CRISIS ON TRITIUM IV

A dangerous biohazard material is discovered within a dead volcano on Tritium IV and has the potential to radically impact the colony's health safety.

As the senior thermal safety engineering officer and responsible for colony, the security council request you to provide the attached analysis and modelling.

HDE #5

Dr. K. J. Berry
ASME FELLOW

BIOHAZARD INTERNAL HEAT GENERATION THREAT

$$\dot{S}_{gen}(r,t) = S_0\left(1+\left(\frac{r}{r_1}\right)^{2.5}\right)\left(1-e^{-\beta t}\right), 0 \le r \le r_1, t \ge 0, r_1 = 100mm, \Delta_{wall} = 30mm$$

where the time constant $\beta = (10 \text{ days})^{-1}$, and,

$$\mathrm{k}_g = 0.7\frac{W}{m-K}, \alpha_g = 3.5x10^{-4}\frac{m^2}{s}, S_0 = 50k\left[\frac{W}{m^3}\right], T_i(r,t=0) = 800K$$

To protect the colony from the dangerous material discovered within a dead volcano on **TRITIUM IV** it is further enclosed within layer of 20 mm thick Carbon Steel followed by a layer of 40mm thick Aluminum.

The containment system is placed within a valley where the wind air velocity varies from 2 MPH to 52 MPH at 80C depending upon the time of days due to massive weather patterns driven by irregular solar winds from the bi-solar star system.

Steady State Surface Temperature vs Cooling Velocity

$$\dot{S}_{gen}(r,t)=S_0\left(1+\left(\frac{r}{r_1}\right)^{2.5}\right)\left(1-e^{-\beta t}\right),0\le r\le r_1,t\ge 0,r_1=100mm,\Delta_{wall}=30mm$$

STEADY STATE $\rightarrow$ t $\rightarrow \infty \rightarrow e^{-\beta t}\rightarrow 0$

OVERALL ENERGY BALANCE

$\dot{E}_{gen}[W]=\dot{E}_{out}[W]\rightarrow$ TOTAL ENERGY GENERATED

$$\dot{E}_{gen}[W]=\int_0^{r_1}\dot{S}_{gen}(r,t)\left[\frac{W}{m^3}\right]dV,V_{sphere}=\frac{4}{3}\pi r^3,\frac{dV}{dr}=4\pi r^2,dV=4\pi r^2dr$$

$$\dot{E}_{gen}[W]=S_0 4\pi\int_0^{r_1}\left(1+\left(\frac{r}{r_1}\right)^{2.5}\right)r^2\left[\frac{W}{m^3}\right]dr=S_0 4\pi\int_0^{r_1}\left(r^2+\left(\frac{r^{4.5}}{r_1^{2.5}}\right)\right)\left[\frac{W}{m^3}\right]dr$$

$$\dot{E}_{gen}[W]=S_0 4\pi\left[\frac{r^3}{3}+\frac{r^{5.5}}{5.5r_1^{2.5}}\right]_{0-r_1}=S_0 4\pi r_1^3\left(\frac{1}{3}-\frac{1}{5.5}\right)=1.9S_0\left[\frac{W}{m^3}\right]r_1^3[m^3]$$

$$\dot{E}_{gen}[W]=1.9S_0\left[\frac{W}{m^3}\right]r_1^3[m^3]=\dot{E}_{out}[W]=\bar{h}(u_\infty)A_0(T_s(u_\infty,r_1)-T_\infty)$$

$$T_s(r_1,u_\infty)=\frac{1.9S_0\left[\frac{W}{m^3}\right]r_1^3[m^3]}{\bar{h}(u_\infty)A_0}+T_\infty$$

$$A_0=4\pi r_0^2$$

$$r_0=\frac{(100+30+20+40)mm}{1000mm/m}=\frac{190}{1000}=0.19m$$

HEAT TRANSFER COEFFICIENT VS VELOCITY

SPHERE IN CROSS FLOW

$$\overline{NU}_D = \frac{\bar{h}D}{k_{fluid}} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left[\frac{\mu(T_\infty)}{\mu(T_s)}\right]^{1/4}$$

$$\mathrm{Re}_D = \frac{\rho \bar{V} D}{\mu}$$

All Other Properties Evaluated at T_∞

PROPERTIES

$$\text{GUESS } T_s = \frac{T_i + T_\infty}{2} = \frac{800+80}{2} = 440C = 713K$$

TABLE A.4

SUEFACE

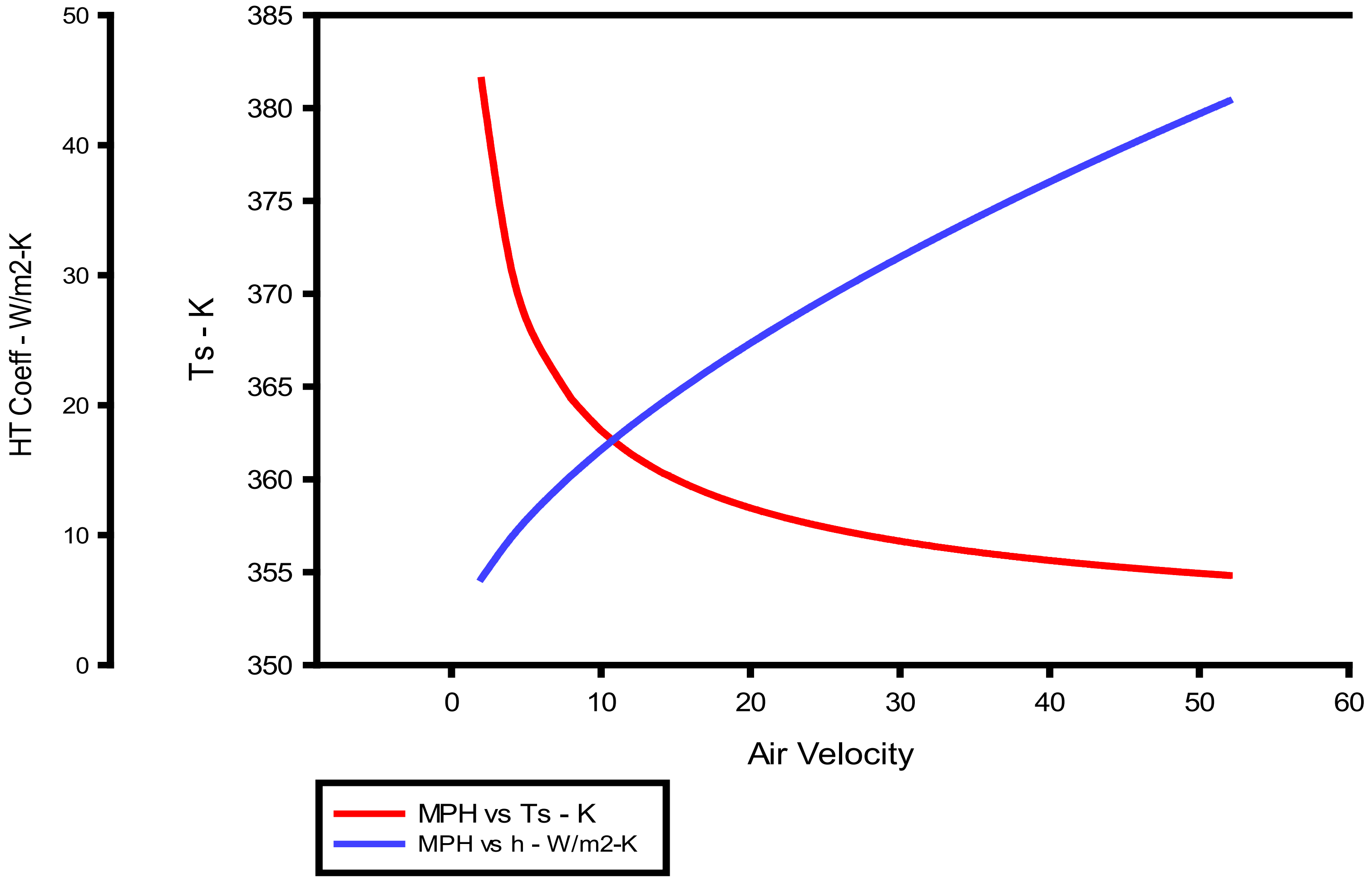
$$700\mathrm{K}, \mu \approx 338.8\mathrm{x}10^{-7}\,\frac{m^2}{s}$$

AMBIENT

$$350K, \mu \approx 208.2\mathrm{x}10^{-7}\,\frac{m^2}{s}, \nu \approx 20.92x10^{-6}\,\frac{m^2}{s}, k \approx 30.0x10^{-3}W/m-K, \mathrm{Pr} \approx 0.700$$

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{u_\infty 2r_0}{\nu}$$

Biohazard Cooling Model
Air Velocity vs Ts and Heat Transfer Coefficient



NOTE: Ts was over-estimated at 713K for any velocity. We should re-set Ts to determine the impact to the VISCOSITY ratio between Ts and Tf. If major change, re-do process with new values.

OVERALL THERMAL RESISTANCE

OVERALL THERMAL RESISTANCE

$$UA\left[\frac{K}{W}\right]=\frac{1}{\sum R_{th}}$$

$$=\frac{1}{\dfrac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi k_{Lead}}+\dfrac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi k_{SS}}+\dfrac{\frac{1}{r_3}-\frac{1}{r_4}}{4\pi k_{AL}}+\dfrac{1}{\overline{h}A_0}}$$

$$U_0\left[\frac{W}{m^2-K}\right]=\frac{1}{A_0\sum R_{th}}=\frac{1}{A_0\left[\dfrac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi k_{Lead}}+\dfrac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi k_{SS}}+\dfrac{\frac{1}{r_3}-\frac{1}{r_4}}{4\pi k_{AL}}\right]+\dfrac{1}{\overline{h}}}$$

OVERALL HEAT TRANSFER COEFFICIENT

$$UA\left[\frac{K}{W}\right]=U_0A_0=\frac{1}{\sum R_{th}}$$

INTERFACE TEMPERATURES --Thermal Circuits, T3

$$q[W] = E_{gen}[W] = UA\Delta T = \frac{\Delta T}{\sum R_{th}} = \frac{\Delta T}{\dfrac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k_{Lead}} + \dfrac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi k_{SS}} + \dfrac{\frac{1}{r_3} - \frac{1}{r_4}}{4\pi k_{AL}} + \dfrac{1}{\bar{h}A_0}}$$

$$E_{gen}[W] = \frac{T_3 - T_4}{\dfrac{\frac{1}{r_3} - \frac{1}{r_4}}{4\pi k_{AL}}} = \frac{T_3 - T_s(u_\infty)}{\dfrac{\frac{1}{r_3} - \frac{1}{r_4}}{4\pi k_{AL}}}$$

$$T_3(u_\infty) = E_{gen}[W] \times \frac{\frac{1}{r_3} - \frac{1}{r_4}}{4\pi k_{SS}} + T_s(u_\infty)$$

INTERFACE TEMPERATURES --Thermal Circuits, T2

$$q[W]=E_{gen}[W]=UA\Delta T=\frac{\Delta T}{\sum R_{th}}=\frac{\Delta T}{\dfrac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi k_{Lead}}+\dfrac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi k_{SS}}+\dfrac{\frac{1}{r_3}-\frac{1}{r_4}}{4\pi k_{AL}}+\dfrac{1}{\bar{h}A_0}}$$

$$E_{gen}[W]=\frac{T_2-T_3(u_\infty)}{\dfrac{\frac{1}{r_3}-\frac{1}{r_4}}{4\pi k_{ss}}}$$

$$T_2(u_\infty)=E_{gen}[W]\times\frac{\frac{1}{r_2}-\frac{1}{r_3}}{4\pi k_{ss}}+T_3(u_\infty)$$

INTERFACE TEMPERATURES --Thermal Circuits, T1

$$q[W] = E_{gen}[W] = UA\Delta T = \frac{\Delta T}{\sum R_{th}} = \frac{\Delta T}{\frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k_{Lead}} + \frac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi k_{SS}} + \frac{\frac{1}{r_3} - \frac{1}{r_4}}{4\pi k_{AL}} + \frac{1}{\bar{h}A_0}}$$

$$E_{gen}[W] = \frac{T_1 - T_2(u_\infty)}{\frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k_{Lead}}}$$

$$T_1(u_\infty) = E_{gen}[W] \times \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k_{lead}} + T_2(u_\infty)$$

INTERFACE TEMPERATURES

				W/m2-K	K	K	K	K
MPH	m/s	ReD	NUD	h (u)	Ts	T3	T2	T1
2	0.8000	1.4532E+04	8.4208E+01	6.6480E+00	381.5003	381.5451	381.6732	382.1674
4	1.6000	2.9063E+04	1.2432E+02	9.8147E+00	371.3368	371.3816	371.5098	372.004
6	2.4000	4.3595E+04	1.5657E+02	1.2360E+01	366.9423	366.9870	367.1152	367.6094
8	3.2000	5.8126E+04	1.8460E+02	1.4574E+01	364.3690	364.4137	364.5419	365.0361
10	4.0000	7.2658E+04	2.0989E+02	1.6571E+01	362.6378	362.6825	362.8107	363.3049
12	4.8000	8.7189E+04	2.3319E+02	1.8410E+01	361.3750	361.4198	361.5480	362.0422
14	5.6000	1.0172E+05	2.5497E+02	2.0129E+01	360.4037	360.4485	360.5766	361.0709
16	6.4000	1.1625E+05	2.7551E+02	2.1751E+01	359.6278	359.6726	359.8008	360.295
18	7.2000	1.3078E+05	2.9505E+02	2.3293E+01	358.9904	359.0352	359.1633	359.6576
20	8.0000	1.4532E+05	3.1372E+02	2.4768E+01	358.4551	358.4999	358.6281	359.1223
22	8.8000	1.5985E+05	3.3167E+02	2.6184E+01	357.9977	358.0425	358.1707	358.6649
24	9.6000	1.7438E+05	3.4897E+02	2.7550E+01	357.6013	357.6460	357.7742	358.2684
26	10.4000	1.8891E+05	3.6570E+02	2.8871E+01	357.2535	357.2983	357.4264	357.9207
28	11.2000	2.0344E+05	3.8192E+02	3.0152E+01	356.9454	356.9901	357.1183	357.6125
30	12.0000	2.1797E+05	3.9769E+02	3.1396E+01	356.6700	356.7148	356.8429	357.3372
32	12.8000	2.3250E+05	4.1304E+02	3.2609E+01	356.4221	356.4668	356.5950	357.0892
34	13.6000	2.4704E+05	4.2802E+02	3.3791E+01	356.1974	356.2421	356.3703	356.8645
36	14.4000	2.6157E+05	4.4265E+02	3.4946E+01	355.9925	356.0373	356.1655	356.6597
38	15.2000	2.7610E+05	4.5696E+02	3.6076E+01	355.8049	355.8497	355.9778	356.472
40	16.0000	2.9063E+05	4.7097E+02	3.7182E+01	355.6322	355.6769	355.8051	356.2993
42	16.8000	3.0516E+05	4.8471E+02	3.8266E+01	355.4725	355.5173	355.6455	356.1397
44	17.6000	3.1969E+05	4.9819E+02	3.9331E+01	355.3244	355.3692	355.4974	355.9916
46	18.4000	3.3423E+05	5.1143E+02	4.0376E+01	355.1866	355.2314	355.3595	355.8537
48	19.2000	3.4876E+05	5.2444E+02	4.1403E+01	355.0579	355.1027	355.2308	355.725
50	20.0000	3.6329E+05	5.3725E+02	4.2414E+01	354.9374	354.9821	355.1103	355.6045
52	20.8000	3.7782E+05	5.4985E+02	4.3409E+01	354.8242	354.8690	354.9972	355.4914

$$T_s(u_\infty, r_1) = \frac{1.9 S_0 \left[\dfrac{W}{m^3}\right] r_1^3 [m^3]}{\bar{h}(u_\infty) A_0} + T_\infty$$

MGS SOLUTION – LEAD SPHERE

LEAD INSULATION

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dT}{dr})+\frac{\dot{S}_{gen}(r,t)}{k_r}=\frac{\rho c_p}{k_r}\frac{dT}{dt}=0, r_1\le r\le r_2$$

BC

$$T(r=r_1)=T_1(u_\infty), T(r=r_2)=T_2(u_\infty)$$

MGS

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

EXACT SOLUTION – LEAD SPHERE

$$T(r,u_\infty) = T_1(u_\infty) + \frac{T_2(u_\infty) - T_1(u_\infty)}{(\frac{1}{r_1} - \frac{1}{r_2})}(\frac{1}{r_1} - \frac{1}{r});$$

$$r_1 \le r \le r_2$$

$$q''(r,u_\infty)\left[\frac{W}{m^2}\right] = -k_{Lead}\frac{T_2(u_\infty) - T_1(u_\infty)}{(\frac{1}{r_1} - \frac{1}{r_2})}\left(\frac{1}{r^2}\right)$$

STEADY STATE CENTER TEMPERATURE VS. VELOCITY

$$\dot{S}_{gen}(r,t) = S_0\left(1+\left(\frac{r}{r_1}\right)^{2.5}\right)\left(1-e^{-\beta t}\right), 0 \le r \le r_1$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{S}_{gen(r)}}{k_r} = -\frac{S_0\left(1+\left(\frac{r}{r_1}\right)^{2.5}\right)}{k_r}$$

$\bullet r^2$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{S_0\left(1+\left(\frac{r}{r_1}\right)^{2.5}\right)r^2}{k_r} = -\frac{S_0\left(r^2+\left(\frac{r^{4.5}}{r_1^{2.5}}\right)\right)}{k_r}, \text{Integrate}$$

$$r^2\frac{dT}{dr} = -\frac{S_0\left(\frac{r^3}{3}+\left(\frac{r^{5.5}}{5.5r_1^{2.5}}\right)\right)}{k_r} + C_1, \div r^2$$

$$\frac{dT}{dr} = -\frac{S_0\left(\frac{r^1}{3}+\left(\frac{r^{3.5}}{5.5r_1^{2.5}}\right)\right)}{k_r} + \frac{C_1}{r^2}$$

Integrate

$$\text{T(r)} = -\frac{S_0\left(\frac{r^2}{6}+\left(\frac{r^{3.5}}{5.5r_1^{2.5}}\right)\right)}{k_r} - \frac{C_1}{r} + C_2 \rightarrow MGS$$

BC

$@r=0, T(r=0) \rightarrow FINITE \rightarrow C_1 = 0$

$@r=r_1, T(r=r_1) = T_1(u_\infty)$

$$T_1 = -\frac{S_0\left(\frac{r_1^2}{6}+\left(\frac{r_1^{3.5}}{5.5r_1^{2.5}}\right)\right)}{k_r} + C_2$$

$$T_1(\text{u}_\infty) + \frac{S_0\left[\frac{W}{m^3}\right]r_1^2[m^2]}{k_r\left[\frac{W}{m-K}\right]}\left(\frac{1}{6}+\frac{1}{5.5}\right) = C_2[K]$$

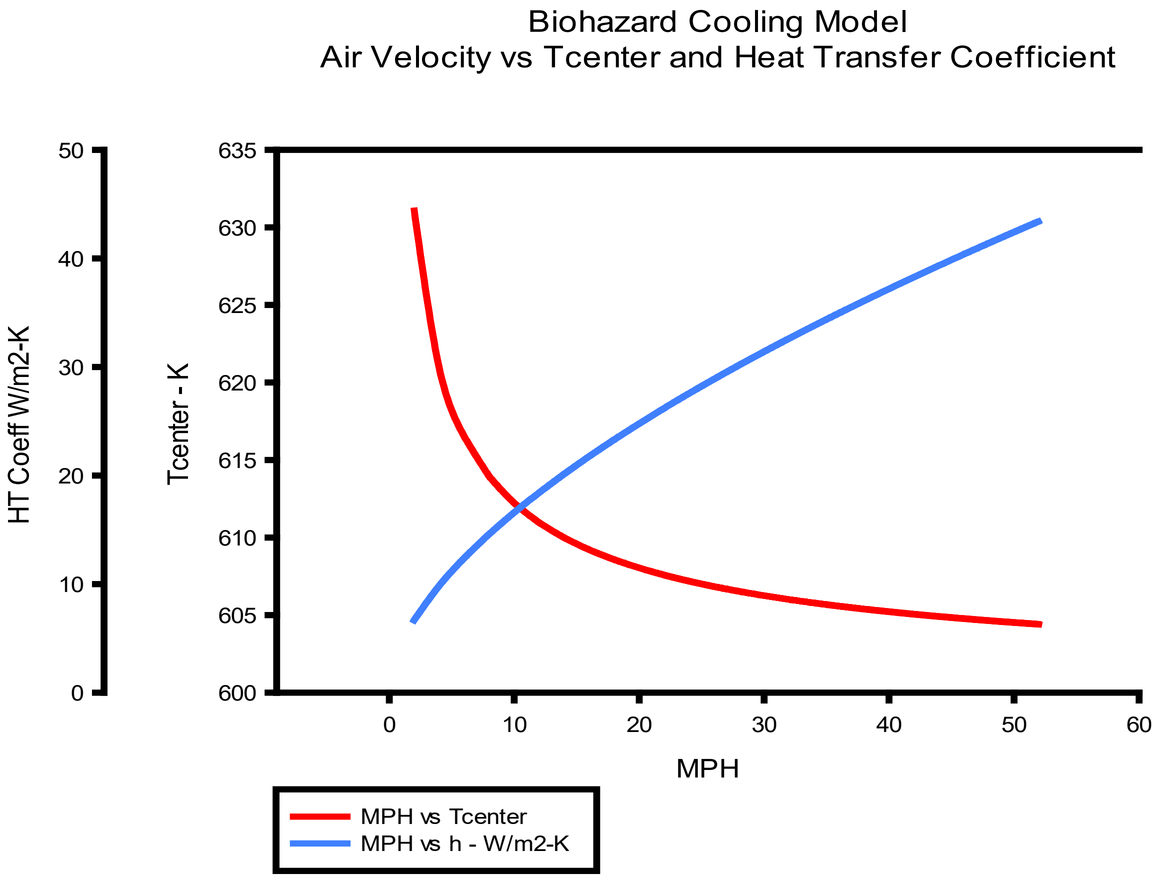
CENTER TEMPERATURE

$$T(\mathrm{r},\mathrm{u}_\infty) = -\frac{S_0\left(\frac{r^2}{6}+\left(\frac{r^{3.5}}{5.5r_1^{2.5}}\right)\right)}{k_r} + C_2(\mathrm{u}_\infty)$$

$$T_{center} = T(r=0) = C_2$$

$$T_1(u_\infty) + \frac{S_0\left[\frac{W}{m^3}\right]r_1^2[m^2]}{k_r\left[\frac{W}{m-K}\right]}\left(\frac{1}{6}+\frac{1}{5.5}\right) = C_2[K]$$

$$T_{center}(u_\infty) = T_1(u_\infty) + \frac{S_0 r_1^2}{k_r}\left(\frac{1}{6}+\frac{1}{5.5}\right)$$



TIME FOR CENTER TO REACH 200C IF LUMPED

$$\text{let: a}=\frac{h_c(u_\infty)A_c}{\rho\forall c_p}, b=\frac{E_{gen}(t)}{\rho\forall c_p}, \theta(t)=T(t)-T_\infty$$

$$\frac{d\theta}{dt}+a\theta=b(t)\rightarrow\text{First Order ODE}$$

$$\theta(t)=e^{-at}\int b(t)e^{+at}dt+Ce^{-at}; a>0$$

$$\dot{E}_{gen}(t)[W]=1.9S_0\left[\frac{W}{m^3}\right]r_1^3[m^3]\left(1-e^{-\beta t}\right)$$

$$\int b(t)e^{+at}dt=\frac{1.9S_0r_1^3}{\rho\forall c_p}\int\left(1-e^{-\beta t}\right)e^{+at}dt$$

$$=\frac{1.9S_0r_1^3}{\rho\forall c_p}\left[\frac{e^{+at}}{a}-\frac{e^{(a-\beta)t}}{a-\beta}\right]_{0-t^*}=\frac{1.9S_0r_1^3}{\rho\forall c_p}\left[\frac{e^{+at^*}}{a}-\frac{e^{(a-\beta)t^*}}{a-\beta}\right]$$

$$a=\frac{h_c(u_\infty)A_c}{\rho\forall c_p}, b=\frac{E_{gen}(t)}{\rho\forall c_p}, \theta(t,u_\infty)=T(t,u_\infty)-T_\infty$$

$$\theta(t,u_\infty)=e^{-at}\int b(t)e^{+at}dt+Ce^{-at}; a>0$$

$$\int b(t)e^{+at}dt=\frac{1.9S_0r_1^3}{\rho\forall c_p}\int\left(1-e^{-\beta t}\right)e^{+at}dt=\frac{1.9S_0r_1^3}{\rho\forall c_p}\left[\frac{e^{+at^*}}{a}-\frac{e^{(a-\beta)t^*}}{a-\beta}\right]$$

$$\theta(t,u_\infty)=e^{-at^*}\left[\frac{1.9S_0r_1^3}{\rho\forall c_p}\left[\frac{e^{+at^*}}{a}-\frac{e^{(a-\beta)t^*}}{a-\beta}\right]\right]+Ce^{-at^*}, t^*\geq 0$$

INITIAL CDN

$$\theta(t^*=0,u_\infty)=\theta_i-\left[\frac{1.9S_0r_1^3}{\rho\forall c_p}\left[\frac{1}{a}-\frac{1}{a-\beta}\right]\right]=C$$

NOTE: Since “a” is a function of the “h” and “h” is a function of convective velocity, the time to reach any temperature at any location is a function of a specified air flow velocity over the sphere.