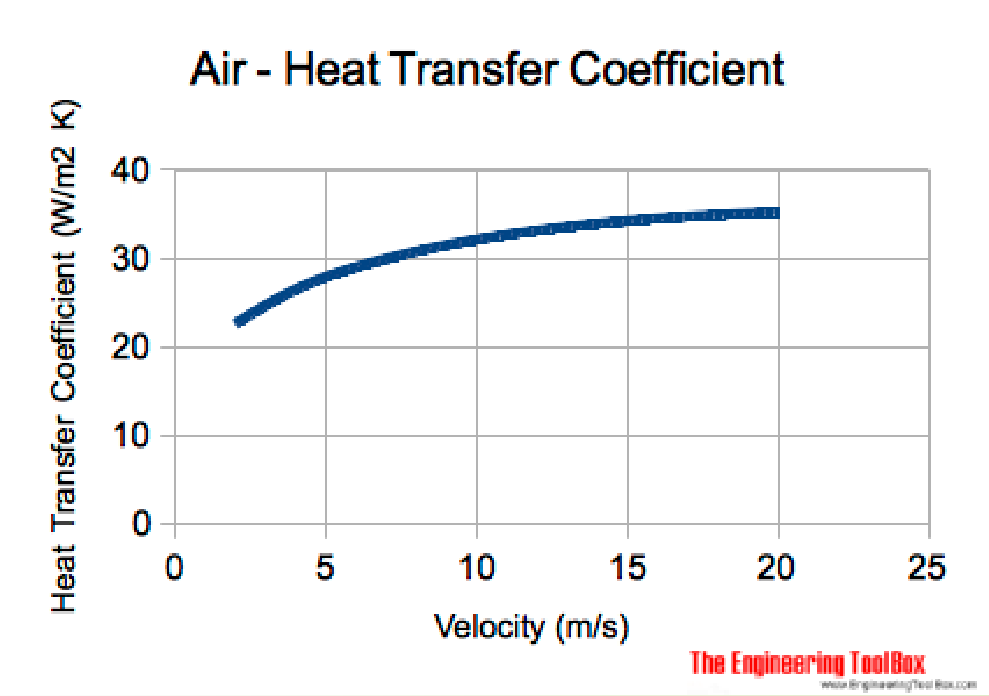


MECH-420 Heat Transfer Chapters 6-8

Introduction to Convective Transfer – External/Internal Flow

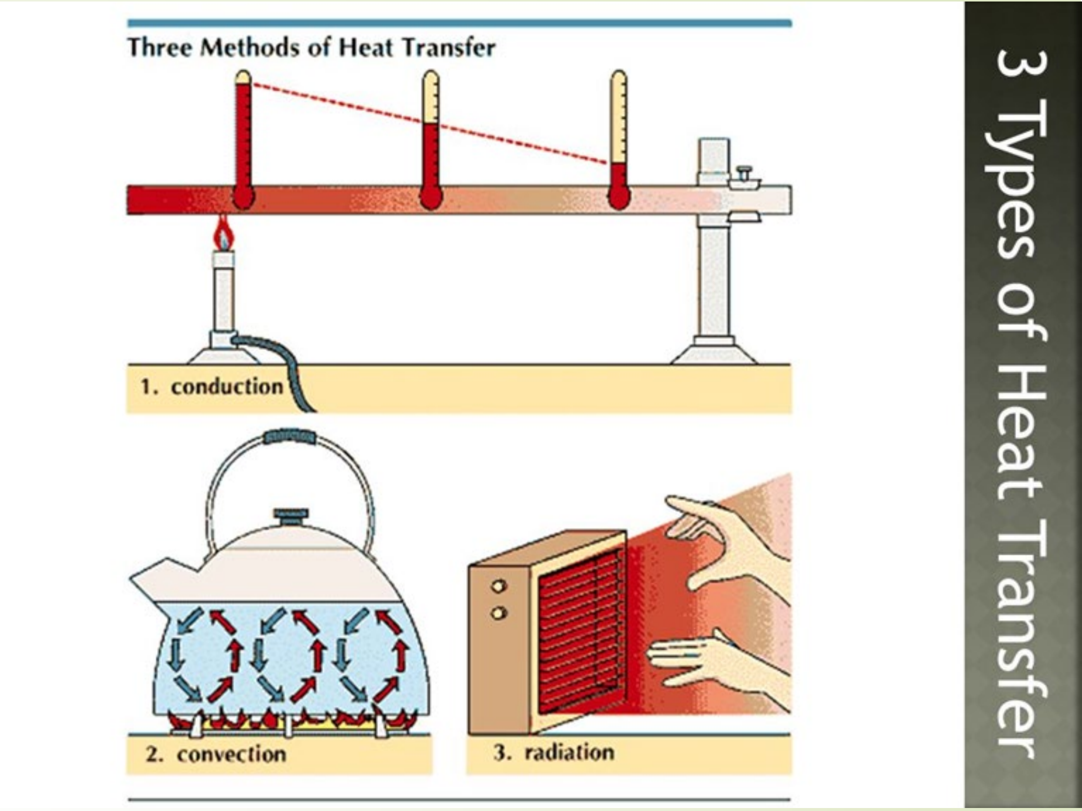
How the heck do we find "h"?

ENGINEERING STUDENT MISPERCEPTIONS (CLICK HERE)

- **GOAL !!!!!!!**
- **FIND CONVECTIVE HEAT TRANSFER COEFFICIENT h= ?**
- **FIND DRAG COEFFICIENT cf=?**

Heat Transfer Modes

- **Conduction**: Energy transfer due to molecular interactions caused by temperature.
- **Radiation**: Energy transfer due to propagation of electromagnetic waves/photons due to surface temperature.
- **Convection**: Energy transfer due to bulk fluid motion and temperature difference between fluid and surface.
- **Boiling and Condensation:** If the fluid is being transformed from liquid to vapor through heat addition, then the process is boiling or evaporation. If vapor is being transformed to liquid by heat removal, then the process is condensation. ... The density difference between a vapor and a liquid is quite large.



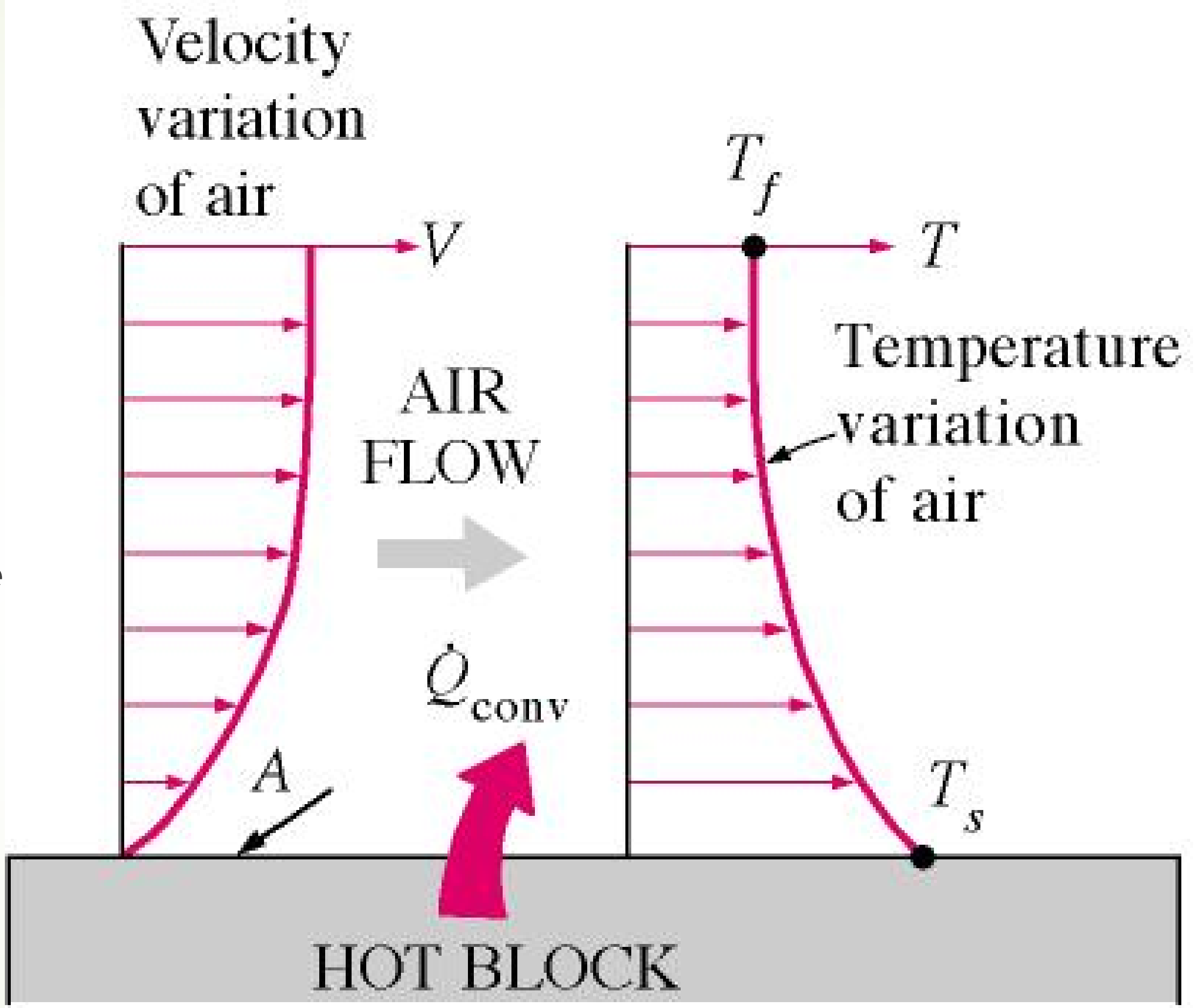
Forced Convection

- Convection: Describes energy transfer between a surface and a fluid moving over the surface at a different temperature.
- Because the surface velocity is zero and the surface temperature is different than the fluid temperature, the result is the development of velocity and thermal “boundary layers” close to the surface shown here.
- The transfer of momentum and energy occurs within these boundary layers.
- Local heat flux may be expressed as:

$$q'' = h(x)(T_s - T_\infty)$$

$h(x) \rightarrow$ local heat transfer convection coefficient

Since the flow and thermal conditions vary from point to point, both q'' and h also vary along the surface.



Total Heat Transfer Rate

- Since flow conditions change along surface, the total heat transfer rate is expressed as:

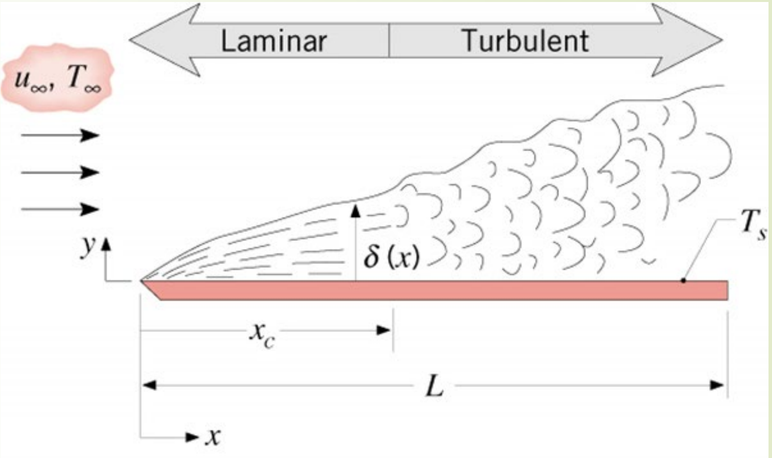
$$q_{Total}[W] = \int_{A_s} q''(s)\,dA_s \rightarrow s \text{ is measured along surface}$$

or defining an average $\bar{h}$ for the entire surface

$$q_{Total}[W] = \bar{h}A_s(T_s - T_\infty) = (T_s - T_\infty)\int_{A_s} h(s)\,dA_s$$

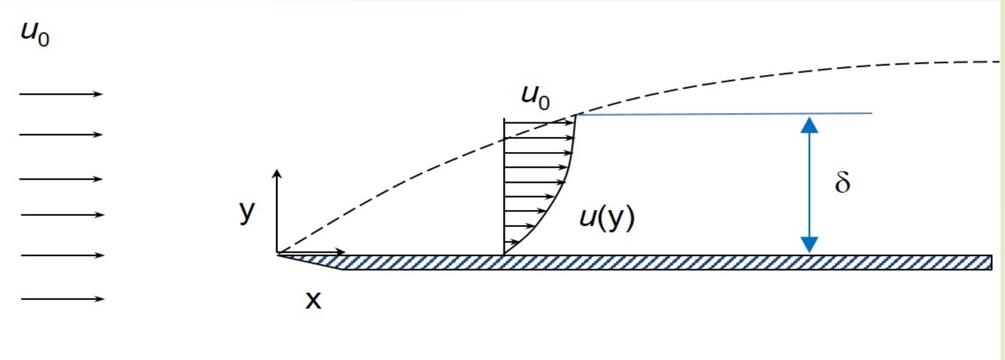
or $\bar{h} \rightarrow$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h(s)\,dA_s$$



- Heat flux requires the determination of the HT coefficient.
- HT Coefficient is a function of numerous fluid properties such as density, viscosity, thermal conductivity and specific heat, and, depend upon the surface geometry and flow conditions.
- This dependency is a result of boundary layers that develop on the surface.

Convective Boundary Layers



- The velocity boundary layers (BL), where:

  $u(y)$ = velocity distribution in boundary layer

  δ(y) = boundary layer thickness, where u(y)=0.99u_∞

- When fluid particles make contact with the surface, they assume zero velocity. These particles act to retard the motion of particles in the adjoining fluid layer and so on…
- This retardation of fluid motion is associated with the shear stress, $\tau[Pa]$
- Due to BL growth, the fluid flow is characterized by two distinct regions:
  - 1. **A thin fluid layer (the boundary layer) where velocity gradients and shear stress are large. BL is a result of fluid VISCOSITY which causes a shear stress at the wall.**
    - BL growth (thickness) increases with increasing "X" from the leading edge.
  - 2. **A region outside the BL where velocity gradients and shear stresses are small (Potential Flow)**
  - **The effects of viscosity penetrates further into free stream as BL grows.**

Velocity BL Relationships

$$\tau_s \equiv \text{wall shear stress} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$\text{where } \mu \rightarrow \text{fluid viscosity [Pa-s]}$$

$$\frac{\partial u}{\partial y} \rightarrow \text{velocity gradient at wall}$$

$$\text{c}_f \equiv \text{friction coefficient} = \frac{\text{shear stress}}{\text{dynamic pressure}} = \frac{\tau_s}{\frac{\rho U_\infty^2}{2}}$$

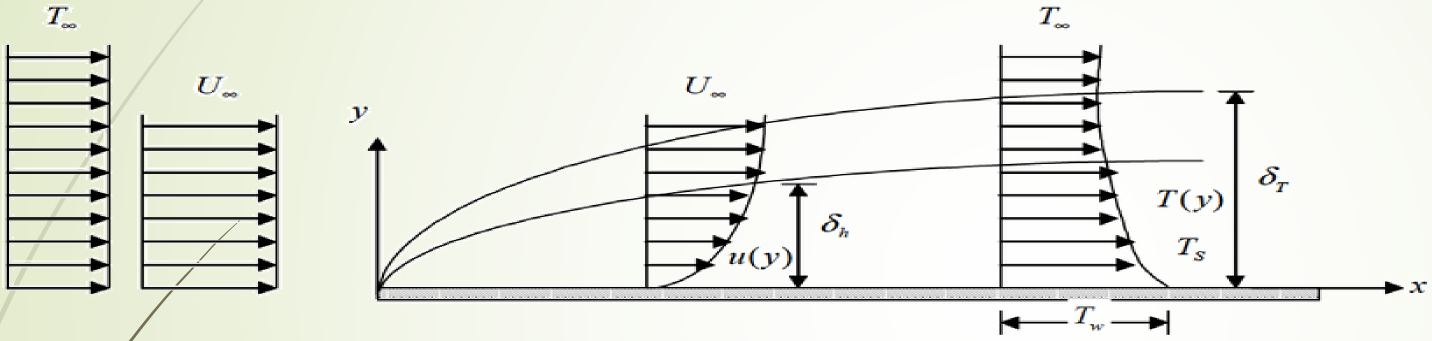
$$D_f \equiv \text{Drag Force} = (\tau_s \bullet A_s)[N]$$

$$A_s = \textit{Area exposed to the fluid shear}$$

$$\text{P} \equiv \text{POWER} = \text{Drag Force} \bullet \text{Velocity[W]}$$

Thermal Boundary Layers

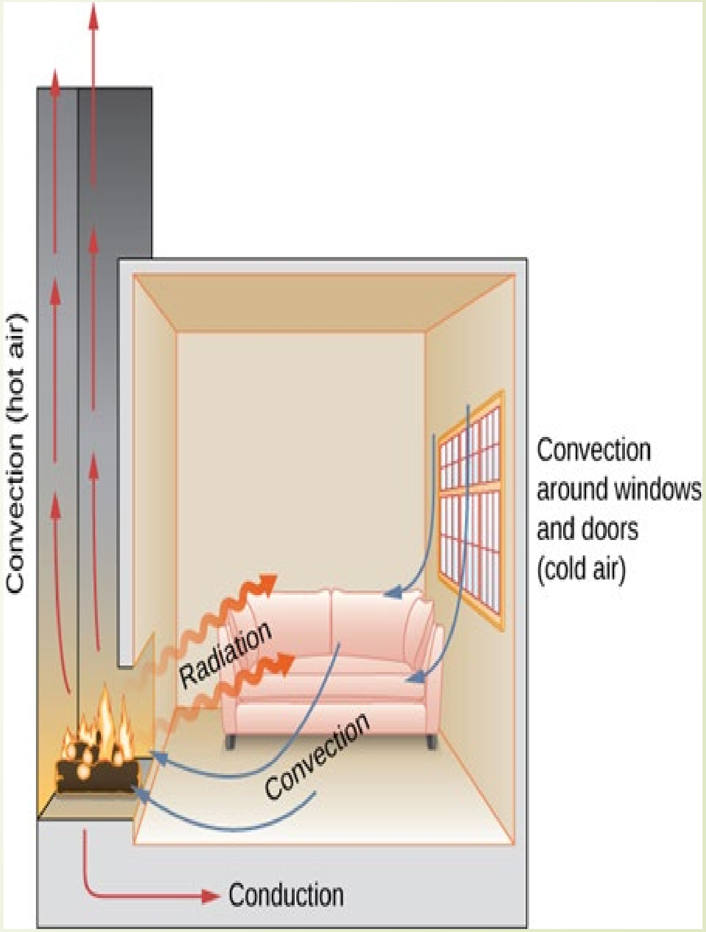
- A Thermal Boundary Layer (TBL) "***MUST***" develop if the fluid free stream and the surface temperature are different.



where:

$T(y) \equiv$ BL Temperature Distribution

$$\delta_T(y) \equiv \text{Thermal Boundary Layer Thickness} \rightarrow \frac{T_s - T}{T_s - T\infty} = 0.99$$



- Due to temperature difference between surface and fluid, the particles in contact with plate achieve thermal equilibrium, and transfer energy to particles in adjoining fluid layers, and so on throughout the thermal BL.
- This results in a temperature gradient in the fluid. With increasing distance from the leading edge, the effects of HT penetrate further into the free stream as the thermal BL grows.

Thermal BL Relationships

LOCAL HEAT FLUX:

$$q_s''\left[\frac{W}{m^2}\right] = -k_{fluid}\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} \rightarrow \text{temperature gradient at wall (conduction)}$$

Energy Balance at Wall

$$q''_{s\,CONDUCTION} = q''_{s\,CONVECTION}$$

Energy Balance at Wall

$$q''_{s\,CONDUCTION} = q''_{s\,CONVECTION}$$

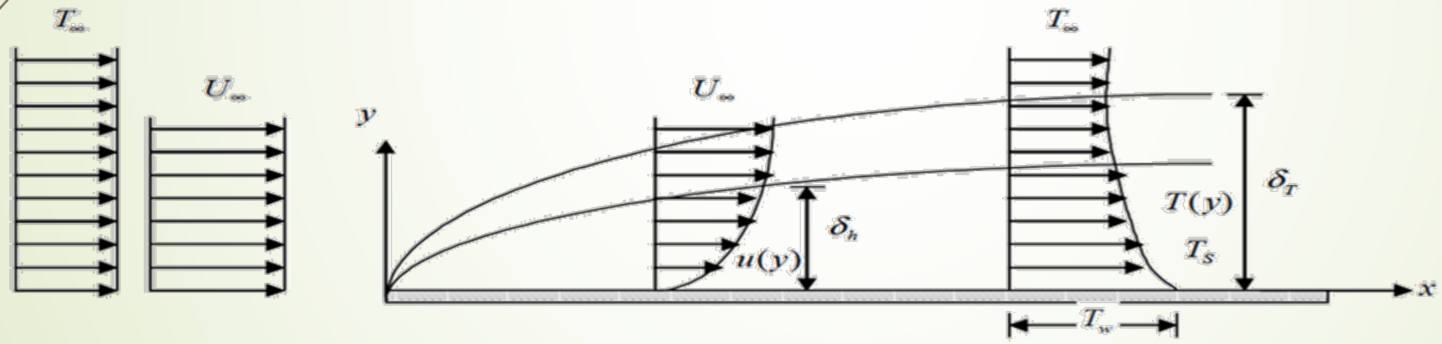
$$-k_{fluid}\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T(x)_{WALL} - T_\infty)$$

$$h(x) = \frac{-k_{fluid}\left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s(x) - T_\infty)} = \frac{NU \bullet k_{fluid}}{x} \rightarrow \text{Local Heat Transfer Coefficient}$$

$$NU_x \equiv \text{LOCAL } Nusselt\,\# = \frac{h(x) \bullet x}{k_{fluid}} \rightarrow \text{DIMENSIONLESS Heat Transfer Relationship}$$

KEY IDEAS!!!

- Conditions in the ***THERMAL BL*** **determines** the rate of HT (q(x)) across the BL, i.e.: $\left.\frac{\partial T}{\partial y}\right|_{y=0} \rightarrow @wall$
- Since **(Ts-Tf) is constant**, temperature gradient in BL MUST decrease with increasing X.
- i.e. $\left.\frac{\partial T}{\partial y}\right|_{y=0} \rightarrow @wall$ **DECREASES WITH INCREASING X**, **Q(X) AND H(X) DECREASES WITH INCREASING X**.
- The ***KEY*** BL parameters are ***FRICTION COEFFICIENT*** AND ***HEAT TRANSFER COEFFICIENT***.



$$\frac{dT}{dy} \downarrow \text{ with INCREASING X}$$

MAX GRADIENT at LEADING EDGE of PLATE

Navier Stokes

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \qquad (1)$$

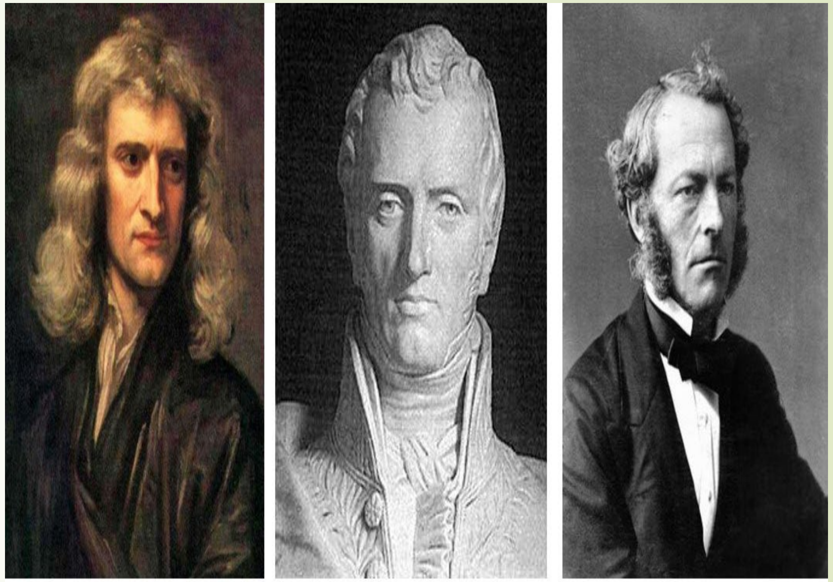
$$\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho u v)}{\partial y} + \frac{\partial (\rho u w)}{\partial z} = -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \qquad (2)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho u v)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial (\rho v w)}{\partial z} = -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \qquad (3)$$

$$\frac{\partial \rho w}{\partial t} + \frac{\partial (\rho u w)}{\partial x} + \frac{\partial (\rho v w)}{\partial y} + \frac{\partial (\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \qquad (4)$$

$$\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho u E)}{\partial x} + \frac{\partial (\rho v E)}{\partial y} + \frac{\partial (\rho w E)}{\partial z} = -\frac{\partial p u}{\partial x} - \frac{\partial p v}{\partial y} - \frac{\partial p w}{\partial z} + S \qquad (5)$$

where ρ is the air density, u, v, w are the components of the air's velocity, E is measure of the air's internal energy (which allows us to compute its temperature) and p is the air pressure.



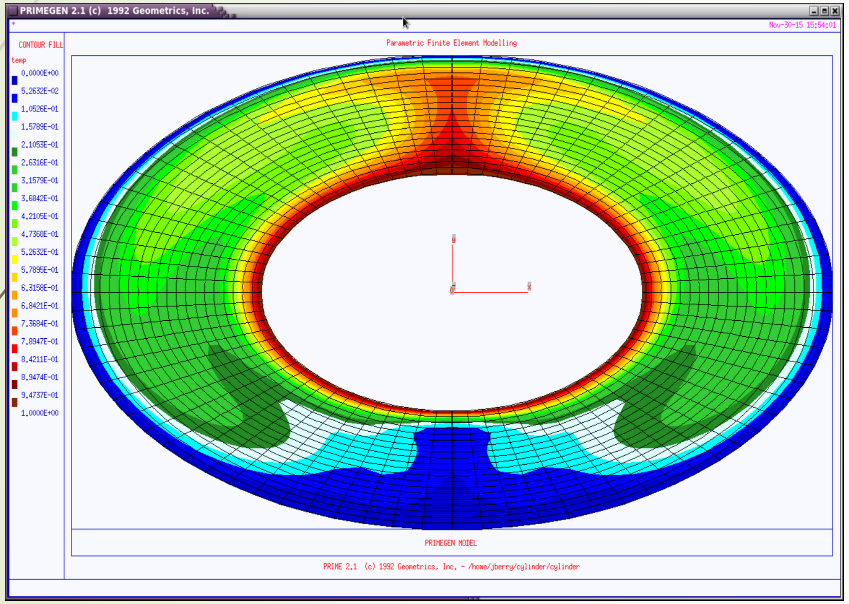
Isaac Newton 1642-1727 | Claude-Louis Navier 1785-1836 | Sir George Stokes 1819-1903

NO SOLUTION POSSIBLE FOR 50+ YEARS

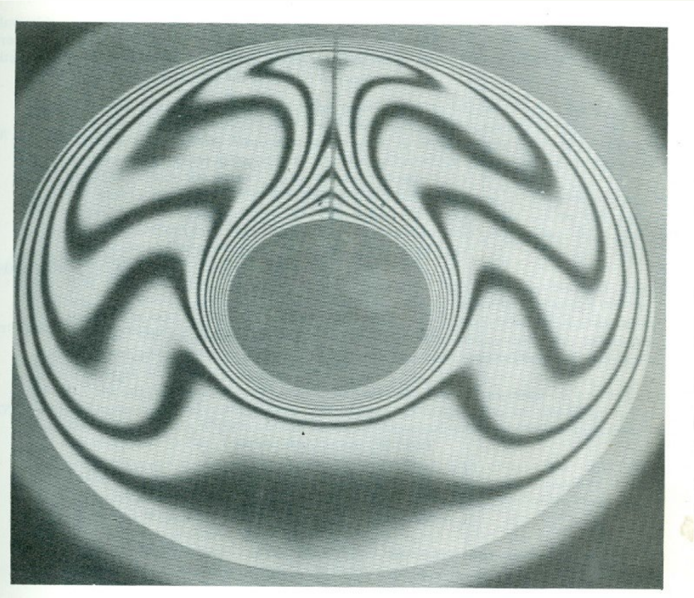
Conservation of Mass, Momentum, and Energy – Navier Stokes Equations

Free/Natural Convective Heat Transfer Concentric cylinders

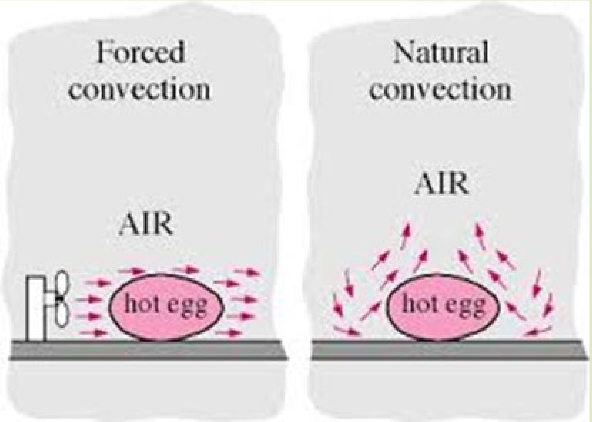
- **The Power of Numerical Methods: Temperature Contours**



Numerical: Dr. K. J. Berry

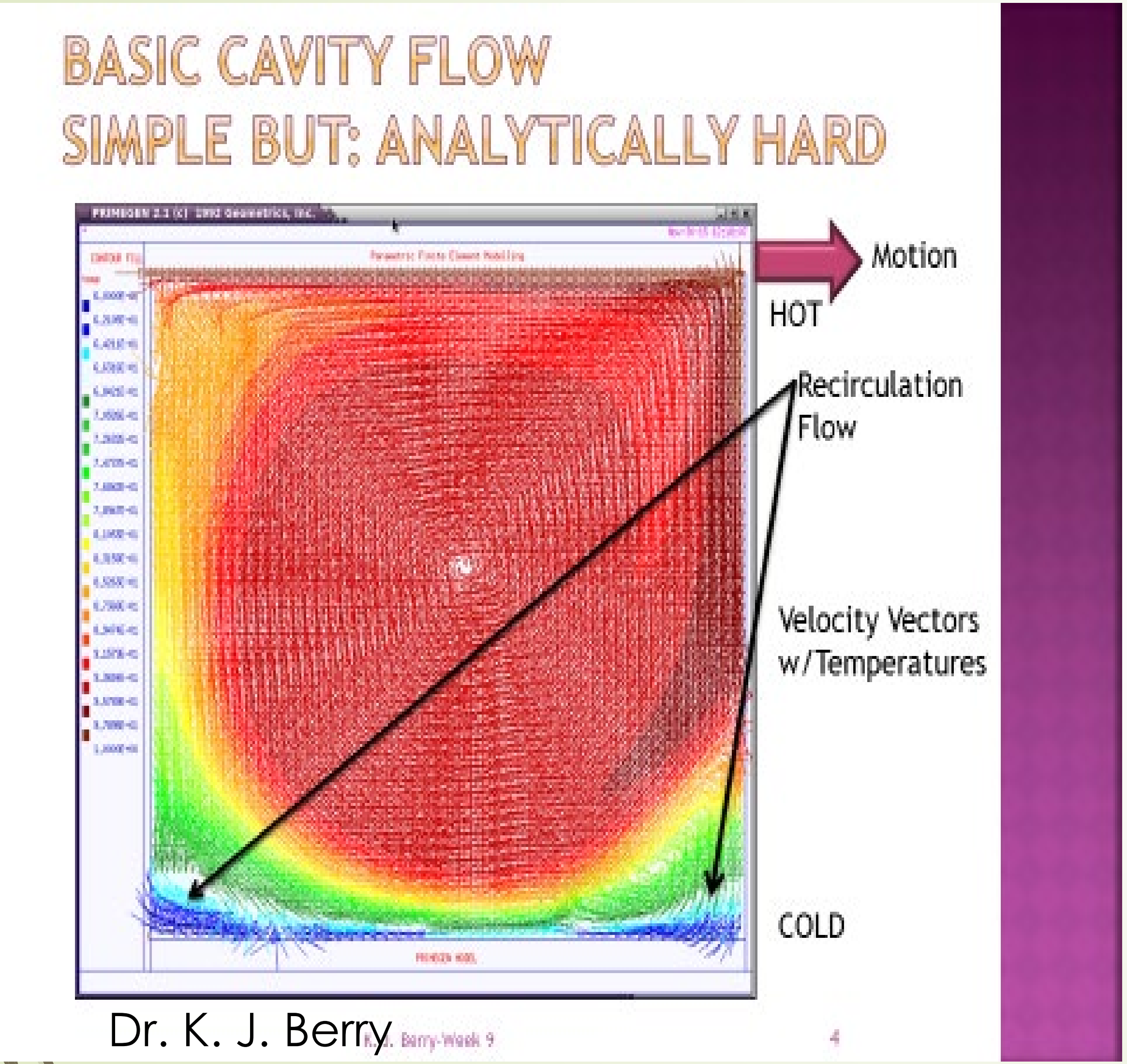


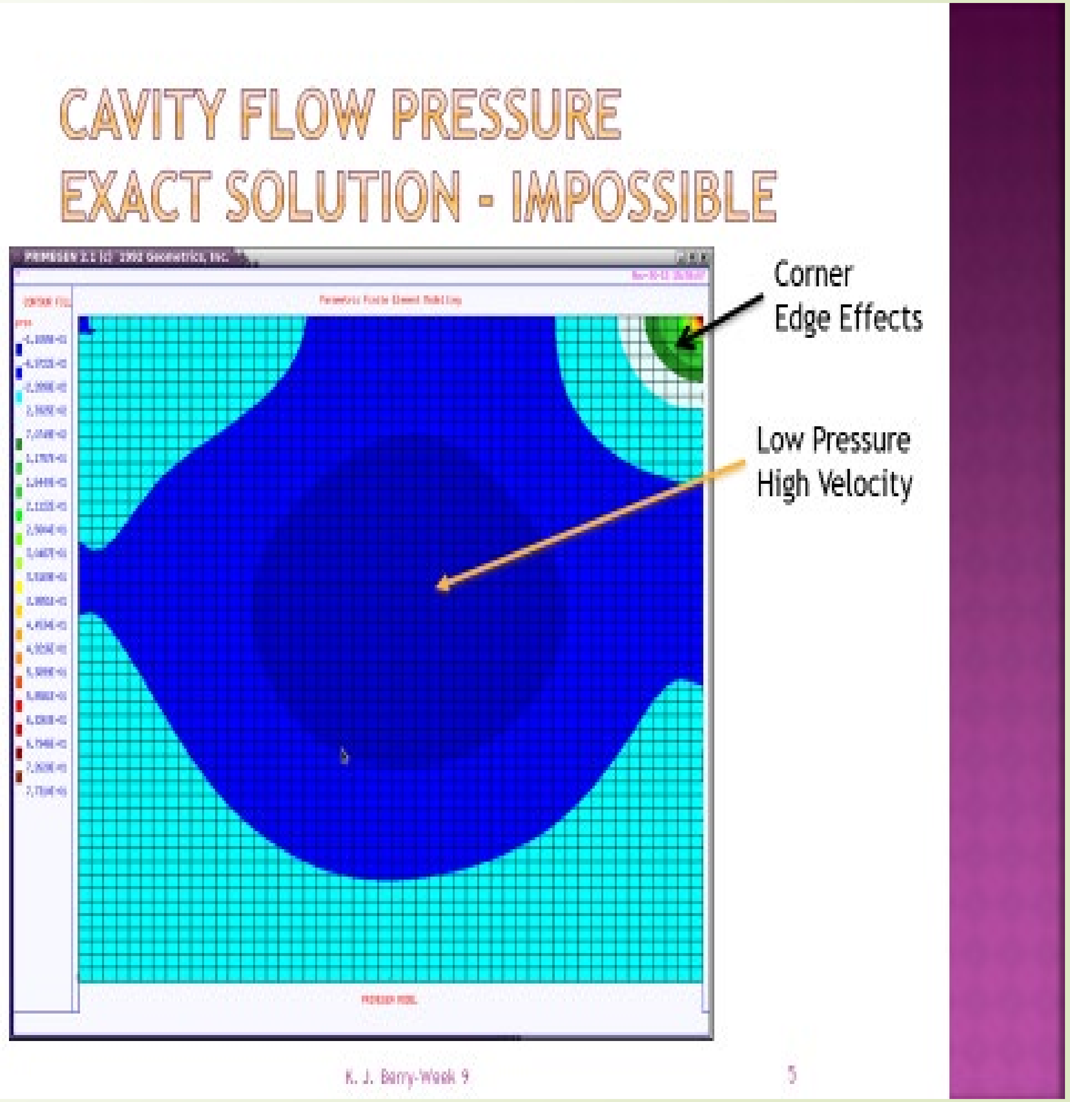
Experimental



- **Conservation of Mass, Momentum, and Energy – Navier Stokes Equations**

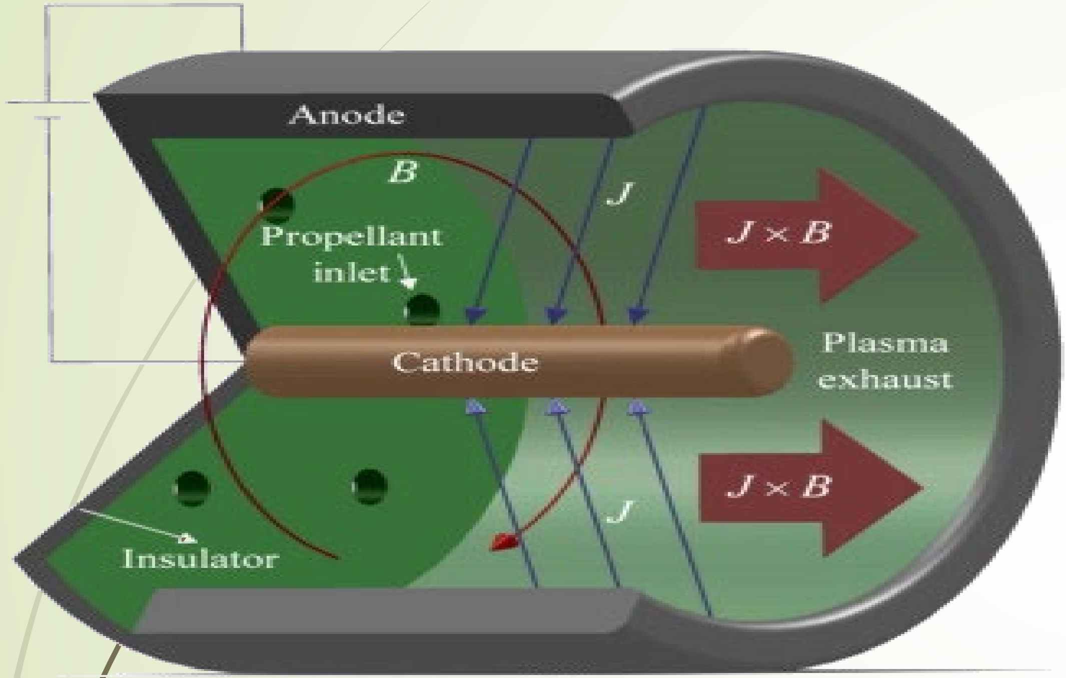
Forced Convection Lid Cavity Flow



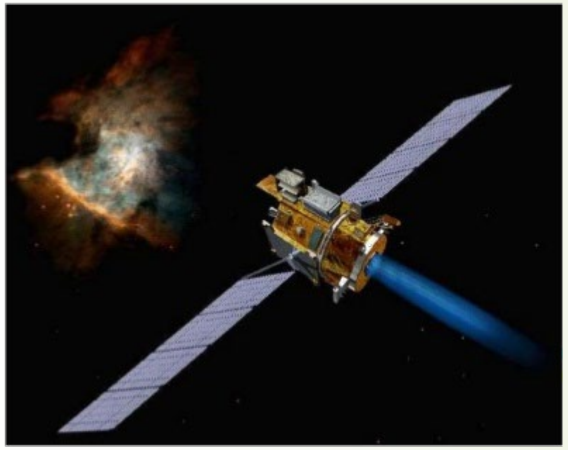


Dr. K. J. Berry

MAGNETOPLASMADYNAMICS + Heat and Fluid Flow, CHAOS: Computational MHD Analysis



Electric Propulsion



ENGINEERING

+

PHYSICS

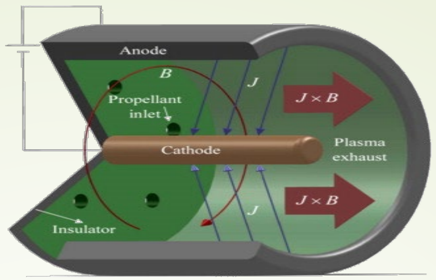
+

MATHEMATICS

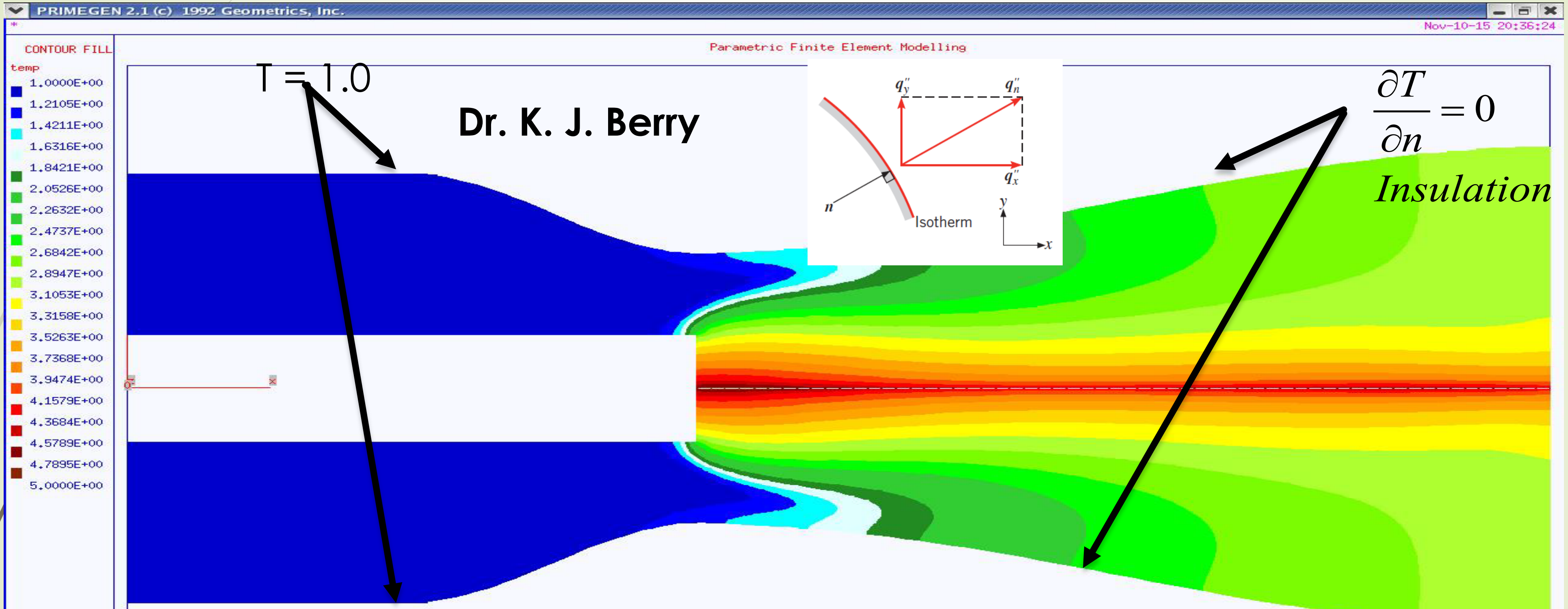
Conservation of Mass, Momentum, Energy, & Maxwell Electromagnetics, Navier Stokes Equations

Conducting fluids passing through electric fields induce magnetic fields that couple with electric currents to produce electromagnetic THRUST force.

Magnetoplasmadynamics Heat and Fluid Flow, CHAOS: Computational MHD Analysis



Electric Propulsion

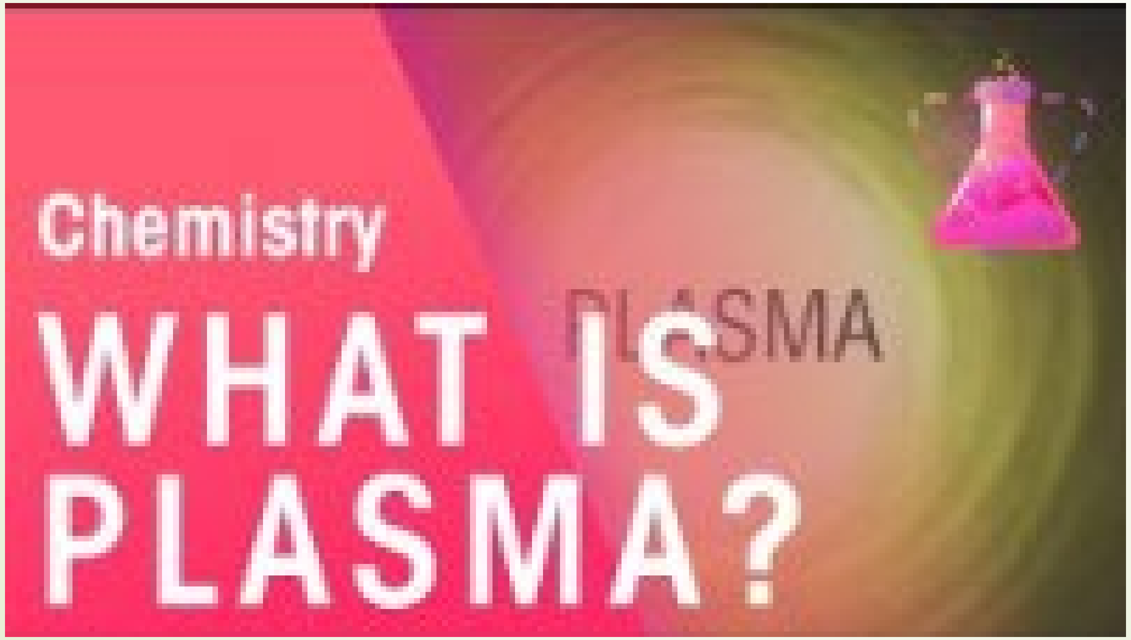


Conservation of Mass, Momentum, Energy, & Maxwell Electromagnetics, Navier Stokes Equations

Conducting fluids passing through electric fields induce magnetic fields that couple with electric currents to produce electromagnetic THRUST force.

PLASMA

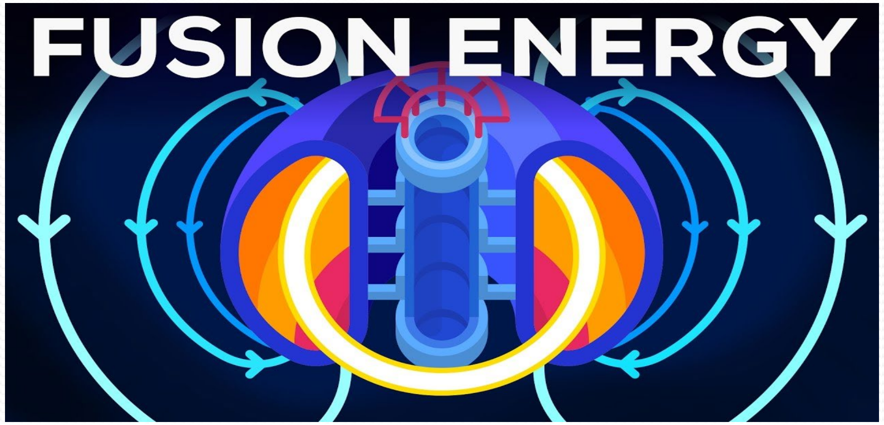
CLICK HERE



FUSION ENERGY (CLICK HERE)

FUSION ENERGY HEAT TRANSER

$150,000,000\ C \rightarrow -269\ C$ (HELIUM CHILLER)



https://www.youtube.com/watch?v=k3zcmPmW6dE

External Forced Convection—FLAT PLATE

FRICTION RELATIONSHIPS

$$Re_x \equiv \text{Renolyds\#} = \frac{\rho U_\infty x}{\mu}$$

LAMINAR FLOW

$$Re_x < 5x10^5$$

$$\delta(x) \equiv \text{Boundary Layer}=\frac{5x}{\sqrt{Re_x}}$$

LOCAL FRICTION COEFF.

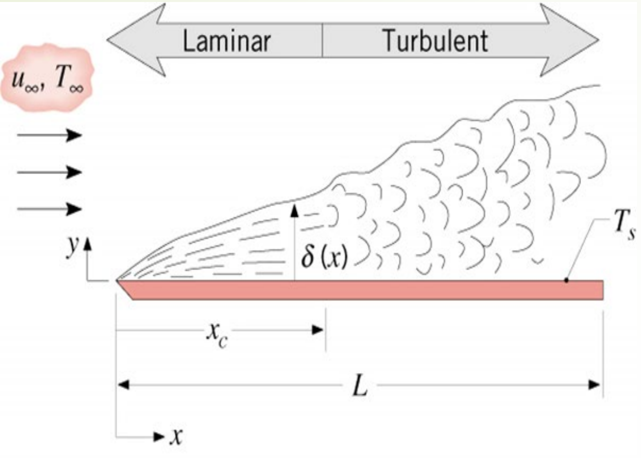
$$c_{f,x} = \frac{\tau_{s,x}}{\frac{\rho U_\infty^2}{2}} = 0.664\,Re_x^{-1/2}$$

AVERAGE FRICTION COEFF.

$$\overline{c_{f,x^*}} = \frac{1}{x^*}\int_0^{x^*} c_{f,x}dx = 1.328\,Re_{x^*}^{-1/2} = \frac{\overline{\tau_{s,x}}}{\frac{\rho U_\infty^2}{2}}$$

DRAG FORCE

$$D_f = \overline{\tau_{s,x}}\left[\frac{N}{m}\right] \bullet A_s\left[m^2\right]$$



$$Re_x \equiv \text{Renolyds\#} = \frac{\rho U_\infty x}{\mu}$$

TURBULENT FLOW

$$Re_x > 5x10^5, x_c \equiv \text{transition location}=\frac{5x10^5\mu}{\rho U_\infty}$$

$$\delta(x) \equiv \text{Boundary Layer}=0.37 \bullet x \bullet Re_x^{-1/5}$$

LOCAL FRICTION COEFF.

$$c_{f,x,TURB} = \frac{\tau_{s,x}}{\frac{\rho U_\infty^2}{2}} = 0.0592\,Re_x^{-1/5}; Re_{x,c} \le Re_x \le 10^8$$

AVERAGE FRICTION COEFF.

$$\overline{c_{f,x^*}} = \frac{1}{x^*}\left[\int_0^{x_c} c_{f,x,LAM}dx + \int_{x_c}^{x^*} c_{f,x,TURB}dx\right] = \frac{\overline{\tau_{s,x}}}{\frac{\rho U_\infty^2}{2}}$$

$$= 0.074\,Re_{x^*}^{-1/5} - \frac{2A}{Re_{x^*}}, Re_{x,c} \le Re_x \le 10^8, A = 0.037\,Re^{4/5}{}_{x,c} - 0.664\,Re^{1/2}{}_{x,c}$$

Note: For a completely TURBULENT FLOW, tripped at leading edge, A=0.

DRAG FORCE

$$D_f = \overline{\tau_{s,x}}\left[\frac{N}{m}\right] \bullet A_s\left[m^2\right]$$

Heat Transfer Relations – Flat Plate

$Re_x \equiv Renolyds\# = \frac{\rho U_\infty x}{\mu}$

LAMINAR FLOW--ISO THERMAL PLATE

$Re_x < 5x10^5$

$NU_x = \frac{h_x x}{k_{fluid}} = 0.332\, Re_x^{1/2}\, Pr^{1/3}, 0.6 \le Pr \le 50,$

$NU_x = \frac{h_x x}{k_{fluid}} = \frac{0.3387\, Re_x^{1/2}\, Pr^{1/3}}{\left[1 + \left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}}; Pr \ge 100$

LAMINAR FLOW-CONSTANT HEAT FLUX PLATE

$h_x = \frac{NU_x \bullet k_{fluid}}{x} = 0.453\, Re_x^{1/2}\, Pr^{1/3}, Pr \ge 0.6$

$\rightarrow$ LOCAL HEAT TRANSFER COEFF.

$Pr \equiv Prandtl\# = \frac{\mu c_p}{k_{fluid}} = \nu/\alpha \equiv \frac{\text{Diffusivity of Momentun}}{\text{Diffusivity of Heat}}$

Thermal Boundary Layer

$\delta_t(x) \approx \frac{\delta(x)}{Pr^{1/3}}$

PROPERTIES

$T_{film} = \frac{T_\infty + T_s}{2}$

AVERAGE (ISOTHERMAL/UNIFORM HEAT FLUX)

$\bar{h}_{x^*} = \frac{1}{x^*}\int_0^{x^*} h_x dx = 2h_x \rightarrow \overline{NU}_{x^*}(average\ \text{value}) = 2NU_x(local\ \text{value})$

TURBULENT FLOW--ISOTHERMAL PLATE

$Re_x > 5x10^5$

$NU_x = \frac{h_x x}{k_{fluid}} = 0.029\, Re_x^{4/5}\, Pr^{1/3}; 0.6 \le Pr \le 60 \rightarrow LOCAL$

TURBULENT FLOW--CONSTANT HEAT FLUX

$NU_x = \frac{h_x x}{k_{fluid}} = 0.0308\, Re_x^{4/5}\, Pr^{1/3}; 0.6 \le Pr \le 60 \rightarrow LOCAL$

$\delta(x) = 0.37\, Re_x^{-1/5}$

*Due to enhanced mixing, the turbulent boundary layer grows more rapidly and has LARGER friction and convection coefficiencts (i.e. more heat transfer and more friction)

MIXED CONDITIONS - LAMINAR and TURBULENT

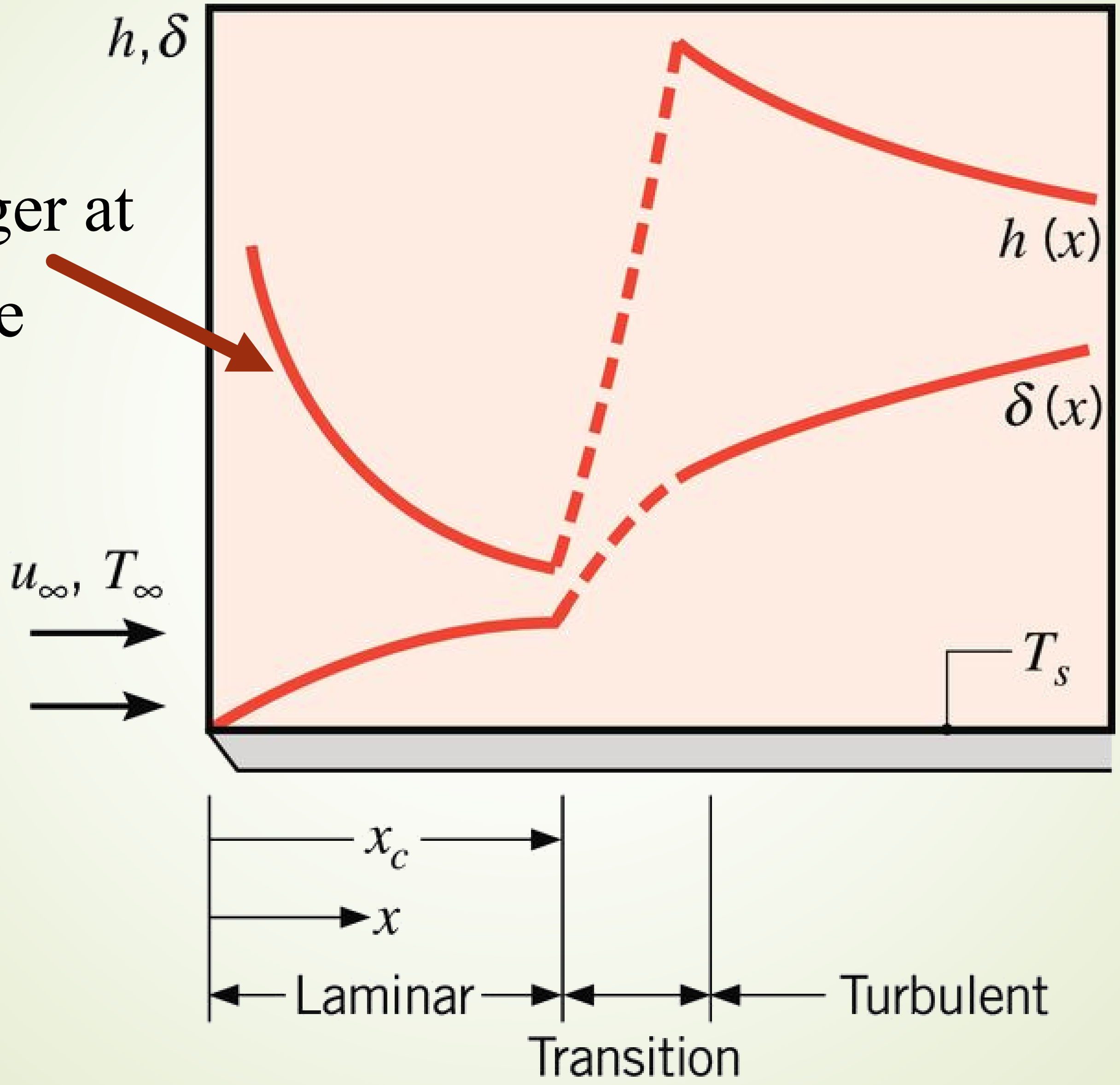
$\overline{NU}_{x^*} = (0.037\, Re^{4/5} - A)\, Pr^{1/3} = \frac{\bar{h}_{x^*} x^*}{k_{fluid}}; 0.6 \le Pr \le 60, 5x10^5 \le Re_{x^*} \le 10^8$

$A = 0.037\, Re^{4/5}{}_{x,c} - 0.664\, Re^{1/2}{}_{x,c} \rightarrow$ FOR TRIPPED TURB BOUNDARY, A=0.0

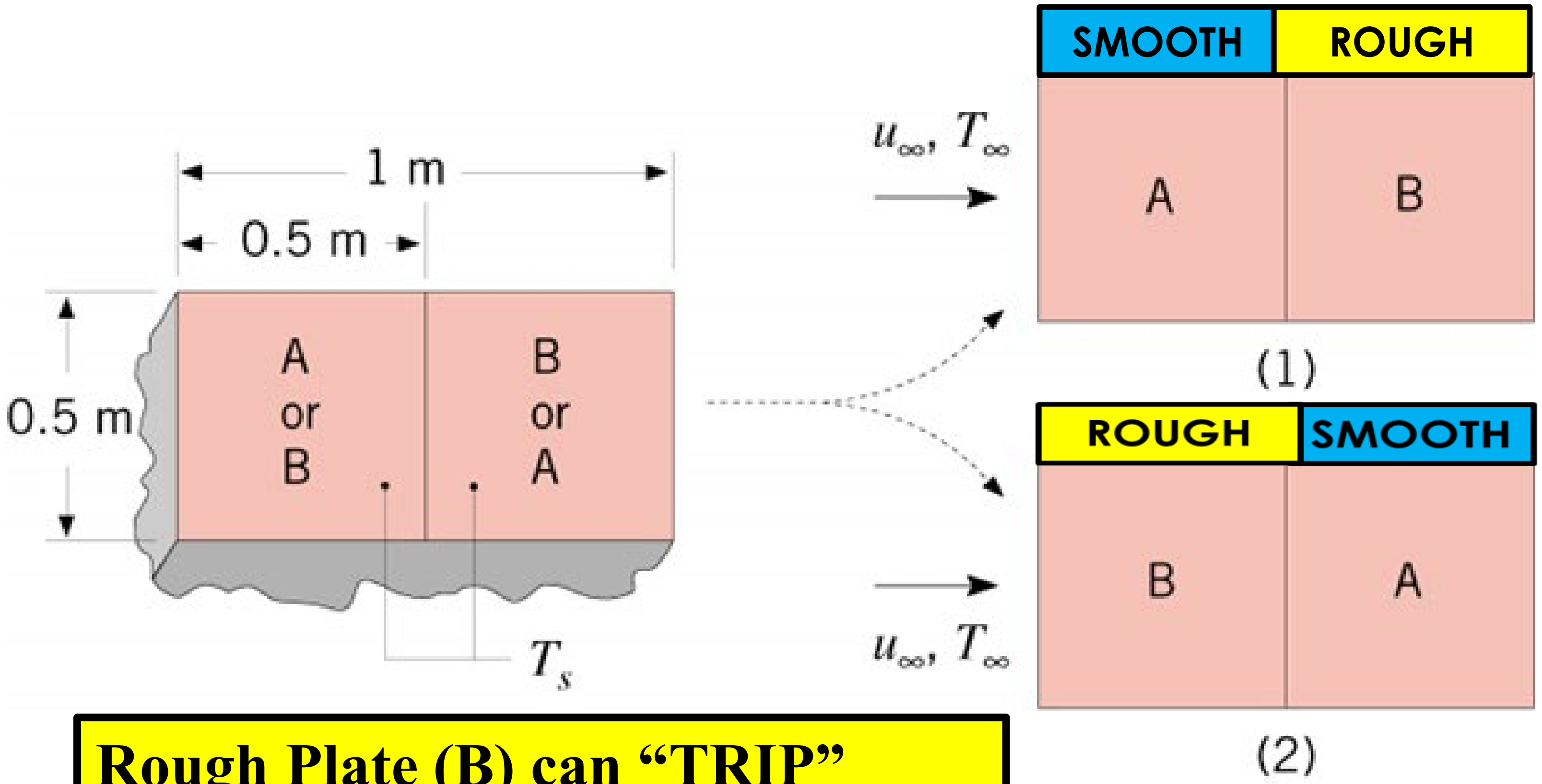
$Re_{x,c} = 5x10^5$

Heat Transfer During Boundary Layer Growth

Heat Transfer Larger at Leading Plate Edge



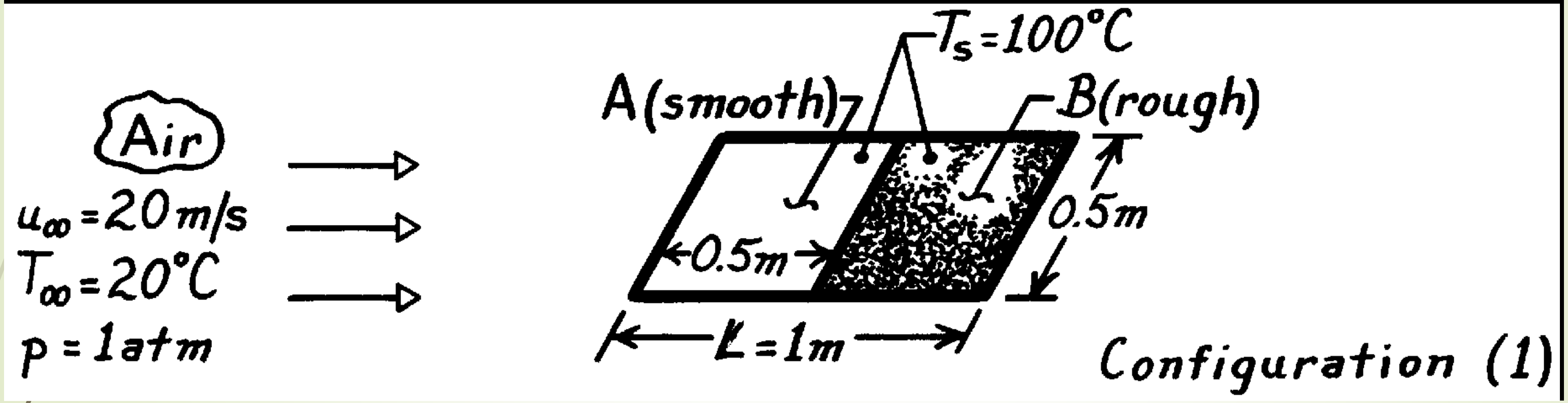
Problem 7.21: Preferred ORIENTATION (corresponding to *HIGHER HEAT LOSS*) and the corresponding HEAT RATE for a surface with adjoining SMOOTH and ROUGHENED sections.



Rough Plate (B) can "TRIP" TURBULENT Boundary at Start.

ROAD MAP
Configuration #1

SCHEMATIC:



THOUGHT #1: What is the FLOW CONDITION @ END

FLOW CONDITION @ END
Laminar vs Turbulent vs MIXED

PROPERTIES: *Table A-4*, Air (T_f = 333K, 1 atm): $\nu = 19.2 \times 10^{-6}$ m²/s, $k = 28.7 \times 10^{-3}$ W/m·K, $Pr = 0.7$.

ANALYSIS: $\rightarrow T_f = \dfrac{T_\infty + T_s}{2} = 333K$

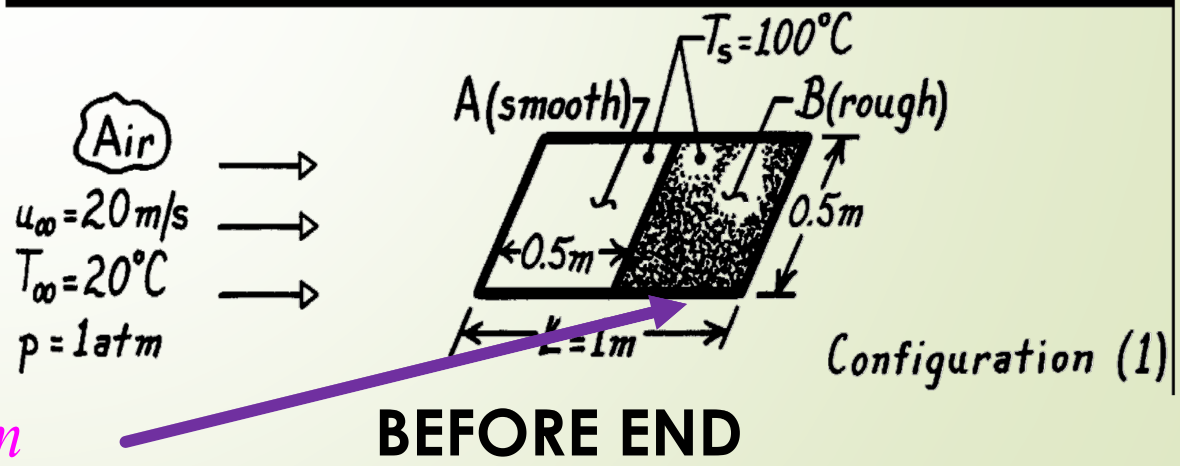
1) $RE_L = \dfrac{U_\infty L}{\frac{\mu}{\rho} = \nu} = \dfrac{20m/s \bullet 1m}{19.2x10^{-6}\frac{m^2}{s}} = 1.04x10^6 > 5x10^5$ (critical transition)

FULLY TURBULENT @ END

2) Where is TRANSITION, "x_c"

$$x_c = \frac{RE_c \bullet \nu}{U_\infty} = \frac{500,000 \bullet 19.2x10^{-6}\frac{m^2}{s}}{20\frac{m}{s}} = 0.84m$$

SCHEMATIC:



CASE 1: LAMINAR + TURBULENT PLATE

MIXED CONDITIONS - LAMINAR and TURBULENT

$$\overline{\mathrm{NU}}_{L^*} = (0.037\,\mathrm{Re}^{4/5} - A)\,\mathrm{Pr}^{1/3} = \frac{\bar{h}_{L^*} L}{k_{fluid}};\ 0.6 \le \mathrm{Pr} \le 60,\ 5x10^5 \le \mathrm{Re}_{x^*} \le 10^8$$

$$A = 0.037\,\mathrm{Re}^{4/5}{}_{x,c} - 0.664\,\mathrm{Re}^{1/2}{}_{x,c} \rightarrow A = 871$$

$$\mathrm{Re}_{x,c} = 5x10^5$$

$$\overline{\mathrm{NU}}_{L^*_1} = (0.037 \bullet \left(1.04x10^6\right)^{4/5} - A) \bullet 0.7^{1/3} = 1366$$

$$\bar{h}_{L^*_1} = \frac{\overline{\mathrm{NU}}_{L^*} \bullet k_{fluid}}{L(\text{entire length})} = \frac{1366\left(28.7x10^{-3}\,\dfrac{W}{m-K}\right)}{1m} = 39.2\frac{W}{m^2-K}$$

$$q_1 = \bar{h}_{L^*} \bullet A_s(T_s - T_\infty) = 39.2\frac{W}{m^2-K} \bullet 0.5m x 1m \bullet (100-20)K$$

$$= 1568W \rightarrow$$ LAMINAR/TURBULENT HEAT TRANSFER

CASE 2: TURBULENT @ START

CONDITIONS - TURBULENT 100%

$$\overline{\mathrm{NU}}_{L^*} = (0.037\,\mathrm{Re}^{4/5}{}_{x,c} - A)\,\mathrm{Pr}^{1/3} = \frac{\bar{h}_{L^*} L}{k_{fluid}}; 0.6 \le \mathrm{Pr} \le 60, 5x10^5 \le \mathrm{Re}_{x^*} \le 10^8$$

$A \Rightarrow \rightarrow 0.0$ (TRIP TURBULENT BL @ LEADING EDGE)

$$\overline{\mathrm{NU}}_{L^*_2} = (0.037 \bullet \left(1.04x10^6\right)^{4/5}) \bullet 0.7^{1/3} = 2139$$

$$\bar{h}_{L_2^*} = \frac{\overline{\mathrm{NU}}_{L_2^*} \bullet k_{fluid}}{L(\text{entire length})} = \frac{2139\left(28.7x10^{-3}\,\frac{W}{m-K}\right)}{1m} = 61.38\frac{W}{m^2-K}$$

$$q_2 = \bar{h}_{L_2^*} \bullet A_s(T_s - T_\infty) = 61.38\frac{W}{m^2-K} \bullet 0.5mx1m \bullet (100-20)K$$

$= 2,455W \rightarrow$ TURBULENT HEAT TRANSFER

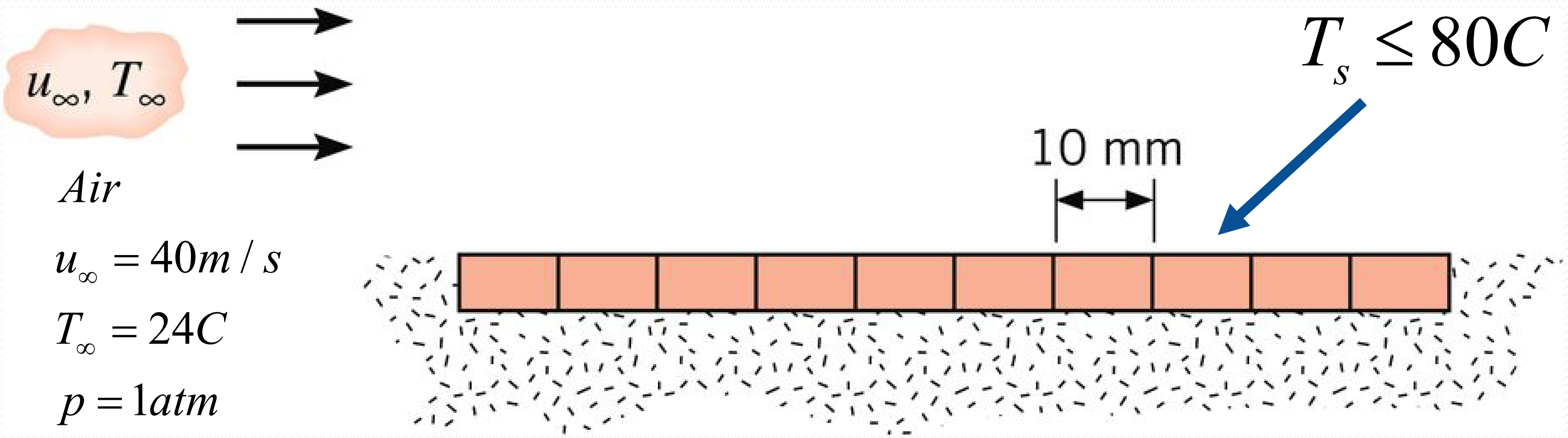
$=$ BETTER OPTION

THEORY COMPLETE!!!

EQUATIONS COMPLETE!!!

$$\overline{q} = \overline{h}(u_\infty, T_\infty, T_s, \Pr, \mu) A_s (T - T_\infty)$$

An array of 10 silicon chips of Length L = 10mm on a side, is insulated on one surface and cooled on th other by air in parallel at 24C at 40 m/s. When in use , the same electrical power is dissipated in each chip, maintaining a uniform heat flux. If the temperature of each chip may not eceed 80C, what is the maximum allowable power per chip? What is the a turbulence promotor is use to trip the boundary layer? What about orientation normal, rather than parallel to the flow?



Assumptions: Steady-State, Neglect Radiation, No Heat Loss via Insulation, Uniform Heat Flux

PROPERTIES

$$T_{film} = \frac{T_\infty + T_s}{2} = \frac{24+80}{2} = 52C$$

$$P = 1atm$$

$$\nu = 18.4x10^{-6}\frac{m^2}{s}, k = 0.0282\frac{W}{m-K}, \Pr = 0.703$$

ANALYSIS

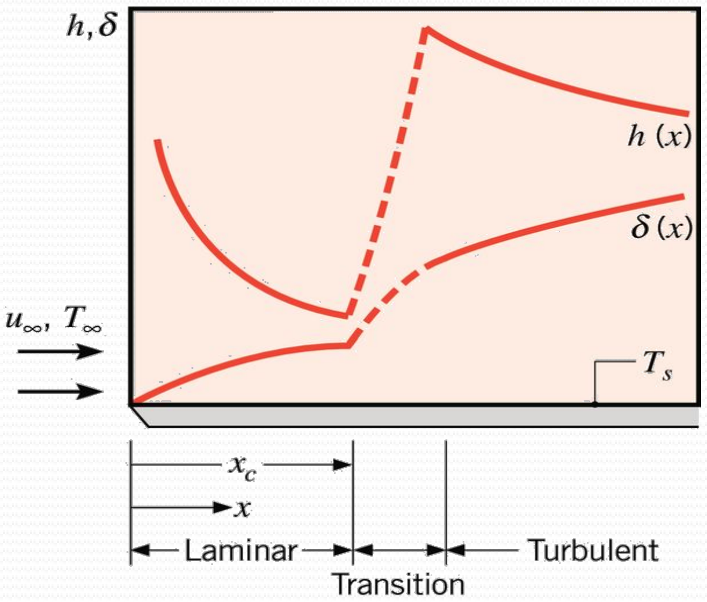
$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{40m/sx0.1m}{18.4x10^{-6}m^2/s} = 2.174x10^5$$

$\leq 500{,}000$

$\rightarrow$ Laminar over all chips

$\rightarrow$ minimum h_x exist over last chip

$\rightarrow$ Approximately the average coefficienct for Chip_{10} as the local coefficienct at x=95mm, $\bar{h}_{10} = h_{x=0.095m}$



Laminar Flow: Constant Heat Flux: Flat Plate

$$NU_x = \frac{h_x x}{k_{fluid}} = 0.453\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3},\ \mathrm{Pr} \geq 0.6$$

$$h_x(x=0.095) = \bar{h}_{10} = 0.453\frac{k}{x}\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Re}_x = \frac{u_\infty(x=0.095m)}{\nu} = 2.065x10^5$$

$$h_x(x=0.095) = \bar{h}_{10} = 0.453\frac{0.0282\frac{W}{m-k}}{0.095m}\left(2.065x10^5\right)^{1/2} 0.703^{1/3}$$

$$= 54.3\frac{W}{m^2-K}$$

$$q_{10} = \bar{h}_{10}A(T_s - T_\infty) = 54.3\frac{W}{m^2-K}x(0.01m)^2(80-24)C = 0.30W$$

Hence if all chips are to dissipate the same power and T_s is not to exceed 80C:

$q_{\max} = 0.30W$

TURBULENT BOUNDARY LAYER AT START

$$h_x(x = 0.095) = \bar{h}_{10} = 0.0308\frac{k}{x}\text{Re}_x^{4/5}\,\text{Pr}^{1/3} = 145\frac{W}{m^2 - K}$$

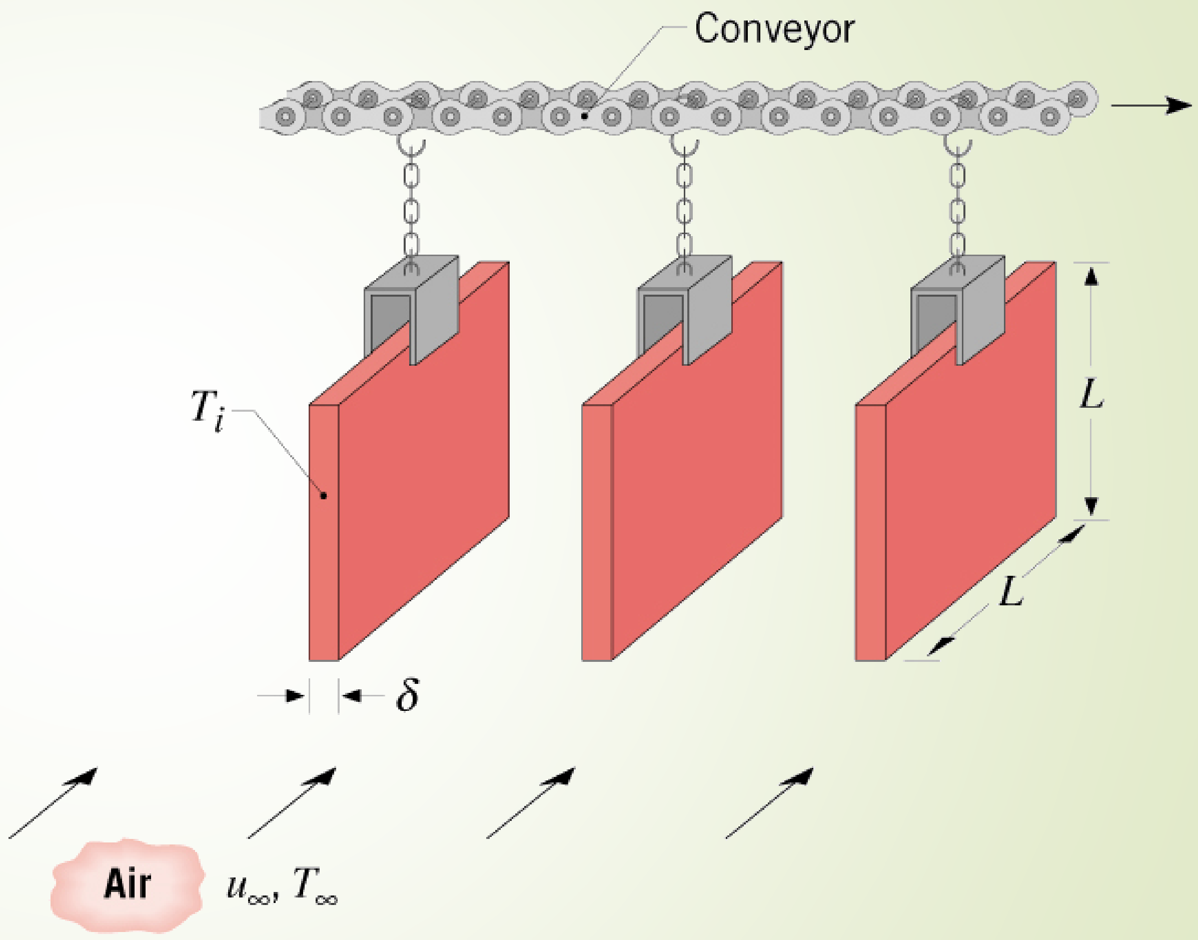
$$q_{10} = \bar{h}_{10}A(T_s - T_\infty) = 145\frac{W}{m^2 - K}x(0.01m)^2(80 - 24)C = 0.81W$$

$$q_{\max} = 0.81W$$

It is better to orient array normal to the air flow.

Since $\bar{h}_1 > \bar{h}_{10}$, more heat could be dissipated per chip, and the same heat could be dissipated from each chip.

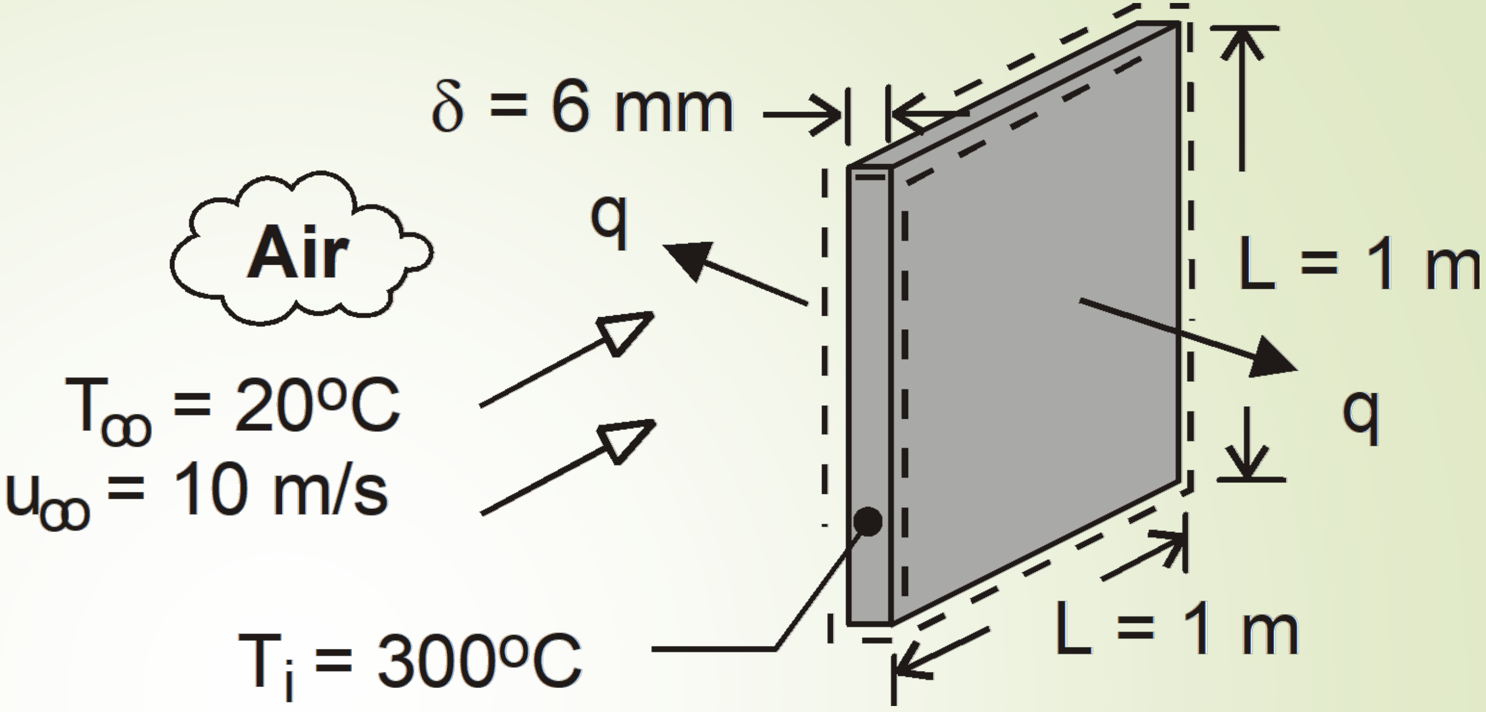
Convection Cooling of Steel Plate in Parallel Flow



Find:

1. **Initial rate of Heat Transfer**

2. **Initial rate of change of Plate Temperature.**



ROAD MAP

$$1)\ q_{initial} = \bar{h}A_s(T_i - T_\infty)$$

$$2)\ \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} = \rho_{solid}\forall c_{solid}\frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{\dot{E}_{out}}{\rho_{solid}\forall c_{solid}} = -\frac{\bar{h}A_s(T_i - T_\infty)}{\rho_{solid}\forall c_{solid}}$$

SOLUTION STEPS

ASSUMPTIONS

-No Radiation

-No Effect of Velocity on BL Growth

-Isothermal

-No Edge Heat Transfer

-Constant Properties

PROPERTIES

$$T_{film} = \frac{T_s + T_\infty}{2} = 433K$$

STEEL

$k = 492W/m-K, \rho = 7832kg/m^3, c = 549J/kg-K$

AIR

$\nu = 30.4x10^{-6} m^2/s, k = 0.0361W/m-K, \Pr = 0.688$

STEP ONE

$$1)\ RE_L = \frac{U_\infty L}{\frac{\mu}{\rho} = \nu} = \frac{10m/s \bullet 1m}{30.4x10^{-6}\frac{m^2}{s}} = 3.29x10^5 < 5x10^5 \text{ (critical transition)}$$

LAMINAR OVER ENTIRE LENGTH

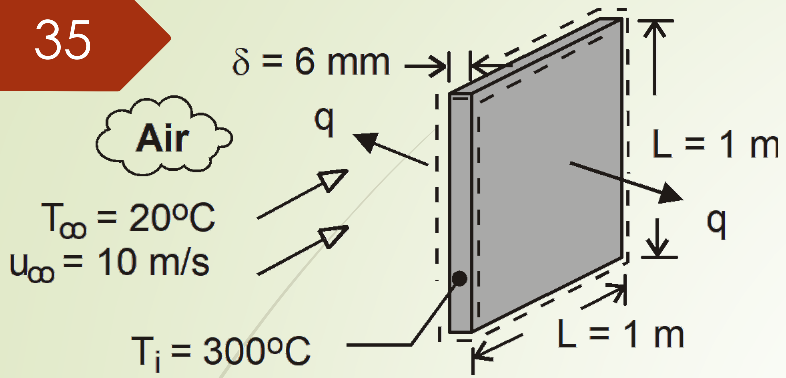
$$2)\ \overline{NU}_L = \frac{\bar{h}_L L}{k_{fluid}} = 0.664\,\text{Re}_L^{1/2}\Pr^{1/3}, 0.6 \le \Pr \le 50$$

$$\bar{h}_L = \frac{\overline{NU}_L \bullet k_{fluid}}{L} = \frac{0.664\,\text{Re}_L^{1/2}\Pr^{1/3} \bullet k_{fluid}}{L}$$

$$= \frac{0.664\left(3.29x10^5\right)^{1/2} 0.688^{1/3} \bullet 0.0361W/m-K}{1m}$$

$$= 12.1W/m^2-K$$

ROAD MAP



$$1)\, q_{initial} = \bar{h} A_s (T_i - T_\infty)$$

$$= (2 sides) 12.1 W / m^2 - K (1x1) m^2 \bullet (300 - 20) K$$

$$= 6780 W @ t = 0$$

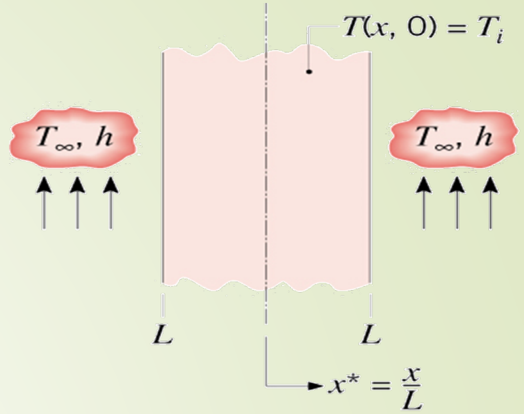
$$2)\, \cancel{\dot{E}_{in}} - \dot{E}_{out} + \cancel{\dot{E}_{gen}} = \dot{E}_{st} = \rho \forall c \frac{dT}{dt} \rightarrow \forall olume = A_s \bullet \Delta thick$$

$$\left.\frac{dT}{dt}\right|_{@t=0} = -\frac{\dot{E}_{out}}{\rho_{solid} \forall c_{solid}} = -\frac{\bar{h} A_s (T_i - T_\infty)}{\rho_{solid} \forall c_{solid}}$$

$$= \frac{-\left(12.1 W / m^2 - K\right) \bullet 2 \bullet (300 - 20) K}{7832 kg / m^3 \bullet 549 J / kg - K \bullet (\Delta thick = 0.006 m)}$$

$$= -0.26 \frac{{}^0 C}{s} @ t = 0$$

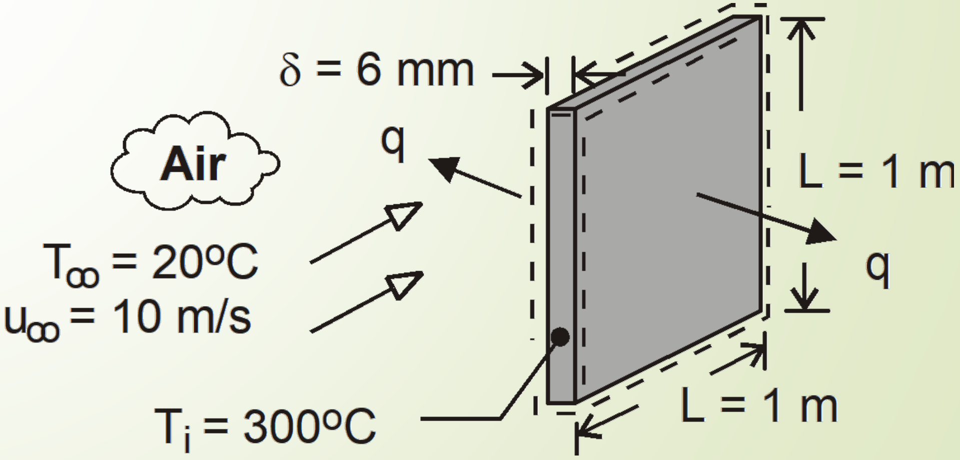
LUMPED or SPATIAL



$$Bi = \frac{UL_c}{k_{solid}} = \frac{\bar{h} \bullet \frac{\Delta thick}{2}}{49.2W/m-K} \rightarrow \Delta thick = 0.006m$$

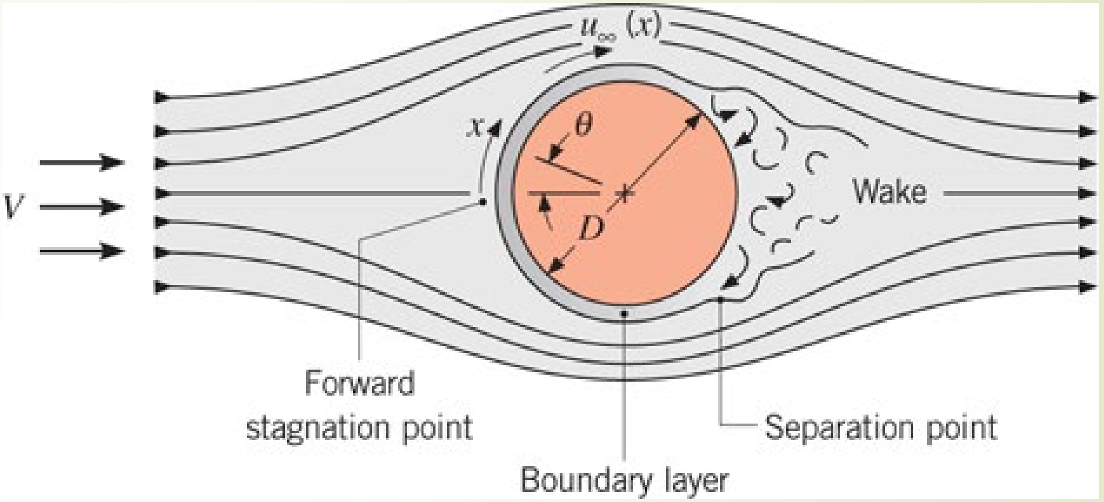
$$= 7.4x10^{-4} < 0.1$$

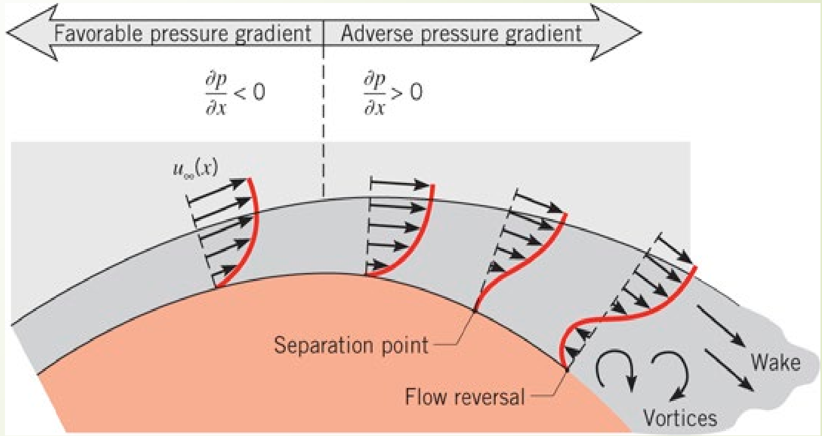
$= $ LUMPED



Cylinder in Cross Flow

- Another common external flow involves fluid motion normal to the axis of a circular cylinder as shown.
- The free stream is brought to rest at the forward stagnation point, with a rise in pressure.
- From this point the pressure decreases with increasing "x", and the boundary layer develops under the influence of a favorable pressure gradient (**dp/dx<0**).
- However, the pressure must reach a minimum, and toward the rear of the cylinder, further BL development occurs in the presence of an adverse pressure gradient (**dpdx >0**).
- At some point the boundary layer separates and resulting in reversed flow.





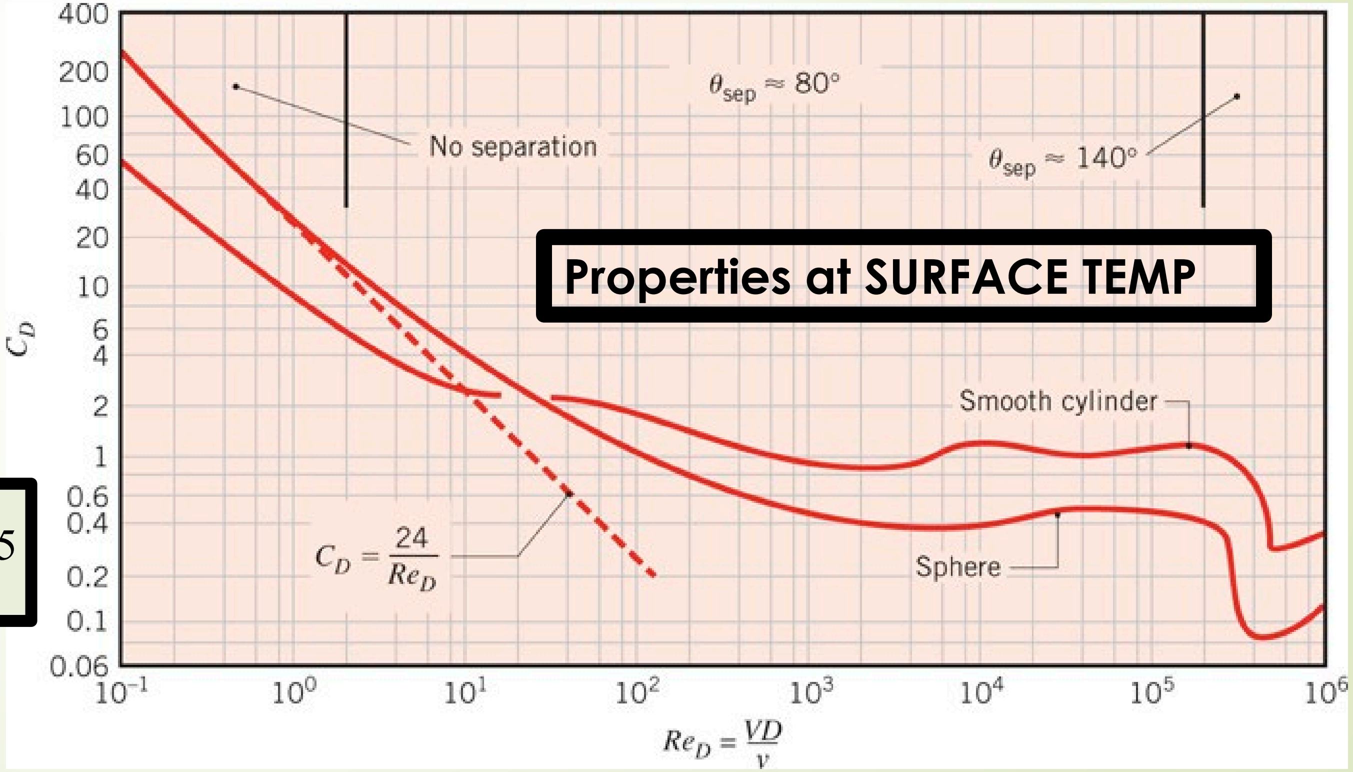
Relations--DRAG

- **Drag Coefficient**

$$\mathrm{Re}_D = \frac{\rho V D}{\mu_{fluid}}$$

$$C_D = \frac{F_D}{A_f \dfrac{\rho V^2}{2}}$$

$$C_D = \frac{24}{\mathrm{Re}_D} \rightarrow \text{CREEPING FLOWS} \rightarrow \mathrm{Re}_D \leq 0.5$$



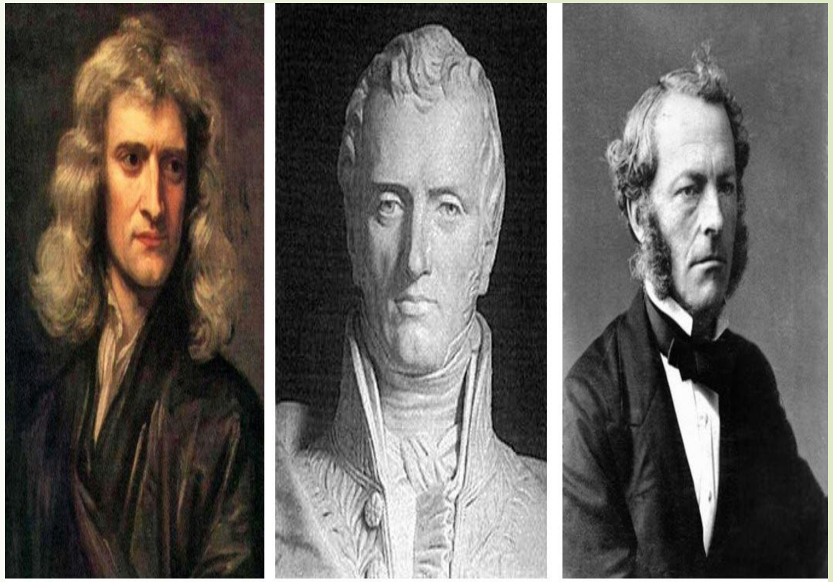
FLUID/SOLID CONVECTIVE HEAT TRANSFER

"REAL SOLUTIONS"

Navier Stokes

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} &= 0 & (1)\\
\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho u v)}{\partial y} + \frac{\partial (\rho u w)}{\partial z} &= -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) & (2)\\
\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho u v)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial (\rho v w)}{\partial z} &= -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) & (3)\\
\frac{\partial \rho w}{\partial t} + \frac{\partial (\rho u w)}{\partial x} + \frac{\partial (\rho v w)}{\partial y} + \frac{\partial (\rho w^2)}{\partial z} &= -\frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) & (4)\\
\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho u E)}{\partial x} + \frac{\partial (\rho v E)}{\partial y} + \frac{\partial (\rho w E)}{\partial z} &= -\frac{\partial p u}{\partial x} - \frac{\partial p v}{\partial y} - \frac{\partial p w}{\partial z} + S & (5)
\end{aligned}
$$

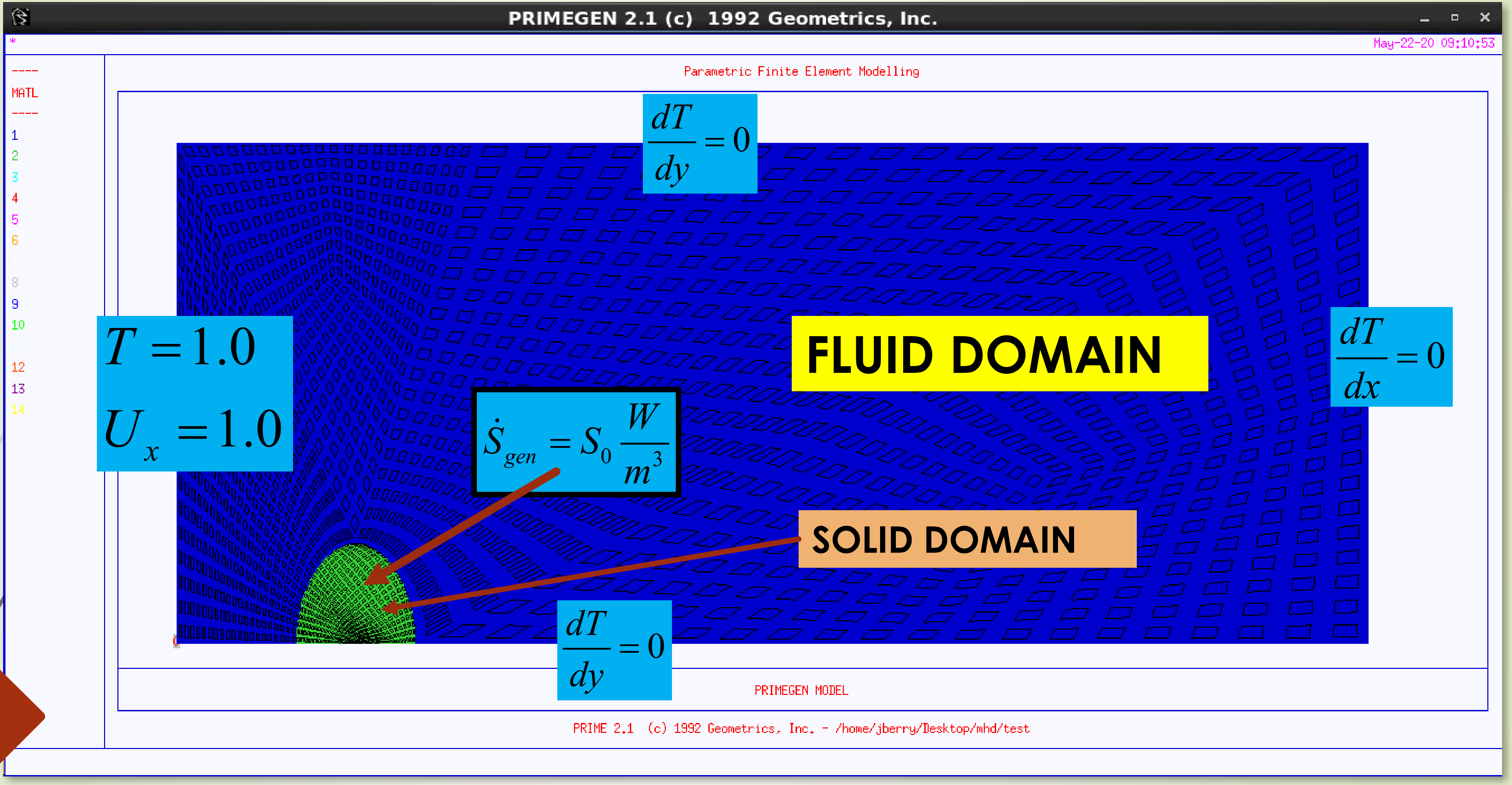
where ρ is the air density, u, v, w are the components of the air's velocity, E is measure of the air's internal energy (which allows us to compute its temperature) and p is the air pressure.



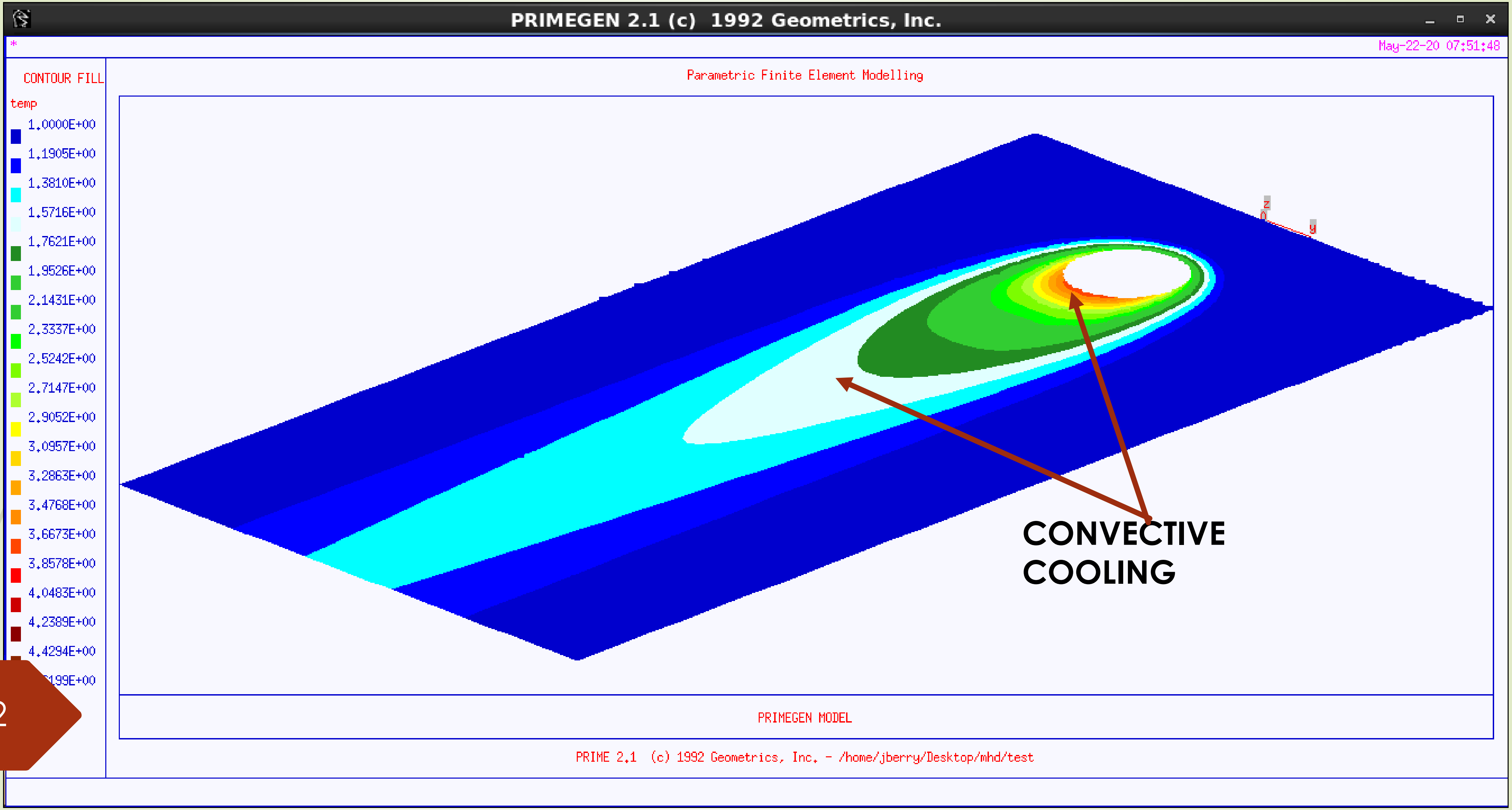
Isaac Newton 1642-1727 | Claude-Louis Navier 1785-1836 | Sir George Stokes 1819-1903

NO SOLUTION POSSIBLE FOR 50+ YEARS

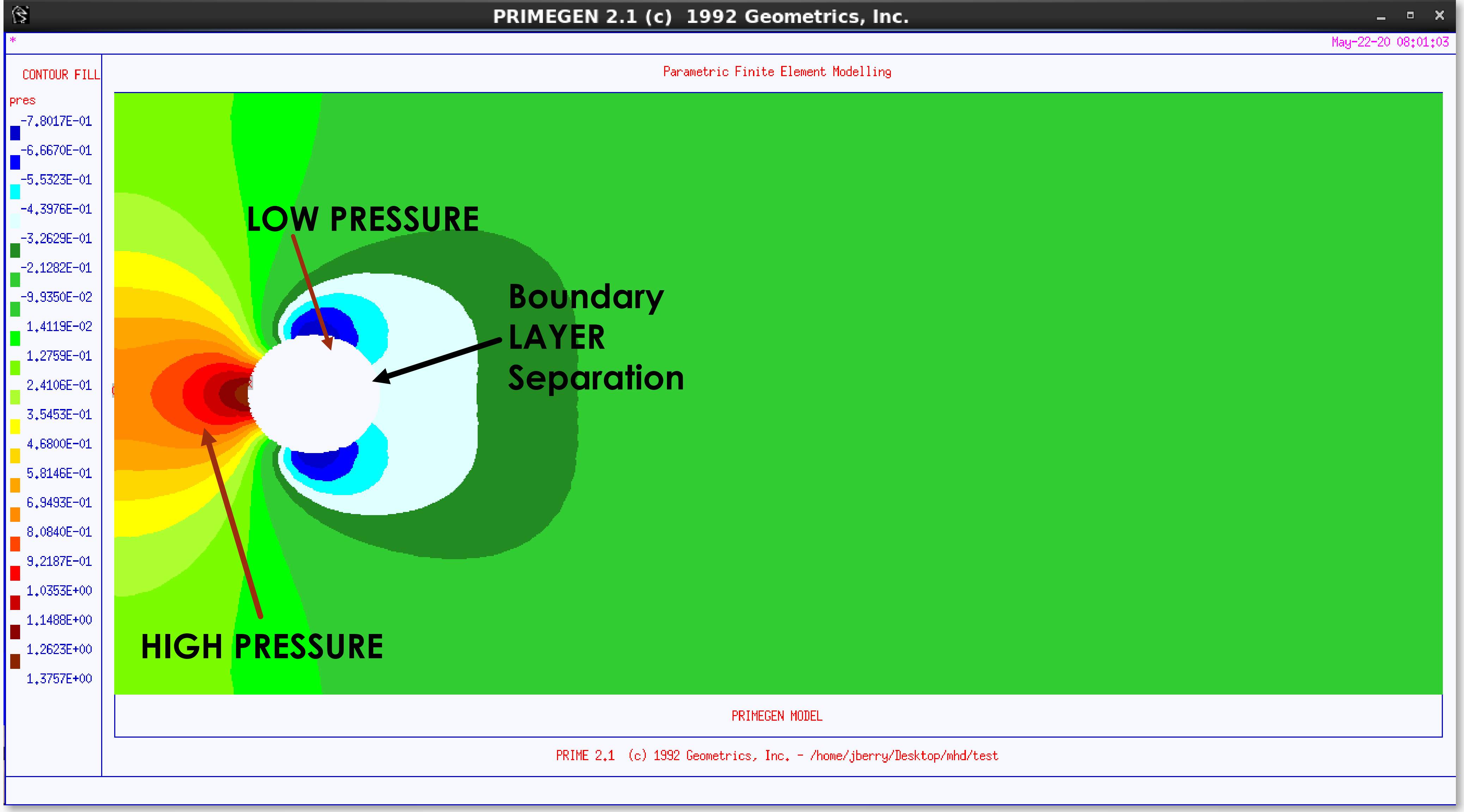
Conservation of Mass, Momentum, and Energy – Navier Stokes Equations



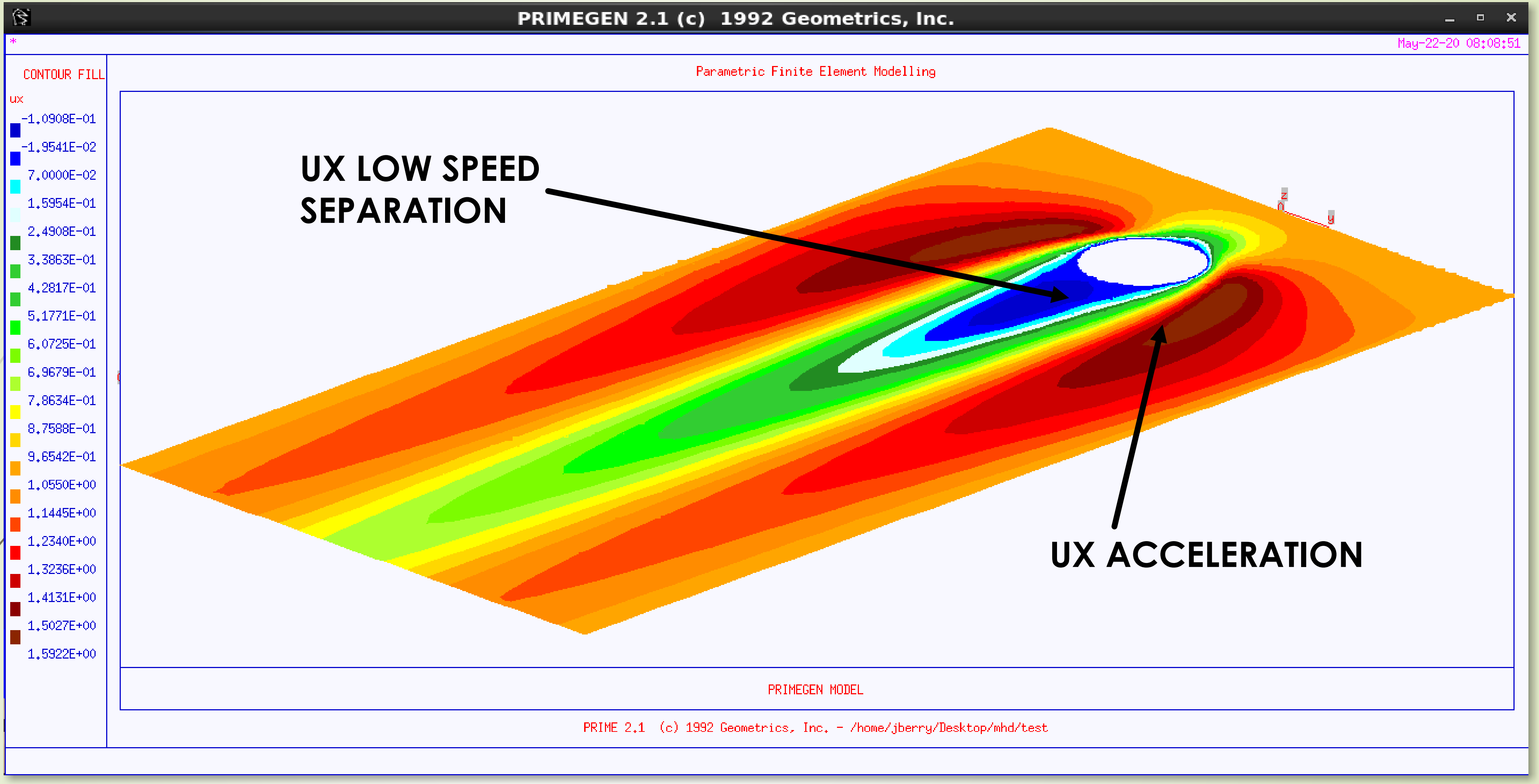
2D Flow: Finite Element Mesh



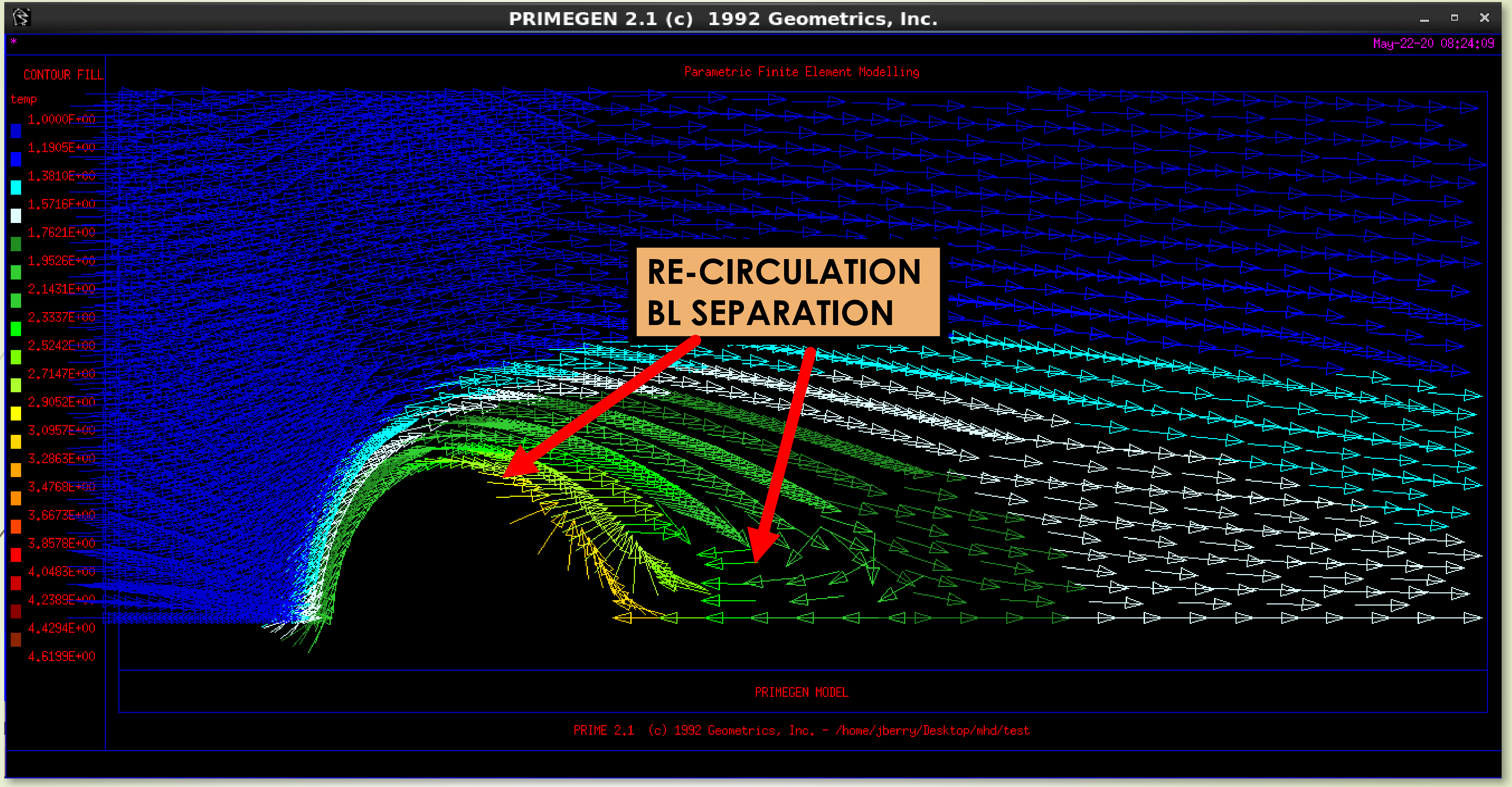
2D Flow Sphere: TEMPERATURE



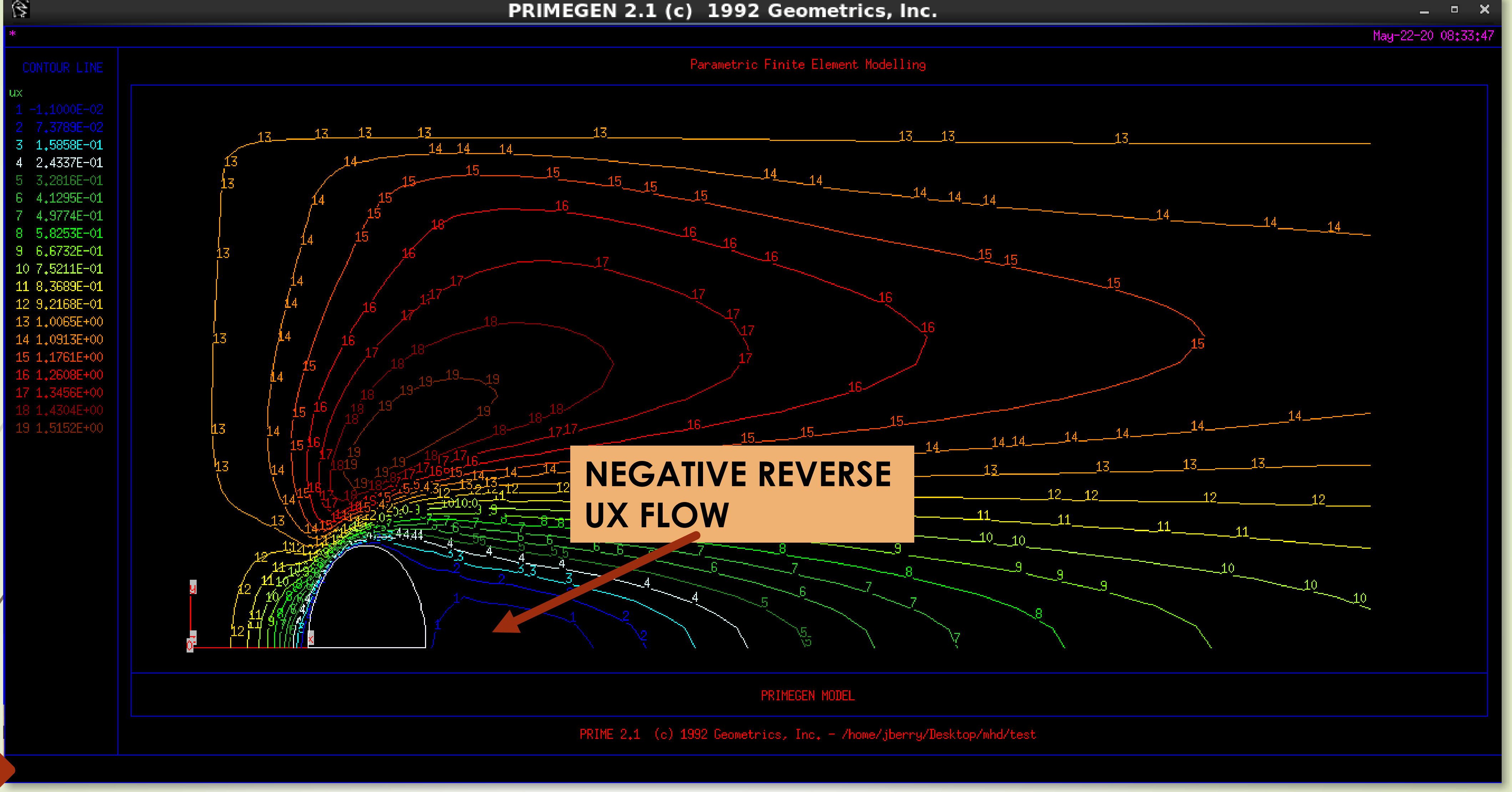
2D Flow Sphere: PRESSURE



2D Flow Sphere: Re=150 VELOCITY UX



2D Flow: Vectors w/Temperature



2D Flow: UX Contour Line

Relations—Cylinder Heat Transfer

- Heat Transfer

$$\overline{NU}_D = \frac{\bar{h}_D D}{k_{fluid}} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Re}_D = \frac{\rho \bar{V} D}{\mu}$$

$$\mathrm{Pr} \geq 0.7$$

TABLE 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

PROPERTIES @ T_{FILM}

MORE ACCURATE

$\mathrm{Re_D Pr} \geq 0.2$

$$\overline{NU}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5} = \frac{\bar{h}_D D}{k_{fluid}}$$

SPHERE in Cross Flow

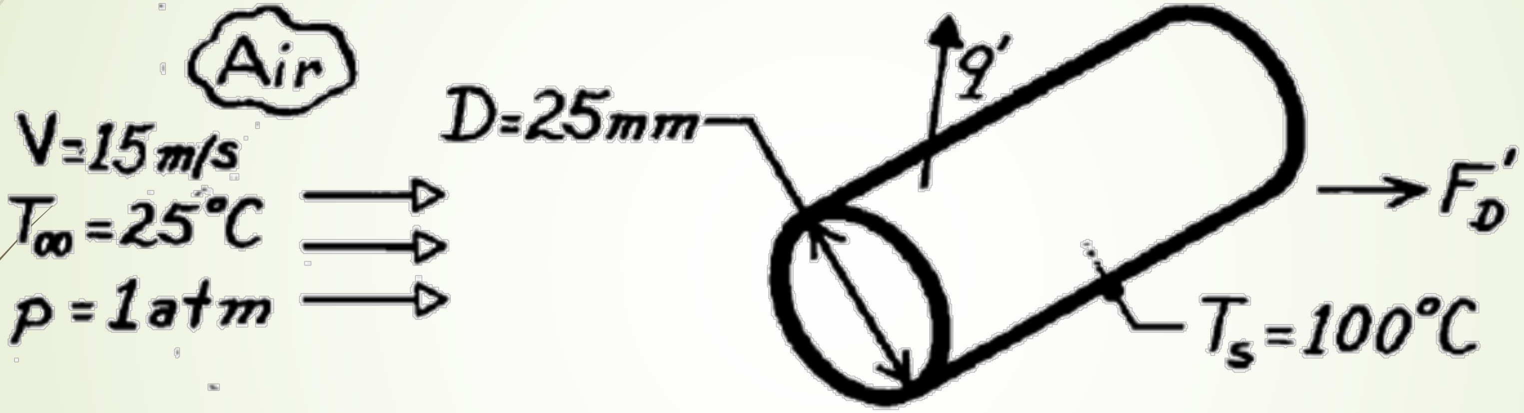
$$\overline{NU}_D = \frac{\bar{h}_D D}{k_{fluid}} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left[\frac{\mu(T_\infty)}{\mu(T_s)}\right]^{1/4}$$

$$\mathrm{Re}_D = \frac{\rho \bar{V} D}{\mu}$$

All Other Properties Evaluated at T_∞

Cylinder in Cross Flow. Find Drag Force & Heat Transfer per Unit Length

SCHEMATIC:



Film Temperature and Properties

$$T_f = \frac{T_s + T_\infty}{2} = \frac{100+25}{2} = 62.5C = 335K$$

Table A.4--AIR

$$\nu = \frac{\mu}{\rho} = 19.31x10^{-6}\frac{m^2}{s}, \rho = 1.048\frac{kg}{m^3}, k = 0.0288\frac{W}{m-K}, \Pr = 0.702$$

DRAG FORCE

$$F_{DRAG} = C_D A_{Frontal\ Area} \frac{\rho V^2}{2}$$

$$A_{Frontal\ Area} = \text{Area Projected Normal To The Free Steam Velocity}$$

$$= D \bullet L$$

$$F'_{Drag} = \frac{F_{DRAG}}{L} = C_D D \frac{\rho V^2}{2}$$

$$Re_D = \text{Reynolds \#} = \frac{VD}{\nu} = \frac{15m/s \bullet (0.025m)}{19.31x10^{-6} \frac{m^2}{s}} = 1.942x10^4$$

$$\text{FIG. 7.9} \rightarrow C_D \approx 1.1$$

$$F'_{Drag} = \frac{F_{DRAG}}{L} = C_D D \frac{\rho V^2}{2} = \frac{1.1(0.025m)}{2} 1.048 \frac{kg}{m^3} (15m/s)^2$$

$$= 3.24 \frac{\frac{kg-m}{s^2}}{m} = 3.24 \frac{N}{m}$$

Power to Overcome Drag

$$P = F_{Drag}[N] \bullet V[m/s] = J/s = W$$

$$= 3.24N/m \bullet 15m/s = 48.6W/m$$

HEAT TRANSFER

HILPERT's Relation

$$\overline{\text{Nu}}=\frac{\bar{h}D}{k_{FLUID}}=C\,\text{Re}_D^m\,\text{Pr}^{1/3}$$

$$\bar{h}=\frac{k_{FLUID}}{D}C\,\text{Re}_D^m\,\text{Pr}^{1/3}$$

$$TABLE\ 7.2\rightarrow \text{C=0.193,m=0.618}$$

$$\bar{h}=\frac{0.0288W/m-K}{0.025m}0.193\left(1.942x10^4\right)^{0.618}(0.702)^{1/3}$$

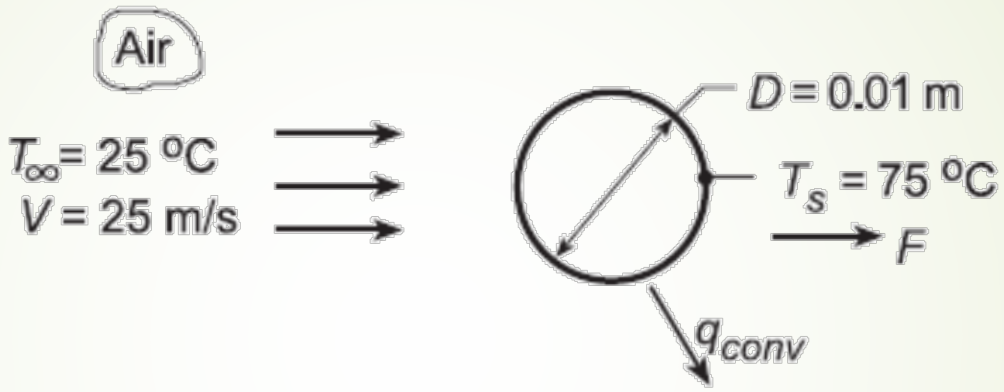
$$=88\frac{W}{m^2-K}$$

$$q'\left[\frac{W}{m}\right]=\frac{q}{L}=\frac{\bar{h}\left(\pi DL\right)(T_s-T_\infty)}{L}$$

$$=88\frac{W}{m^2-K}\pi D(100-25)=520W/m$$

Sphere in Cross Flow. Find Drag Force and Heat Rate.

SCHEMATIC:



Film Temperature and Properties

$T_\infty = 298K$

Table A.4--AIR

$$\nu = \frac{\mu}{\rho} = 15.71x10^{-6}\frac{m^2}{s}, \mu = 184x10^{-7}\,Pa - s, k = 0.0261\frac{W}{m-K}, \Pr = 0.71$$

$T_s = 323K$

$$\nu = \frac{\mu}{\rho} = 18.2x10^{-6}\frac{m^2}{s}, \mu = 208x10^{-7}\,Pa - s, \Pr = 0.71, \rho = 1.085\frac{kg}{m^3}$$

DRAG FORCE

Properties at T_s

$$Re_D = \frac{VD}{\nu_s} = \frac{25m/s(0.01)}{18.2x10^{-6}\dfrac{m^2}{s}} = 1.37x10^4$$

$$Fig.7.9 \rightarrow C_D \approx 0.4$$

$$F_D = C_D A_{frontal} \frac{\rho V^2}{2} = 0.4 \frac{\pi D^2}{4} \frac{1.085\dfrac{kg}{m^3}(25m/s)^2}{2} = 0.011N$$

HEAT RATE

$$\overline{NU}_D = \frac{\bar{h}D}{k_{fluid}} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left[\frac{\mu(T_\infty)}{\mu(T_s)}\right]^{1/4}$$

$$\mathrm{Re}_D = \frac{\rho \bar{V} D}{\mu}$$

All Other Properties Evaluated at T_∞

$$\mathrm{Re}_D = \frac{VD}{\nu_s} = \frac{25m/s(0.01)}{15.71x10^{-6}\,\frac{m^2}{s}} = 1.59x10^4$$

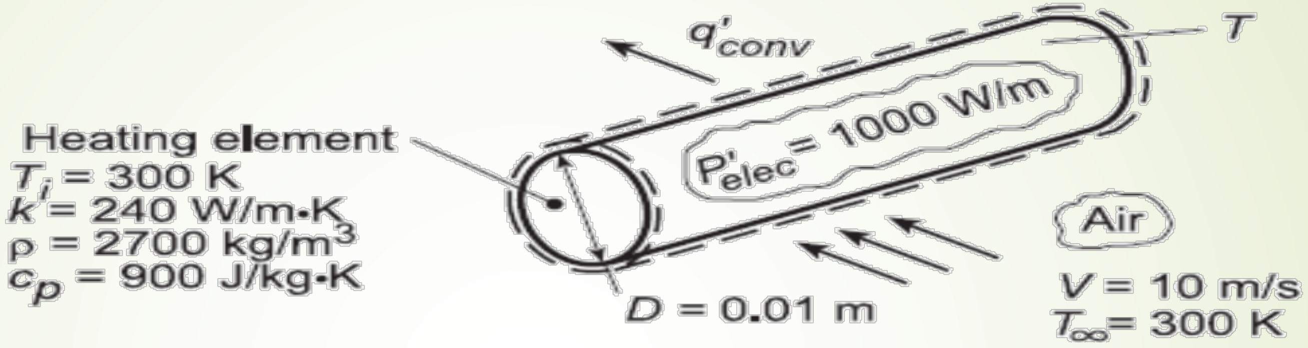
$$\overline{NU}_D = \frac{\bar{h}D}{k_{fluid}} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})0.71^{0.4}\left[\frac{184}{208}\right]^{1/4} = 76.7$$

$$\bar{h} = \overline{NU}_D \frac{k_{fluid}}{D} = 76.7\frac{0.0261W/m-K}{0.01m} = 200W/m^2-K$$

$$q = \bar{h}(A_{sphere} = 4\pi r^2)(T_s - T_\infty) = \bar{h}(4\pi r^2)(75-25) = 3.14W$$

For enclosed cylindrical heater below exposed to cross flow at initial temperature Ti, find steady state temperature, and time to come to 10C of steady state temperature.

SCHEMATIC:



Film Temperature and Properties

$$T_f = \frac{T_s + T_\infty}{2} \rightarrow \text{GUESS} \rightarrow \approx 450\text{K}$$

Table A.4--AIR

$$\nu = \frac{\mu}{\rho} = 32.39x10^{-6}\frac{m^2}{s}, \rho = 1.048\frac{kg}{m^3}, k = 0.0373\frac{W}{m-K}, \Pr = 0.686$$

NOTE:

Initiall all temperatures are at T_∞, and heater is energized.

Surface temperature MUST RISE over time and be > T_∞

How to CHECK T_{FILM} ?

1. Guess T_s
2. Compute $T_{FILM} = \frac{T_s + T_\infty}{2}$
3. Obtain Fluid Properties and compute Re
4. Solve Problem for "TRUE" T_s
5. Compute $T_{FILM_{TRUE}} = \frac{T_{s_{TRUE}} + T_\infty}{2}$
6. COMPUTE NEW Re_{TRUE}
7. TEST: $\frac{|\text{Re} - \text{Re}_{TRUE}|}{\text{Re}_{TRUE}} > 0.30 \rightarrow GOTO\ 4 \rightarrow$ REPEAT
8. $\frac{|\text{Re} - \text{Re}_{TRUE}|}{\text{Re}_{TRUE}} <= 0.30 \rightarrow$ CONVERGED

STEADY STATE TEMPERATURE

Overall Energy Balance

$$\dot{E}_{gen}[W] = \dot{E}_{out}[W]$$

$$P'_{elec}\left[\frac{W}{m}\right]\not{L} = \bar{h}(A_s = \pi D \not{L})(T_s - T_\infty)$$

$$T_s = \frac{P'_{elec}\left[\frac{W}{m}\right]}{\bar{h}(\pi D)} + T_\infty$$

$$Re_D = \frac{VD}{\nu} = \frac{10m/s(0.01m)}{32.39x10^{-6}\frac{m^2}{s}} = 3{,}087$$

HILPERT's Relation

$$\overline{Nu} = \frac{\bar{h}D}{k_{FLUID}} = C\,Re_D^m\,Pr^{1/3}$$

$$\bar{h} = \frac{k_{FLUID}}{D}C\,Re_D^m\,Pr^{1/3}$$

$$TABLE\ 7.2 \rightarrow C=0.683, m=0.466$$

$$\bar{h} = \frac{0.0373W/m-K}{0.01m}0.683(3{,}087)^{0.466}(0.686)^{1/3}$$

$$= 94.99\frac{W}{m^2-K}$$

$$q'\left[\frac{W}{m}\right] = \frac{q}{L} = \frac{\bar{h}(\pi D L)(T_s - T_\infty)}{L}$$

$$= 94.99\frac{W}{m^2-K}\pi D(?-300) = ?W/m$$

$$T_s = \frac{P'_{elec}\left[\frac{W}{m}\right]}{\bar{h}(\pi D)} + T_\infty$$

$$= \frac{1000W/m}{94.99\frac{W}{m^2-K}\pi 0.01m} + 300K$$

$$= 635K$$

CHECK Ts GUESS Validity

$$T_{s\,TRUE} = \frac{P'_{elec}\left[\frac{W}{m}\right]}{\bar{h}(\pi D)} + T_\infty$$

$$= \frac{1000W/m}{94.99\frac{W}{m^2-K}\pi 0.01m} + 300K$$

$$= 635K$$

How to CHECK T_{FILM} ?

1. Guess T_s
2. Compute $T_{FILM} = \frac{T_s + T_\infty}{2}$
3. Obtain Fluid Properties and compute Re
4. Solve Problem for "TRUE" T_s
5. Compute $T_{FILM_{TRUE}} = \frac{T_{s_{TRUE}} + T_\infty}{2}$
6. COMPUTE NEW Re_{TRUE}
7. TEST: $\frac{|Re - Re_{TRUE}|}{Re_{TRUE}} > 0.30 \rightarrow GOTO\ 4 \rightarrow$ REPEAT
8. $\frac{|Re - Re_{TRUE}|}{Re_{TRUE}} <= 0.30 \rightarrow$ CONVERGED

Time to Reach Ts-10K= 625K

TRANSIENT CONDUCTION--CYLINDER IN CROSSFLOW

$$\text{Bi}=\frac{UL_c}{k_{SOLID}}=\frac{\bar{h}\frac{r_0}{2}}{k_{SOLID}}=\frac{94.99\frac{W}{m^2-K}0.005m}{105.2\frac{W}{m-K}}=0.0045<0.1\rightarrow \text{LUMPED}$$

$$\ln\left[\frac{T(t)-T_\infty-b/a}{T_i-T_\infty-b/a}\right](-\tau)=t$$

$$a=\frac{1}{\tau};\tau\equiv\frac{\rho\forall c}{\text{h}A_{s,c}}=\text{R}_t\text{C}_t$$

$$\text{b}=\frac{q_s''A_s+\dot{E}_g}{\rho\forall c}$$

LUMPED ANALYSIS

$$a=\frac{1}{\tau};\tau\equiv\frac{\rho\forall c}{\bar{h}A_{s,c}}=\frac{\rho c\left[\frac{\pi D^2}{4}\not{L}\right]}{\bar{h}\pi D\not{L}}=$$

$$\tau=\frac{\rho c\left[\frac{\pi D^2}{4}\not{L}\right]}{\bar{h}\pi D\not{L}}=\frac{2700\frac{kg}{m^3}900\frac{J}{kg-K}0.01m}{4\bar{h}\frac{W}{m^2-K}}=64\text{s}$$

$$b=\frac{\not{q}''_s\not{A}_s[W]+\dot{E}_g[W]}{\rho\forall c}=\frac{P_{elec}\left[\frac{W}{m}\right]\not{L}}{\rho c\left[\frac{\pi D^2}{4}\not{L}\right]}=$$

$$b=\frac{1000W/m}{2700\frac{kg}{m^3}900\frac{J}{kg-K}\frac{\pi 0.01^2m^2}{4}}=5.24\frac{K}{s}$$

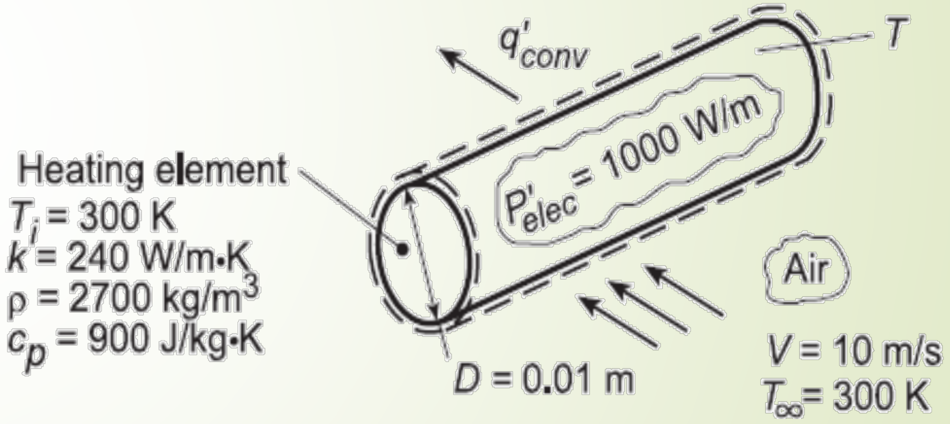
$$b/a=\frac{5.24\frac{K}{s}}{\frac{1}{64\text{s}}}=335.4K$$

$$\ln\left[\frac{T(t)-T_\infty-b/a}{T_i-T_\infty-b/a}\right](-\tau)=t$$

$$\ln\left[\frac{625-300-335.4}{300-300-335.4}\right](-64)=t$$

$$222.3s=t$$

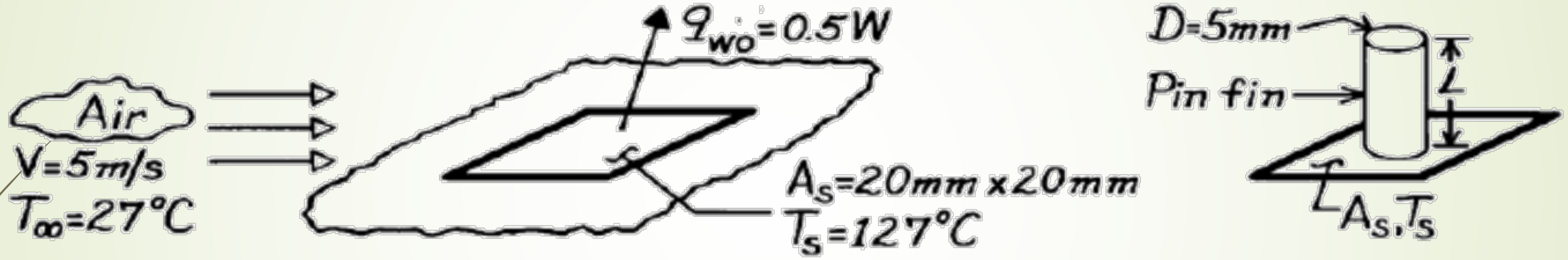
SCHEMATIC:



Find the CENTER Temperature of the Cylindrical Heater?

Air passes over area at 127C with 0.5W removed. To increase heat transfer a steel pin fin is affixed. Find max possible heat removal and fin effectiveness.

SCHEMATIC:



Assume: Steady State, uniform flow, pin in cross flow.

Properties:

Air: Table A-4

$$T_{film} = \frac{T_\infty + T_s}{2} = 350K$$

$$\nu = 20.92x10^{-6}\frac{m^2}{s}, k = 30.0x10^{-3}, \Pr = 0.700$$

Stainless Steel: Table A-1

$$k = 15.8\frac{W}{m-K}$$

Solution: MAX POSSIBLE HEAT REMOVAL

Maximum heat rate from fin ocuurs when fin is INFINITELY long.

$$q_f = M = \sqrt{\bar{h}PkA_c}\,\theta_b$$

$$Re_D = \frac{VD}{\nu} = \frac{5m/s(0.005m)}{20.92x10^{-6}\frac{m^2}{s}} = 1195$$

HILPERT CORRELATION, EQN. 744, TABLE 7.2

$$\overline{Nu} = \frac{\bar{h}D}{k_{FLUID}} = C\,Re_D^m\,Pr^{1/3}$$

$$\bar{h} = \frac{k_{FLUID}}{D}C\,Re_D^m\,Pr^{1/3}$$

$$TABLE\ 7.2 \rightarrow C=0.683, m=0.466$$

$$\bar{h} = \frac{0.030W/m-K}{0.005m}0.683(1195)^{0.466}(0.700)^{1/3}$$

$$= 98.9\frac{W}{m^2-K}$$

FIN VARIABLES

$$P=\pi D, A_c = \frac{\pi D^2}{4}, \theta_b = T_s - T_\infty$$

$$q_f = M = \sqrt{\bar{h}PkA_c}\,\theta_b$$

$$= \left(98.9\frac{W}{m^2-K}\bullet\pi(0.005)\bullet 15.8\frac{W}{m-K}\bullet\frac{\pi(0.005)^2}{4}\right)^{0.50}(127-25)K = 2.20W$$

$$\varepsilon = \frac{q_f}{q_{nofin}} = \frac{q_f}{\bar{h}A_{c,b}\theta_b} = \frac{2.20W}{98.9\frac{W}{m^2-K}\frac{\pi(0.005)^2}{4}(127-25)K}$$

$$= \frac{2.20W}{0.198W} = 11.1$$

Max Heat w/FIN ARRAY

$$q_{\text{TOTAL}}[W] = q_{fin} + q_{wall-exposed}$$

q_{fin} = Heat transfer for all fins

$q_{wall-exposed}$ = Heat transfer for exposed wall with NO Fins

$$q_{\text{TOTAL}}[W] = N\eta_{fin} q_{MAXIMUM} + hA_0\theta_b$$

A_0 = Wall exposed surface area

N = Number of Fins

$$\theta_b = T_b - T_\infty$$

$$\eta_{fin} = \text{Fin Efficiency} = \frac{q_f}{q_{max}}$$

$$q_{max} = h \bullet A_{\text{FIN TOTAL}} \bullet \theta_b$$

Number of fins per side with 0.25D spacing between fins

$$N = \frac{\frac{20}{1000}}{0.25D + D/2} \approx 6/side \rightarrow \text{Total Fins} = 6\text{x}6 = 36 \text{ fins}$$

$$A_0 = \frac{20x20}{1000x1000} - 36x\frac{\pi D^2}{4} = 1.11x10^{-3} m^2$$

$$L \approx L_\infty \text{(example 3.9)} = 2.65\sqrt{\frac{kA_c}{\bar{h}P}} = 37.4mm$$

$$q_{max} = \bar{h}A_{fin}\theta_b = 98.9\frac{W}{m^2-K}(\pi DL)(127-27) = 5.8W$$

$$\eta_{fin} = \frac{q_f}{q_{max}} = \frac{2.20W}{5.8W} = 0.38$$

$$q_{\text{TOTAL}}[W] = N\eta_{fin} q_{MAXIMUM} + \bar{h}A_0\theta_b$$

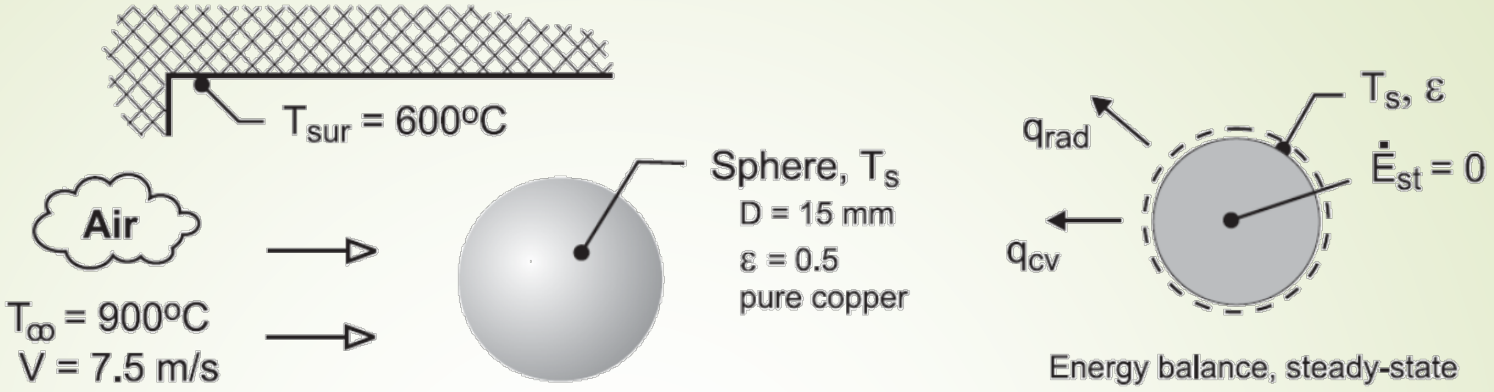
$$q_{\text{TOTAL}}[W] = (36)(0.38)(5.8W) + (98.9\frac{W}{m^2-K})(1.10686^{-3} m^2)(100K)$$

$$q_{\text{TOTAL}}[W] = 90W$$

UNDERSTAND YOUR CONNECTIONS

HOMEWORK

7.1,2,8,9,20,35,44, 47,49,54,74,76,79



ASSUMPTIONS: (1) Flow over a smooth sphere, (2) Sphere behaves as spacewise isothermal object; lumped capacitance method is valid, (3) Sphere is small object in large, isothermal surroundings, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air (T_∞ = 1173 K, 1 atm): $\mu = 4.665 \times 10^{-5}$ N·s/m^2, $\nu = 0.0001572$ m^2/s, k = 0.075 W/m·K, Pr = 0.728; Air (T_s = 1010 K, 1 atm): $\mu_s = 4.268 \times 10^{-5}$ N·s/m^2.

Sphere initially at 25C large furnace as shown.
1. Find Steady State Temperature.
2. Plot T(t) and q(t) for 0-300 sec ignoring radiation.

Transient Solution

STEADY STATE TEMPERATURE

Energy Balance

$$\dot{E}_{in} - \dot{E}_{out} - \dot{E}_{gen} = \dot{E}_{st}$$

$$-\bar{h}A_s(T_s - T_\infty) - \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = 0$$

m	m2		W/m2-K	K	K	W/m2-K^4
r0	As	emiss	h	Tf	Tsurr	sigma
0.0075	0.000706858	0.5	79.8	1173	873	5.67E-08

0.5

K		C
Ts	BALANCE	Ts
1000	1.35874414	727
1000.5	1.290431612	727.5
1001	1.222058906	728
1001.5	1.153625961	728.5
1002	1.085132718	729
1002.5	1.016579115	729.5
1003	0.947965094	730
1003.5	0.879290593	730.5
1004	0.810555552	731
1004.5	0.74175991	731.5
1005	0.672903609	732
1005.5	0.603986586	732.5
1006	0.535008782	733
1006.5	0.465970136	733.5
1007	0.396870588	734
1007.5	0.327710077	734.5
1008	0.258488543	735
1008.5	0.189205924	735.5
1009	0.119862161	736
1009.5	0.050457193	736.5
1010	-0.01900904	737

$TRANSIENT -$ NEED Bi#

$$\overline{NU}_D(\mathrm{Re}_D) = \frac{\bar{h}_D D}{k_{fluid}} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left[\frac{\mu(T_\infty)}{\mu(T_s)}\right]^{1/4}$$

m/s			W/m^2-K
U_∞	Re_D	$\overline{NU}_D$	$\bar{h}_D$
7.5	715.6	15.96	79.8

$$k_{copper} = 401 W/m-K, r_0 = 0.0075m$$

$$\mathrm{Bi} = \frac{UL_c}{k_{solid}} = \frac{\bar{h}_D \frac{r_0}{3}}{k_{solid}} = 4.9x10^{-4} < 0.1 \rightarrow LUMPED$$

$T(time) \rightarrow$ TEMPERATURE FUNCTION OF TIME *ONLY*

$$T_s \sim 737C \rightarrow \text{STEADY STATE TEMPERATURE}$$

LUMPED TRANSIENT NO RADIATION

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=\exp\left(-\frac{t}{\tau}\right)$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=\exp\left(-\frac{t}{\tau}\right)+\frac{b/a}{T_i-T_\infty}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]$$

$$a=\frac{1}{\tau};\tau\equiv\frac{\rho\forall c}{UA} \qquad b\equiv\left(q_s''A_s+E_{gen}\right)/\rho\forall c$$

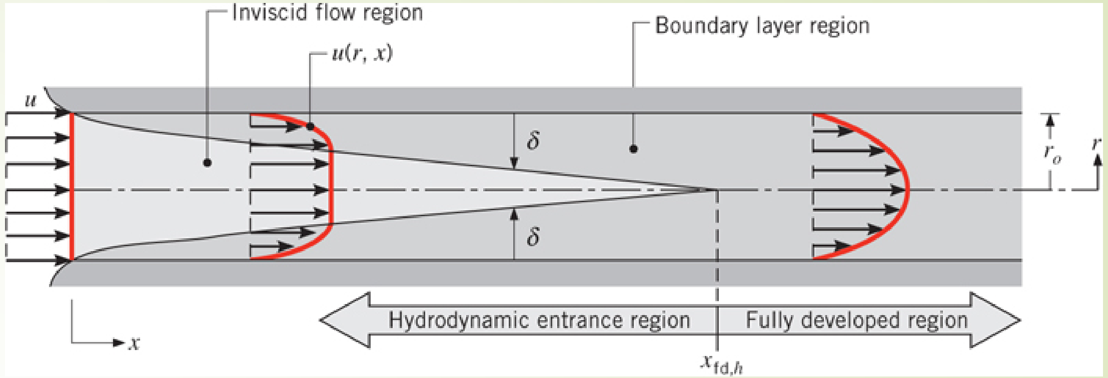
$$q(t)=\bar{h}A_s\left(T_s(t)-T_\infty\right)\rightarrow \text{HEAT TRANSFER RATE}$$

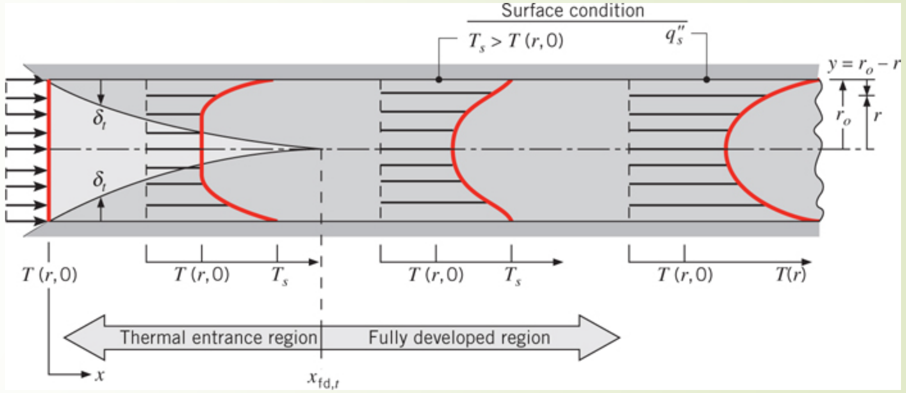
THOUGHT?

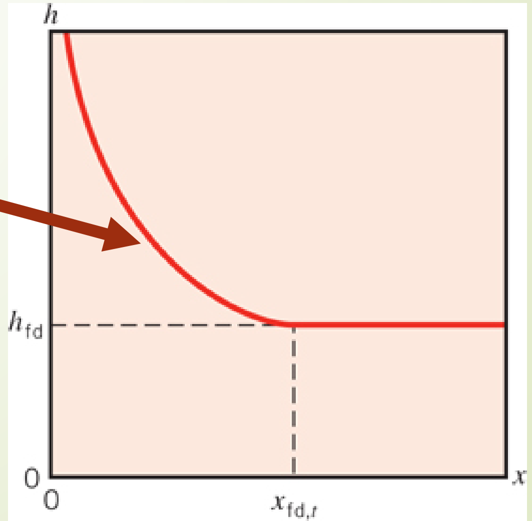
- How would analysis be updated if sphere was incased within 4mm of ANSI 316 steel?
- Hmm???

INTERNAL FLOW CHAPTER 8

- When fluid enter tube with uniform velocity, the fluid makes contact with the surface and viscous effects become important.
- A boundary layer develops with increasing X.
- This development occurs at the expense of a shrinking inviscid flow region and concludes with ***velocity/thermal*** boundary layer merger at the centerline.
- Following this merger, viscous effects extend over the entire cross section.
- The ***velocity/thermal*** profile NO LONGER changes with increasing X. This is called ***FULLY DEVELOPED FLOW (x/D>10).***
- ***The distance from the entry for full developed flow is denoted as X,fd, and the velocity is parabolic.***
- ***For TURBULENT flow, the profile is "FLATTER" due to turbulent mixing in the radial direction.***
- ***The heat transfer coefficient (h) decreases from a maximum at the inlet to a constant value for FULLY DEVELOPED FLOW.***







Hydrodynamic Considerations Fully Developed Flow

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu_{fluid}}, u_m \equiv \text{mean velocity}$$

$$\dot{m} \equiv \text{mass flow rate} = \rho u_m A_c$$

$$A_c \equiv \text{duct cross section area:} \frac{\pi D^2}{4}$$

Pressure Drop & Friction Coefficient

$$\Delta P = \mathrm{f}\, \frac{\rho u^2{}_m}{2} \frac{\Delta x}{D}, \mathrm{c}_f \equiv \frac{\tau_s}{\frac{\rho u^2{}_m}{2}} = \frac{f}{4}$$

Power

$$P = \frac{\dot{m}\Delta P}{\rho} = Q\Delta P$$

LAMINAR

$$0 \le \mathrm{Re}_D \le 2300$$

Friction Factor

$$\mathrm{f} = \frac{64}{\mathrm{Re}_D}$$

TURBULENT

$$\mathrm{Re}_D > 2300$$

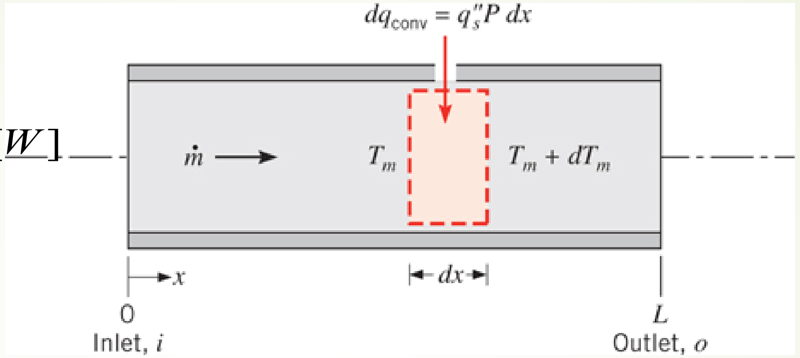
HALLAND

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right)$$

Thermal Considerations
Fully Developed Flow

Newton's Law of Cooling

$$q_s = hA_s(T_s - T_m)[W] = \dot{m}c_p(T_{m,out} - T_{m,in})[W]$$



Energy Balance

$$dq_{conv} = q_s''Pdx = \dot{m}c_p dT_m$$

Combining

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m)$$

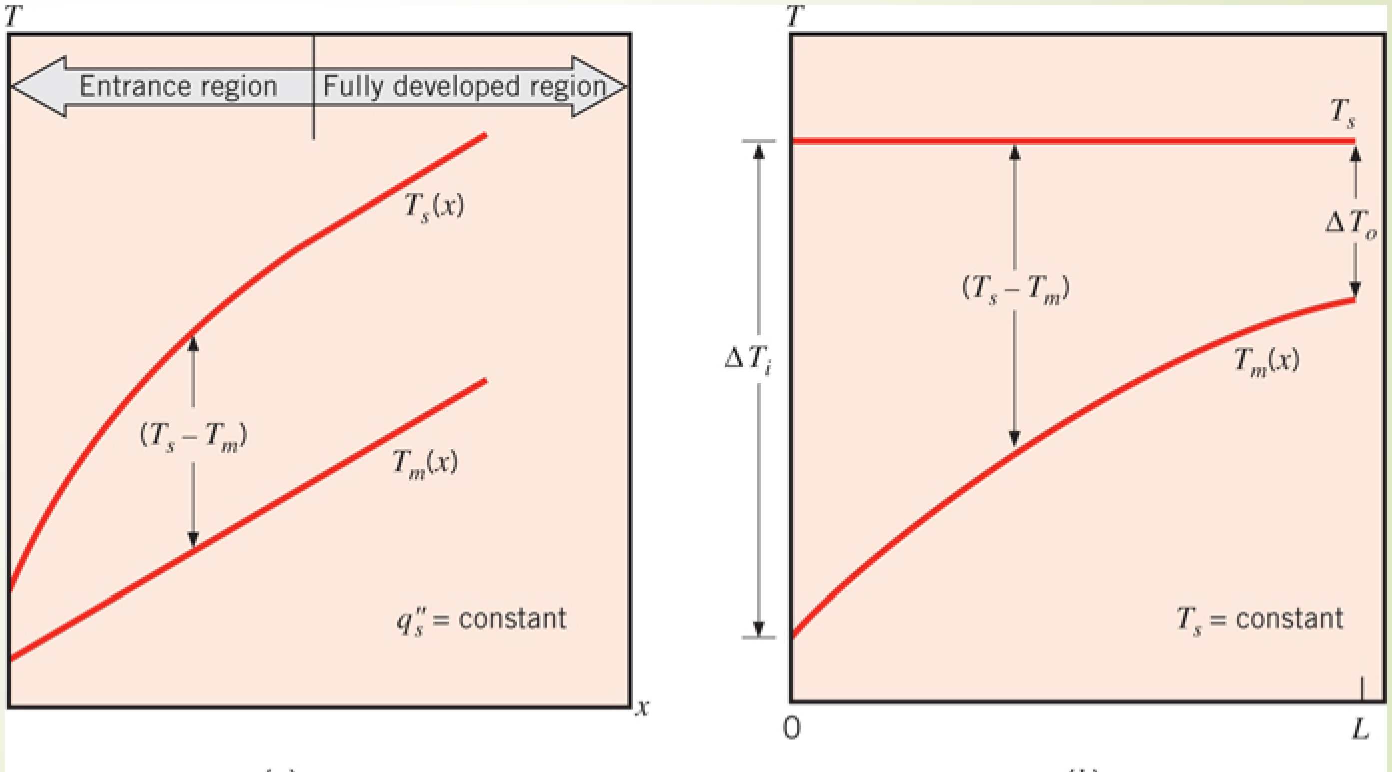
Constant Surface Heat Flux

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} \neq f(x) \rightarrow \text{Full Developed Flow}$$

$P \equiv$ PERIMETER$=\pi$D

INTEGRATING

$$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p} \bullet x \quad \rightarrow q_s'' = \text{constant}$$



Thermal Considerations
HEAT TRANSFER

CONSTANT SURFACE TEMPERATURE

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h \Delta T$$

Seperating Variables

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}c_p}\int_0^L h dx$$

$$(1): \ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left[\frac{1}{L}\int_0^L hdx\right] = -\frac{PL}{\dot{m}c_p}\overline{h_L} = -\frac{A_s}{\dot{m}c_p}\overline{h_L} \rightarrow T_s = \text{CONSTANT}$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left[-\frac{A_s\overline{h_L}}{\dot{m}c_p}\right] \rightarrow \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{R_t}\right] \rightarrow T_s = \text{CONSTANT}$$

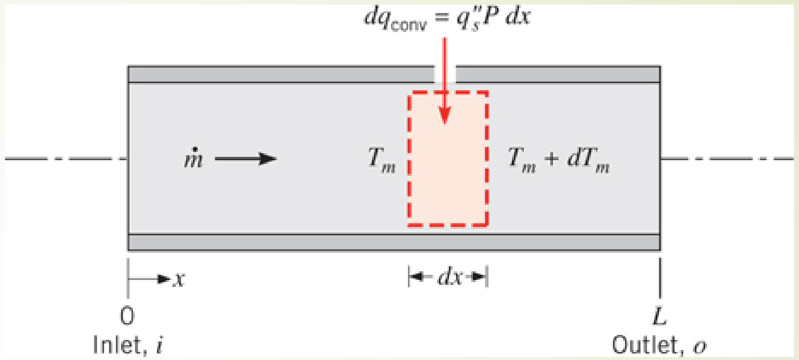
Heat Transfer

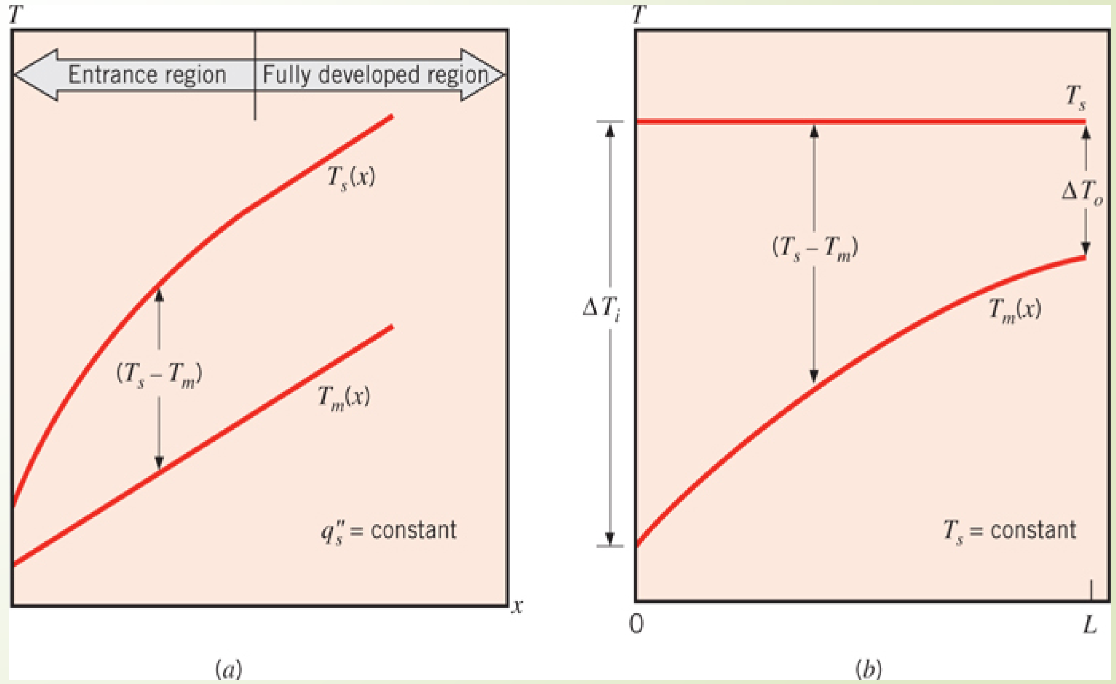
$$(2): q_{conv} = \dot{m}c_p[(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

BUT:

$$\dot{m}c_p = -\frac{A_s\overline{h_L}}{\ln\frac{\Delta T_o}{\Delta T_i}} \text{(From 1:)} \rightarrow SUB \text{ INTO } (2)$$

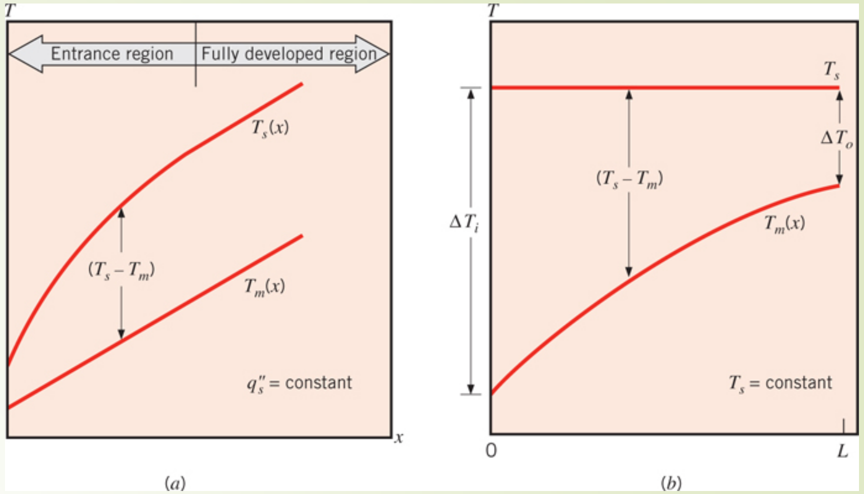
$$q_{conv} = \dot{m}c_p(\Delta T_i - \Delta T_o) = A_s\overline{h_L}\frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}} = A_s\overline{h_L}\Delta T_{LM} = \frac{\Delta T_{LM}}{R_t}$$





BIG PICTURE

Summary
Heat Transfer



CONSTANT HEAT FLUX

Newton's Law of Cooling

$$q_s = hA_s(T_s - T_m)[W] = \dot{m}c_p(T_{m,out} - T_{m,in})[W]$$

$$\mathrm{T}_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m}c_p} \bullet x$$

CONSTANT TEMPERATURE

$\Delta T_{LM} \equiv$ LOG MEAN TEMPERATURE DIFFERENCE

$$\mathrm{q}_{conv} = \dot{m}c_p(T_{m,out} - T_{m,in}) = \dot{m}c_p(\Delta T_i - \Delta T_o) = A_s \overline{h_L} \bullet \left[\frac{\Delta T_o - \Delta T_i}{\ln \frac{\Delta T_o}{\Delta T_i}} \right] = A_s \overline{h_L} [\Delta T_{LM}] = \frac{\Delta T_{LM}}{R_t}$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left[-\frac{P \bullet x}{\dot{m}c_p} \overline{h_x} \right] = \exp\left[-\frac{A_s \overline{h_x}}{\dot{m}c_p} \right] = \exp\left[-\frac{1}{\dot{m}c_p} \frac{1}{R_t} \right]$$

$$P = \pi D, PL = \mathrm{AREA} \rightarrow \mathrm{A}_s$$

NUSSELT Number
Fully Developed Flow

LAMINAR

$$0 \le \text{Re}_D \le 2300$$

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 4.36 \rightarrow \text{Constant Heat Flux}$$

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 3.66 \rightarrow \text{Constant Surface Temperature}$$

Evaluate Properties at T_{mean}

TURBULENT

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 0.023\,\text{Re}_D^{4/5}\,\text{Pr}^n \rightarrow \text{DITTUS-BOELTER}$$

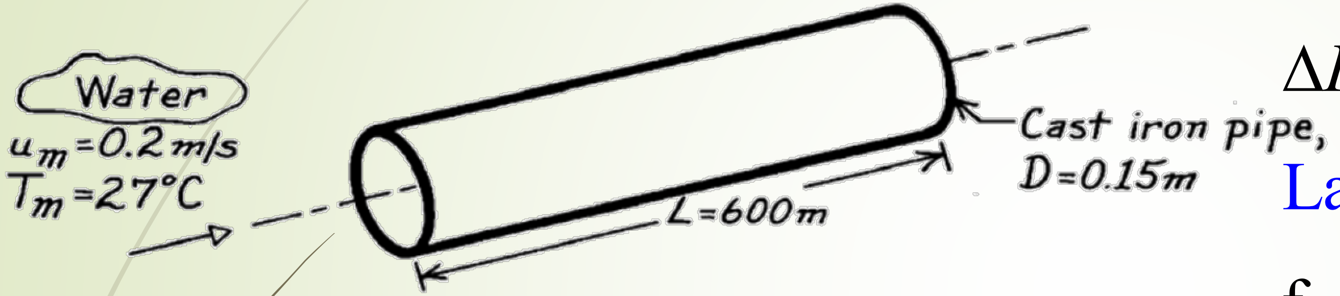
$n=0.4 \rightarrow$ Heating $(T_s > T_m)$

$n=0.3 \rightarrow$ Cooling $(T_s < T_m)$

Evaluate Properties at T_{mean}

Known temperature and mean velocity through duct. Find PRESSURE DROP.

SCHEMATIC:



$$T_{film} = T_m = 300K$$

$$\rho = 997\frac{kg}{m^3}, \mu = 855x10^{-6}\frac{N-s}{m^2}$$

Pressure Drop & Friction Coefficient

$$\Delta P = f\frac{\rho u^2{}_m}{2}\frac{\Delta x}{D},$$

Laminar

$$f = \frac{64}{Re_D}$$

Turbulent

$$Re_D > 2300$$

HALLAND

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$$

SOLUTION

REYNOLDS

$$Re_D = \frac{\rho u_m D}{\mu_f} = \frac{997 kg/m^3 \bullet 0.2 m/s \bullet 0.15 m}{855x10^{-6} N-s/m^2} = 3.5x10^4 \rightarrow TURBULENT$$

HALLAND

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$$

$$\frac{\varepsilon}{D} \rightarrow Fig.8.3 \rightarrow \varepsilon = 2.6x10^{-4}$$

$$\frac{\varepsilon}{D} = 1.73x10^{-3}$$

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{1.73x10^{-3}}{3.7}\right)^{1.11} + \frac{6.9}{3.5x10^4}\right) \rightarrow f = 0.027$$

$$\Delta P = f\frac{\rho u_m^2}{2}\frac{\Delta x}{D}$$

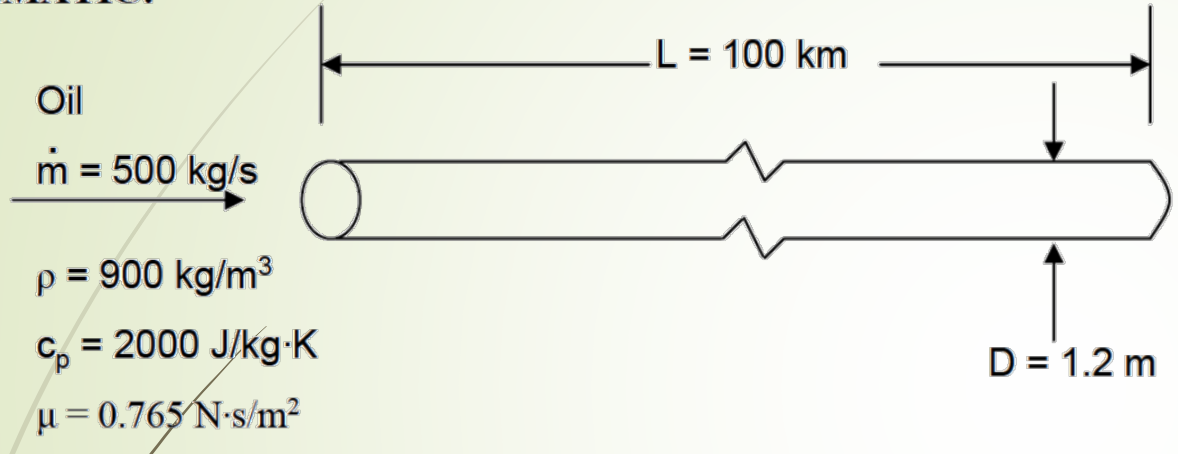
$$= 0.027\frac{997 kg/m^3 (0.02 m/s)^2}{2x0.15m}(600m)[\frac{kg}{m-s^2} = N/m^2]$$

$$= 2154 N/m^2 [Pa]$$

$$= 0.0215 \text{ Bar}$$

Find pressure drop, flow work, temperature rise.

SCHEMATIC:



Assume: steady, incompressible, no other than flow work

$\dot{m} = \rho A_c u_m \rightarrow$ mass flow rate(kg/s)

$$u_m = \frac{\dot{m}[kg/s]}{\rho\left[kg/m^3\right] A_c\left[m^2\right]} = 0.491 m/s$$

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu_f} = 693 \rightarrow LAMINAR \rightarrow f = \frac{64}{\mathrm{Re}_D} = 0.09235$$

$$f = \frac{64}{\mathrm{Re}_D} = 0.09235$$

$$\Delta P = \mathrm{f}\,\frac{\rho u_m^2}{2}\frac{\Delta x}{D}$$

$$= 0.09235 \frac{900 kg/m^3 (0.491 m/s)^2}{2} \frac{100{,}000 m}{1.2 m} = 8.4x10^5 N/m^2$$

$$= 0.84 MPa$$

FLOW WORK

$$\dot{W}_{flow} = \frac{\dot{m}\Delta P}{\rho} = \frac{\cancel{kg}/s \bullet N/m^2}{\cancel{kg}/m^3} = \frac{N-m}{s} = J/s = W$$

$$= \frac{500 kg/s \bullet 8.4x10^5 N/m^2}{900 kg/s} = 4.7x10^5 W = 0.47 MW$$

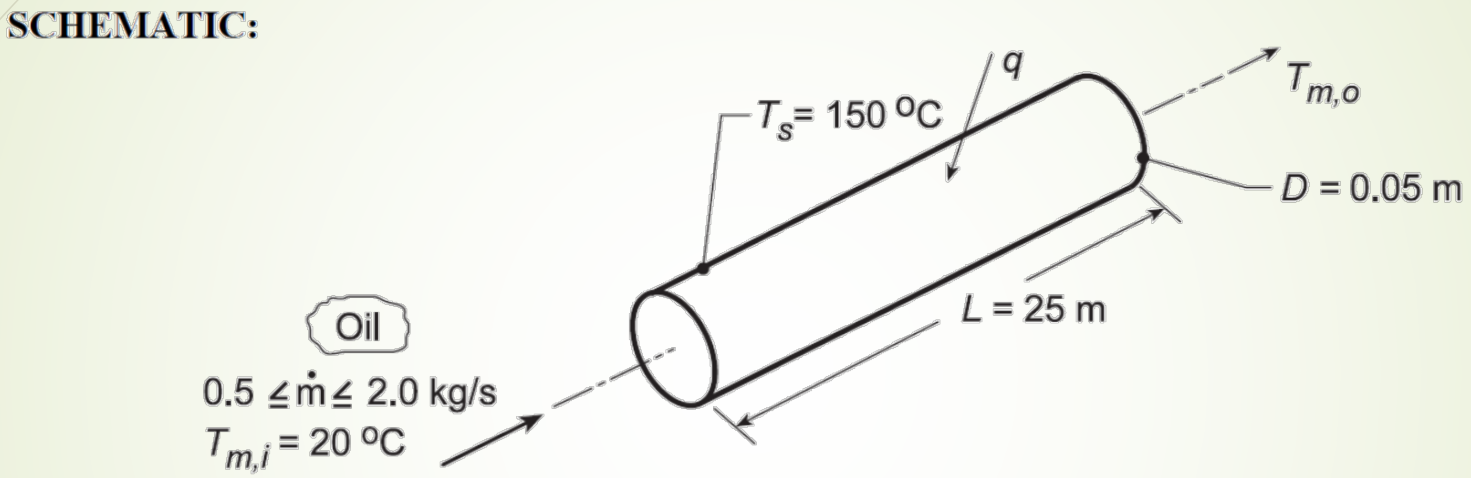
ΔT RISE

$$\dot{W}_{flow} = \frac{\dot{m}\Delta P}{\rho} = \dot{m} c_p \Delta T \quad (1^{st}\ \text{Law Thermodynamics})$$

$$\Delta T = \frac{\dot{W}_{flow}[W = J/s]}{\dot{m} c_p \left[kg/s \bullet \frac{J}{kg-K}\right]} = \frac{4.7x10^5 W}{500 kg/s \bullet 2000 J/kg-K} = 0.46 C$$

Despite the long pipe, high viscosity, and large DP, DT is quite small.

Find oil outlet temperature and total heat transfer rate.



$$T_{film} = T_m = \frac{T_{m,i} + T_{m,o}}{2} = \frac{20 + T_{m,o}(\approx 140C)}{2} = 80C = 353K$$

$$\rho = 852kg/m^3, \nu = 37.5x10^{-6} m^2/s, k = 138x10^{-3} W/m-K, \Pr = 490, cp = 2131J/kg-K$$

Assume FULLY DEVELOPED FLOW, Steady, Incompressible

SOLUTION
Constant Temperature DUCT

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left[-\frac{P \bullet x}{\dot{m}c_p}\overline{h_x}\right] = \exp\left[-\frac{A_s \overline{h_x}}{\dot{m}c_p}\right] = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{R_t}\right]$$

$$P = \pi D, PL = \text{AREA} \rightarrow \text{A}_s$$

$$\frac{T_s - T_m(x = L)}{T_s - T_{m,i}} = \exp\left[-\frac{P \bullet L}{\dot{m}c_p}\overline{h_x}\right]$$

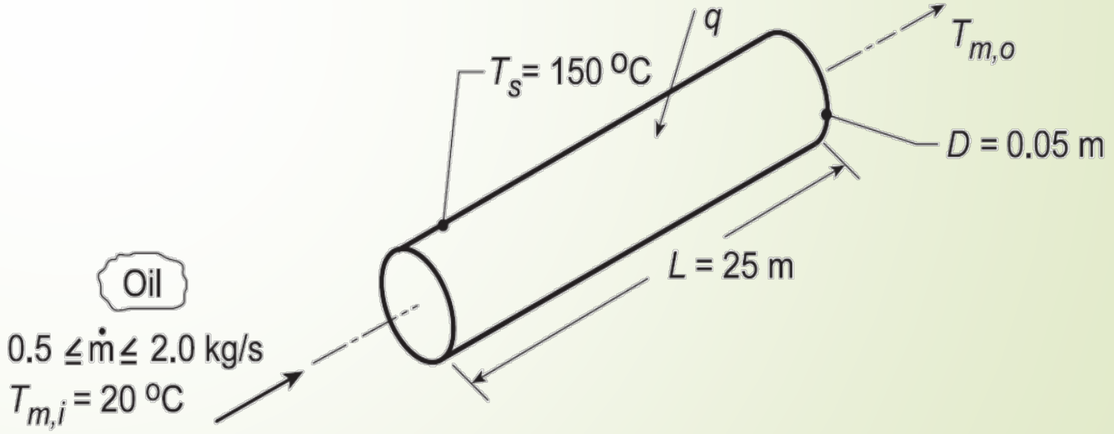
$$T_m(x = L) = T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left[-\frac{P \bullet L}{\dot{m}c_p}\overline{h_x}\right]$$

$$\dot{m} = \rho \text{A}_c u_m \rightarrow \text{ mass flow rate(kg/s)}$$

$$u_m = \frac{\dot{m}[kg/s]}{\rho\left[kg/m^3\right]\text{A}_c\left[m^2\right]} = 0.299 m/s$$

$$\text{Re}_D = \frac{u_m D}{\nu} = 398 < 2300 \rightarrow \text{Laminar}$$

SCHEMATIC:



EXIT TEMPERATURE

$$\mathrm{Re}_D = \frac{u_m D}{\nu} = 398 < 2300 \rightarrow \text{Laminar}$$

$$NU_D = \frac{\bar{h}D}{k_{fluid}} = 3.66 \text{ (constant wall temperature)}$$

$$\bar{h} = 3.66 \bullet \frac{k_{fluid}}{D} = 10.1 W/m^2 - K$$

$$T_m(x = L) = T_{m,o} = T_s - (T_s - T_{m,i})\exp\left[-\frac{P \bullet L}{\dot{m}c_p}\overline{h_x}\right]$$

$$T_m(x = L) == 150C - (150C - 20C)\exp\left[-\frac{\pi(0.05m)\bullet(25m)}{0.5\text{kg/s} \bullet 231\text{J/kg-K}}\overline{h_x}\right]$$

$$T_{m,o} = 58C$$

OVERALL ENERGY BALANCE

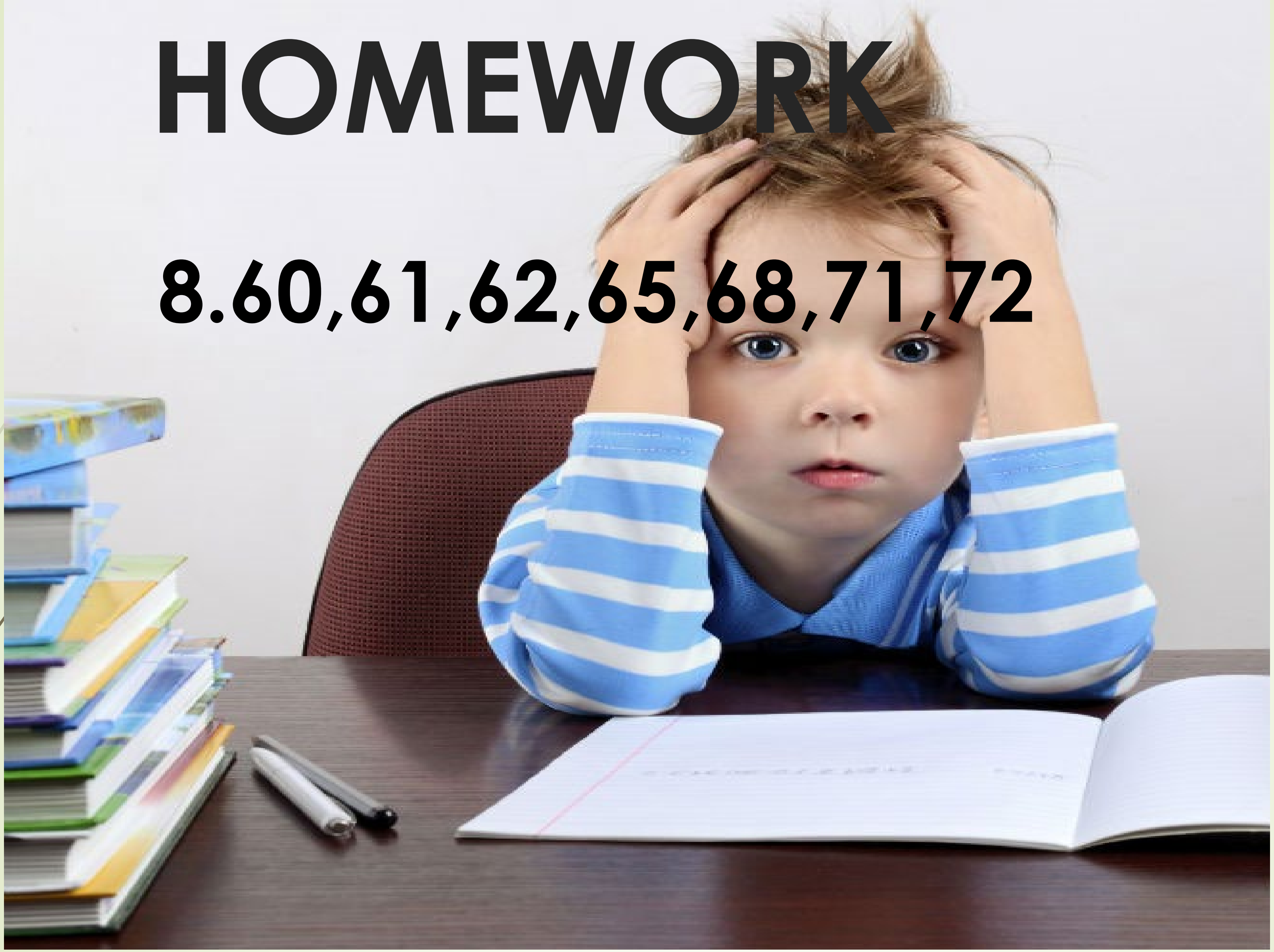
$$q = \dot{m}c_p(T_{m,o} - T_{m,i}) = 0.5kg/s \times 2131J/kg - K \times (58 - 20)$$

$$= 40,259W$$

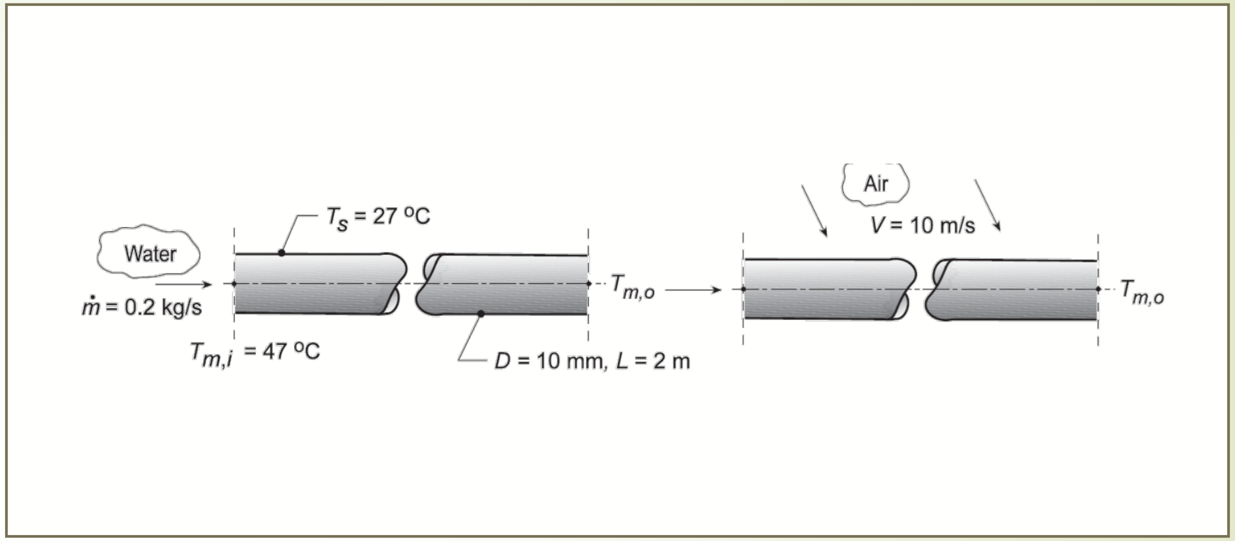
The value of $T_{m,o}$ has been grossly overestimated. The properties should be re-evaluated at $T_{film} = \frac{20+58}{2} = 39C = 312K$ and the calculations repeated.

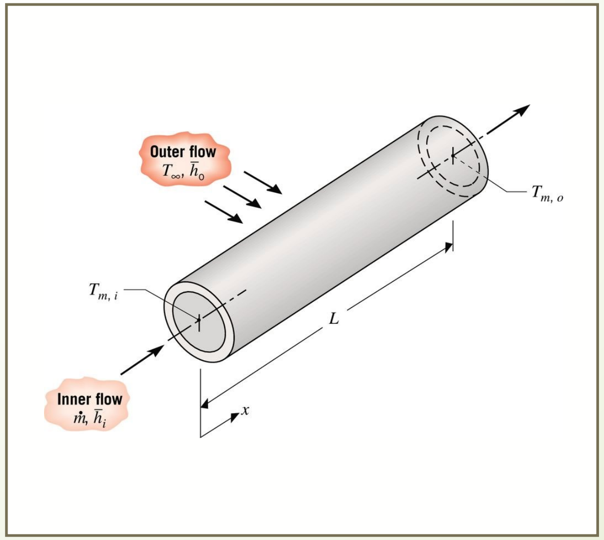
HOMEWORK

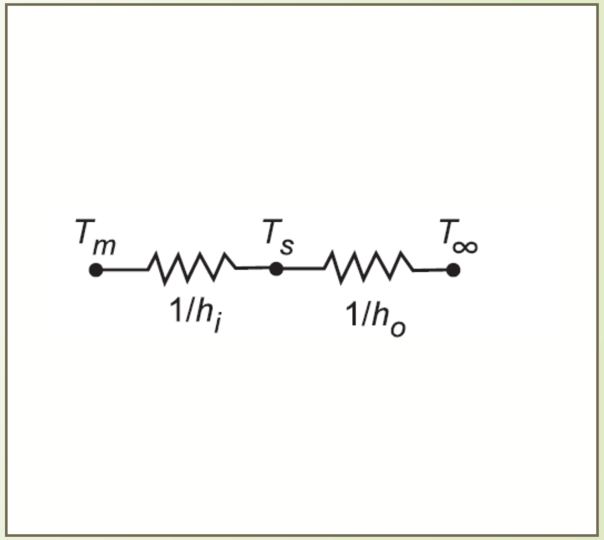
8.60,61,62,65,68,71,72

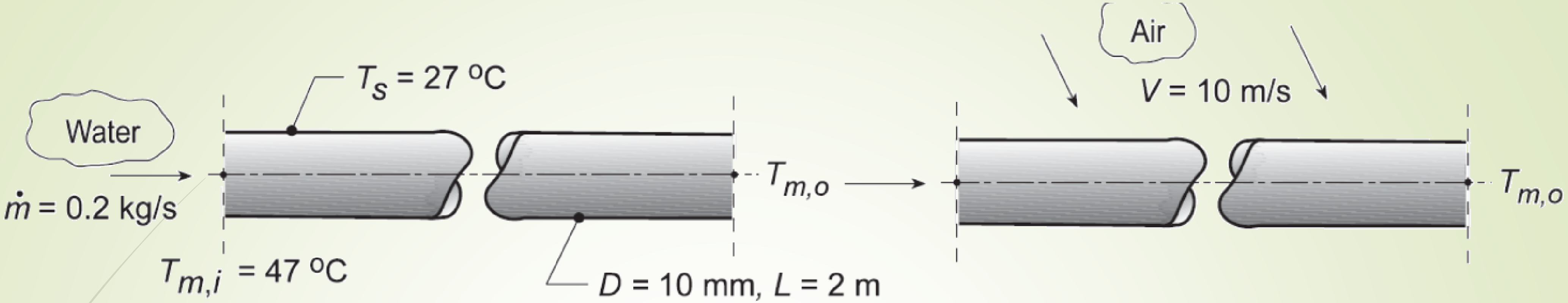


Convective Heating by Air at 10m/s and 100C in Cross Flow









Find "**mean**" water exit temperature for two cases for ***THIN WALL TUBE***.

A) Constant surface temperature, Ts=27C (**cooling**).

B) External Air at 10m/s and 100C (**heating**)

SOLUTION PROCESS ROAD MAP

1. Guess Tmean
2. Get Properties
3. Compute Actual Tmean
4. Compare to Initial Guess
5. Adjust Properties (if necessary) Based Upon New Tmean
6. Repeat Analysis if "BIG" Change in Properties ($\pm$30%)

PROPERTIES

$$T_s = 27C$$

$$T_{m_{water}} = \frac{T_{m,i} + T_{m,o}}{2} = \frac{47 + ?}{2}$$

$$T_{m,o}(guess) = \frac{27+47}{2} = 37C \rightarrow 27C \le T_{m,o} \le 47C \text{ (logical)}$$

$$T_{m_{water}} = \frac{47+37}{2} = 42C = 315K$$

MUST COMPUTE and CHECK

VALIDITY OF GUESS

FLUID PROPERTIES MAY NOT BE STRONG FUNCTION OF TEMPERATURE. GUESS MAY NOT BE VITAL AND MAY NOT CHANGE ANSWER. (Especially Water)

WATER

$$T_{m,o_{guess}} = 37C$$

$$\rho = 997 kg/m^3$$

$$c = 4179 J/kg-K$$

$$k = 0.613 W/m-K$$

$$\mu = 855x10^{-6} Pa-s$$

AIR

$$T_{m,o_{guess}} = \frac{47+100}{2} = 73.5C,$$

$$T_m = \frac{47+73.5}{2} = 60.25C(333.25K)$$

$$\nu = 20.92x10^{-6} m^2/s$$

$$k = 0.030 W/m-K$$

$$\Pr = 0.70$$

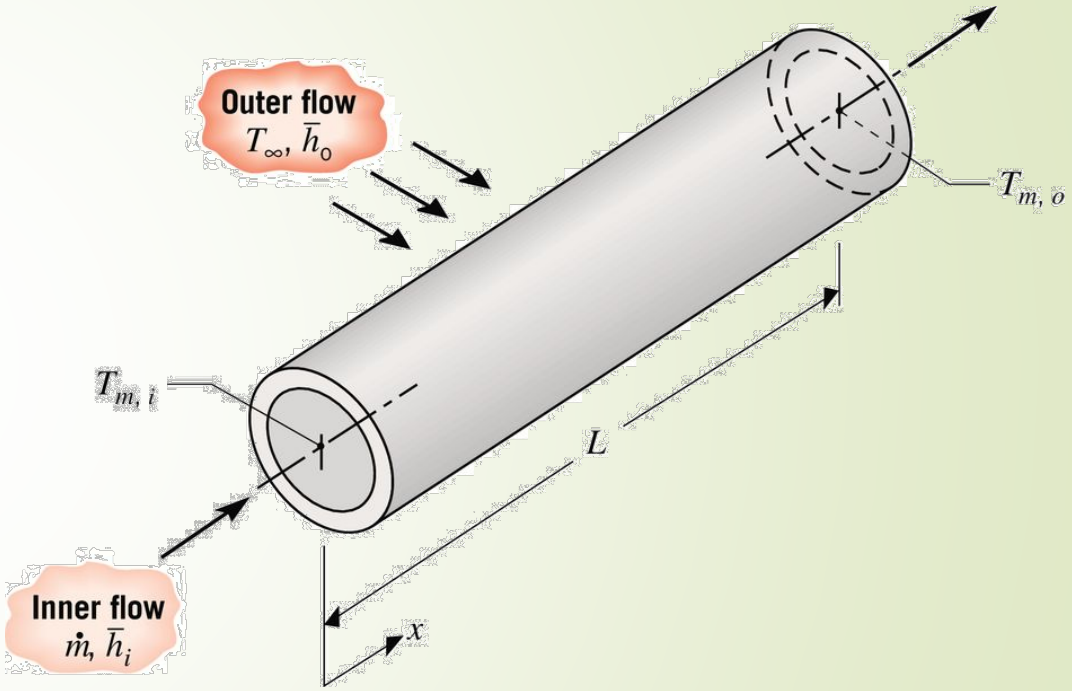
Internal Duct Flow w/Constant SURFACE Temperature

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left[-\frac{P \bullet x}{\dot{m}c_p}\overline{h_x}\right]$$

$$@\, x = L$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left[-\frac{P \bullet L}{\dot{m}c_p}\overline{h_i}\right];\, P = \pi D$$

$$T_{m,o} = T_s - \left\langle (T_s - T_{m,i})\exp\left[-\frac{P \bullet L}{\dot{m}c_p}\overline{h_i}\right]\right\rangle$$



Nusselet

$$Re_D = \frac{\rho V D}{\mu}$$

$$= \frac{4\dot{m}}{\pi D \mu}$$

$$= \frac{4 \bullet 0.2 kg/s}{\pi \bullet 0.10m \bullet 855x10^{-6} N-s/m^2}$$

$$= 29,783 > 2300$$

FULL DEVELOPED TURBULENT FLOW

TURBULENT

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 0.023 Re_D^{4/5} Pr^n \rightarrow \text{DITTUS-BOELTER}$$

$n=0.4 \rightarrow$ Heating ($T_s > T_m$)

$n=0.3 \rightarrow$ Cooling ($T_s < T_m$)

Evaluate Properties at T_{mean}

$$\bar{h} = k_{fluid} \frac{0.023 Re_D^{4/5} Pr^n}{D}; n = 0.3$$

$$= 0.613 W/m-K \frac{0.023 Re_D^{4/5} 5.83^{0.3}}{0.01m}$$

$$= 9,080 W/m^2 - K$$

SOLUTION, Tm,o

$$T_{m,o} = T_s - \left\langle (T_s - T_{m,i}) \exp\left[-\frac{P \bullet L}{\dot{m} c_p} \bar{h}_i \right] \right\rangle$$

$$\bar{h}_i = k_{fluid} \frac{0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n}{D} ; n = 0.3$$

$$= 0.613W/m - K \frac{0.023\,\mathrm{Re}_D^{4/5}\,5.83^{0.3}}{0.01m}$$

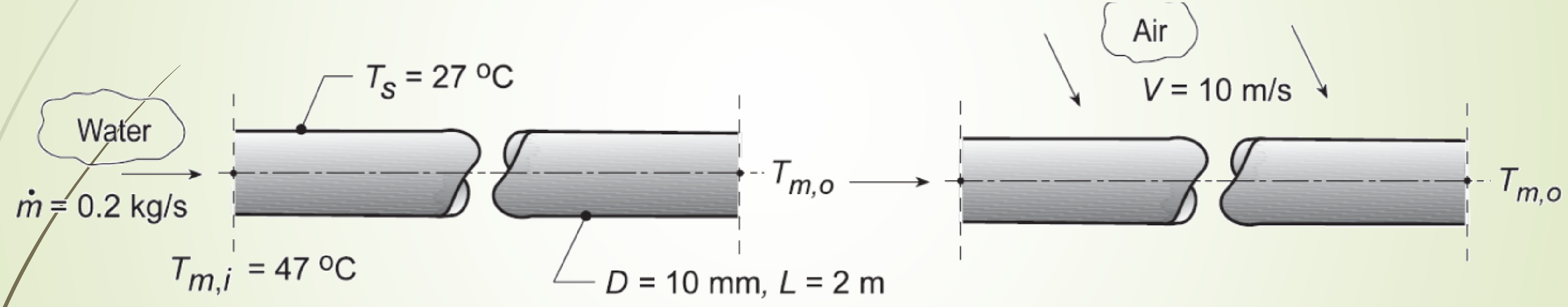
$$= 9{,}080W/m^2 - K$$

$$L = 2m, P = \pi D, cp = 4179J/kg - K, T_s = 27C, T_{m,i} = 47C$$

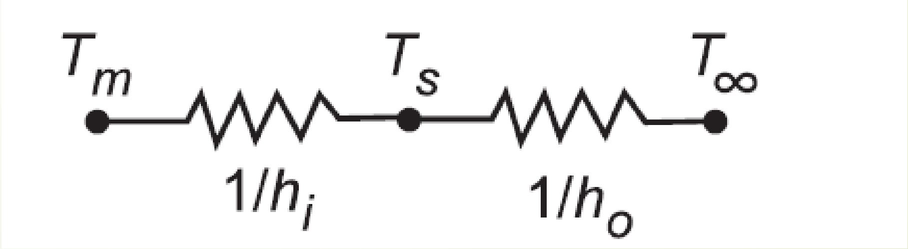
$$T_{m,o} = 37.1C$$

$$T_{m,guess} = 37C \rightarrow GOOD\ \mathrm{GUESS}$$

CASE B



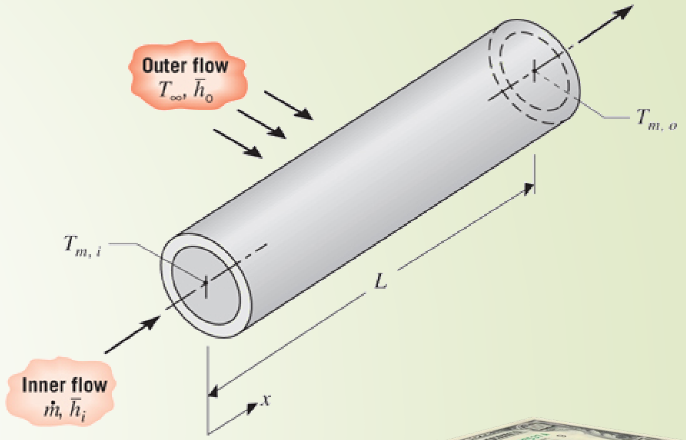
Water
T_s = 27 °C
$\dot{m}$ = 0.2 kg/s
$T_{m,i}$ = 47 °C
$T_{m,o}$
D = 10 mm, L = 2 m
Air
V = 10 m/s
$T_{m,o}$



T_m
T_s
T_∞
$1/h_i$
$1/h_o$

SPECIAL CASE
INTERNAL PIPE FLOW/EXTERNAL CONVECTION

- In some applications, the temperature of the external fluid, rather than the surface temperature is **FIXED**.
- If so, we can define the Heat flow as follows:



CONSTANT TEMPERATURE-T_∞

$$q_{conv} = \bar{U}A\frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}} = \bar{U}A\Delta T_{LM} = \frac{\Delta T_{LM}}{\sum R_{th}}$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{\sum R_{th}}\right] = \exp\left[-\frac{\bar{U}A}{\dot{m}c_p}\right]$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{\bar{U}A}{\dot{m}c_p}\right]; U \rightarrow \text{Overvall Heat Transfer Coefficient}$$

FORMER CASE

CONSTANT SURFACE TEMP

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}}$$

SOLUTION (special case)

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{\overline{U}A}{\dot{m}c_p}\right]$$

$@\,x = L$

$$T_{m,o} = T_\infty - (T_\infty - T_{m,i})\exp\left[-\frac{\overline{U}A}{\dot{m}c_p}\right]$$

$$\overline{U}A = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{\overline{h}_i A_i} + \frac{1}{\overline{h}_0 A_0}};\ A_i = \pi D_i L,\ A_o = \pi D_o L,$$

$D_i = D_o = D \rightarrow$ THIN WALL

WATER

$$T_{m,o_{guess}} = 37C$$

$$\rho = 997 kg/m^3$$

$$c = 4179 J/kg-K$$

$$k = 0.613 W/m-K$$

$$\mu = 855x10^{-6} Pa-s$$

AIR

$$T_{m,o_{guess}} = \frac{47+100}{2} = 73.5C,$$

$$T_m = \frac{47+73.5}{2} = 60.25C(333.25K)$$

$$\nu = 20.92x10^{-6} m^2/s$$

$$k = 0.030 W/m-K$$

$$\Pr = 0.70$$

EXTRENAL CYLINDER CROSS FLOW HEATING (THIS CASE: THIN WALL TUBE)

MORE ACCURATE

$\mathrm{Re_D Pr} \geq 0.2$

$$\overline{NU}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282,000}\right)^{5/8}\right]^{4/5} = \frac{\bar{h}_o D}{k_{fluid}}$$

$$\mathrm{Re}_D = \frac{VD}{\nu_{air}} = \frac{10m/s x 0.010m}{20.92x10^{-6} m^2/s} = 4760$$

$$\overline{NU}_D = 35.76$$

$$\bar{h}_o = k_{fluid}\frac{\overline{NU}_D}{D} = \frac{0.030W/m-K \bullet 35.76}{0.010m} = 107W/m^2-K$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{\bar{U}A}{\dot{m}c_p}\right]$$

RECOMPUTE "h_i" – *HEATING*

$$\bar{h}_i = k_{fluid}\frac{0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n}{D}; n = 0.4$$

$$= 10,800 W/m2-K$$

$$\bar{U}A = \frac{1}{\sum R_{th}} = \frac{1}{\left(\frac{1}{h_i A_i} + \frac{1}{h_0 A_0}\right)}; A_i = \pi D_i L, A_o = \pi D_o L,$$

$D_i = D_o = D$

$$\bar{U} = \frac{1}{A\sum R_{th}} = \frac{1}{\frac{A}{\pi DL}\left(\frac{1}{h_i} + \frac{1}{h_0}\right)} = 106W/m2-K$$

$$T_{m,o} = T_\infty - (T_\infty - T_{m,i})\exp\left[-\frac{\bar{U}A}{\dot{m}c_p}\right]; A = \pi DL$$

$= 47.4C$ (73.5C Guess)

Constant Temperature Special Case – EXTERNAL CROSS FLOW

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{\sum R_{th}}\right] = \exp\left[-\frac{UA}{\dot{m}c_p}\right]$$

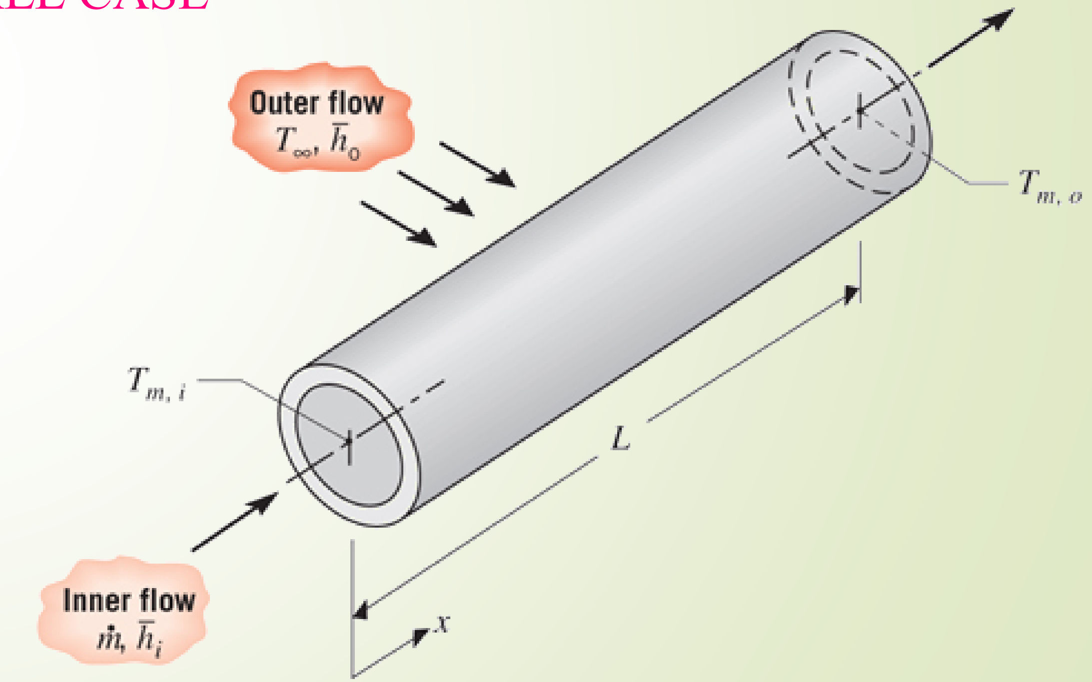
$UA[W/K] \equiv$ OVERALL HEAT TRASFER THERMAL RESISTANCE

$$UA = \frac{1}{\sum R_t} = \frac{1}{\frac{1}{h_i A_i} + \frac{\ln(\frac{r_0}{r_i})}{2\pi k_{wall} L} + \frac{1}{h_o A_o}} \rightarrow \text{THICK WALL CASE}$$

THIN WALL TUBE

$$\frac{\ln(\frac{r_0}{r_i})}{2\pi k L} \rightarrow 0 \rightarrow A_i = 2\pi r_i L, A_o = 2\pi r_o L$$

$$UA = U_0 A_0 = U_i A_i = \frac{1}{\sum R_t} = \frac{1}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o}}$$



GENERAL THERMAL RESISTANCES

CONSTANT WALL/EXTERNAL FLUID TEMPERATURE

$$q_{conv} = \overline{U}A\frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}} = \overline{U}A\Delta T_{LM} = \frac{\Delta T_{LM}}{\sum R_{th}}$$

OVERALL THERMAL RESISTANCE $\rightarrow W/K$

$$\overline{U}A = U_0 A_0 = U_i A_i = \frac{1}{\sum R_t} = \frac{1}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o} + R_{t_{CONDUCTION}}}$$

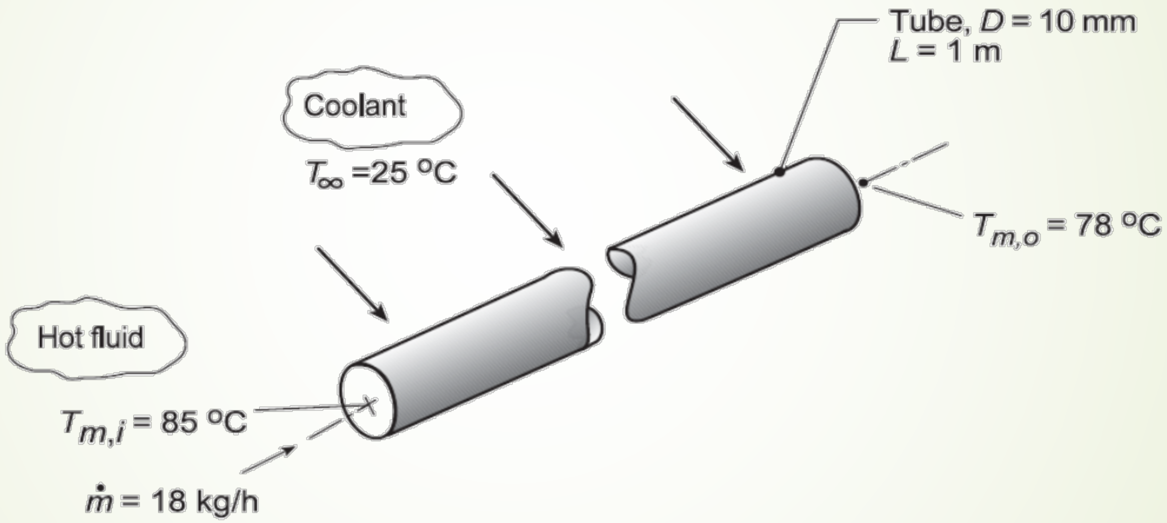
OVERALL HEAT TRANSFER COEFFICIENT $\rightarrow W/m^2 - K$

$$\overline{U}_0 = \frac{1}{A_o\left[\frac{1}{h_i A_i} + \frac{1}{h_o A_o} + R_{t_{CONDUCTION}}\right]} = \frac{1}{\left[\frac{A_o}{h_i A_i} + \frac{1}{h_o} + A_o R_{t_{CONDUCTION}}\right]}$$

$$\overline{U}_i = \frac{1}{A_i\left[\frac{1}{hA_i} + \frac{1}{h_o A_o} + R_{t_{CONDUCTION}}\right]} = \frac{1}{\left[\frac{1}{h} + \frac{A_i}{h_o A_o} + A_i R_{t_{CONDUCTION}}\right]}$$

Hot fluid enters this wall tube at 85C and is in cross flow with air at 25C as shown. Find outlet mean temperature if flow rate is increased by 2.

SCHEMATIC:



Given

$$\rho = 1079 kg/m3, cp = 2637 J/kg-K, \mu = 0.0034 N-s/m^2, k = 0.261 W/m-K$$

FIRST FIND UA (GIVEN EXIT TEMP) OVERALL THERMAL RESISTANCE

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{\sum R_{th}}\right] = \exp\left[-\frac{UA}{\dot{m}c_p}\right]$$

$$-\ln\left[\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}}\right]\dot{m}c_p = UA$$

$$-\ln\left[\frac{25-78}{25-85}\right][18\text{kg/hr} \times 1\text{hr/3600s}]2637 J/kg-K = UA[W/K]$$

$$1.64[W/K] = UA \rightarrow \text{Overall Thermal Resistance}$$

$$\frac{1.64W/K}{A(=\pi DL)}\left[\frac{W}{m^2-K}\right] = 52.1 = U \rightarrow \text{Overall Heat Transfer Coefficienct}$$

INTERNAL CONVECTIVE HEAT TRANSFER COEFFICIENT, hi.

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 18 kg/h \times 1h/3600s}{\pi \times 0.01 \times 0.0034 N-s/m^2} = 187 \rightarrow \text{LAMINAR}$$

NU=3.66(constant surface temprature)

$$= \frac{\bar{h}_i D}{k_{fluid}}$$

$$\frac{3.66 k_{fluid}}{D} = \bar{h}_i = 1,402 \frac{W}{m^2-K}$$

DOUBLE MASS FLOW

$$Re_D = 2 \bullet 187 = 374 \rightarrow \text{LAMINAR}$$

$$\bar{h}_i = 1,402 \frac{W}{m^2-K} \rightarrow \text{UA DOES NOT CHANGE}$$

$$UA = U_0 A_0 = U_i A_i = \frac{1}{\sum R_t} = \frac{1}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o}}$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{UA}{\dot{m}c_p}\right]$$

NEW EXIT TEMPERATURE

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{\sum R_{th}}\right] = \exp\left[-\frac{UA}{\dot{m}c_p}\right]$$

$$T_{m,o} = T_\infty - \exp\left[-\frac{UA}{\dot{m}c_p}\right]\left(T_\infty - T_{m,i}\right)$$

$$T_{m,o} = 25C - \exp\left[-\frac{1.64W/K}{(18\times 2\times 1/3600)\text{kg/s}\bullet 2637\text{J/kg-K}}\right](25-85)$$

$$= 81.4C$$

NOTE: FASTER FLOW RATE LEADS TO HIGHER OUTLET TEMPERATURE ????

THOUGHT?

- How would analysis be updated if CYLINDER had **wall thickness of 4mm** of ANSI 316 steel?
- Hmm???