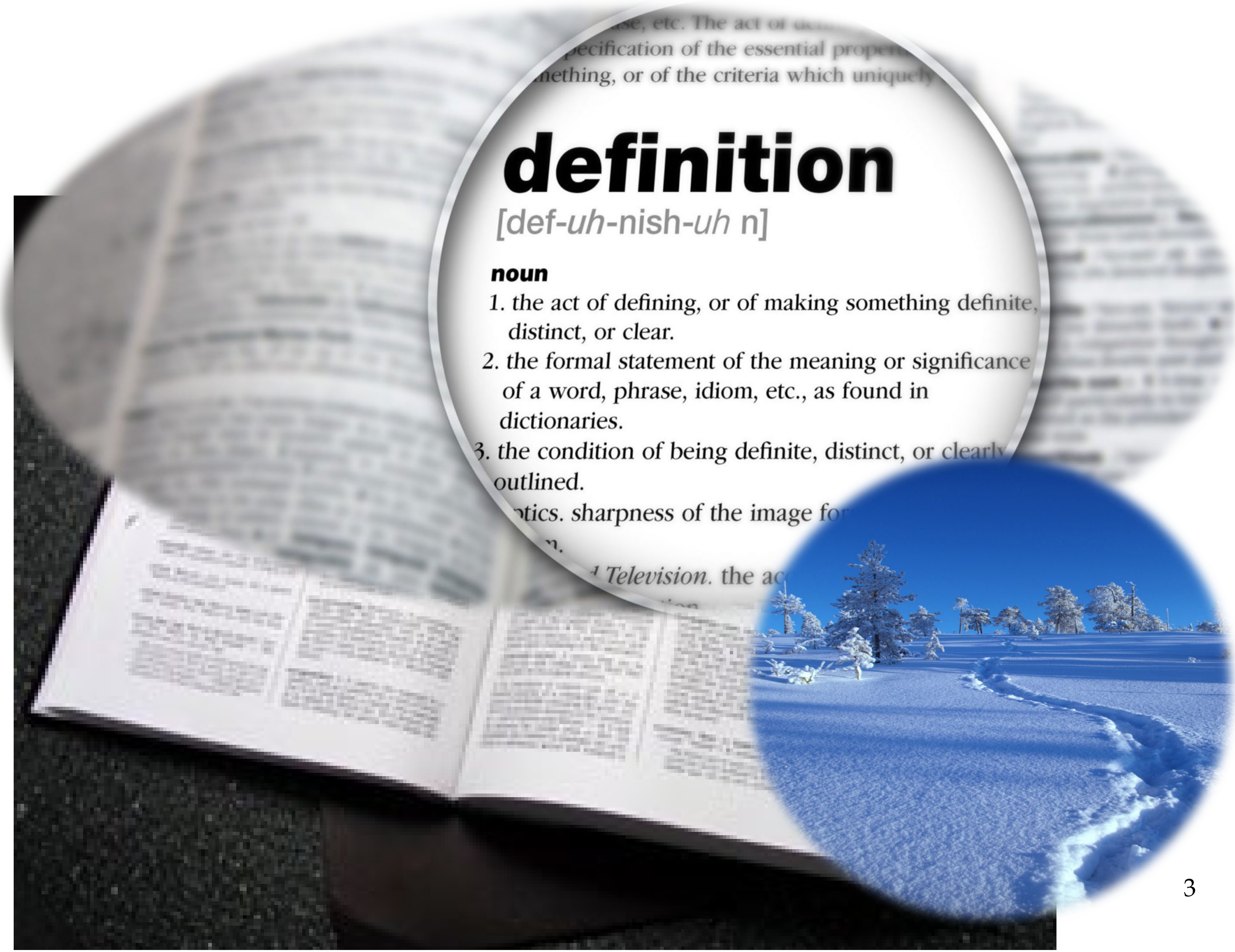
Chapter 3
Elementary Fluid Dynamics
Bernoulli Equation

PART 1

"Fluid Mechanics has helped me build a stronger problem-solving skill by helping me not focus on the problem itself, but on the path that must be followed."

Former student, Winter 2022


definition
[def-uh-nish-uh n]
noun
1. the act of defining, or of making something definite, distinct, or clear.
2. the formal statement of the meaning or significance of a word, phrase, idiom, etc., as found in dictionaries.
the condition of being definite, distinct, or clearly outlined.

Educational Objectives

- **Explain the development, uses, and limitations of the Bernoulli Equations.**
- **Use the Bernoulli equation (stand-alone or in combination with the simple continuity) to solve flow problems.**
- **Apply the concepts of static, stagnation, dynamic, and total pressures.**

Elementary Fluid Dynamics

- **To understand fluid dynamics, fundamental laws that govern motion of fluid particles must be considered**

- **Newton's second law (F=ma) can be applied to motion of fluid particles**

- **Bernoulli's equation is one of the oldest fluid mechanics equation that can be effectively used to predict and analyze flow**

- **Bernoulli's equation is also "*the most used and most abused equation in fluid mechanics*"**

Bernoulli
(1667-1748)

Elementary Fluid Dynamics

- When a fluid particle moves from one point to another, it experiences an acceleration or deceleration
- The net force on the fluid particle is: F=ma
- In this chapter we assume fluid with zero viscosity (INVISCID FLUIDS)
- In reality there is no inviscid fluid since every fluid supports shear stress
- In many flow situations, the viscous effect is smaller than other effects and can be neglected

Newton
(1642-1727)

Fluid Dynamics- Visual representation of a flow

Streamline: **Streamlines are lines drawn in the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field. Consequently,**

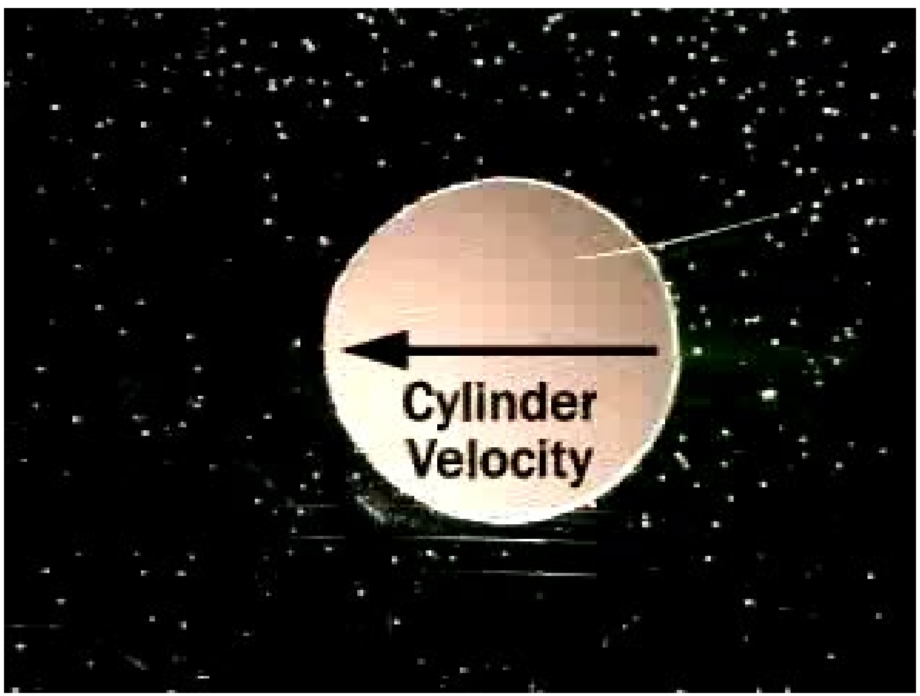
$$\left.\frac{dy}{dx}\right|_{streamline} = \frac{v}{u}$$

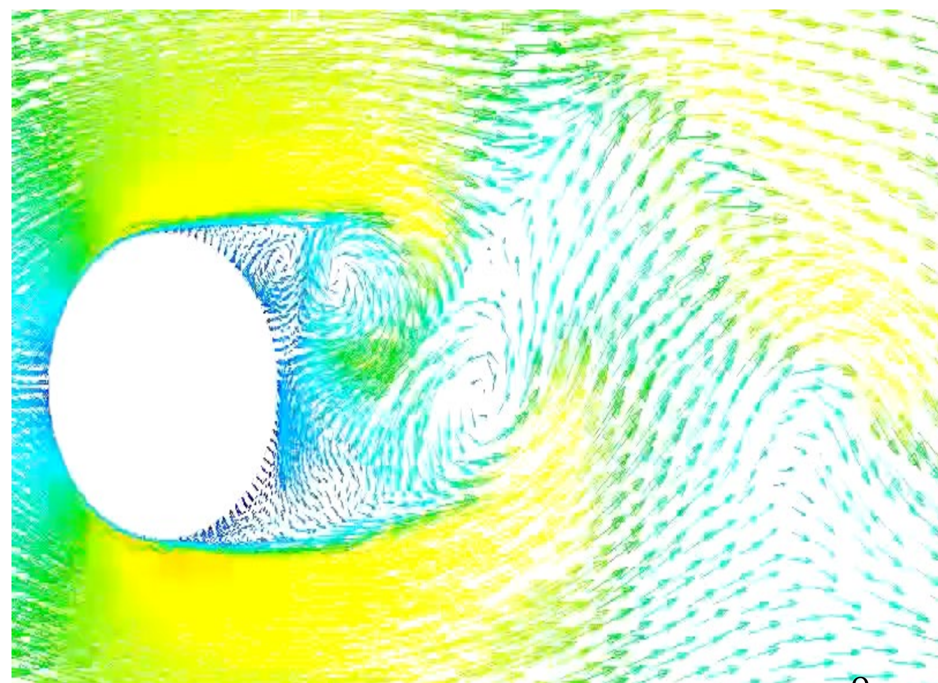
Since the streamlines are tangent to the velocity vector at every point in the flow field, there can be no flow across a streamline. Streamlines are the most commonly used visualization technique in fluid mechanics.

Visual representation of fluid flow

Boundary Layer Separation and Vortex Shedding

Streamline: **Commonly used method of flow visualization**

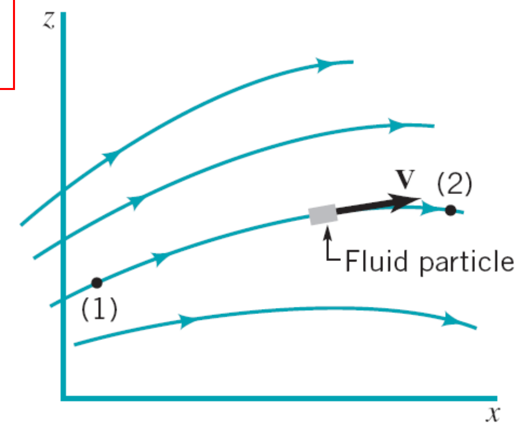




Flow Induced Vibration

Elementary Fluid Dynamics

- **The motion of each fluid particle is described in terms of its velocity vector V**

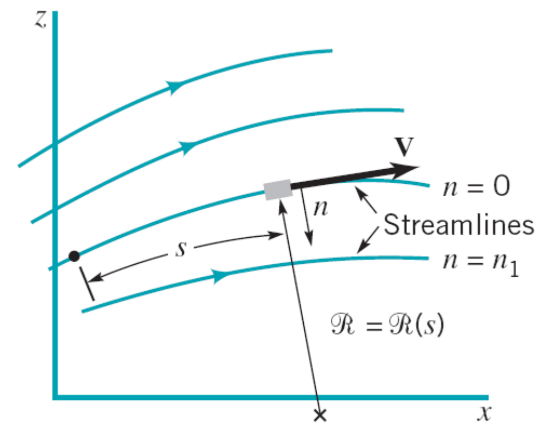


(a)

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = a_s\hat{s} + a_n\hat{n}$$

- **If nothing changes with time at a given location in flow field, it is called STEADY FLOW**

- **For steady flow, each particles slides along its path, the velocity vector is tangent to the path**

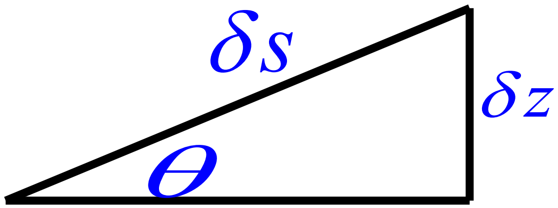


(b)

- **The lines that are tangent to the velocity vectors are called STREAMLINES**

$$a_s = \frac{\partial V}{\partial t} = V\frac{\partial V}{\partial s};\ a_n = \frac{V^2}{R}$$

Bernoulli's Equation



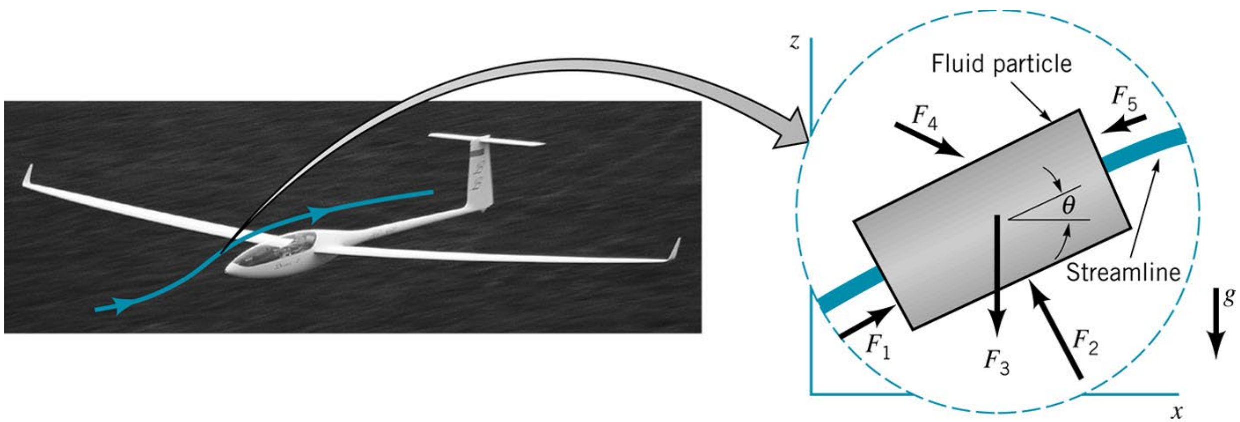
$$\sum \delta F_s = (\delta m)\, a_s$$

$$\sum \delta F_s = \delta F_{gravity} + \delta F_{pressure} + \delta F_{viscous} (=0)$$

$$\Rightarrow \sum \delta F_s = \left(-\gamma(\delta\forall)\sin\theta\right) + \left(-\frac{\partial p}{\partial s}(\delta\forall)\right) + 0$$

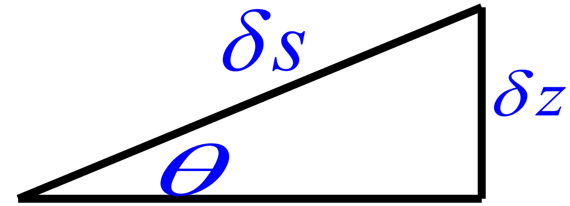
$$\delta m = \rho(\delta\forall)$$

$$a_s = \frac{\partial V}{\partial t} = V\frac{\partial V}{\partial s}$$



Bernoulli's Equation

Put it all together



$$\sum \delta F_s = (\delta m) a_s$$

$$\Rightarrow (-\gamma \delta\forall \sin\theta) + \left(-\frac{\partial p}{\partial s}\delta\forall\right) = \rho(\delta\forall)\cdot V\frac{\partial V}{\partial s}$$

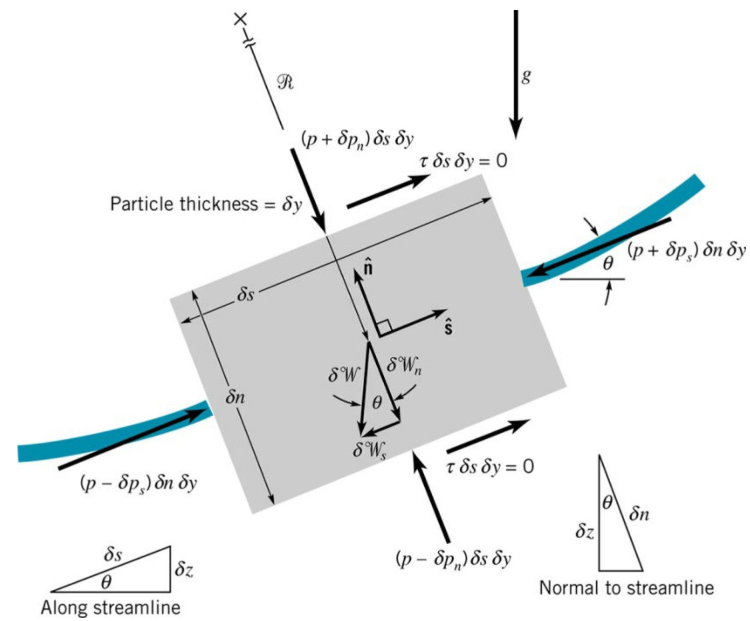
$$\Rightarrow -\gamma \sin\theta + \left(-\frac{\partial p}{\partial s}\right) = \rho V \frac{\partial V}{\partial s}$$

$$\sin\theta = \frac{\partial z}{\partial s}$$

$$\rho V \frac{\partial V}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2}\rho V^2\right)$$

$$\Rightarrow -\gamma\frac{\partial z}{\partial s} + \left(-\frac{\partial p}{\partial s}\right) = \frac{\partial}{\partial s}\left(\frac{1}{2}\rho V^2\right)$$

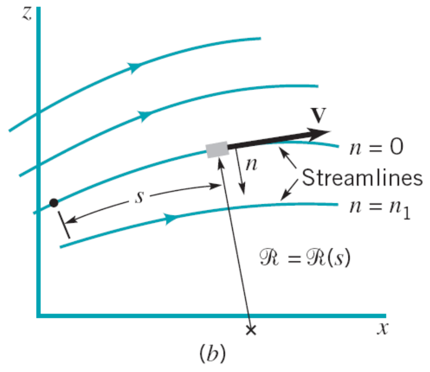
$$\boxed{p + \gamma z + \frac{1}{2}\rho V^2 = C}$$



Class 05: Bernoulli's Equation

Bernoulli Equation

$$p + \gamma z + \frac{1}{2}\rho V^2 = C$$



Only applicable if:

- **Density constant along streamline (Incompressible)**
- **Steady Flow**
- **Viscous force negligible compare to the force due to gravity and Pressure (Invisid)**
- **Only good along a Streamline**
- **No pump or work**

ISIS+ holds ⟶ Apply Bernoulli

Bernoulli's Equation - Interpretation

- **In terms of pressure**

$$p + \gamma z + \frac{1}{2}\rho V^2 = C, \text{ along a streamline}$$

p — **Static Pressure**

γz — **Hydrostatic Pressure – Related to potential energy variation due to elevation changes**

$\frac{1}{2}\rho V^2$ — **Dynamic Pressure**

Bottomline: Sum of the pressures is called total pressure

Bernoulli's Equation - Interpretation

- **In terms of heights - called heads**

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C, \text{ along a streamline}$$

$$\frac{p}{\gamma}$$

Pressure head – represents the height of a column of the fluid that is needed to produce the pressure p.

$$z$$

Elevation head – elevation term, related to the potential energy of the fluid particle.

$$\frac{V^2}{2g}$$

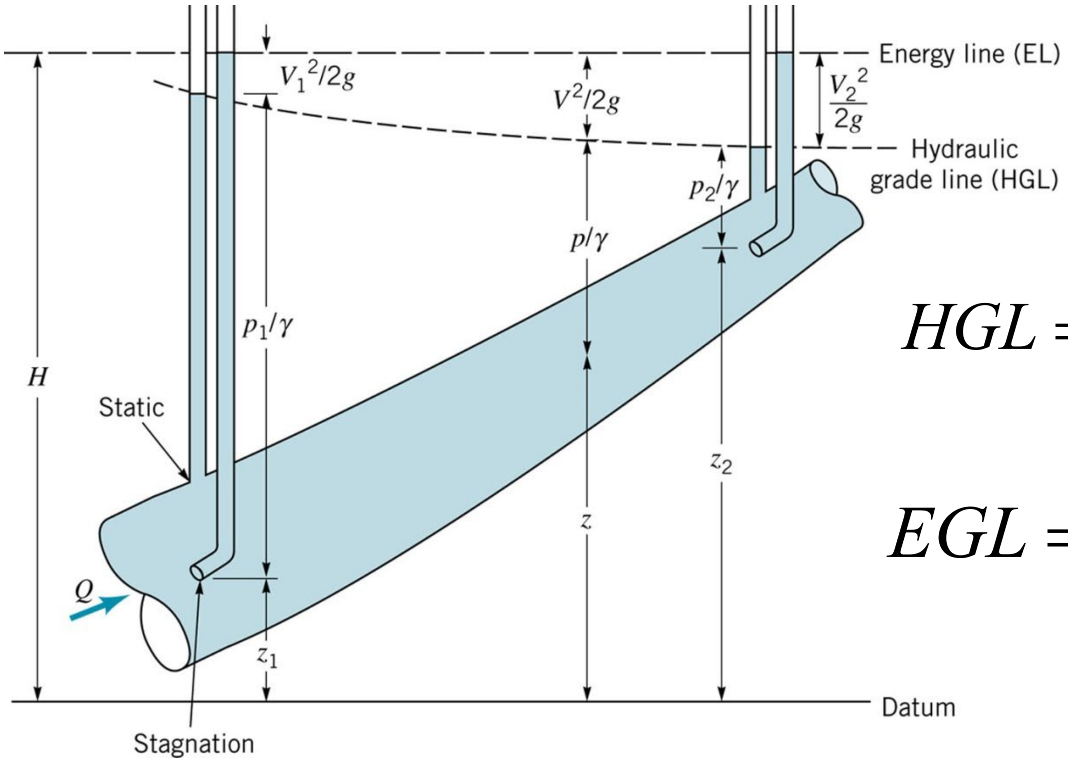
Velocity head – represent the vertical distance needed for the fluid to fall freely (neglecting friction) if it is to reach velocity V from rest.

Bottomline: Sum of the heads is constant along a streamline

Hydraulic & Energy Grade lines

❑ In terms of Energy

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C, \text{ along a streamline}$$



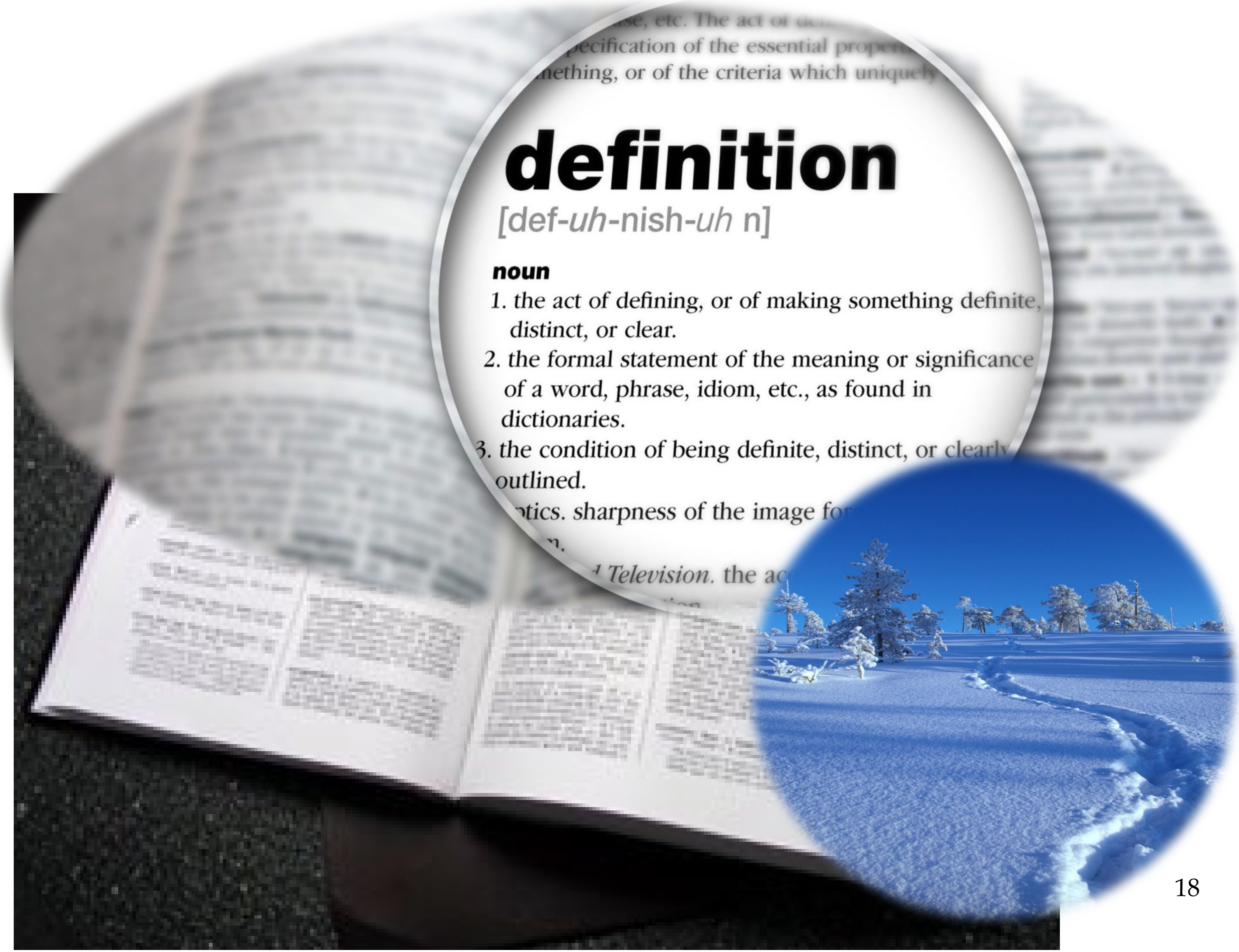
Piezometric Head

$$HGL = \frac{P}{\gamma} + Z$$

Potential Energy?

$$EGL = HGL + \frac{V^2}{2g}$$

Kinetic Energy?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the condition of being definite, distinct, or clearly
outlined.
sharpness of the image
Television. the

DEFINITION

BERNOULLI represents the CONSERVATION of ENERGY or "***TOTAL PRESSURE*** " along a STREAMLINE between "ANY" two points.

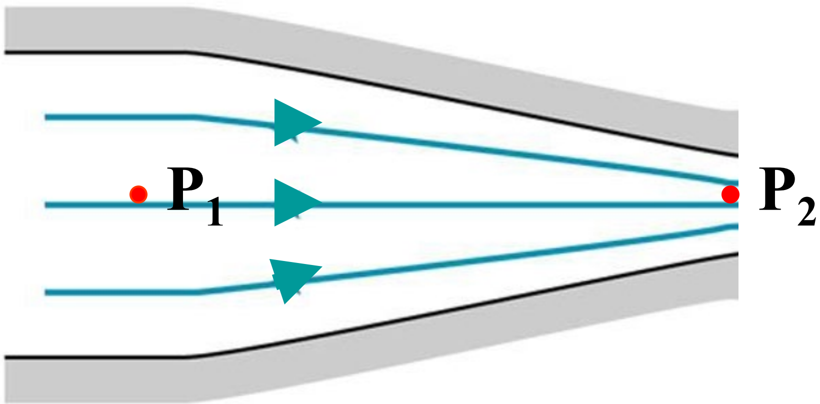
It can be applied for the following conditions:

Incompressible
Steady Flow
INVISCID (Ignore Friction)
Streamline
NO SHAFT WORK Between Points

Bernoulli's Equations - Example

Problem # 1: Air flows through a nozzle and exits to the atmosphere as shown in the Figure below. If the pressure at the upstream location is read as 100kPa and the velocity at the upstream is known to be about 30mph (13m/s), what is the velocity at the nozzle exit?

Solution: Consider 2 points along a streamline – one point at the upstream and another one is at the exit.



Given: $P_1 = 100kPa$, $V_1 = 13m/s$, $P_2=0$

Apply Bernoulli at P_1 and P_2:

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma Z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma Z_2;\ Z_1 = Z_2 \text{ - same elevation; } \rho=1.23\,kg/m^3$$

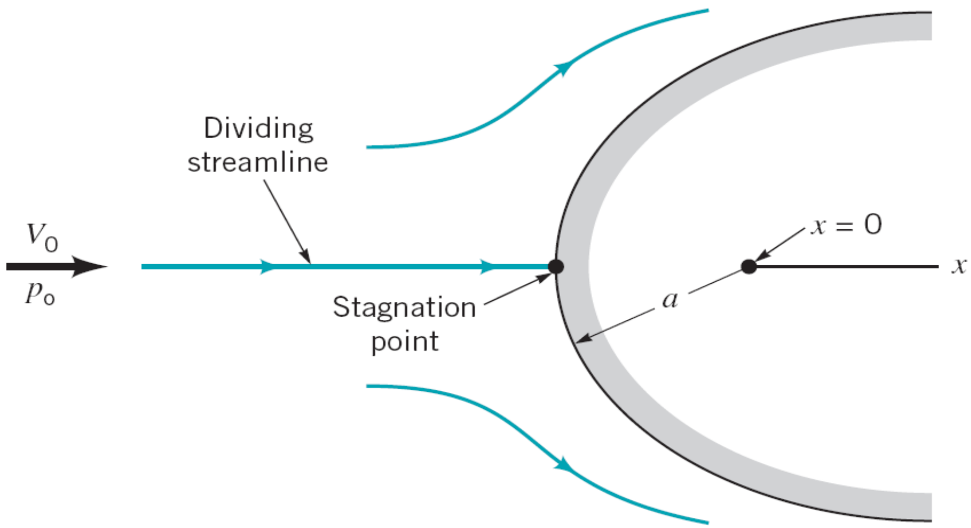
$$\Rightarrow V_2 = \sqrt{V_1^2 + 2\left(P_1 - P_2\right)/\rho} = 443.45\,m/s$$

Stagnation Point

Stagnation point: **The point where the fluid particle velocity become stationary, i.e. <u>ZERO VELOCITY</u>. There is a stagnation point on any stationary body placed into a flowing fluid. Some of the fluid flows 'over' and some 'under' the object.**

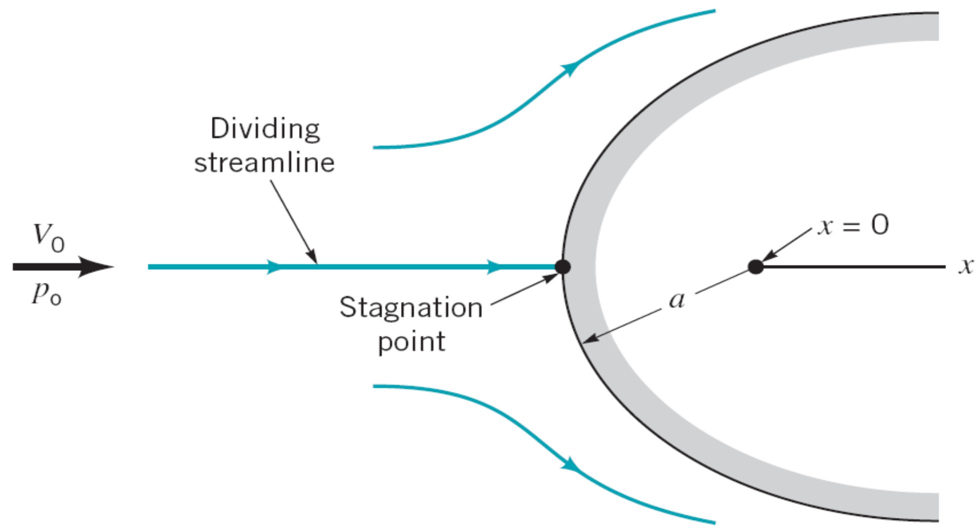
The streamline of the fluid particle has to divide at the stagnation point. The divided line is called stagnation streamline.

Stagnation Pressure:



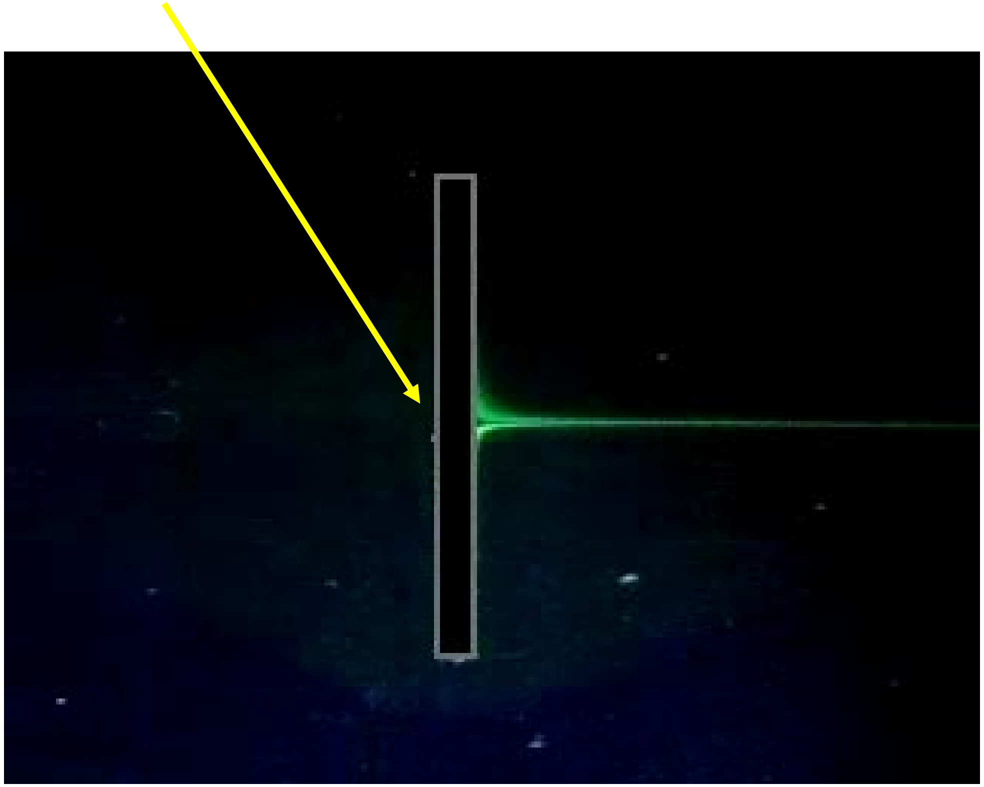
$$\overbrace{P_{stag}}^{TOTAL} = \overbrace{P_0}^{STATIC} + \overbrace{\frac{1}{2}\rho V_0^2}^{DYNAMIC}$$

$$V_0 = \sqrt{\frac{2\left(P_{stag} - P_0\right)}{\rho}}$$



Visual representation of fluid flow

Stagnation point and dividing streamlines



Stagnation Point

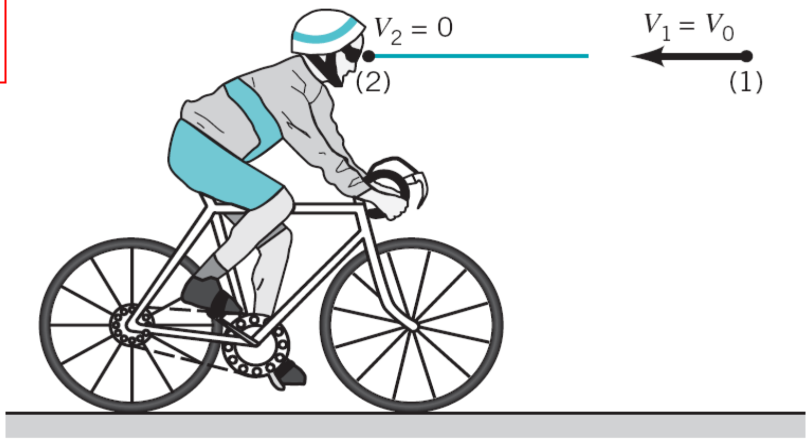
Example: Stagnation point

- To determine the difference in pressure between point 1 and 2, Bernoulli's equation can be applied along the streamline that passes through point 1 and point 2

- Point (1) to be free stream so that $V_1 = V_0$

- Assuming point (1) and (2) are at same elevation $z_1 = z_2$

- By measuring the differential pressure $p_2 - p_1$, the speed V_0 can be determined

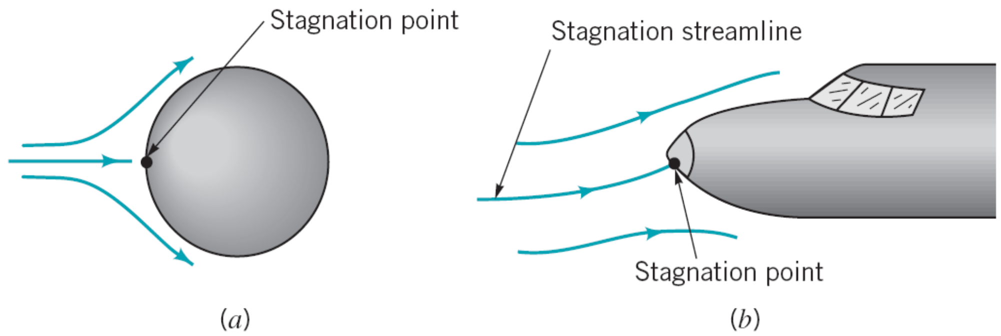


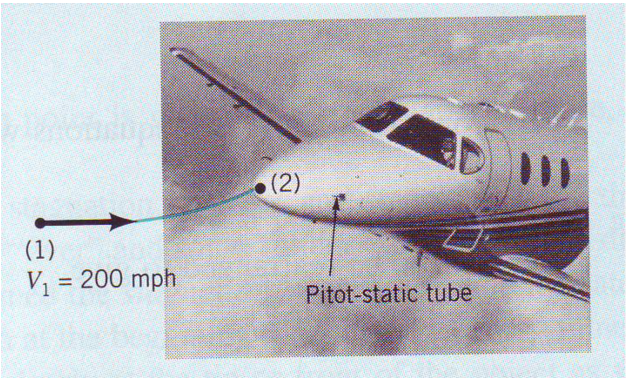
$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

$$p_2 - p_1 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V_0^2$$

If the bicyclist was accelerating or decelerating, the flow would be unsteady ($V_2 \neq 0$) and the above analysis would be incorrect.

Stagnation Point





Stagnation point & Stagnation Pressure

- **There is a stagnation point on any stationary body that is placed into a flowing fluid**

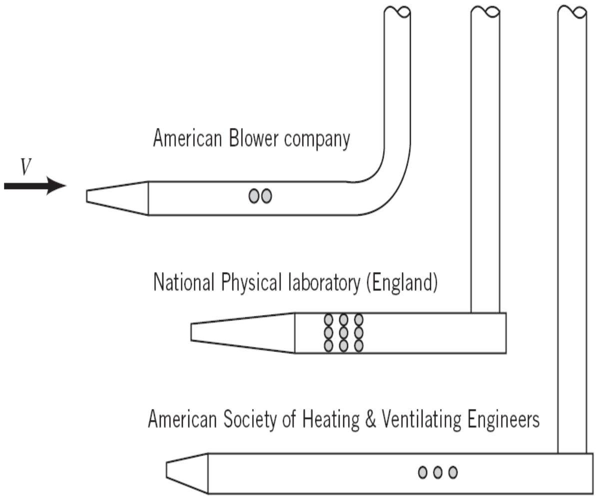
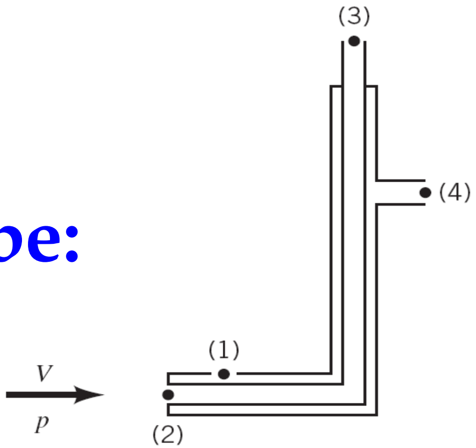


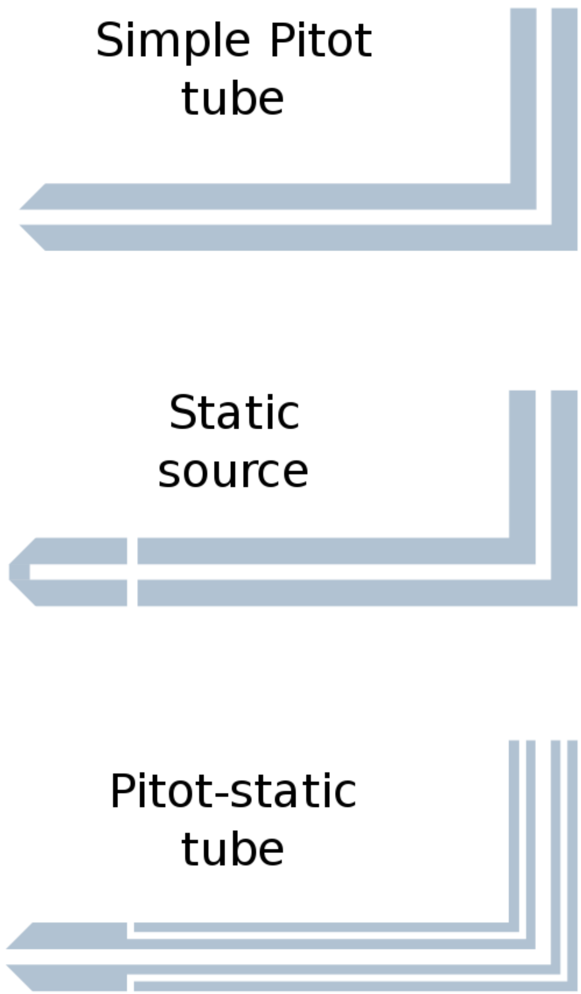
FIGURE 3.7 Typical Pitot-static tube designs.

Pitot/Static Tube:



PITOT/STATIC TUBE

A **pitot tube** is used in wind tunnel experiments and on airplanes to **measure** flow speed. It's a slender **tube** that has two holes on it. The front hole is placed in the airstream to **measure** what's called the stagnation **pressure**. The side hole **measures** the static **pressure**.



Application of the Bernoull's Equation – Pitot Tube

Pitot Tube:

Apply Bernoulli's equation between two points – point 1 & point 2

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

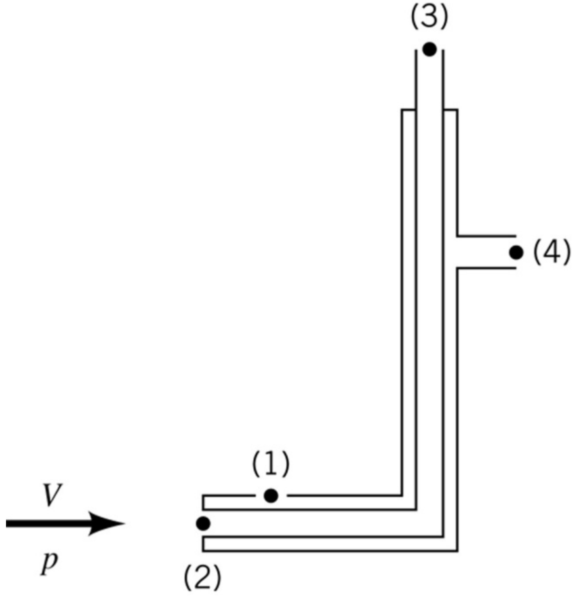
At pt. 2, $V_2=0$ (stagnation point)

$$V_1=V=\sqrt{\frac{2}{\rho}\left(p_{z,1}-p_{z,2}\right)}$$

If a differential pressure gage is connected across the taps,

$$V=\sqrt{2\Delta p/\rho}$$

Δp **pressure difference measured by the gage**



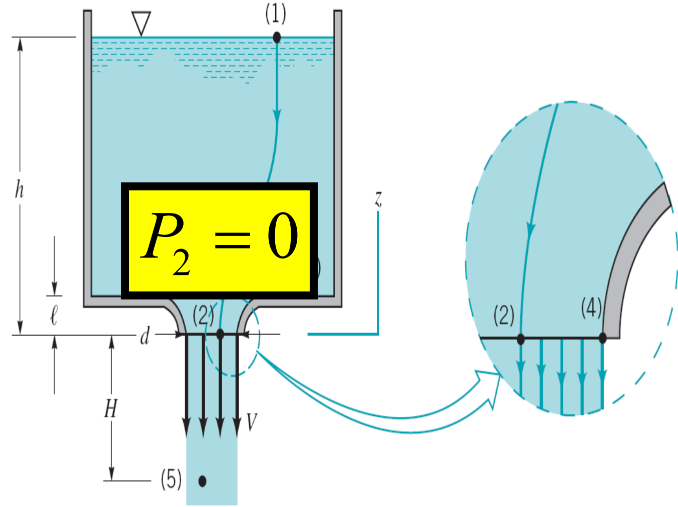
Free Jet

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

FREE SURFACE LARGE TANK

P=0, V=0

Free Jet



$$P_2=0$$

- The fluid leaves as free jet (p_2=0)
- The exit pressure for an incompressible fluid jet is equal to the surrounding pressure (p_2=0)
- The speed increases between point (1) and (5) according to the last equation on the right
- Where H is the distance - the fluid has fallen outside the nozzle

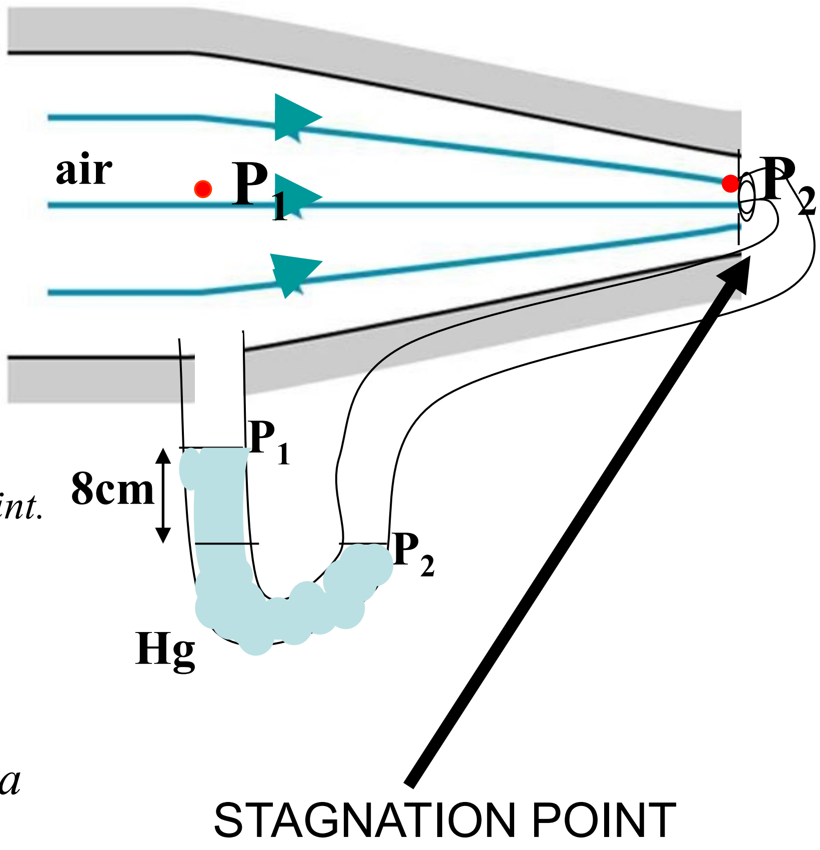
$$\gamma h=\frac{1}{2}\rho V^2$$

$$V=\sqrt{2\frac{\gamma h}{\rho}}=\sqrt{2gh}$$

$$V=\sqrt{2g(h+H)}$$

Bernoulli's Equations - Example

Problem # 2: **Air flows through a nozzle and static pressure tap placed just before converging section of the nozzle as shown in the Figure below. Determine the velocity of the fluid upstream of the entrance?**



Solution: **Apply Bernoulli along a streamline through P_1 and P_2.**

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma Z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma Z_2$$

$(Z_1 = Z_2$ *- same elevation;* $\rho = 1.23\,kg/m^3)$

$$\Rightarrow V_1 = \sqrt{2(P_2 - P_1)/\rho};\; V_2 = 0 \;@P_2 \text{ because of stagnation point.}$$

To determine P_2-P_1, use manometry:

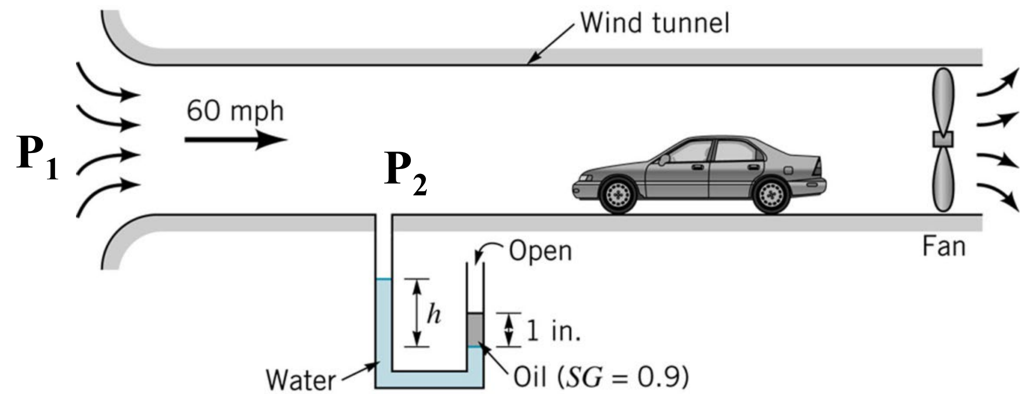
$$P_1 + \gamma_{Hg}(0.08m) = P_2$$

$$\Rightarrow P_2 - P_1 = 133\times10^3\,(N/m^2)(0.08m) = 10{,}640\,Pa$$

$$\Rightarrow V_1 = \sqrt{(2\times10{,}640\,Pa)/(1.23\,kg/m^3)} = 131.5\,m/s$$

Bernoulli's Equations - Example

Problem # 3.48: **Air is drawn into a wind tunnel used for testing automobiles as shown in the Figure below. (a) Determine the manometer reading, *h*, when the velocity in the test section is 60mph. Note that there is a 1-inch column of oil on the water in the manometer. (b) Determine the difference between the stagnation pressure on the front of the automobile and the pressure in the test section.**



Solution: **Apply Bernoulli along a streamline through P_1 and P_2.**

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma Z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma Z_2$$

$$(Z_1 = Z_2 \text{ - same elevation; } P_1 = 0;\ V_1 = 0;\ V_2 = 60mph = 88\ ft/s)$$

$$\Rightarrow P_2 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}\left(0.00238\,slugs/ft^3\right)\left(88\ ft/s\right)^2 = -9.22\,lbf/ft^2$$

Bernoulli's Equations - Example

USING manometry we have (IGNORE AIR ABOVE FLUID):

$$P_2 + \gamma_{H_2O} h - \gamma_{oil}\left(1/12\,ft\right) = P_0 = 0; \qquad \gamma_{oil} = S.G. \times \gamma_{H_2O} = 0.9 \times \left(62.4\,lb/ft^3\right) = 56.2\,lb/ft^3$$

$$\Rightarrow -9.22\,lbf/ft^2 + \left(62.4\,lb/ft^3\right)(h) - \left(56.2\,lb/ft^3\right)\left(1/12\,ft\right) = 0$$

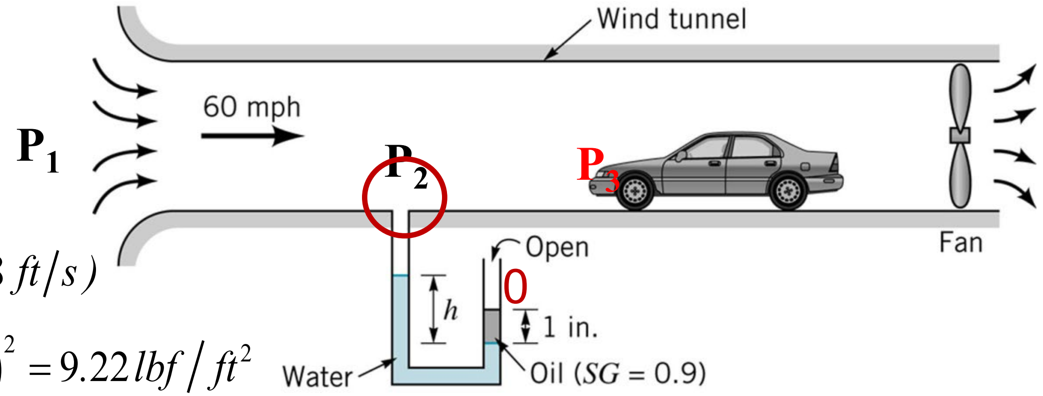
$$\Rightarrow \boxed{h = 0.223\,ft}$$

(b) Apply Bernoulli along a streamline through P_2 and P_3.

$$P_2 + \frac{1}{2}\rho V_2^2 + \gamma Z_2 = P_3 + \frac{1}{2}\rho V_3^2 + \gamma Z_3$$

$$(Z_2 = Z_3 \text{ - same elevation; } V_3 = 0;\ V_2 = 60mph = 88\,ft/s)$$

$$\Rightarrow P_3 - P_2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}\left(0.00238\,slugs/ft^3\right)\left(88\,ft/s\right)^2 = 9.22\,lbf/ft^2$$



HOMEWORK

3-44,48,57,59,64,65,
71,72,74,75,90,96,97,99

Chapter 3
Elementary Fluid Dynamics
Bernoulli Equation

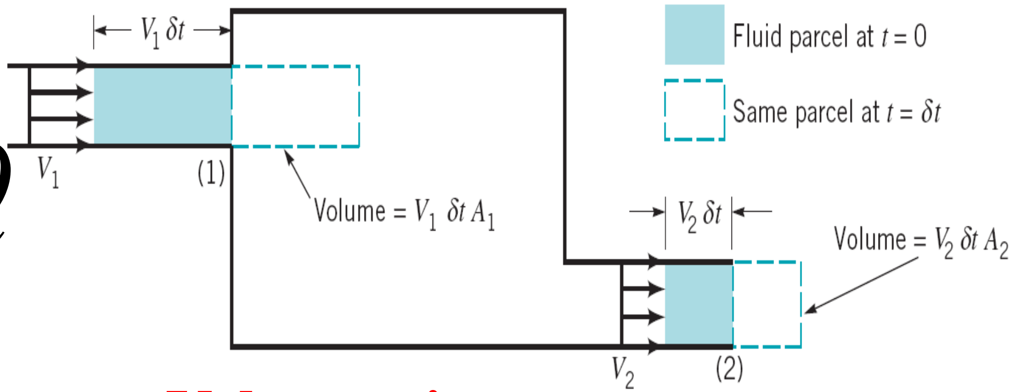
PART 2

Confined Flows – Mass Conservation

Conservation of Mass: If the fluid flow is steady so that there is no additional accumulation of fluid within the volume, the rate at which the fluid flows into the volume must equal the rate at which it flows out of the volume.

Mass Flow rate

$$\dot{m} = \rho * V * A = \rho * Q$$



Mass conservation:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\Rightarrow \sum \rho * (A * V)_{in} = \sum \rho * (A * V)_{out}$$

$$\Rightarrow \sum Q_{in} = \sum Q_{out}$$

Volumetric Flow rate

$$Q = V * A$$

gallons/s, m^3/s

MASS CONSERVATION

FIXED MASS SYSTEM:

Time rate of change of system mass = 0

$$\frac{DM_{sys}}{Dt} = 0$$

where the system mass, $M_{sys} = \rho\forall$ is generally expressed as:

$$M_{sys} = \int_{sys} \rho\left[\frac{kg}{m^3}\right] d\forall[m^3]; or:$$

$$\frac{D}{Dt}\left[\int_{sys} \rho\left[\frac{kg}{m^3}\right] d\forall[m^3]\right] = 0$$

and the integration is over the volume of the system.

MASS CONTINUITY

CONTINUITY – –NON-DEFORMING CONTROL VOLUME :

$$\overbrace{\frac{\partial}{\partial t}\int_{cv}(\rho d\forall)}^{\text{MASS STORAGE INSIDE}} + \overbrace{\sum \dot{m}_{out} - \sum \dot{m}_{in}}^{\text{MASS NET TRANSPORT AT CONTROL SURFACE}} = 0$$

$\rho d\forall \rightarrow MASS$

$\frac{\partial}{\partial t}\int_{cv}(\rho d\forall) \rightarrow$ Mass Storage Over Time

STEADY STATE $\rightarrow$ NO MASS STORAGE

$$\frac{\partial}{\partial t}\left[\int_{cv}(\rho d\forall)\right] = 0 \rightarrow \rho \equiv CONSTANT \rightarrow \frac{d}{dt}(\rho\forall) = 0$$

$$\sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

$\dot{m} = \rho A V_n = \rho Q$: MASS FLOW RATE (mass/time)

V_n = NORMAL VELOCITY @ SURFACE

UNITS
Definitions + Concepts
Manometry & Hydrostatics (WHY ?)
Pressure Conservation: Bernoulli (WHY ?)
Mass Conservation/Continuity (WHY ?)

Fluid Mechanics is NOT the big stuff, but rather it is doing the little things right, over and over, and over. We learn small tools each week that are combined with DEINITIONS and concepts to solve more involved problems, week-by-week.

"...one can not build a house until one learns 'how' and 'when' to use a hammer.."
Dr. K. J. Berry
ASME FELLOW

Determine “ha”

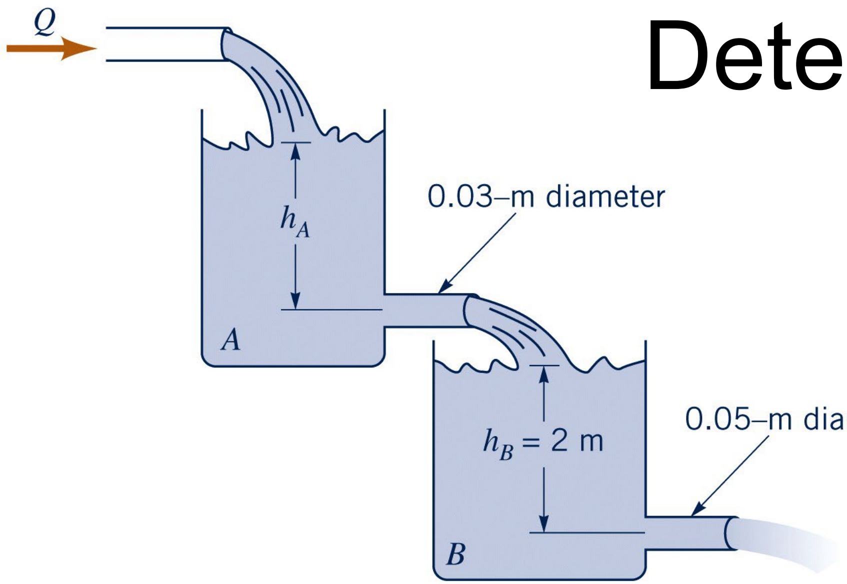


Figure P3.44

Determine “ha”

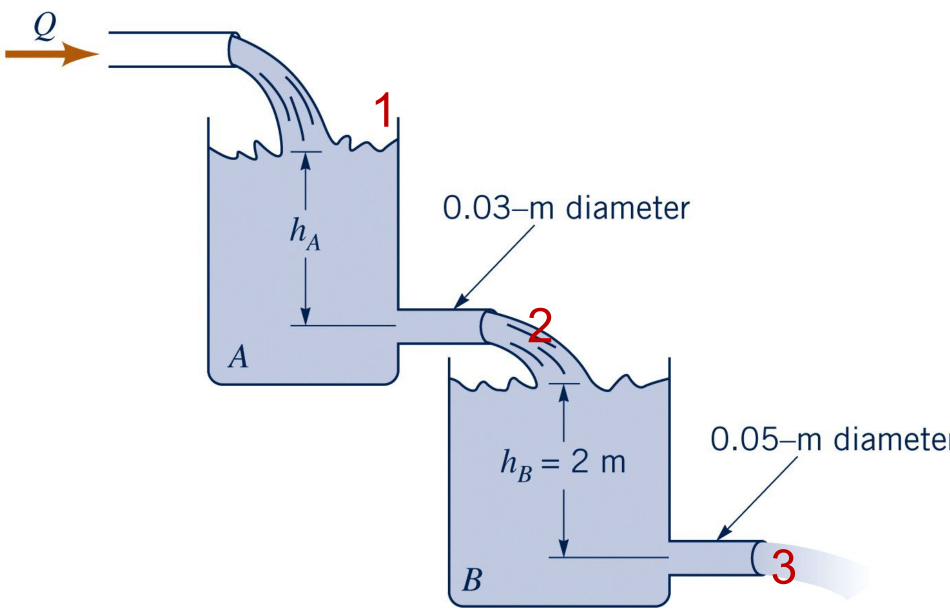


Figure P3.44

IDENTIFY CRITICAL POINTS

Free Surfaces

Inlet/Exit

Free Jets

Static / Stagnation

Change in Diameters

MISSING DIMENSIONS

MANOMETERS----NO

BERNOULI 2-3

$$\overbrace{\frac{P_2}{\gamma_f}}^{0} + \overbrace{z_2}^{0} + \frac{V_2^2}{2g} = \overbrace{\frac{P_3}{\gamma_f}}^{0} + \overbrace{z_3}^{-h_b} + \frac{V_3^2}{2g}$$

$$\frac{V_2^2}{2g} - \frac{V_3^2}{2g} = -h_b = -2m$$

$$\frac{V_2^2}{2g} - \frac{V_3^2}{2g} = -h_b = -2m$$

MASS CONSERVATION

$$A_2V_2 = A_3V_3$$

$$V_3 = \frac{A_2V_2}{A_3}$$

COMBINE

$$\frac{V_2^2}{2g}\left(1 - \left(\frac{A_2}{A_3}\right)^2\right) = -h_b$$

SOLVE

$$V_2 = \sqrt{\left\{\frac{-h_b[m] \bullet 2g[\frac{m}{s^2}]}{\underbrace{1 - \left(\frac{A_2}{A_3}\right)^2}_{<0}}\right\}[m^2/s^2]} = 1.47m/s$$

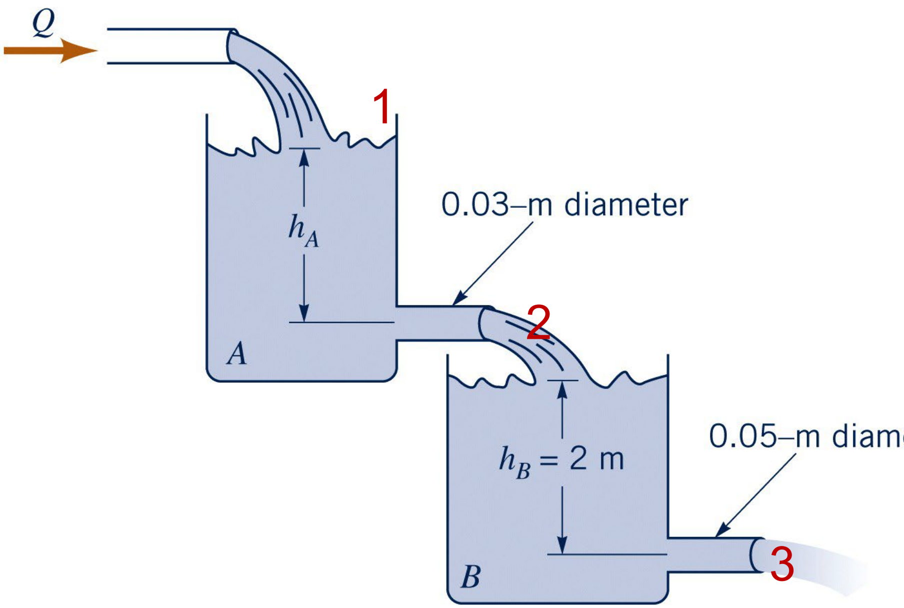


Figure P3.44

$$Q_2 \equiv \text{VOLUME FLOW RATE} = V_2[m/s]\overbrace{A_2}^{\frac{\pi D_2^2}{4}}[m^2] = 0.001\frac{m^3}{s}$$

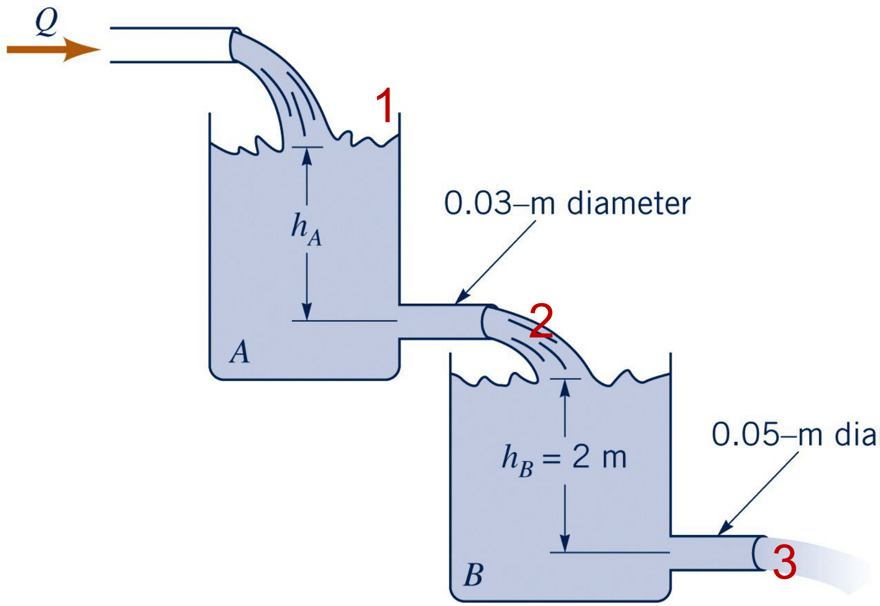
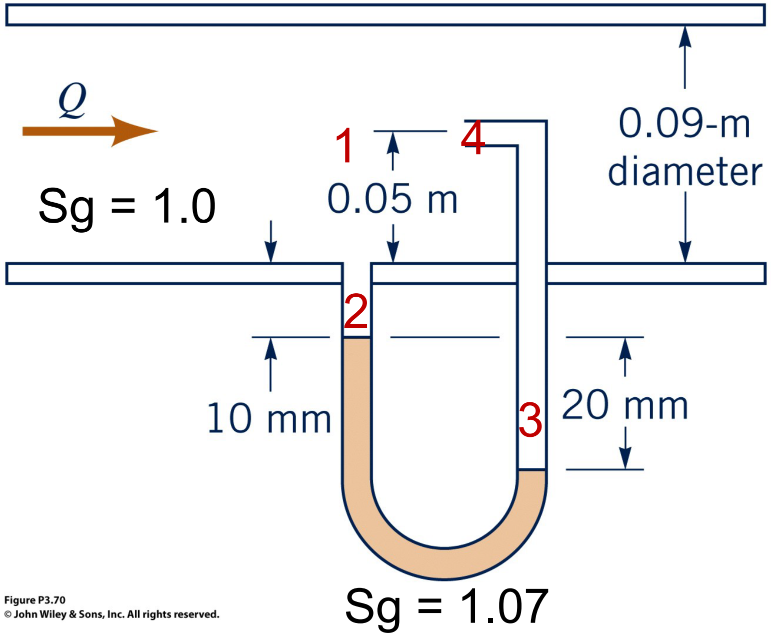


Figure P3.44

BERNOULLI 1-2

$$\overbrace{\frac{P_1}{\gamma_f}}^{0} + \overbrace{z_1}^{h_a} + \overbrace{\frac{V_1^2}{2g}}^{0} = \overbrace{\frac{P_2}{\gamma_f}}^{0} + \overbrace{z_2}^{0} + \frac{V_2^2}{2g}$$

$$h_a = \frac{V_2^2}{2g} = \frac{\left(1.47\frac{m}{s}\right)^2}{2 \bullet 9.81\frac{m}{s^2}} = 0.11m$$


Q
Sg = 1.0
1
4
0.05 m
0.09-m
diameter
2
10 mm
3
20 mm
Sg = 1.07
Figure P3.70
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FIND Q

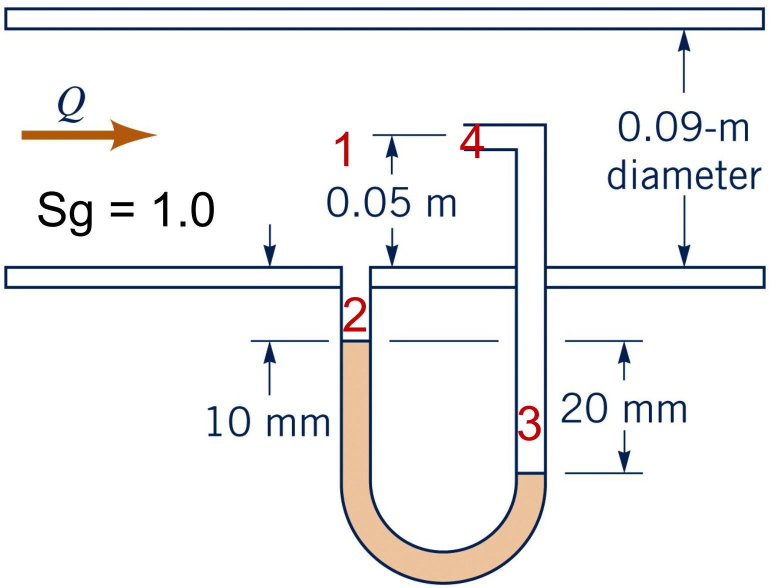


Figure P3.70

FIND Q

IDENTIFY CRITICAL POINTS

- Free Surfaces
- Inlet/Exit
- Free Jets
- *Static / Stagnation*
- Change in Diameters
- MISSING DIMENSIONS

MANOMETRY--YES

1-4

$$P_1 + \gamma_f(0.05 + 10/1000)m + \gamma_m(20/1000) - \gamma_f(20/1000 + 0.05 + 10/1000) = P_4$$

$$P_1 + \frac{20}{1000}(\gamma_m - \overbrace{\gamma_f}^{water}) = P_4$$

$$\frac{P_1 - P_4}{\gamma_f} = \frac{20}{1000}(1 - \frac{\gamma_m}{\gamma_f}) = 0.02(1 - S_{g_f})$$

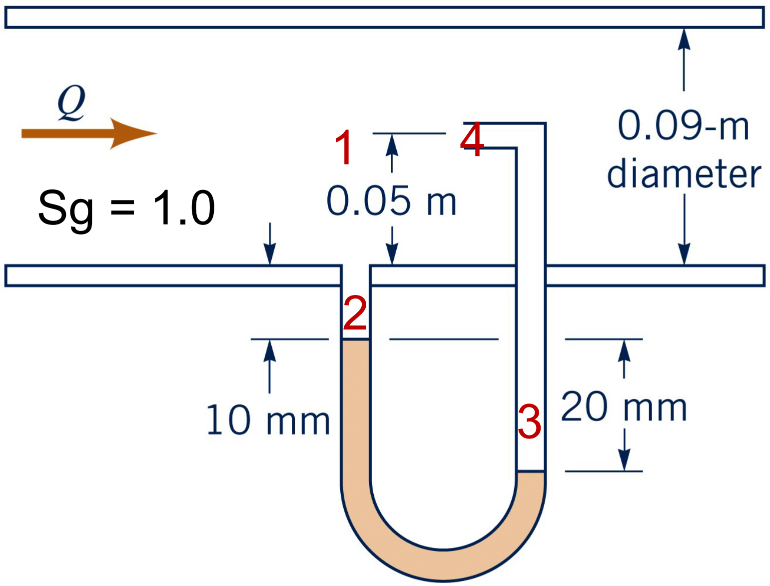


Figure P3.70

MANOMETRTY:1-4

$$\frac{P_1 - P_4}{\gamma_f} = 0.02(1 - S_{g_f})$$

BERNOULLI: 1-4

$$\frac{P_1}{\gamma_f} + \not{z}_1 + \frac{V_1^2}{2g} = \frac{P_4}{\gamma_f} + \not{z}_4 + \frac{V_4^2}{2g}$$

$$\frac{P_1}{\gamma_f} - \frac{P_4}{\gamma_f} = \overbrace{\frac{V_4^2}{2g}}^{0\,g:stagnation} - \frac{V_1^2}{2g}$$

$$0.02[m](1 - S_{g_f}) = -\frac{V_1^2}{2g}$$

$$0.02[m](S_{g_f} - 1) = \frac{V_1^2}{2g}$$

$$\sqrt{0.02[m](S_{g_f} - 1)2g\left[\frac{m}{s^2}\right]} = V_1 = 0.1657 m/s$$

$$Q = V_1 \bullet A_1 = V_1[m/s] \bullet \frac{\pi 0.09^2}{4}[m^2] = 0.0010 m^3/s$$

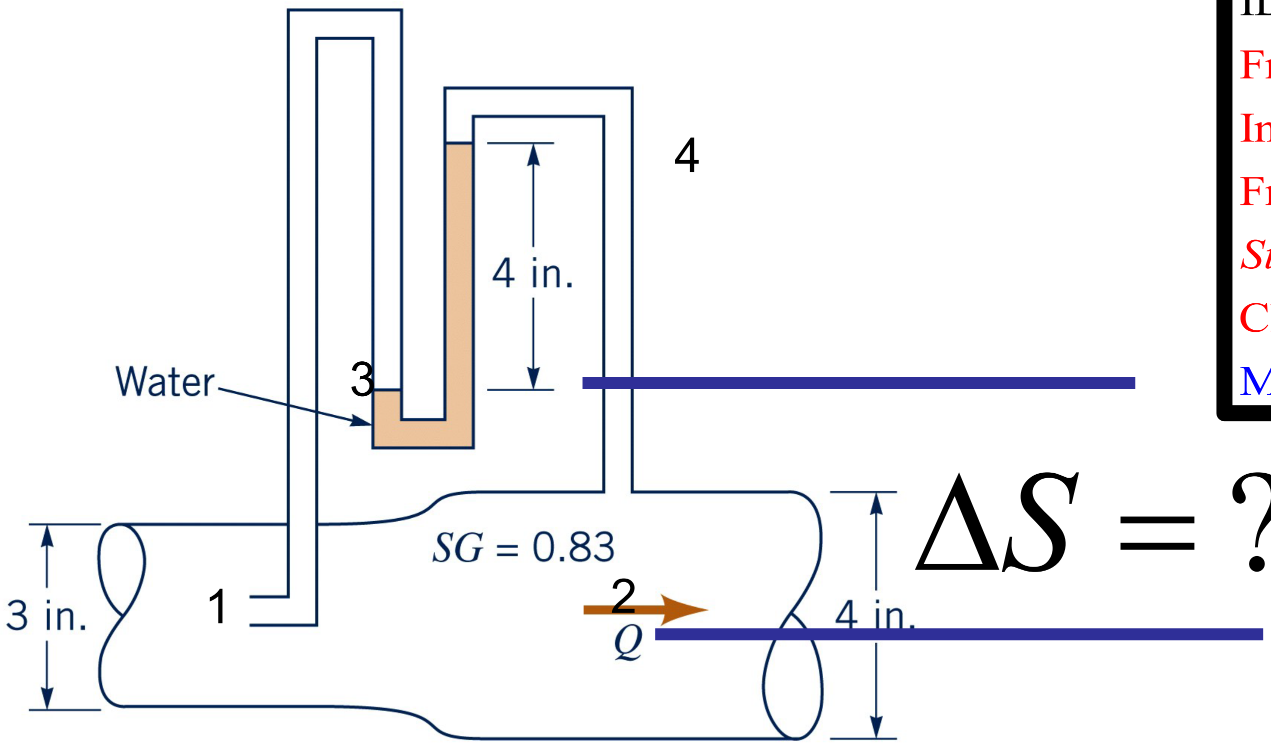
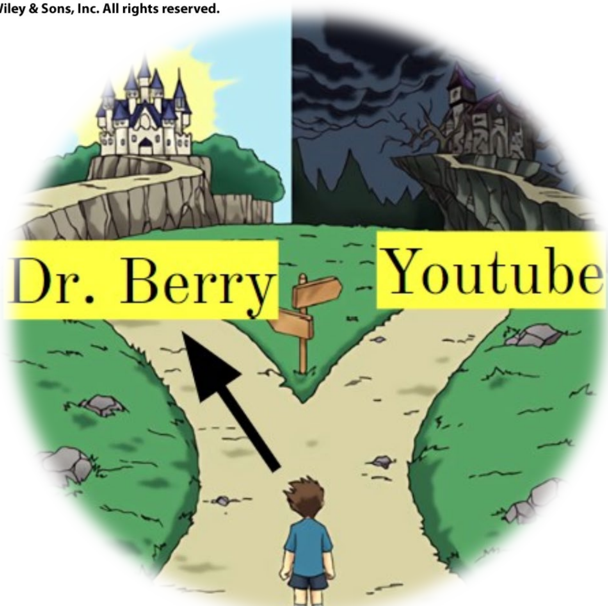


Figure P3.67

IDENTIFY CRITICAL POINTS

- Free Surfaces
- Inlet/Exit
- Free Jets
- *Static / Stagnation*
- Change in Diameters
- MISSING DIMENSIONS



MANOMETRY → YES

1-2

$$P_1 - \cancel{\gamma_f(\Delta S)} - \frac{4}{12}\gamma_m + \gamma_f\left(\frac{4}{12} + \cancel{\Delta S}\right) = P_2$$

$$P_1 - P_2 = \frac{4}{100}(\gamma_m - \gamma_f)$$

$$\frac{P_1 - P_2}{\gamma_f} = \frac{4}{12}\left(\frac{\gamma_{h20}}{\gamma_f} - 1\right) = \frac{4}{12}\left(\frac{1}{S_{g_f}} - 1\right)$$

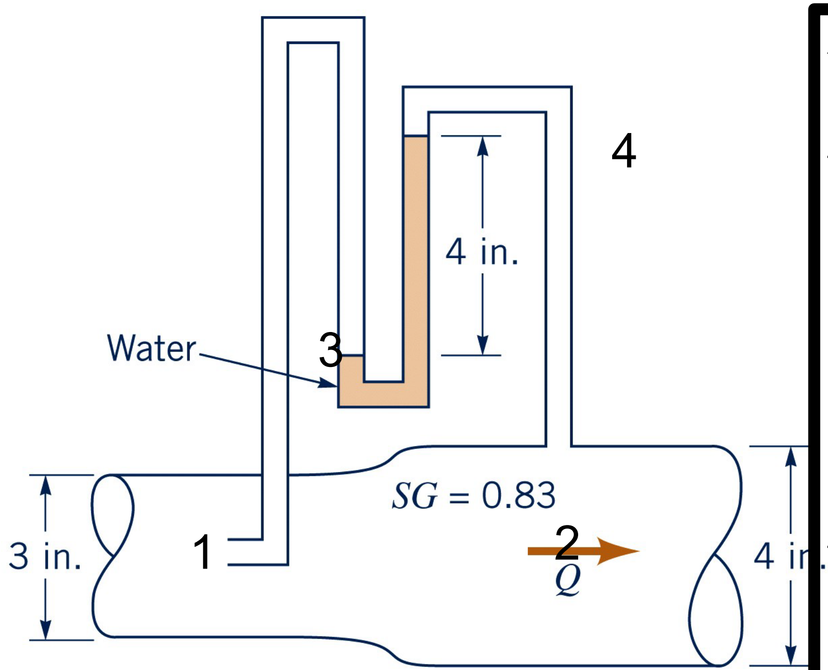


Figure P3.67

$$MANOMETRY: 1-2$$

$$\frac{P_1 - P_2}{\gamma_f} = \frac{4}{12}\left(\frac{1}{S_{g_f}} - 1\right) = 0.0683\,ft$$

$$P_1 - P_2 = 0.0683\,ft \bullet 0.83 \bullet 62.4\,lbf/ft^3 = 3.536\,\frac{lbf}{ft^2}$$

$$BERNOULLI: 1-2$$

$$\frac{P_1}{\gamma_f} + \cancel{z_1} + \overbrace{\frac{V_1^2}{2g}}^{0\,g:STAGNATION} = \frac{P_2}{\gamma_f} + \cancel{z_1} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\gamma_f} - \frac{P_2}{\gamma_f} = \frac{V_2^2}{2g} = \frac{4}{12}\left(\frac{1}{S_{g_f}} - 1\right)$$

$$V_2 = \sqrt{\frac{4}{12}[m]\left(\frac{1}{S_{g_f}} - 1\right) 2g[m/s^2]} = 2.097\,m/s$$

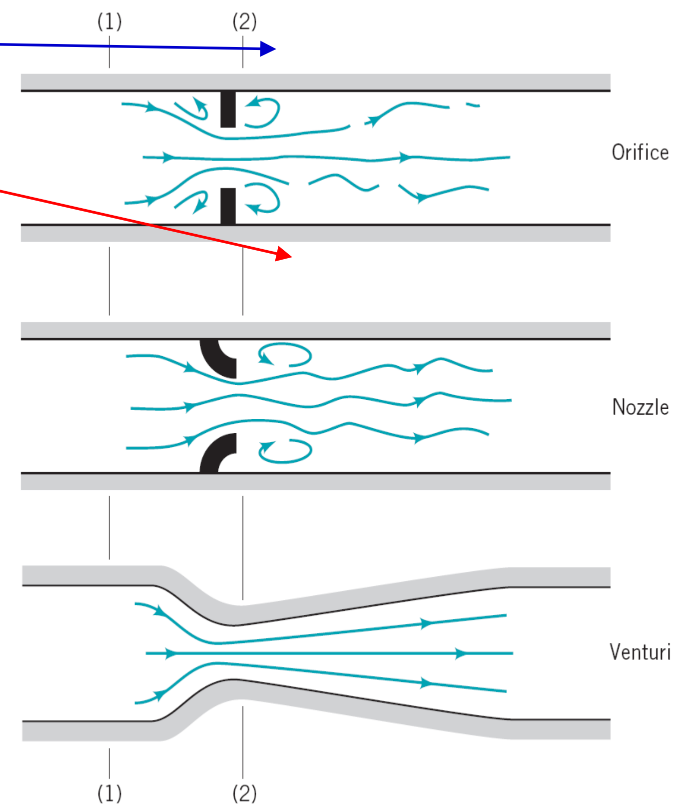
$$Q = V_2 A_2 = V_2\,[ft/s]\,\frac{\pi\left(\frac{4}{12}\right)^2}{4}\,\left[ft^2\right] = 0.182998\,\frac{ft^3}{s}$$

Flow rate Measurement

- **Flowrate through a pipe can be measured by placing some types of restrictions**

- **Common types flow meters are**
  - ***Orifice meters***
  - ***Nozzle meter***
  - ***Venturi meter***

- **For a given geometry (A_1 and A_2), the flowrate can be determined if the pressure difference p_1-p_2 is measured**



BERNOULLI

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$p_1 - p_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2$$

$$\frac{2(p_1 - p_2)}{\rho} = V_2^2 - V_1^2$$

MASS CONSERVATION

$\sum \dot{m}_{IN} = \sum \dot{m}_{OUT} \rightarrow$ CRITICAL PATH FOR AREA CHANGE $$$$$

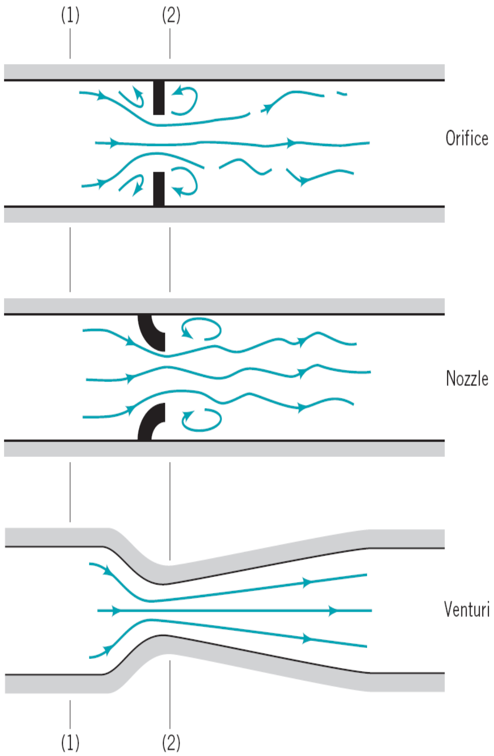
$\not\rho A_1 V_1 = \not\rho A_2 V_2 \rightarrow Q \equiv$ VOLUME FLOW RATE $\left[\frac{vol}{time}\right]$

$V_1 = \frac{A_2 V_2}{A_1} \rightarrow$ COMBINE WITH BERNOULI $\rightarrow$ CRITICAL PATH

$$\frac{2(p_1 - p_2)}{\rho} = V_2^2 - V_1^2 = V_2^2 - \left(\frac{A_2 V_2}{A_1}\right)^2$$

$$\frac{2(p_1 - p_2)}{\rho} = V_2^2 \left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$$

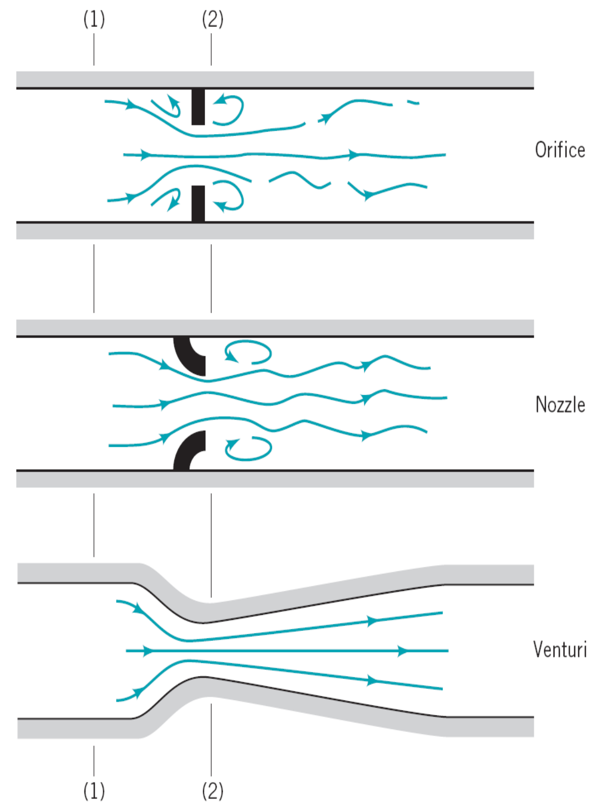
$$\frac{\frac{2(p_1 - p_2)}{\rho}}{\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)} = V_2^2$$



$$V_2 = \sqrt{\frac{2\left(p_1 - p_2\right)\left[\dfrac{N = \dfrac{\cancel{kg} - m}{s^2}}{m^2}\right]}{\dfrac{\rho\left[\dfrac{\cancel{kg}}{m^3}\right]}{\left(1 - \left(\dfrac{A_2}{A_1}\right)^2\right)}}} = \sqrt{\left[\frac{m^2}{s^2}\right]} = V_2\left[\frac{m}{s}\right]$$

$Q = A_2 V_2 \rightarrow$ VOLUME FLOW RATE DEFINITION

$$Q\left[\frac{m^3}{s}\right] = A_2[m^2]\sqrt{\frac{2(p_1 - p_2)\left[\dfrac{N = \dfrac{\cancel{kg} - m}{s^2}}{m^2}\right]}{\rho\left[\dfrac{\cancel{kg}}{m^3}\right]\left[1 - \left(A_2 / A_1\right)^2\right]}} \left[m^2\sqrt{\frac{m^4}{m^2 - s^2}} = \frac{m^3}{s}\right]$$




FOLLOW
THE
PATH

ENERGY FLOW UNITS

$$POWER[Watts] = \dot{m}\left[\frac{kg}{s}\right] \bullet g\left[\frac{m}{s^2}\right] \bullet h[m]$$

$$= \frac{N-m}{s}$$

$$= \frac{J}{s}$$

$$= Watts$$

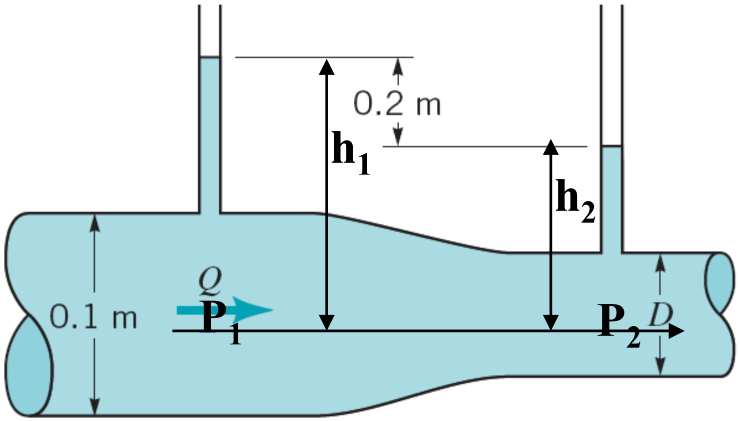
$$POWER[\frac{ft-lbf}{s}] = \dot{m}\left[\frac{slugs = \frac{lbf - \cancel{s^2}}{\cancel{ft}}}{s}\right] \bullet g\left[\frac{ft}{\cancel{s}^2}\right] \bullet h[\cancel{ft}]$$

$$= \frac{ft-lbf}{s}$$

$$POWER[hp] = \frac{ft-lbf}{s} \bullet \frac{1hp}{550\frac{ft-lbf}{s}}$$

Conservation of Mass - Example

Problem # 3.52: Water flows through the pipe contraction shown in Figure below. For the given 0.2m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.



Solution: Apply Bernoulli at P_1 and P_2.

$$\dot{m}_1\left(P_1+\frac{1}{2}\rho V_1^2+\gamma Z_1\right)=\dot{m}_2\left(P_2+\frac{1}{2}\rho V_2^2+\gamma Z_2\right)$$

$$SINGLE\ STREAM \rightarrow \dot{m}_1=\dot{m}_2$$

$$(Z_1 = Z_2;\ same\ elevation)$$

$$\Rightarrow \frac{P_1-P_2}{\gamma}=\frac{V_2^2-V_1^2}{2g}$$

Now

$$Q_1 = Q_2 \rightarrow MASS\ CONSERVATION$$

$$V_1 A_1 = V_2 A_2 \rightarrow CRITICAL\ FOR\ AREA\ CHANGE$$

$$V_2=\frac{A_1}{A_2}V_1=\frac{\left(\pi/4\,D_1^2\right)}{\left(\pi/4\,D_2^2\right)}V_1=\left(\frac{0.1}{D}\right)^2 V_1$$

Mass Conservation is required when there is a change in flow AREA.

Conservation of Mass - Example

Mass Conservation

$$V_2 = \left(\frac{0.1}{D}\right)^2 V_1;$$

Bernoulli

$$\frac{P_1 - P_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g}$$

Manometery

$$P_1 - \gamma h_1 = 0 \;\&\; P_2 - \gamma h_2 = 0 \rightarrow (POINT - TO - POINT)$$

$$\Rightarrow \frac{P_1 - P_2}{\gamma} = h_1 - h_2 = 0.2m$$

Combine

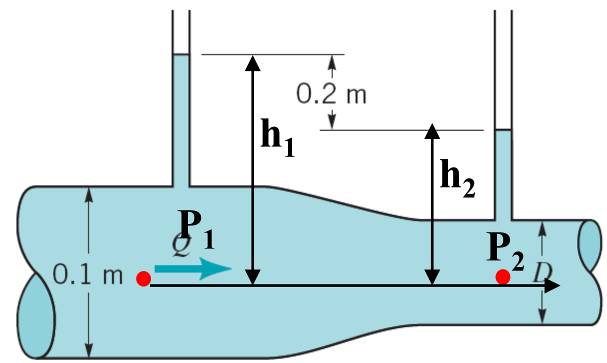
$$0.2m = \frac{(0.1m/D)^4 V_1^2 - V_1^2}{2g}$$

$$\Rightarrow V_1(D) = \sqrt{\frac{0.2m(2g)}{\left[(0.1m/D)^4 - 1\right]}}$$

FINAL

$$Q_1 = V_1 A_1 = \frac{\pi}{4}(0.1m)^2 \sqrt{\frac{0.2m\left[2 \times (9.81m/s^2)\right]}{\left[(0.1m/D)^4 - 1\right]}}$$

$$\Rightarrow Q(D) = \frac{0.0156 D^2}{\sqrt{\left[(0.1)^4 - D^4\right]}} \frac{m^3}{s}$$

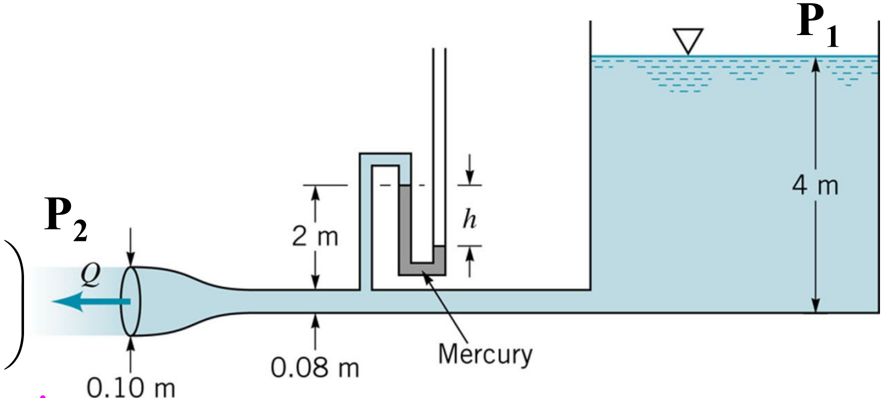


DEFINITIONS
Manometry
Bernoulli
Mass Conservation

Confined Flows – Mass Conservation

Problem # 3.68: Water flows steadily from the large open tank shown in the Figure below. If the viscous effects are negligible, determine (a) the flowrate, *Q*, and (b) the manometer reading, *h*.

Solution: **Apply Bernoulli along a streamline through P_1 and P_2.**



Bernoulli

$$\dot{m}_1\left(P_1+\frac{1}{2}\rho V_1^2+\gamma Z_1\right)=\dot{m}_2\left(P_2+\frac{1}{2}\rho V_2^2+\gamma Z_2\right)$$

$$(Z_1=4m;\ Z_2\approx 0;\ P_1=0;\ P_2=0;\ V_1=0,\dot{m}_1=\dot{m}_2)$$

$$\Rightarrow V_2=\sqrt{2gZ_1}=\sqrt{2\left(9.8\,m/s^2\right)\left(4m\right)}=8.86\,m/s$$

VOLUME FLOW RATE → DEFINITION

$$Q=V_2A_2=\left(8.86\,m/s\right)\times\frac{\pi}{4}\left(0.1m\right)^2=0.0696\,m^3/s$$

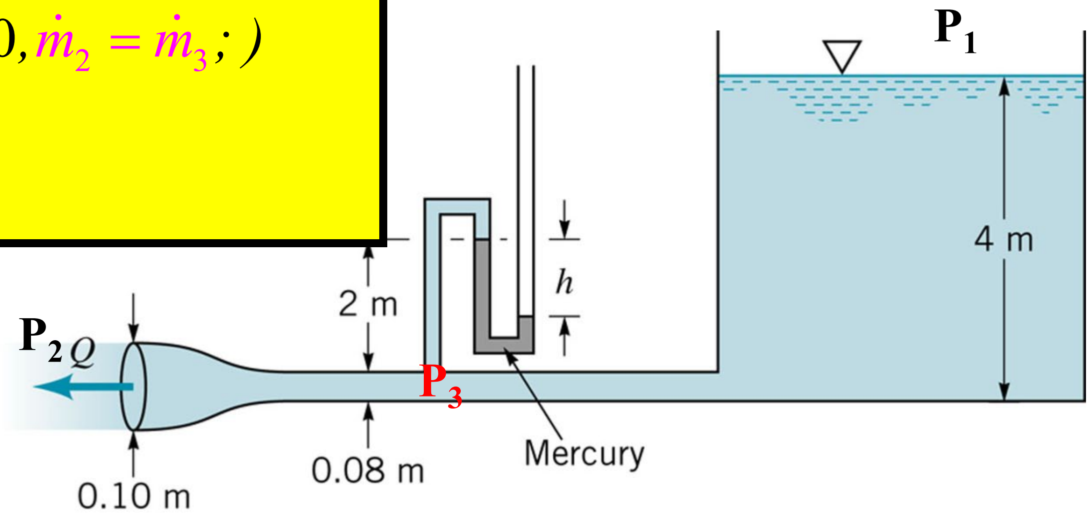
Confined Flows – Mass Conservation

(b) **Consider point P_3 at the same elevation of P_2. Apply Bernoulli along a streamline through P_2 and P_3.**

$$\dot{m}_2\left(P_2+\frac{1}{2}\rho V_2^2+\gamma Z_2\right)=\dot{m}_3\left(P_3+\frac{1}{2}\rho V_3^2+\gamma Z_3\right)$$

$$(\, Z_3 = Z_2,\ same\ elevation;\ P_2 = 0, \dot{m}_2 = \dot{m}_3;\,)$$

$$\Rightarrow P_3=\frac{1}{2}\rho\left(V_2^2-V_3^2\right)$$



Now

$$Q_2 = Q_3$$

$$\Rightarrow V_2A_2 = V_3A_3$$

$$\Rightarrow V_3=\frac{A_2}{A_3}V_2=\left(\frac{D_2}{D_3}\right)^2 V_2=\left(\frac{0.1m}{0.08m}\right)^2(8.86\,m/s)=13.84\,m/s$$

Therefore

$$P_3=\frac{1}{2}\left(1000\,kg/m^3\right)\left[\left(8.86\,m/s\right)^2-\left(13.84\,m/s\right)^2\right]=-56{,}523\,N/m^2$$

Confined Flows – Mass Conservation

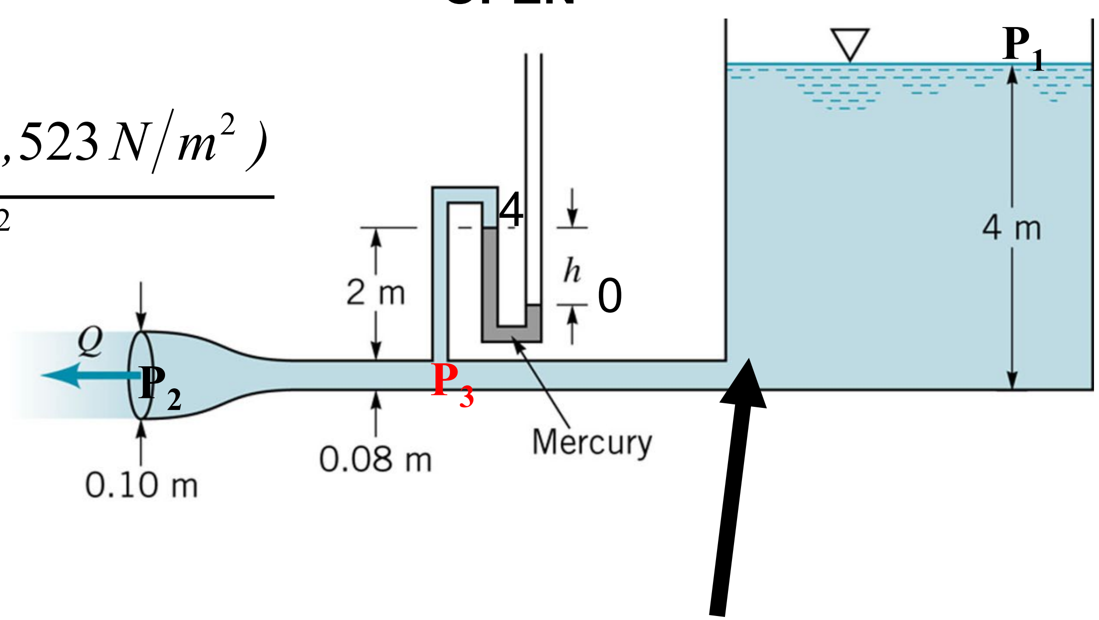
(b) Use manometry (APPLY BERRY METHOD CORRECTLY):

$$P_3 - \gamma_{H_2O}\left(2m + \left(0.08/2\right)m\right) + \gamma_{Hg} h = P_0 = 0$$

$$h = \frac{\gamma_{H_2O}\left(2m + \left(0.08/2\right)m\right) - P_3}{\gamma_{Hg}}$$

$$= \frac{\left(9800\, N/m^2\right)\left(2.04m\right) - (-56{,}523\, N/m^2)}{13.55 \bullet 9800\, N/m^2}$$

$$\Rightarrow h = 0.575m$$



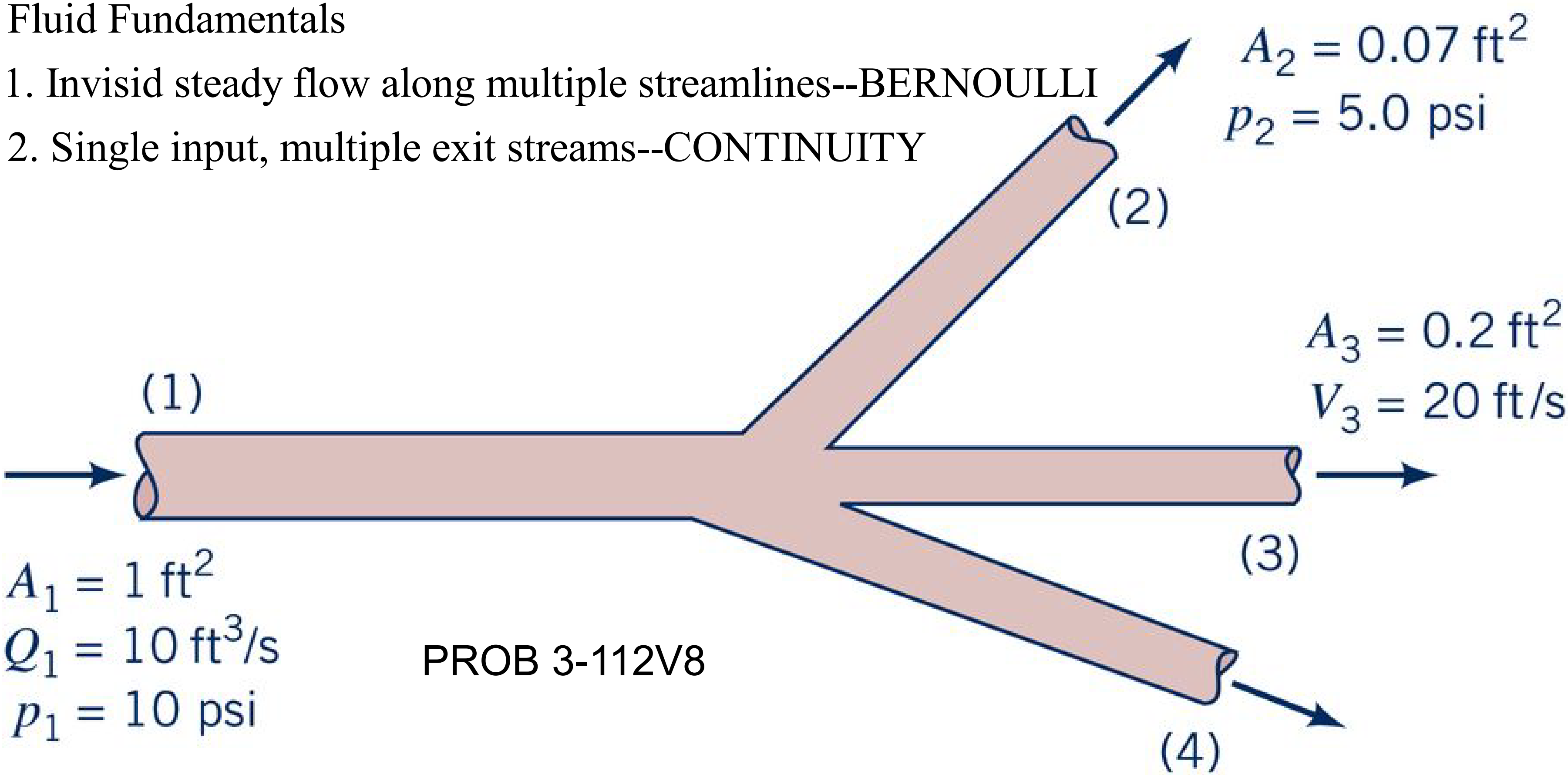
WHAT IS PRESURE HERE ? (5 Points)

CONTINUITY & BRANCH FLOWS

Water flows steadily through the horizontal pipe. If viscous effects are neglected determine water speed at section 2, pressure at 3, and flow rate at 4.

Fluid Fundamentals

1. Invisid steady flow along multiple streamlines--BERNOULLI
2. Single input, multiple exit streams--CONTINUITY

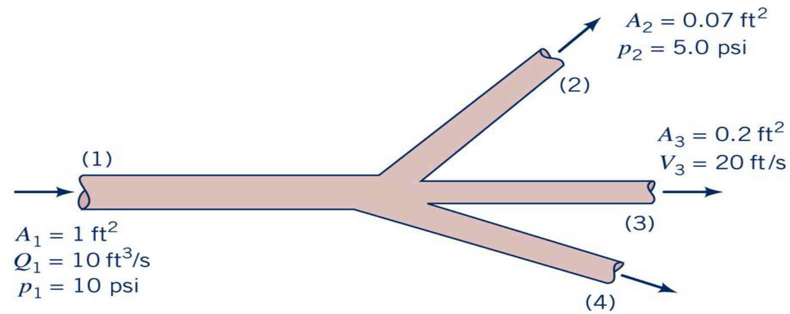


PROB 3-112V8

SOLUTION

FIND V2?

Apply Bernoulli to 1-2 STREAMLINE, $Z_1=Z_2$



$$\dot{m}_1\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)=\dot{m}_2\left(\frac{P_2}{\gamma_{H20}}+\frac{V_2^2}{2g}\right)$$

$$\frac{\dot{m}_1}{\dot{m}_2}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)-\frac{P_2}{\gamma_{H20}}=\frac{V_2^2}{2g}$$

$$\frac{A_1V_1}{A_2V_2}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)=\frac{P_2}{\gamma_{H20}}+\frac{V_2^2}{2g}$$

$$\frac{A_1V_1}{A_2}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)=\frac{P_2V_2}{\gamma_{H20}}+\frac{V_2^3}{2g}$$

$$V_2^3+V_2\left(\frac{P_2}{\gamma_{H20}}\bullet 2g\right)-\frac{A_1V_1}{A_2}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)\bullet 2g=0$$

$$V_1=\frac{Q}{A_1}$$

$$P_2=5\frac{lbf}{in^2}\bullet\frac{144in^2}{ft^2}$$

$$P_1=10\frac{lbf}{in^2}\bullet\frac{144in^2}{ft^2}$$

$$\gamma_{H_2O}=62.4\frac{lbf}{ft^3}$$

$$V_2^3+\left(0V_2^2\right)+V_2q+r=0\rightarrow \text{ONE REAL ROOT}$$

$$a=\left(\frac{P_2}{\gamma_{H20}}\bullet 2g\right), b=-\frac{A_1V_1}{A_2}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)\bullet 2g$$

$$A=\left(\frac{-b}{2}+\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}\right)^{\frac{1}{3}};B=\left(\frac{-b}{2}-\sqrt{\frac{b^2}{4}+\frac{a^3}{27}}\right)^{\frac{1}{3}}$$

root $=V_2 = A + B$

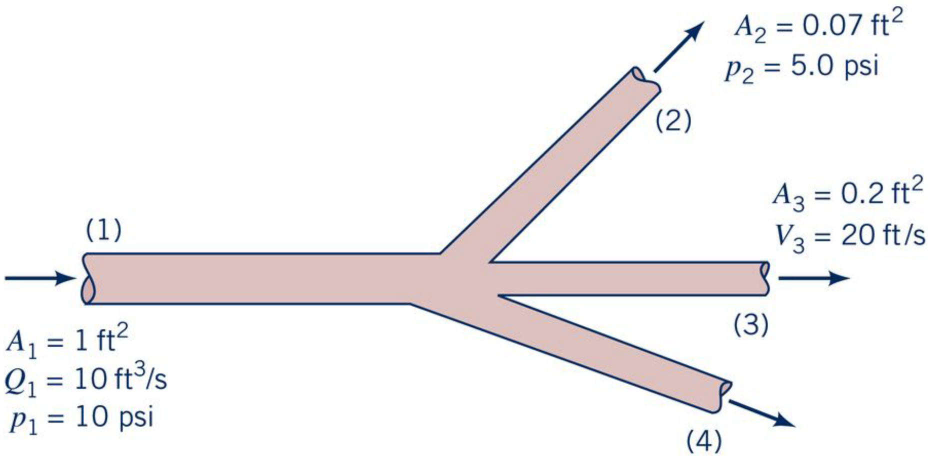
$$Q_2\left[\frac{ft^3}{s}\right]=V_2\left[\frac{ft}{s}\right]\bullet A_2\left[ft^3\right]$$

CARDANO'S FORMULA

(1501-1576)

ANOTHER INSPIRING ENGINEERING MOVIE

FIND P3



FIND P3?

Apply Bernoulli to 1-3 STREAMLINE, $Z_1=Z_3$

$$\dot{m}_1\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)=\dot{m}_3\left(\frac{P_3}{\gamma_{H20}}+\frac{V_3^2}{2g}\right)$$

$$\left\{\frac{\dot{m}_1}{\dot{m}_3}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)-\frac{V_3^2}{2g}\right\}\gamma_{H20}=P_3$$

$$\left\{\frac{A_1V_1}{A_3V_3}\left(\frac{P_1}{\gamma_{H20}}+\frac{V_1^2}{2g}\right)\left[\frac{\frac{lbf}{ft^2}}{\frac{lbf}{ft^3}}=ft\right]-\frac{V_3^2}{2g}\right\}\gamma_{H20}\left[\frac{lbf}{ft^3}\right]=P_3\left[\frac{lbf}{ft^2}\right]$$

Mass Conservation $\rightarrow$ $V_1=\frac{Q_1}{A_1}=10\,ft/s$

FIND Q4

STEADY STATE

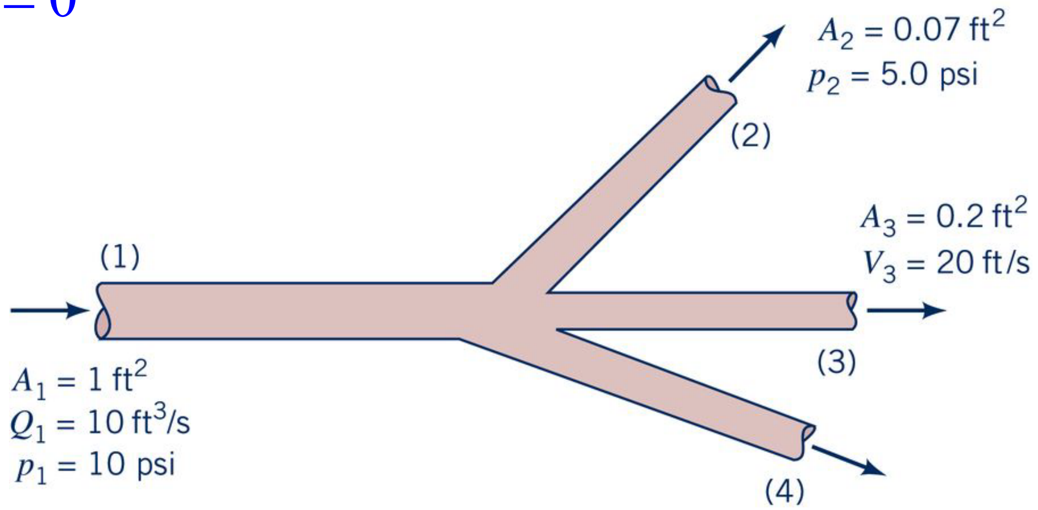
$$\sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

$$\sum (\rho AV)_{out} - \sum (\rho AV)_{in} = 0$$

$$\sum (\rho Q)_{out} - \sum (\rho Q)_{in} = 0$$

$$\sum (Q)_{out} - \sum (Q)_{in} = 0$$

$$Q_2 + Q_3 + Q_4 - Q_1 = 0$$



Solution

$$Q_4 = Q_1 - Q_2 - Q_3$$

$$Q_4 = 10\frac{ft^3}{s} - 0.70\frac{ft^3}{s} - 4\frac{ft^3}{s}$$

$$Q_4 = 5.3\frac{ft^3}{s}$$

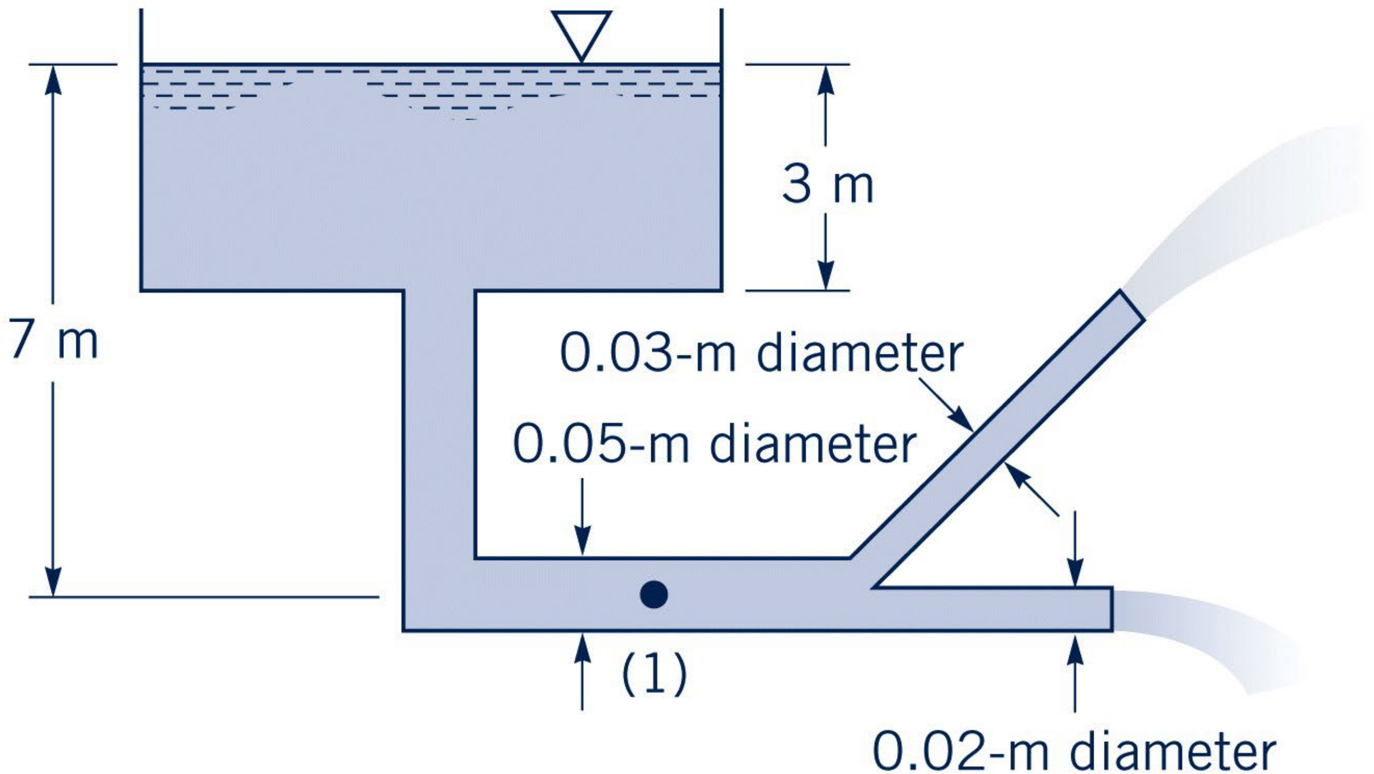


Figure P3.104

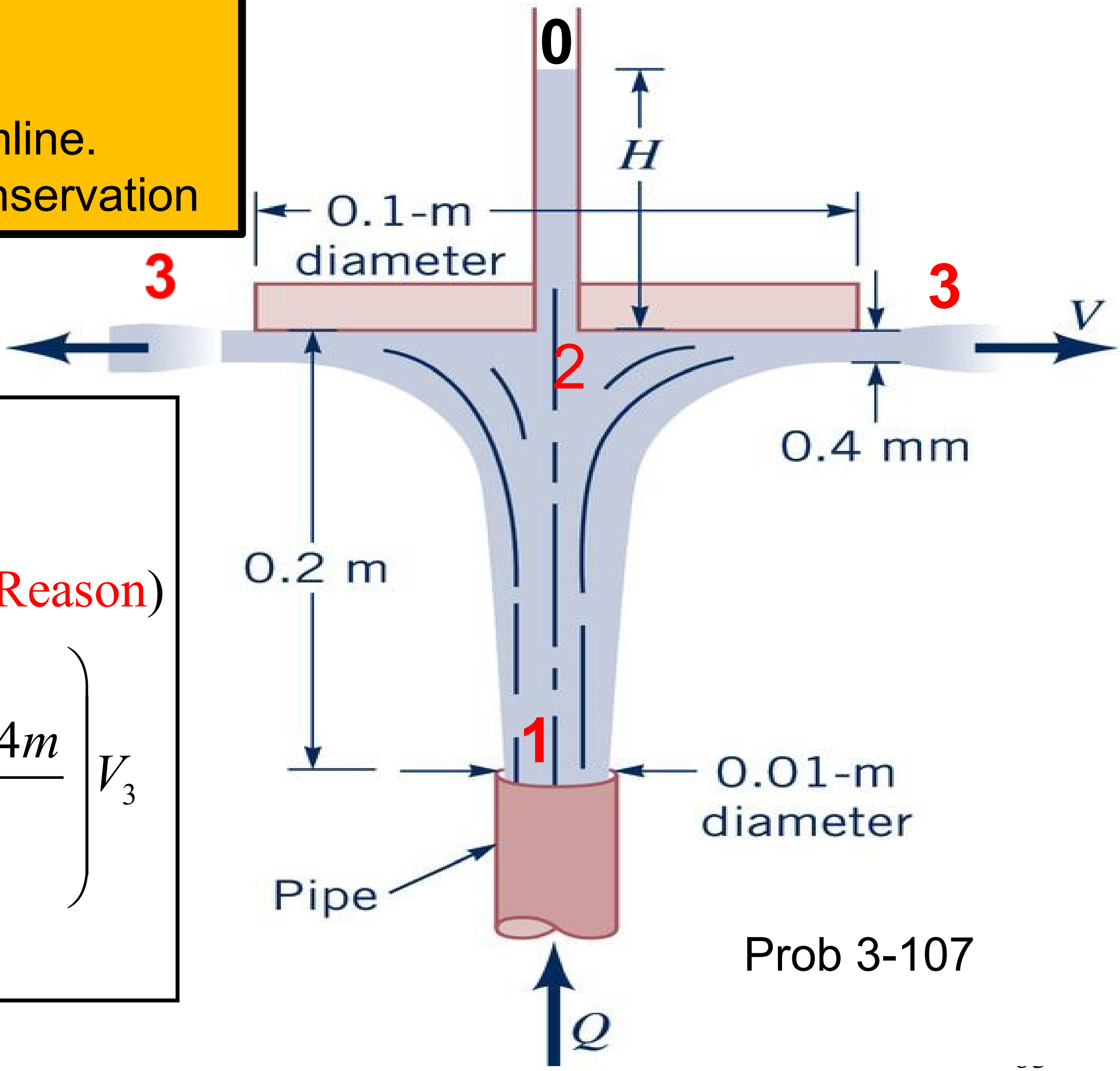
FIND FLOW RATE and PRESSURE at Point 1

Inviscid and Steady Flow

3-113V8

Water flows from the pipe as "free jet" and strikes a circular flat plate. Determine the flowrate and the **manometer** reading H

Fluid Fundamentals
0. Definitions
1. Manometer
2. Bernoulli flow along streamline.
3. Change in Area: Mass Conservation



Prob 3-107

Mass Conservation

$A_1V_1=A_3V_3$

$A_3 = \pi D \Delta$ (Think About This Reason)

$$V_1 = \frac{A_3V_3}{A_1} = \left(\frac{\pi \bullet 0.1m \bullet 0.0004m}{\frac{\pi 0.01^2}{4}} \right) V_3$$

$V_1 = 1.6V_3$

BERNOULLI

Bernoulli 1-3

$$\dot{m}_1\left(\frac{P_1}{\gamma_{H20}}+Z_1+\frac{V_1^2}{2g}\right)=\dot{m}_3\left(\frac{P_3}{\gamma_{H20}}+Z_3+\frac{V_3^2}{2g}\right)$$

$V_1 = 1.6V_3$ (Mass Conservation), $\dot{m}_1 = \dot{m}_3$

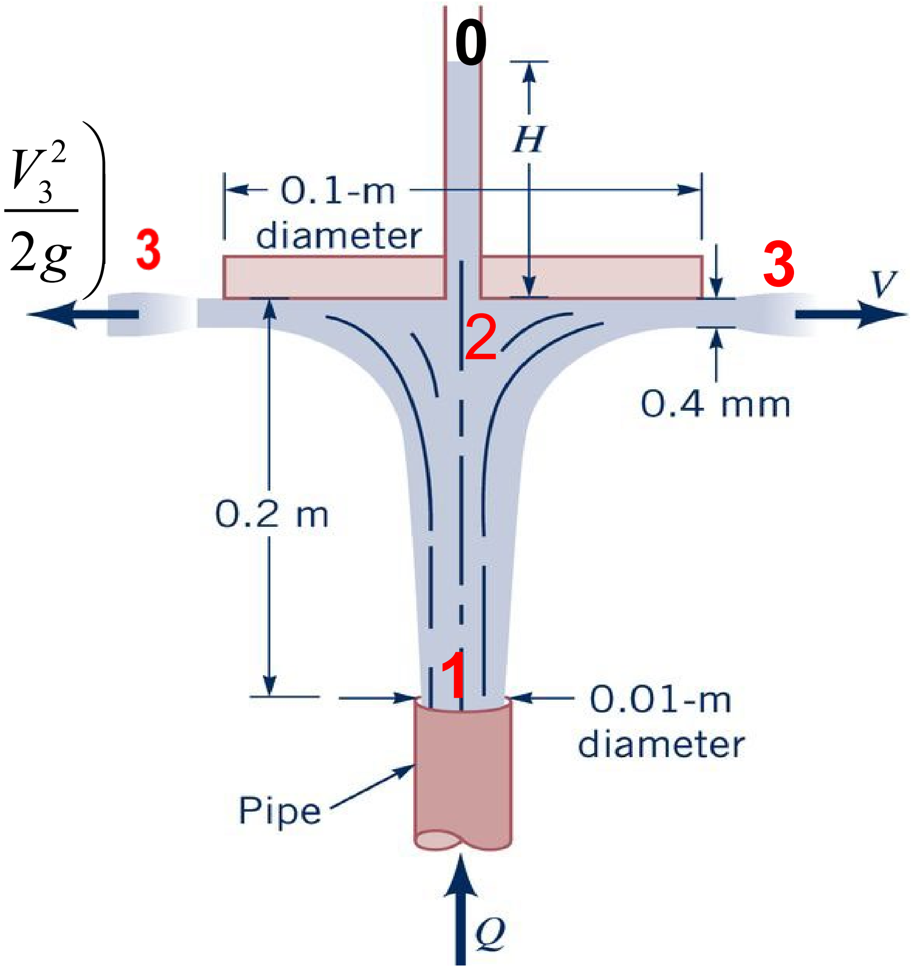
$$\frac{(1.6V_3)^2}{2g}-\frac{V_3^2}{2g}=Z_3-Z_1$$

$$\frac{V_3^2}{2g}(1.6^2-1)=0.2,\ g=9.81m/s^2$$

$1.59m/s = V_3 \rightarrow V_1 = 1.6V_3 = 2.5m/s$

Volume Flow Rate

$$\text{Q=VA=}V_1A_1 = 2.5m/s\,\frac{\pi 0.01^2}{4}=0.0002m^3/s$$



MANOMETER

Bernoulli 1-2

$$\dot{m}_1\left(\frac{P_1}{\gamma_{H20}}+Z_1+\frac{V_1^2}{2g}\right)=\dot{m}_2\left(\frac{P_2}{\gamma_{H20}}+Z_2+\frac{V_2^2}{2g}\right)$$

$$P_2=P_{STAG}=\left(\frac{V_1^2}{2g}+Z_1-Z_2\right)\gamma_{H20},\ \dot{m}_1=\dot{m}_2$$

$$V_1=1.6V_3=2.5m/s$$

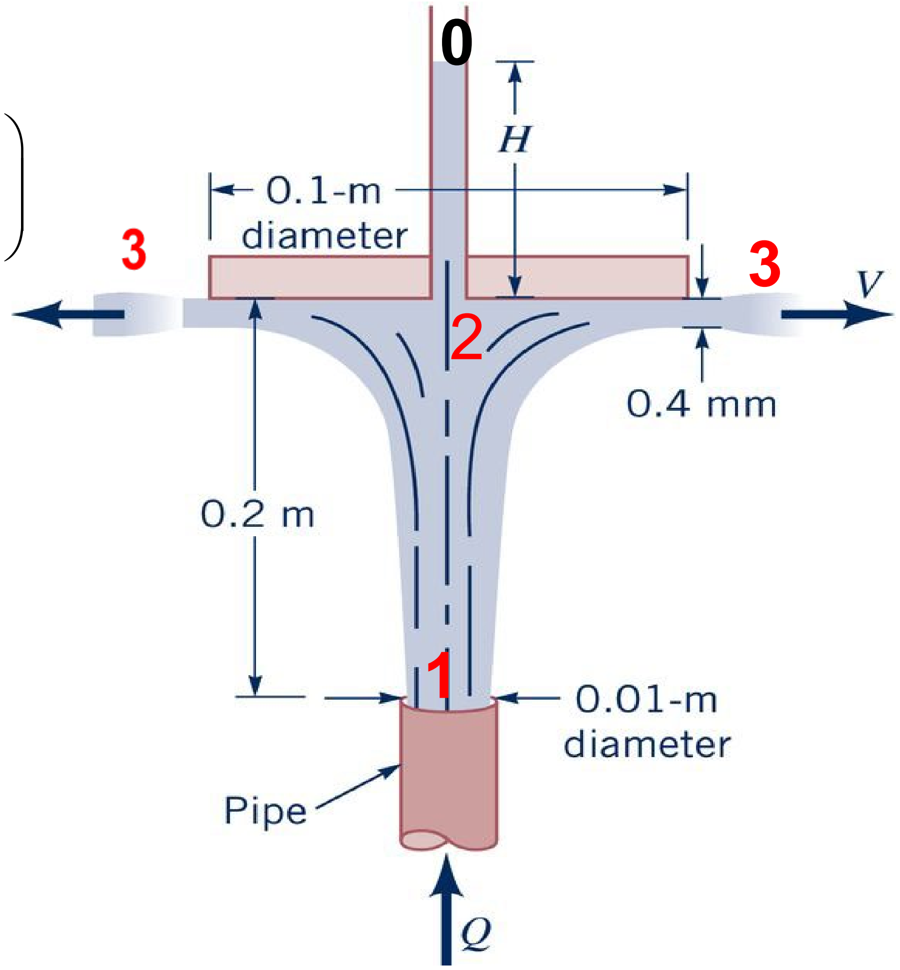
$$P_2=P_{STAG}=\left(\frac{2.54^2 m/s}{2\bullet 9.81m/s^2}-0.20m\right)\gamma_{H20}\frac{N}{m^3}$$

$$=1263PA$$

Manometer 2-0

$$P_2-\gamma_{H20}H=P_0=0gauge$$

$$H=\frac{P_2}{\gamma_{H20}}=\frac{1263N/m^2}{9800N/m^3}=0.13m$$



HOMEWORK

3-96,97,99,109,111,113,108,109

End of Chapter 3

任何人都可以记住事情，但重要的是要了解它。