

*Seek Wisdom Do You? Do,
or do not, there is no try.*

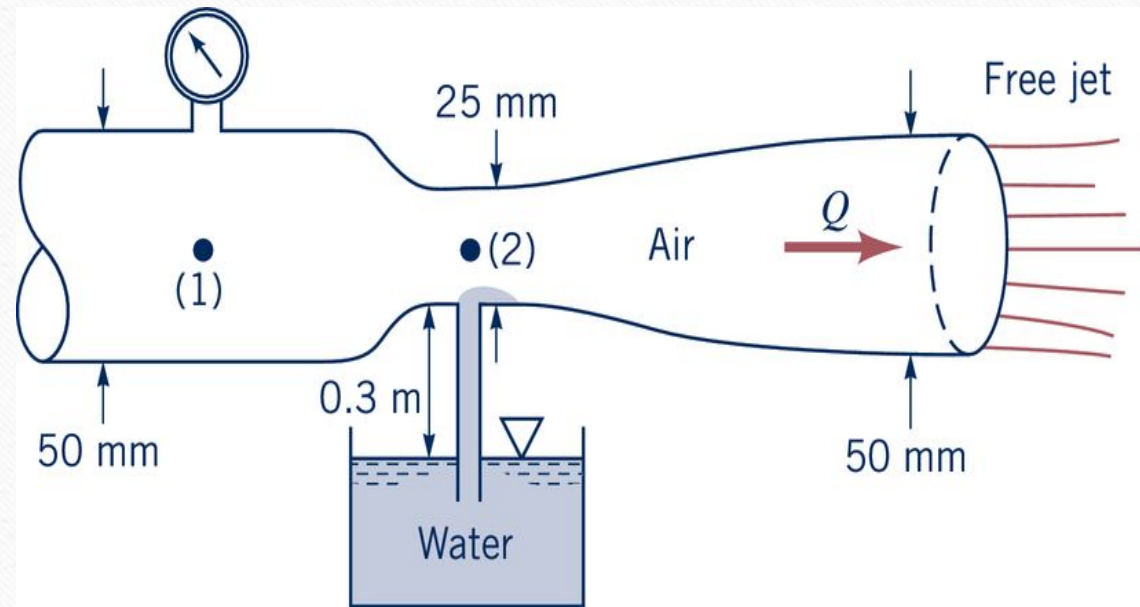
STUDY AID

Bernoulli & Manometry

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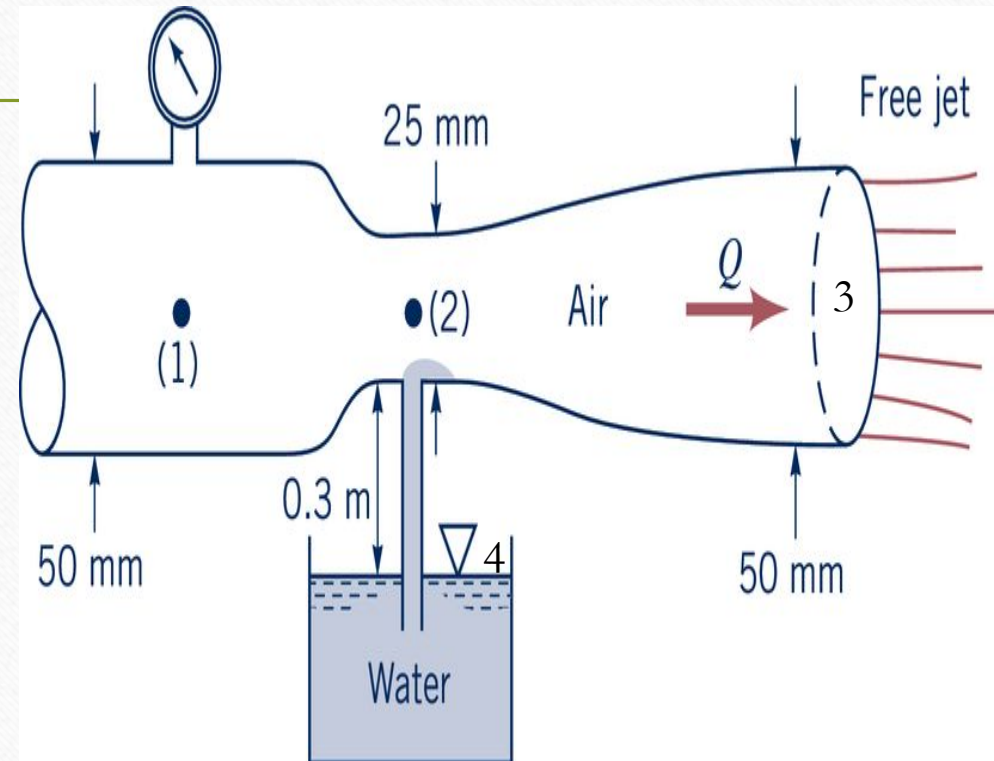
Bernoulli & Manometer

- Air flows steady and frictionless as shown. If flowrate is large enough, the pressure in the constriction will be low enough to draw water into the tube. Determine the volume flow rate Q and the inlet pressure requirements.



CRITICAL STEP #1

- What fundamentals are important?
 - Bernoulli (Fluid Flow)
 - Mass Conservation (Diameter Change)
 - Manometry (Hydro Statics)
 - Provides Pressure Differential
- Important to identify points:
 - Inlet/Exit (1/3)
 - Static Pressure Points (2)
 - Stagnation Pressure Points
 - Manometer Fluid Interfaces (4)



Step #2

- Apply Manometry First To Identify Knowns and Unknowns.
- Be Consistent

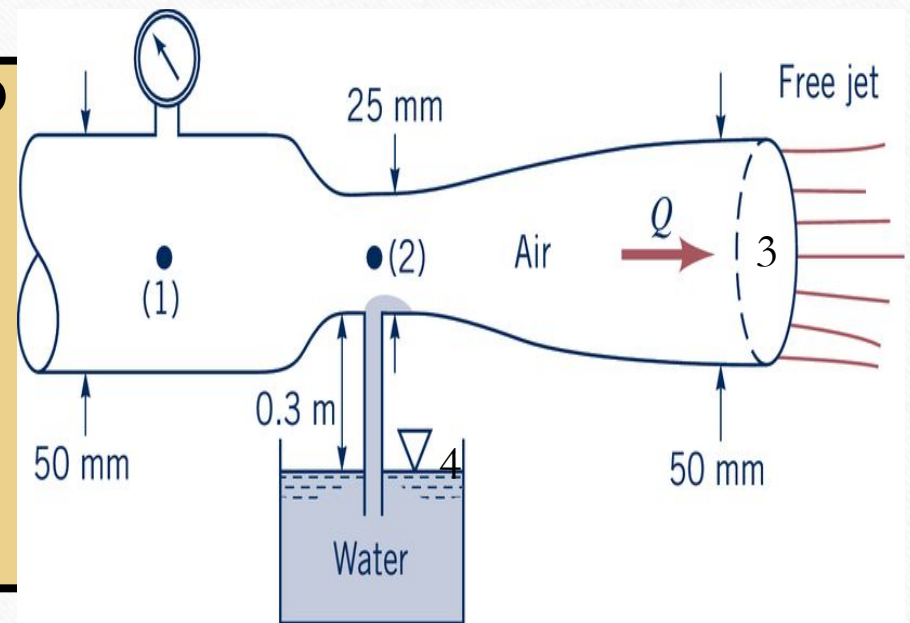
IGNORE AIR PRESSURE CHANGE ABOVE FLUID

$$P_2 + \gamma_m \Delta z = P_4 = 0(\text{open})$$

$$P_2 = -\gamma_m \Delta z$$

$$P_2 = -9800 \text{ N / m}^3 \cdot 0.3 \text{ m} = 2.94 \text{ kPa}$$

$$\frac{P_2}{\gamma_{\text{air}}} = -\frac{2940 \text{ Pa}(\text{N / m}^2)}{12 \text{ N / m}^3} = -245 \text{ m}$$



Step 3

- Apply Bernoulli to points associated with manometer.
- Combined with Mass Conservation if change in diameter and find velocity
- Determine Volume Flow Rate

2-3

$$z_2 = z_3$$

$$\frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g} = \frac{P_3}{\gamma_{air}} + \frac{V_3^2}{2g}, P_3 = 0 \rightarrow \text{FREE JET}$$

Mass Conservation

$$V_2 = \frac{A_3 V_3}{A_2} = \left(\frac{50}{25}\right)^2 V_3 = 4V_3$$

$$\frac{V_2^2}{2g} - \frac{V_3^2}{2g} = -\frac{P_2}{\gamma_{air}}$$

$$\frac{V_3^2}{2g} (16 - 1) = -\frac{P_2}{\gamma_{air}}$$

$$V_3 = \sqrt{\frac{-\frac{P_2}{\gamma_{air}} \cdot 2g}{15}}$$

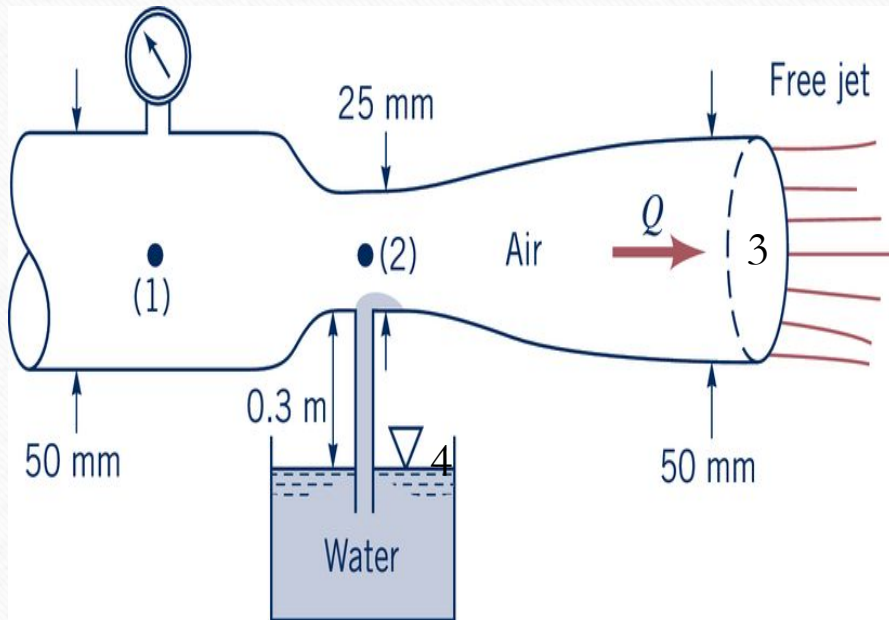
$$\text{Now, } \frac{P_2}{\gamma_{air}} = -\frac{2940 \text{ Pa} (N/m^2)}{12 \text{ N/m}^3} = -245 \text{ m}$$

$$V_3 = \sqrt{\frac{245 \text{ m} \cdot 2 \cdot 9.81 \text{ m/s}^2}{15}} = \sqrt{320.41 \frac{\text{m}^2}{\text{s}^2}} = 17.9 \frac{\text{m}}{\text{s}}$$

$$Q = A_3 V_3 = \pi \frac{D_3^2}{4} 17.9 \frac{\text{m}}{\text{s}} = 0.0351 \frac{\text{m}^3}{\text{s}}$$

Step 4

- Solve for other unknown velocities and pressures



$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$V_2 = \frac{Q}{A_2} = 71.5 \text{ m/s}$$

$$V_1 = \frac{Q}{A_1} = 17.9 \text{ m/s} = V_3 \rightarrow \text{Constant Diameter Pipe}$$

1-2

$$\frac{P_1}{\gamma_{air}} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g}$$

$$P_1 = \left(\frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) \gamma_{air}$$

$$P_1 = \left(-245 + \frac{1}{2g} (71.5^2 - 17.9^2) \right) \gamma_{air}$$

$$P_1 = (-245 + 245) \gamma_{air}$$

$$P_1 = 0$$

1-3, OR ALTERNATIVE METHOD

$$\frac{P_1}{\gamma_{air}} + \frac{V_1^2}{2g} = \cancel{\frac{P_2}{\gamma_{air}}} + \frac{V_3^2}{2g}$$

$$\frac{P_1}{\gamma_{air}} = \frac{V_3^2}{2g} - \frac{V_1^2}{2g} = 0$$