac{r_{o}}{r_{i}}\right)}+h_{o}\left(\pi D_{o}\right)\right)$

SPECIAL CASE: Internal Flow, External Convection
$\frac{T_{\infty}-T_{m, 0}}{T_{\infty}-T_{m, i}}=\exp \left[-\frac{U A}{\bar{m} c_{p}}\right]$
The thermal resistance equation is plugged in for UA, and both sides are multiplied by natural log:
$\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right)=-\frac{L\left(h_{i}\left(\pi D_{i}\right)+\frac{2 \pi k_{\text {wal }}}{\ln \left(\frac{r_{0}}{r_{i}}\right.}+h_{o}\left(\pi D_{o}\right)\right)}{\bar{m} c_{p}}$
Now $-\bar{m} c_{p}$ is multiplied to get out of the denominator:
$-\bar{m} c_{p} * \ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right)=L\left(h_{i}\left(\pi D_{i}\right)+\frac{2 \pi k_{\text {wall }}}{\ln \left(\frac{r_{o}}{r_{i}}\right)}+h_{o}\left(\pi D_{o}\right)\right)$
The equation can now be rewritten to solve of Length:

Within the attached excel sheet, length is solved for.


## Pressure Drop

$\Delta P=f \frac{\rho u_{m}^{2}}{2} \frac{L}{D}$
$\frac{1}{\sqrt{f}}=-1.8 \log _{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)$
In the problem description: "Overcome friction within smooth tube"; This allows for us to proceed with the following expression: $\varepsilon / D=0$.
In order to solve for the pressure drop, the friction needs to be found first
$f=\frac{1}{-1.8 \log _{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)}{ }^{2}$
$u_{m}[m / s]=Q / A_{c}$
$A_{c}[m]=\pi r_{i}^{2}$

## Fan Power to Overcome Friction

$$
P[W]=\frac{\bar{m} \Delta P}{\rho_{\text {air }}}=Q \Delta P
$$

## Annual Operating Costs

Cost $[\$]=\frac{P[W]}{1000}[k W] * 0.15\left[\frac{\$}{k W h}\right] *(24 * 365)[h r]$
The equation is dependent upon the fan power to overcome friction calculation, which was performed previously in the report.

## Heat Transfer Rate

$q[W]=\bar{m} c_{p}\left(T_{m, o u t}-T_{m, i}\right)$
$\bar{m}\left[\frac{\mathrm{~kg}}{\mathrm{~s}}\right]=\rho Q$

## Plots

## Length Graph



Pressure Drop Graph
Delta Pressure (Pa) vs. Q (m^3/s)


Power Graph

## Power [W] vs. Q (m³/s)



Annual Power Graph
Annual Power [kWh] vs. $Q\left(m^{\wedge} 3 / s\right)$


Cost Graph


## Heat Transfer Rate Graph

$q(W)$ vs. $Q\left(m^{\wedge} 3 / s\right)$


## Analysis

Through the above equations and workflow, you can see how we got to our excel sheet conclusions. There are some concerns we have with the data, primarily due to the length results. It seems as though the length is much smaller than it shouldoe. Just given the type of problem, and the usage of this, I'd expeet to see a length greater than 100 meters given the fact that the pipe has to go down 50 meters to the depth where we are analyzing convection, and then another 50 meters back up to the height of sea level. With our length of less than 1 meter, it wouldn't make sense to run the pipe to that depth to benefit from the cold temperatures-due to pipe losses in the up and down sections that we didn'tanalyze. After we reached out to you, wediscovered that our specific heat was in terms of $\mathrm{KJ} / \mathrm{Kg}-\mathrm{K}$, rather than the standard (units of $\mathrm{J} / \mathrm{Kg}-\mathrm{K}$. Following this change we saw an improvenent in L, but it was still smakler than what would be expected. Because of the issues with the length, many other metrics are off. An example of this is the pressure drop. Pressure drop is dependent on the length, so our small length also skews the result of that equation. Ultimately, the important metrics like the power and cost are very small due to the issues with the length. We spent a great deal of time trying to understand where the errors occurred in our calculations, and are left puzzled. Overall, we believe that the roadmap that we layed out should lead to the right conclusion, but that we may have made algebraic or interpolation errors along the way in the sub calculations that led up to us finding the length. We
were careful in our methods to check our units as well as retyping equations into Excel as well as our calculators to ensure that they were transcribed properly, and believe that none of these are issues resulting in the significantly short L. Hopefully we were effective at displaying our work so that you can follow our steps and identify the section in which we may have made an error. Overall, it is evident that a higher volumetric flow as well as a higher mass flow rate correspondingly results in more expensive operational cost and higher losses.
apure wis

I


In order to find the heat transfer through the .15 m copper pipeline, a thermal circuit can be used. The total thermal resistance of the copper pipe can be determined by using the formula

$$
R_{\text {cond }}=\frac{\ln \left(r^{1} / r^{2}\right)}{2 \pi L_{\text {pipeline }} K_{\text {copper }}}: \text { where } K_{\text {copper }}=386 \mathrm{Wm}^{-1} \mathrm{~K}^{-1}
$$

And the formula

$$
R_{\text {conv }}=\frac{1}{h_{\text {seawater }} A}
$$

Considering boundary layers, temperature, viscosity, and density of seawater, $h_{\text {seawater }}$ can be calculated for the above equation.

Knowing the thermal conductivity of copper and the temperature of the water surrounding the copper pipeline (which according to the plotted variation of temperature with depth should be a steady 13 degrees fahrenheit), the surface temperature of the interior of the pipeline and exterior of the pipeline can be calculated.

The relationships between the variables of the system components are as follows: As pipe length $L$ increases, the overall amount of energy transferred between the air inside the pipe and the surrounding water will increase as well, up until the point of equilibrium (if volumetric flow is to stay constant). In order to reach the desired temperature the equilibrium point must be manipulated by modulating the volumetric flow rate of air through the pipe. The modulation of the volumetric flow rate depends on the pump (fan) power, which is to be determined by the pressure differential inside the smooth pipe. As the depth increases, this pressure differential increases. As the length of the pipe increases, the pump power also increases. For an accurate representation of the volumetric flow the following assumptions are made:

Compressible flow. Since the speed of the air is so high (4.25 times the speed of sound) flow of the air through the pipe can be considered compressible.

Air is an ideal gas.
Airflow is laminar. Air flowing through a significant length of smooth-walled pipe becomes laminar.

Operating cost can be calculated by multiplying the operating wattage of the system (accounting for the efficiency of system components) by the cost of a kWhr , multiplied by the hours per year.

Cost $=W_{\text {ideal }} * \eta_{\text {system }}=W \underset{\text { operating }}{ } \operatorname{Cost}_{k W h r} * 8766$ hours/year

Volumetric flow Q : v equals velocity, A equals the area of the tube
$Q=v * A$

Re could change with depth, from Laminar $<2000<$ Transitional $<3500$ Turbulent, the ocean has changes in flow types as depth changes

The e with equal 0 due to smoothness of the pipe
$\frac{1}{\sqrt{f}}=-1.8 * \log \left[\left(\frac{\varepsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right]$



Define constants and parameters:

```
air.T.in_value = 26; air.T.out_value = 19; air.T.unit = 'C';
Dia.i = 0.20; Dia.o = 0.35; Dia.unit = 'm';
Depth.value = 50; Depth.unit = 'm';
water.v.value = 14.67; water.v.unit = 'm/s';
water.T.value = 13.2; water.T.unit = 'C';
air.T_film.value = (air.T.in_value + air.T.out_value)/2;
air.T_film.unit = 'C';
water.T_film.value = (water.T.value + air.T_film.value)/2;
water.T_film.unit = 'C';
```

Define properties of water at $\mathrm{T}_{\mathrm{f}}$ film.

```
water.mu.value = 1.0518*10^(-3);
water.mu.unit = 'N*s/m^2';
water.dens.value = 998.57;
water.dens.unit = 'kg/m^3';
```

Define properties of air at T_film.

```
air.mu.value = 18.17*10^(-6);
air.mu.unit = 'N*s/m^2';
air.dens.value = 1.198;
air.dens.unit = 'kg/m^3';
air.Cp.value = 1006;
air.Cp.unit = 'J/(kg*k)';
air.k.value = 0.026;
air.k.unit = 'w/(m*k)';
v_dot.value = 0.02:0.02:1; v_dot.unit = 'm^3/s';
m_dot.value = air.dens.value*v_dot.value; m_dot.unit = 'm^3/s';
air.v.value = m_dot.value./air.dens.value./(pi*(0.5*Dia.i)^2);
air.v.unit = 'm/s';
```

Determine flow friction factor (f) using Haaland's equation, given smooth pipe.

## Class 16: Moody diagram

$f=\frac{64}{\mathrm{Re}_{D}}$; Laminar Flow
Wall roughness \& friction factor



TABLE 8.1
Equivalent Roughness for New Pipes [From Moody
(Ref. 7) and Colebrook (Ref. 8)]

|  | Equivalent Roughness, $\boldsymbol{\varepsilon}$ |  |
| :--- | :--- | :--- |
| Pipe | Feet | Millimeters |
| Riveted steel | $0.003-0.03$ | $0.9-9.0$ |
| Concrete | $0.001-0.01$ | $0.3-3.0$ |
| Wood stave | $0.0006-0.003$ | $0.18-0.9$ |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Commercial steel |  |  |
| $\quad$ or wrought iron | 0.00015 | 0.045 |
| Drawn tubing | 0.000005 | 0.0015 |
| Plastic, glass | 0.0 (smooth) | 0.0 (smooth) |
|  |  |  |

```
P_length.value = 100:100:5000;
P_length.unit = 'm';
Reynolds = air.dens.value .* air.v.value .* Dia.o ./ air.mu.value
```

```
Reynolds = 1×50
105 x
```

0.1469
0.2938
0.4407
0.5876
0.7345
0.8815
1.0284 1.1753 $\cdots$
disp('Turbulent flow');
Turbulent flow

```
e_rough.value = 0; e_rough.unit = 'm';
```

friction_factor = (1./(-1.8.*log10((e_rough.value/Dia.i/3.7)^1.11 + 6.9./Reynolds))).^2;
friction_factor = friction_factor.';
Pr = 0.7;
NUD $=0.023 . *$ Reynolds.^(4/5).$* \operatorname{Pr}^{\wedge} 0.3$;
h_bar.in = (air.k.value .* NUD ./Dia.i).';
h_bar.out = (air.k.value .* NUD ./Dia.o).';
UA $=1 . /($ (1./(h_bar.in*P_length.value .*pi.*Dia.i)) + (1./(h_bar.out*P_length.value .*pi.*Dia
delta_Ti = water.T.value - air.T.in_value;
delta_To = water.T.value - air.T.out_value;
q_conv.value = UA .* (delta_To-delta_Ti) ./ log(delta_To/delta_Ti) .*(-1);
q_conv.unit = 'w';
figure(1);
Mesh0 = mesh(P_length.value, v_dot.value, q_conv.value);
title(\{'Heat Transfer Rate vs Volume Flow Rate vs Pipe Length' ...
'- Lin, Riese'\});
xlabel('Pipe Length (m)');
ylabel('Volume Flow Rate (m^3/s)');
zlabel('Heat Transfer Rate (w)');
ylim([0 1]);
$x \lim ([0$ 5000]);
colorbar;
grid on;


Determine pump power at various pipe lengths.

```
delta_p = (friction_factor* P_length.value ) .* air.dens.value .* (air.v.value).'.^2 ./2 ./Di
figure(2);
Mesh1 = mesh(P_length.value, v_dot.value ,delta_p);
title({'Pressure Drop vs Volume Flow Rate vs Pipe Length' ...
    '- Lin, Riese'});
xlabel('Pipe Length (m)');
ylabel('Volume Flow Rate (m^3/s)');
zlabel('Pressure Drop (N/m^2)');
ylim([0 1]);
xlim([0 5000]);
colorbar;
grid on;
```

Pressure Drop vs Volume Flow Rate vs Pipe Length


```
pump_power = m_dot.value.' .* delta_p ./ air.dens.value;
figure(3);
Mesh2 = mesh(P_length.value, v_dot.value ,pump_power);
title({'Pump Power vs Volume Flow Rate vs Pipe Length' ...
    '- Lin, Riese'});
xlabel('Pipe Length (m)');
ylabel('Volume Flow Rate (m^3/s)');
zlabel('Pump Power (w)');
ylim([0 1]);
xlim([0 5000]);
colorbar;
grid on;
```

Pump Power vs Volume Flow Rate vs Pipe Length



Volume Flow Rate ( $\mathrm{m}^{3} / \mathrm{s}$ )

```
cost_1h = pump_power .* 0.15;
figure(4);
Mesh3 = mesh(P_length.value, v_dot.value ,cost_1h);
title({'Energy Cost for 1hr vs Volume Flow Rate vs Pipe Length' ...
    '- Lin, Riese'});
xlabel('Pipe Length (m)');
ylabel('Volume Flow Rate (m^3/s)');
zlabel('Energy Cost ($)');
ylim([0 1]);
xlim([0 5000]);
colorbar;
grid on;
```


## Energy Cost for $\mathbf{1 h r}$ vs Volume Flow Rate vs Pipe Length

- Lin, Riese


Volume Flow Rate ( $\mathrm{m}^{3} / \mathrm{s}$ )

# Heat Transfer Group Project By: Cecilia Linck and Sarah Sawyer <br>  



As engineers for Parametric Design, Inc you are requested to evaluate a facility cooling concept on your clients remote Pacific Ocean resort complex 500 miles west of California. Air at 26C is routed through thick wall cooper tubing, $\mathrm{Di}=0.20 \mathrm{~m}, \mathrm{Do}=0.35 \mathrm{~m}$ that flows through the cool pacific waters at a depth of 50 m at a volume flow rate that is to vary from $0.02 \mathrm{~m} 3 / \mathrm{s}$ to $1 \mathrm{~m} 3 / \mathrm{s}$. The velocity and temperature profiles can be assumed as shown below.


Your engineering staff is requested to study the parametric relationships between the submerged pipe length (L), volume flow rate, pressure drop, fan power to overcome friction within the smooth tube, and heat transfer rates, and annual operating cost at $\$ 0.15 / \mathrm{kWh}$, to ensure a discharge air temperature of 19 C .
You are to use your engineering judgement to determine what data is needed, what calculations are needed, what plots are needed, and what assumptions are needed to complete the engineering analysis report and the ROAD MAP, and to meet Berry's (CEO) expectations as you know his proclivity for details and logic.

Provide a 2-4-page, 1.5 line spacing, 11 pitch font, research essay regarding the threat and impact of global warming and climate change, and your perspective on the engineering challenges your generation must face to save mankind.

All reports ae to be typed professionally as they will be posted online for display and review for future students and professionals.

## Road Map

The first step in solving this problem is identifying what we want to solve first. The project asks for submerged pipe length (L), so we decided to find that first. To do this, we first need to find the heat transfer coefficient (h) for both the air inside of the pipe and the ocean water outside of the pipe. To find h , we need to use the Reynolds number equation, which requires us to know velocity.


## Air Inside Pipe

## Find Velocity

We found velocity of the air inside of the pipe by using the volumetric flow rate given to us in the problem statement. The volumetric flow rate ranged from $0.02 \frac{\mathrm{~m}^{3}}{s}$ to $1 \frac{\mathrm{~m}^{3}}{\mathrm{~s}}$.

$$
V \frac{m}{s}=\frac{Q\left[\frac{m^{3}}{s}\right]}{A_{i}\left[m^{2}\right]}
$$

Thus, using the inner area of the pipe, we were able to use this equation to find a range of velocities for air in the pipe, which can be seen in our spreadsheet. Those values ranged from $\sim 1$ $\mathrm{m} / \mathrm{s}$ to $32 \mathrm{~m} / \mathrm{s}$.

## Find Reynolds Number

We found our Reynolds number by using the air velocity we calculated and some of the problem properties.

$$
\begin{gathered}
R e \#=\frac{U_{m}\left[\frac{m}{s}\right] \times D[m]}{\nu\left[\frac{m^{2}}{s}\right]} \\
R e \#=\frac{0.6366\left[\frac{m}{s}\right] \times 0.2[m]}{0.000015444\left[\frac{m^{2}}{s}\right]}=6982.39
\end{gathered}
$$

Because our Reynolds number is larger than 2300 we know we are working with turbulent flow.

To find those properties, we used the following equation:

$$
T_{m}=\frac{T_{m, i}+T_{m, 0}}{2}=\frac{26+19}{2}=22.5 \mathrm{C}=295.5 \mathrm{~K}
$$

Once we had that value for temperature, we used Table A. 4 to find the values for $v, \rho, c_{p}, k$, and $\mu$. For $v$, we got approximately $15.444 * 10^{-6} \frac{m^{2}}{s}$. We were then able to calculate the Reynolds number for the varying velocities between 1 and $32 \mathrm{~m} / \mathrm{s}$. For the diameter, we used the specified $\mathrm{Di}=0.20 \mathrm{~m}$, since the flow is through the inside of the pipe. See spreadsheet for these values. We found that the Reynolds numbers calculated for this problem are larger than 2300. This means we have turbulent air flow in the pipe.

## Find Nusselt Number

To find the Nusselt number you have to use the air properties, because it is the fluid flowing through the pipe. We used the Dittus-Boelter Equation since we have internal turbulent flow:

$$
N U=0.023(R e)^{4 / 5} \times \operatorname{Pr}^{0.3}
$$

$$
N U=0.023(6982.39)^{4 / 5} \times 0.70752^{0.3}=24.65
$$

The Prandtl number is raised to the 0.3 power because the air inside of the pipe is cooling. If the air inside of the pipe was being heated, the Prandtl number would be raised to the 0.4 power. Thus, we were able to get Nusselt numbers for each of the respective Reynolds numbers.

## Find Heat Transfer Coefficient of Air in Pipe

Once the Nusselt was obtained, we could find the heat transfer coefficient of air flowing through the pipe using the following equation:

$$
\begin{gathered}
h_{\text {air }}\left[\frac{W}{m^{2}-k}\right]=\frac{N U \times k_{\text {air }}\left[\frac{W}{m-k}\right]}{D[m]} \\
h_{\text {air }}\left[\frac{W}{m^{2}-k}\right]=\frac{24.65 \times 0.02614\left[\frac{W}{m-k}\right]}{0.2[m]}=3.22\left[\frac{W}{m^{2}-k}\right]
\end{gathered}
$$

Our $h_{\text {air }}$ values ranged between $\sim 3-84 \frac{W}{m^{2} K}$

## Ocean Water Outside Pipe

To find $h_{\text {water }}$, we performed a similar analysis as that of the $h_{\text {air }}$. We realize that salt water is slightly more dense than freshwater, but there was no saltwater table in the textbook. So, we just chose to use Table A. 6 (Saturated Water). This way, we were also able to get values for cp , k , etc. based on our temperature, which was something we were unable to find online.

## Find Velocity

To find the velocity of the ocean water, we were able to use the velocity curve provided. At the specified depth of 50 m , the curve shows a velocity of approximately $14.67 \mathrm{~m} / \mathrm{s}$.

## Find Reynolds Number

For the Reynolds number, we needed to find the new $v$ to use in the equation:

$$
R e \#=\frac{U_{m}\left[\frac{m}{s}\right] \times D[m]}{v\left[\frac{m^{2}}{s}\right]}
$$

To do so, we used the provided temperature profile curve to approximate the temperature of the water at 50 m . Based on the curve, we assumed that the temperature was around 13 degrees C . $13 \mathrm{C}=286 \mathrm{~K}$, so we used 286 K to find the properties of water at this temperature. We got that $v=1196 * 10^{-6} \frac{\mathrm{~m}^{2}}{\mathrm{~s}}$. We also found $\operatorname{Pr}=8.56$ and $\mathrm{k}=0.5916 \mathrm{~W} / \mathrm{mK}$. We also used the diameter of $\mathrm{Do}=0.35 \mathrm{~m}$, since the flow is now occurring outside the pipe.

## Find Nusselt Number

To find the Nusselt, we had to use the equation corresponding to a cylinder in cross flow:

$$
N U=C R e_{D}{ }^{m} \operatorname{Pr}^{1 / 3}
$$

Where C and m are constants based off of Table 7.2. For us, we had a Reynolds number of 4293, so our $\mathrm{C}=0.193$ and our $\mathrm{m}=0.618$. Therefore, our equation looked like:

$$
N U=0.193 * 4293^{0.618} * 8.56^{1 / 3}=69.4
$$

## Find Heat Transfer Coefficient of Water over Pipe

Once the Nusselt was obtained, we could find the heat transfer coefficient of water flowing over the pipe using the following equation:

$$
\begin{gathered}
h_{\text {water }}\left[\frac{W}{m^{2}-k}\right]=\frac{N U \times k_{\text {water }}\left[\frac{W}{m-k}\right]}{D[m]} \\
\text { Which gave us: } \\
h_{\text {water }}=\frac{69.4^{*} 0.5916}{0.35}=117.3
\end{gathered}
$$

## Finding the First Relationship

## Pipe Length

To find the length, we then had to use the special case of external convection and internal flow:

$$
\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}=\exp \left[-\frac{1}{m_{d o t}{ }_{p}} \frac{1}{\Sigma R_{t h}}\right]
$$

Where $\Sigma R_{t h}$ is the summation of all the resistances in the system. For us, the resistances are that due to outer convection: $\frac{1}{h_{\text {water }} A_{o}}$, inner convection: $\frac{1}{h_{\text {air }} A_{i}}$, and conduction through the tube (since it is thick wall, cannot neglect resistance): $\frac{\ln \left(\frac{r_{2}}{r_{1}}\right)}{2 \pi L k}$. The areas for the convective resistances are equal to $\pi D L$, so for the outer area: $\pi(0.2) L$, and inner: $\pi(0.35) L$. Therefore, we can rearrange the equation for L :

$$
\begin{aligned}
& -\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right) * m_{d o t} C_{p}=\frac{1}{\frac{1}{L}\left(\frac{1}{h_{\text {water }} \pi D_{o}}+\frac{1}{h_{\text {air }} \pi D_{i}}+\frac{\ln \left(\frac{r_{2}}{r_{1}}\right)}{2 \pi k}\right)} \\
& \text { Which gives: } \\
& L=\frac{-\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m i}}\right) * m_{d o t} c_{p}}{1} \frac{\left(\frac{1}{h_{\text {water }} T_{o}}+\frac{1}{h_{\text {air }}+D_{i}}+\frac{\ln \left(\frac{r_{2}}{r_{1}}\right.}{2 \pi \pi k}\right)}{2 \pi / 2}\left(\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right) * m_{d o t} c_{p}\right)\left(\frac{1}{h_{\text {water }} \pi D_{o}}+\frac{1}{h_{\text {air }} \pi D_{i}}+\frac{\ln \left(\frac{r_{2}}{r_{1}}\right)}{2 \pi k}\right)
\end{aligned}
$$

Therefore, for the first volumetric flow rate value of $0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s},\left(m_{d o t}=Q^{*} \rho_{\text {air }}=0.024\right)$ we get a length of:
$-\left(\ln \left(\frac{13-19}{13-26}\right) * 0.024 * 1007\right)\left(\frac{1}{117.3^{*} \pi^{*} 0.35}+\frac{1}{3.65^{*} \pi^{*} 0.2}+\frac{\ln \left(\frac{0.35 / 2}{0.2 / 2}\right)}{2 \pi^{*} 0.02594}\right)=71.2 m$
Then, we did this for each value of volumetric flow rate so we were able to see a parametric relationship between the volumetric flow rate and the required length for the exit temperature to be maintained at 19 degrees C. See graph below.


Looking at the graph above our data makes sense. The longer the pipe the higher your flow rate needs to be to push the air all the way down the pipe.

## Pressure Drop

## Find Friction Factor

The first step in finding the pressure drop is finding the friction factor, since the equation for pressure drop is as follows:

$$
\Delta P=f \frac{\rho_{a i r}{ }^{*} u_{m}^{2}}{2} * \frac{\Delta x}{D_{i}}
$$

Where $u_{m}$ is our mean velocity (obtained from volumetric flow rate), and $\Delta \mathrm{x}$ is our change in
length. To get our friction factor f , we must use the equation for turbulent flow:

$$
\frac{1}{\sqrt{f}}=-1.8 \log _{10}\left(\left(\frac{\epsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)
$$

Solve for f :

$$
f=\left(\frac{1}{-1.8 \log _{10}\left(\left(\frac{\epsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)}\right)^{2}
$$

For the roughness $\epsilon$, our pipe is smooth.
Therefore, our friction factor for volumetric flow rate of $0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}$ was:

$$
f=\left(\frac{1}{-1.8 \log _{10}\left(\left(\frac{0 / 0.2}{3.7}\right)^{1.11}+\frac{6.9}{8244}\right)}\right)^{2}=0.033
$$

Also, our mean velocity is $u_{m}=\frac{m_{\text {dot }}}{\rho_{\text {air }} A_{c}}$ where $A_{c}=\frac{\pi D^{2}}{4}$
For $\mathrm{Q}=0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}, u_{m}=\frac{0.024}{1.1824^{*}\left(\pi(0.2)^{2} / 4\right)}=0.637 \mathrm{~m} / \mathrm{s}$
So we can find our pressure drop:

$$
\Delta P=0.033 \frac{1.1824^{*} 0.637^{2}}{2} * \frac{71.2}{0.2}=2.78 P a
$$

Our pressure drop ranged from this value all the way up to $129,461 \mathrm{~Pa}$


Looking at the figure above, it shows that as the pipe length increases the pressure drop also increases. This makes sense because the pipe roughness slows the air down in the pipe.
Therefore the pressure in the pipe drops because the speed of the air in the pipe slows down the further it goes.

Fan Power to Overcome Friction

To make air in the duct properly flow a fan was added to the design criteria.
To find the amount of power needed for the fan to overcome the friction in the pipe we used this equation:

$$
\begin{gathered}
\text { Power }=\frac{m_{\text {dot }} \Delta P}{\rho}=Q \Delta P \\
\text { For } \mathrm{Q}=0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}, \Delta \mathrm{P}=2.95 \mathrm{~Pa} \\
\text { Power }=0.02 * 2.95=0.000059 \mathrm{Watts}
\end{gathered}
$$



Our data shows that as the pipe length increases, the more power the fan is going to need. This makes sense because the farther the air has to travel, the more the air has to be pushed harder from the start of the duct.


This table shows that as the friction in the pipe increases, the maximum length the pipe can be is reduced. This makes sense because the more friction there is, the faster the pipe will heat up and thus increase the exit temperature.

## Heat Transfer Rate

To find the heat transfer rate from the ocean water to the air flowing in the pipe we used this equation:

$$
q_{\text {conv }}=m_{\text {dot }} C_{p}\left(T_{m, \text { out }}-T_{m}\right)
$$

Where, $m_{d o t}=0.02 \frac{m^{3}}{s}, C_{p}=1006.9 \frac{J}{k g-k}, T_{m, o u t}=19$ Degrees $C, T_{m, i n}=26$ Degrees $C$

$$
q_{\text {conv }}[\text { watts }]=0.02 * 1006.9(19 \text { Degrees } C-26 \text { Degrees } C)=-166.98 \text { [watts] }
$$

A negative number for this application makes sense because the air in the duct should be losing heat as it travels through the pipe.


Looking at the graph above the data shown makes sense. The longer the air stays in the pipe the more heat will be transferred from it due to the ocean being colder than the air inside of the pipe.

## Annual Operating Cost

To find the annual operating cost of this cooling system we used this equation:

$$
\begin{gathered}
\text { Annual Operating Cost }[\$]=\text { Fan Power }[\text { Watts }] * \frac{\text { hours }}{\text { Day }} * \frac{\$}{k W h} * \frac{\text { Days }}{\text { Year }} \\
\text { Annual Operating Cost }=0.0000589[\text { watts }] * 24 \frac{\text { hours }}{\text { Day }} * \frac{\$ 0.15}{k W h} * \frac{365 \text { Days }}{\text { Year }}=\$ 0.07
\end{gathered}
$$

$\$ 0.07$ is a very low cost for this cooling system. After all of the construction to buid this cooling system is done it makes a little bit of sense because you are only paying før the electricity to yun the fan. You would not be paying to cool the air in a cooling system like an AC. The air would be cool naturally by the water in the ocean. It should also be taken into account that this is the cost to run the system for a very low volumetric flow rate. To make a low volumetric flow rate, the fan will naturally not need to spin very fast, thus requiring less power. For higher volumetric flow rates, such as $1 \mathrm{~m}^{\wedge} 3 / \mathrm{s}$, the required fan power was about 129 kW , which had a yearly cost of about $\$ 170,000$. So the cost to run the system is heavily dependent on h$\varnothing$ w fast the flow rate must be.



The graph above shows that the longer the pipe is underwater the higher the annual operating cost. This is because of how much you would have to pay to supply electricity to power the fan in the duct. The longer the pipe, the more electricity the fan will need to pump the same amount
of volume through the pipe to achieve that exit temperature of 19 degrees C.

# Heat Transfer Group Project By: Cecilia Linck and Sarah Sawyer 

As engineers for Parametric Design, Inc you are requested to evaluate a facility cooling concept on your clients remote Pacific Ocean resort complex 500 miles west of California. Air at 26C is routed through thick wall cooper tubing, $\mathrm{Di}=0.20 \mathrm{~m}, \mathrm{Do}=0.35 \mathrm{~m}$ that flows through the cool pacific waters at a depth of 50 m at a volume flow rate that is to vary from $0.02 \mathrm{~m} 3 / \mathrm{s}$ to $1 \mathrm{~m} 3 / \mathrm{s}$. The velocity and temperature profiles can be assumed as shown below.


Your engineering staff is requested to study the parametric relationships between the submerged pipe length (L), volume flow rate, pressure drop, fan power to overcome friction within the smooth tube, and heat transfer rates, and annual operating cost at $\$ 0.15 / \mathrm{kWh}$, to ensure a discharge air temperature of 19 C .
You are to use your engineering judgement to determine what data is needed, what calculations are needed, what plots are needed, and what assumptions are needed to complete the engineering analysis report and the ROAD MAP, and to meet Berry's (CEO) expectations as you know his proclivity for details and logic.

Provide a 2-4-page, 1.5 line spacing, 11 pitch font, research essay regarding the threat and impact of global warming and climate change, and your perspective on the engineering challenges your generation must face to save mankind.

All reports ae to be typed professionally as they will be posted online for display and review for future students and professionals.

## Road Map

The first step in solving this problem is identifying what we want to solve first. The project asks for submerged pipe length (L), so we decided to find that first. To do this, we first need to find the heat transfer coefficient (h) for both the air inside of the pipe and the ocean water outside of the pipe. To find h , we need to use the Reynolds number equation, which requires us to know velocity.

## Air Inside Pipe

## Find Velocity

We found velocity of the air inside of the pipe by using the volumetric flow rate given to us in the problem statement. The volumetric flow rate ranged from $0.02 \frac{\mathrm{~m}^{3}}{s}$ to $1 \frac{\mathrm{~m}^{3}}{\mathrm{~s}}$.

$$
V \frac{m}{s}=\frac{Q\left[\frac{m^{3}}{s}\right]}{A_{i}\left[m^{2}\right]}
$$

Thus, using the inner area of the pipe, we were able to use this equation to find a range of velocities for air in the pipe, which can be seen in our spreadsheet. Those values ranged from $\sim 1$ $\mathrm{m} / \mathrm{s}$ to $32 \mathrm{~m} / \mathrm{s}$.

## Find Reynolds Number

We found our Reynolds number by using the air velocity we calculated and some of the problem properties.

$$
\begin{gathered}
R e \#=\frac{U_{m}\left[\frac{m}{s}\right] \times D[m]}{\nu\left[\frac{m^{2}}{s}\right]} \\
R e \#=\frac{0.6366\left[\frac{m}{s}\right] \times 0.2[m]}{0.000015444\left[\frac{m^{2}}{s}\right]}=6982.39
\end{gathered}
$$

Because our Reynolds number is larger than 2300 we know we are working with turbulent flow.

To find those properties, we used the following equation:

$$
T_{m}=\frac{T_{m, i}+T_{m, 0}}{2}=\frac{26+19}{2}=22.5 \mathrm{C}=295.5 \mathrm{~K}
$$

Once we had that value for temperature, we used Table A. 4 to find the values for $v, \rho, c_{p}, k$, and $\mu$. For $v$, we got approximately $15.444 * 10^{-6} \frac{m^{2}}{s}$. We were then able to calculate the Reynolds number for the varying velocities between 1 and $32 \mathrm{~m} / \mathrm{s}$. For the diameter, we used the specified $\mathrm{Di}=0.20 \mathrm{~m}$, since the flow is through the inside of the pipe. See spreadsheet for these values. We found that the Reynolds numbers calculated for this problem are larger than 2300. This means we have turbulent air flow in the pipe.

## Find Nusselt Number

To find the Nusselt number you have to use the air properties, because it is the fluid flowing through the pipe. We used the Dittus-Boelter Equation since we have internal turbulent flow:

$$
N U=0.023(R e)^{4 / 5} \times \operatorname{Pr}^{0.3}
$$

$$
N U=0.023(6982.39)^{4 / 5} \times 0.70752^{0.3}=24.65
$$

The Prandtl number is raised to the 0.3 power because the air inside of the pipe is cooling. If the air inside of the pipe was being heated, the Prandtl number would be raised to the 0.4 power. Thus, we were able to get Nusselt numbers for each of the respective Reynolds numbers.

## Find Heat Transfer Coefficient of Air in Pipe

Once the Nusselt was obtained, we could find the heat transfer coefficient of air flowing through the pipe using the following equation:

$$
\begin{gathered}
h_{\text {air }}\left[\frac{W}{m^{2}-k}\right]=\frac{N U \times k_{\text {air }}\left[\frac{W}{m-k}\right]}{D[m]} \\
h_{\text {air }}\left[\frac{W}{m^{2}-k}\right]=\frac{24.65 \times 0.02614\left[\frac{W}{m-k}\right]}{0.2[m]}=3.22\left[\frac{W}{m^{2}-k}\right]
\end{gathered}
$$

Our $h_{\text {air }}$ values ranged between $\sim 3-84 \frac{W}{m^{2} K}$

## Ocean Water Outside Pipe

To find $h_{\text {water }}$, we performed a similar analysis as that of the $h_{\text {air }}$. We realize that salt water is slightly more dense than freshwater, but there was no saltwater table in the textbook. So, we just chose to use Table A. 6 (Saturated Water). This way, we were also able to get values for cp , k , etc. based on our temperature, which was something we were unable to find online.

## Find Velocity

To find the velocity of the ocean water, we were able to use the velocity curve provided. At the specified depth of 50 m , the curve shows a velocity of approximately $14.67 \mathrm{~m} / \mathrm{s}$.

## Find Reynolds Number

For the Reynolds number, we needed to find the new $v$ to use in the equation:

$$
R e \#=\frac{U_{m}\left[\frac{m}{s}\right] \times D[m]}{v\left[\frac{m^{2}}{s}\right]}
$$

To do so, we used the provided temperature profile curve to approximate the temperature of the water at 50 m . Based on the curve, we assumed that the temperature was around 13 degrees C . $13 \mathrm{C}=286 \mathrm{~K}$, so we used 286 K to find the properties of water at this temperature. We got that $v=1196 * 10^{-6} \frac{\mathrm{~m}^{2}}{\mathrm{~s}}$. We also found $\operatorname{Pr}=8.56$ and $\mathrm{k}=0.5916 \mathrm{~W} / \mathrm{mK}$. We also used the diameter of $\mathrm{Do}=0.35 \mathrm{~m}$, since the flow is now occurring outside the pipe.

## Find Nusselt Number

To find the Nusselt, we had to use the equation corresponding to a cylinder in cross flow:

$$
N U=C R e_{D}^{m} \operatorname{Pr}^{1 / 3}
$$

Where C and m are constants based off of Table 7.2. For us, we had a Reynolds number of 4293, so our $\mathrm{C}=0.193$ and our $\mathrm{m}=0.618$. Therefore, our equation looked like:

$$
N U=0.193 * 4293^{0.618} * 8.56^{1 / 3}=69.4
$$

## Find Heat Transfer Coefficient of Water over Pipe

Once the Nusselt was obtained, we could find the heat transfer coefficient of water flowing over the pipe using the following equation:

$$
\begin{gathered}
h_{\text {water }}\left[\frac{W}{m^{2}-k}\right]=\frac{N U \times k_{\text {water }}\left[\frac{W}{m-k}\right]}{D[m]} \\
\text { Which gave us: } \\
h_{\text {water }}=\frac{69.4^{*} 0.5916}{0.35}=117.3
\end{gathered}
$$

## Finding the First Relationship

## Pipe Length

To find the length, we then had to use the special case of external convection and internal flow:

$$
\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}=\exp \left[-\frac{1}{m_{d o t}{ }_{p}} \frac{1}{\Sigma R_{t h}}\right]
$$

Where $\Sigma R_{t h}$ is the summation of all the resistances in the system. For us, the resistances are that due to outer convection: $\frac{1}{h_{\text {water }} A_{o}}$, inner convection: $\frac{1}{h_{\text {air }} A_{i}}$, and conduction through the tube (since it is thick wall, cannot neglect resistance): $\frac{\ln \left(\frac{r_{2}}{r_{1}}\right)}{2 \pi L k}$. The areas for the convective resistances are equal to $\pi D L$, so for the outer area: $\pi(0.2) L$, and inner: $\pi(0.35) L$. Therefore, we can rearrange the equation for $L$ :

$$
\begin{aligned}
& -\ln \left(\frac{T_{\infty}-T_{m, 0}}{T_{\infty}-T_{m, i}}\right) * m_{d o t} C_{p}=\frac{1}{\frac{1}{L}\left(\frac{1}{h_{\text {water }} \pi D_{o}}+\frac{1}{h_{\text {air }} \pi D_{i}}+\frac{\ln \left(\frac{r_{2}}{r_{1}}\right)}{2 \pi k}\right)} \\
& \text { Which gives: } \\
& L=\frac{-\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m i}}\right) * m_{d o t} c_{p}}{1} \frac{1}{\left(\frac{1}{h_{\text {water }} T_{o}}+\frac{1}{h_{\text {air }} \pi D_{i}}+\frac{r_{2}\left(\frac{r_{2}}{r_{1}}\right.}{2 \pi k}\right)} \text { 2mk }\left(\ln \left(\frac{T_{\infty}-T_{m, o}}{T_{\infty}-T_{m, i}}\right) * m_{d o t} c_{p}\right)\left(\frac{1}{h_{\text {water }} \pi D_{o}}+\frac{1}{h_{\text {air }} \pi D_{i}}+\frac{\ln \left(\frac{r_{2}}{r_{1}}\right)}{2 \pi k}\right)
\end{aligned}
$$

Therefore, for the first volumetric flow rate value of $0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s},\left(m_{d o t}=Q^{*} \rho_{\text {air }}=0.024\right)$ we get a length of:
$-\left(\ln \left(\frac{13-19}{13-26}\right) * 0.024 * 1007\right)\left(\frac{1}{117.3^{*} \pi^{*} 0.35}+\frac{1}{3.65^{*} \pi^{*} 0.2}+\frac{\ln \left(\frac{0.35 / 2}{0.2 / 2}\right)}{2 \pi^{*} 0.02594}\right)=71.2 m$
Then, we did this for each value of volumetric flow rate so we were able to see a parametric relationship between the volumetric flow rate and the required length for the exit temperature to be maintained at 19 degrees C. See graph below.


Looking at the graph above our data makes sense. The longer the pipe the higher your flow rate needs to be to push the air all the way down the pipe.

## Pressure Drop

## Find Friction Factor

The first step in finding the pressure drop is finding the friction factor, since the equation for pressure drop is as follows:

$$
\Delta P=f \frac{\rho_{a i r}{ }^{*} u_{m}^{2}}{2} * \frac{\Delta x}{D_{i}}
$$

Where $u_{m}$ is our mean velocity (obtained from volumetric flow rate), and $\Delta \mathrm{x}$ is our change in length. To get our friction factor f , we must use the equation for turbulent flow:

$$
\frac{1}{\sqrt{f}}=-1.8 \log _{10}\left(\left(\frac{\epsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)
$$

Solve for f :

$$
f=\left(\frac{1}{-1.8 \log _{10}\left(\left(\frac{\epsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R e}\right)}\right)^{2}
$$

For the roughness $\epsilon$, our pipe is smooth.
Therefore, our friction factor for volumetric flow rate of $0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}$ was:

$$
f=\left(\frac{1}{-1.8 \log _{10}\left(\left(\frac{0 / 0.2}{3.7}\right)^{1.11}+\frac{6.9}{8244}\right)}\right)^{2}=0.033
$$

Also, our mean velocity is $u_{m}=\frac{m_{\text {dot }}}{\rho_{\text {air }} A_{c}}$ where $A_{c}=\frac{\pi D^{2}}{4}$
For $\mathrm{Q}=0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}, u_{m}=\frac{0.024}{1.1824^{*}\left(\pi(0.2)^{2} / 4\right)}=0.637 \mathrm{~m} / \mathrm{s}$
So we can find our pressure drop:

$$
\Delta P=0.033 \frac{1.1824^{*} 0.637^{2}}{2} * \frac{71.2}{0.2}=2.78 P a
$$

Our pressure drop ranged from this value all the way up to $129,461 \mathrm{~Pa}$


Looking at the figure above, it shows that as the pipe length increases the pressure drop also increases. This makes sense because the pipe roughness slows the air down in the pipe.
Therefore the pressure in the pipe drops because the speed of the air in the pipe slows down the further it goes.

Fan Power to Overcome Friction

To make air in the duct properly flow a fan was added to the design criteria.
To find the amount of power needed for the fan to overcome the friction in the pipe we used this equation:

$$
\begin{gathered}
\text { Power }=\frac{m_{\text {dot }} \Delta P}{\rho}=Q \Delta P \\
\text { For } \mathrm{Q}=0.02 \mathrm{~m}^{\wedge} 3 / \mathrm{s}, \Delta \mathrm{P}=2.95 \mathrm{~Pa} \\
\text { Power }=0.02 * 2.95=0.000059 \mathrm{Watts}
\end{gathered}
$$



Our data shows that as the pipe length increases, the more power the fan is going to need. This makes sense because the farther the air has to travel, the more the air has to be pushed harder from the start of the duct.


This table shows that as the friction in the pipe increases, the maximum length the pipe can be is reduced. This makes sense because the more friction there is, the faster the pipe will heat up and thus increase the exit temperature.

## Heat Transfer Rate

To find the heat transfer rate from the ocean water to the air flowing in the pipe we used this equation:

$$
q_{\text {conv }}=m_{\text {dot }} C_{p}\left(T_{m, \text { out }}-T_{m}\right)
$$

Where, $m_{d o t}=0.02 \frac{m^{3}}{s}, C_{p}=1006.9 \frac{J}{k g-k}, T_{m, o u t}=19$ Degrees $C, T_{m, i n}=26$ Degrees $C$

$$
q_{\text {conv }}[\text { watts }]=0.02 * 1006.9(19 \text { Degrees } C-26 \text { Degrees } C)=-166.98 \text { [watts] }
$$

A negative number for this application makes sense because the air in the duct should be losing heat as it travels through the pipe.


Looking at the graph above the data shown makes sense. The longer the air stays in the pipe the more heat will be transferred from it due to the ocean being colder than the air inside of the pipe.

## Annual Operating Cost

To find the annual operating cost of this cooling system we used this equation:

$$
\begin{gathered}
\text { Annual Operating Cost }[\$]=\text { Fan Power }[\text { Watts }] * \frac{\text { hours }}{\text { Day }} * \frac{\$}{k W h} * \frac{\text { Days }}{\text { Year }} \\
\text { Annual Operating Cost }=0.0000589[\text { watts }] * 24 \frac{\text { hours }}{\text { Day }} * \frac{\$ 0.15}{k W h} * \frac{365 \text { Days }}{\text { Year }}=\$ 0.07
\end{gathered}
$$

$\$ 0.07$ is a very low cost for this cooling system. After all of the construction to build this cooling system is done it makes a little bit of sense because you are only paying for the electricity to run the fan. You would not be paying to cool the air in a cooling system like an AC. The air would be cool naturally by the water in the ocean. It should also be taken into account that this is the cost to run the system for a very low volumetric flow rate. To make a low volumetric flow rate, the fan will naturally not need to spin very fast, thus requiring less power. For higher volumetric flow rates, such as $1 \mathrm{~m}^{\wedge} 3 / \mathrm{s}$, the required fan power was about 129 kW , which had a yearly cost of about $\$ 170,000$. So the cost to run the system is heavily dependent on how fast the flow rate must be.


The graph above shows that the longer the pipe is underwater the higher the annual operating cost. This is because of how much you would have to pay to supply electricity to power the fan in the duct. The longer the pipe, the more electricity the fan will need to pump the same amount
of volume through the pipe to achieve that exit temperature of 19 degrees C.


As engineers for Parametric Design, Inc you are requested to evaluate a facility cooling concept on your clients remote Pacific Ocean resort complex 500 miles west of California. Air at 26C is routed through thick wall cooper tubing, $\mathrm{Di}=0.20 \mathrm{~m}, \mathrm{Do}=0.35 \mathrm{~m}$ that flows through the cool pacific waters at a depth of 50 m at a volume flow rate that is to vary from $0.02 \mathrm{~m} 3 / \mathrm{s}$ to $1 \mathrm{~m} 3 / \mathrm{s}$. The velocity and temperature profiles can be assumed as shown below.


Your engineering staff is requested to study the parametric relationships between the submerged pipe length (L), volume flow rate, pressure drop, fan power to overcome friction within the smooth tube, and heat transfer rates, and annual operating cost at $\$ 0.15 / \mathrm{kWh}$, to ensure a discharge air temperature of 19C.

You are to use your engineering judgement to determine what data is needed, what calculations are needed, what plots are needed, and what assumptions are needed to complete the engineering analysis report and the ROAD MAP, and to meet Berry's (CEO) expectations as you know his proclivity for details and logic.

Provide a 2-4-page, 1.5 line spacing, 11 pitch font, research essay regarding the threat and impact of global warming and climate change, and your perspective on the engineering challenges your generation must face to save mankind.

All reports ae to be typed professionally as they will be posted online for display and review for future students and professionals.


## Roadmap

We need to determine the convective heat transfer coefficient between the thick wall copper pipe and the water flowing over it. This is a special case because the pipe has a thick wall.

- Given: water @ depth of $50 \mathrm{~m}=13 \mathrm{C}$ (from plot)
- The velocity of the external water (in cross flow) $=1467 \mathrm{~cm} / \mathrm{s}=14.67 \mathrm{~m} / \mathrm{s}$
- Assumptions for the water: $\rho=999.46 \mathrm{~kg} / \mathrm{m}^{3}$ and $\mu=0.0010518 \mathrm{~N}^{*} \mathrm{~s} / \mathrm{m}^{2}$

First, the Reynolds Number must be determined:

$$
R_{e D}=\frac{\rho V D}{\mu}=\frac{\left(999.46 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} \times 14.67 \frac{\mathrm{~m}}{\mathrm{~s}} \times 0.35 \mathrm{~m}\right)}{\left(1.0518 \times 10^{-3} \frac{\mathrm{~N} \cdot \mathrm{~s}}{\mathrm{~m}^{2}}\right)}=4.879 \times 10^{6}
$$

The Reynolds Number is greater than 2300. Therefore we have turbulent flow.

Now the Nusselt Number can be found:
$\overline{N U_{D}}=0.023 * R e^{4 / 5} * \operatorname{Pr}^{n}$

Where $\mathrm{n}=0.3$ due to cooling

In order to find the Nusselt Number, the Prandtl Number is needed:
$\operatorname{Pr}=\frac{\mu c_{p}}{k_{\text {fluid }}}=\frac{\left(1.0518 \times 10^{-3} \frac{\mathrm{~N} \cdot \mathrm{~s}}{\mathrm{~m}^{2}}\right) \times(4181 \mathrm{~J} / \mathrm{KgK})}{0.575 \mathrm{~W} / \mathrm{mk}}=7.64$


Now the Nusselt Number can be calculated:
$\overline{N U_{D}}=0.023 *\left(4.879 \times 10^{9}\right)^{4 / 5} * 7.64^{0.3}=9491$


The convective heat transfer coefficient can be calculated as seen below:
$\overline{h_{0}}=k_{\text {fluid }} \frac{\overline{N U_{D}}}{D}=0.575 W / m k\left(\frac{9491}{0.35 m}\right)=15,592 \frac{w}{m^{2} k}$

The convective heat transfer coefficient can be used to find the heat rate per unit length of the pipe:
$q($ per unit length $)=\overline{h_{L}} A_{S} \frac{\Delta T_{0}-\Delta T_{i}}{\ln \frac{\Delta T_{0}}{\Delta T_{i}}}=\left(10,613 \frac{\mathrm{w}}{m^{2} k}\right)(\pi \times 0.35 m \times L[m])\left(\frac{(13-19 \mathrm{C})-(13-26 \mathrm{C})}{\ln \frac{(13-19 \mathrm{C})}{(13-26 \mathrm{C})}}\right)=\frac{-106 \mathrm{~kW}}{m}$

Now we can find the pressure drop:

$$
\frac{1}{\sqrt{f}}=-1.8 \log \left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11}+\frac{6.9}{R_{e}}\right)
$$



Equivalent Roughness for New Pipes [Adapted from Moody (Ref. 7) and Colebrook (Ref. 8)]

|  | Equivalent Roughness, $\varepsilon$ |  |
| :--- | :--- | :--- |
| Pipe | Feet | Millimeters |
| Riveted steel | $0.003-0.03$ | $0.9-9.0$ |
| Concrete | $0.001-0.01$ | $0.3-3.0$ |
| Wood stave | $0.0006-0.003$ | $0.18-0.9$ |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Commercial steel |  |  |
| $\quad$ or wrought iron | 0.00015 | 0.045 |
| Drawn tubing | 0.000005 | 0.0015 |
| Plastic, glass | 0.0 (smooth) | 0.0 (smooth) |

$$
\frac{1}{\sqrt{f}}=-1.8 \log \left(\left(\frac{4.3 \times 10^{-6}}{3.7}\right)^{1.11}+\frac{6.9}{169617}\right)
$$

$f=3.44$
$\Delta P($ per unit length $)=f \frac{\rho u^{2}}{2} \frac{\Delta x}{D}=(3.44)\left(\frac{\left(999.46 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)\left(14.67^{2} \mathrm{~m} / \mathrm{s}\right)}{2}\right)\left(\frac{1 \mathrm{~m}}{0.2 \mathrm{~m}}\right)=1.85 \mathrm{MPa} / \mathrm{m}$

Now we can find the relationship between the internal flow rate and the pipe length:
$\dot{\mathrm{m}} c_{p}=\frac{-A_{s} \overline{h_{L}}}{\ln \frac{\Delta T_{0}}{\Delta T_{i}}}=\frac{-(\pi D L) \overline{h_{L}}}{\ln \frac{\Delta T_{0}}{\Delta T_{i}}}$
$L=\frac{\ln \frac{\Delta T_{0}}{\Delta T_{i}} \times \dot{\mathrm{m}} c_{p}}{-\pi D h_{L}}$


Now the work done to flow the fluid can be calculated per unit length:

$$
\dot{\mathrm{W}}_{f l o w}=\frac{\dot{\mathrm{m}} \Delta P}{\rho}=\frac{\left(\dot{\mathrm{m}} \frac{\mathrm{~kg}}{\mathrm{~s}}\right)(1.85 \mathrm{MPa} / \mathrm{m})}{\left(999.46 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right)}
$$

Flow Work vs Pipe Length



The annual cost of operating this system can also be found by using the flow work and the operating cost in kWh in order to calculate the yearly cost depending on the length of the pipe:

Annual Cost vs Length



Finally, the heat rate can also be found depending on the length of the pipe, using the heat rate per unit length calculation from earlier:

Heat Rate vs Length



