MECH-322
STUDY AID

Work and Energy
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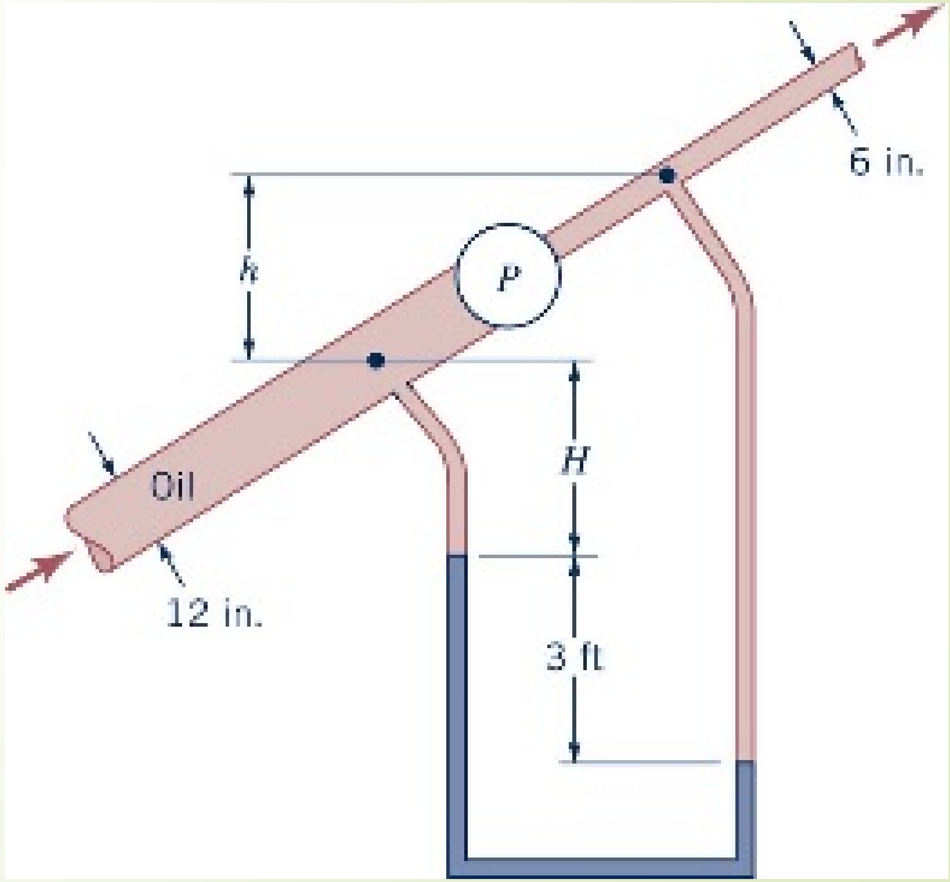
Conservation of Energy Model

•Oil $(SG=0.88)$ flows steadily at a rate of Q ft^3/s, if the system piping flow head loss is $h_l = \frac{ft-lbs}{slug}$

- Determine parametric model to find actual pump work as a function of primary input variables flow rate (Q), pump efficiency and flow head loss (hl).
- Manometer Important:
  - Fluid is mercury
- Mass conservation important:
  - Change in diameter
- Energy conservation important:
  - Flow, Work and Energy



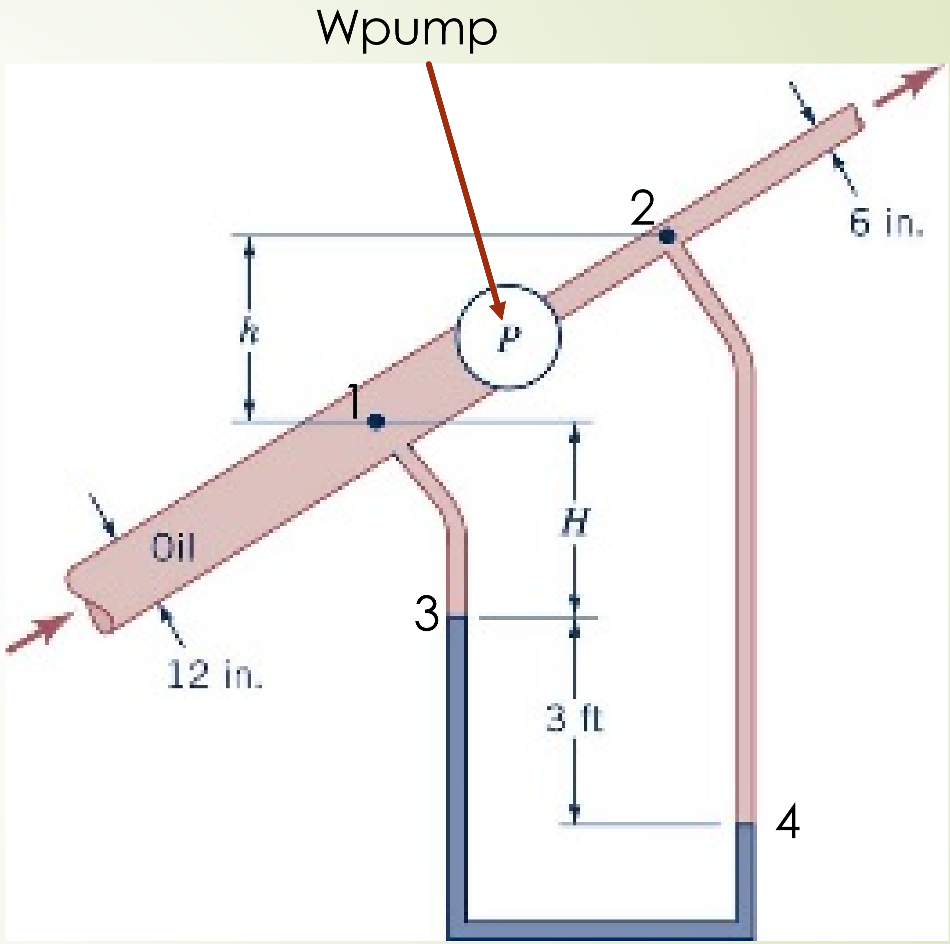
Step 1/2:

- Label Points
- Apply Manometry

$$P_1 + \cancel{\gamma_{oil} H} + \gamma_m 3' - \gamma_{oil}(3 + \cancel{H} + h) = P_2$$

$$P_1 - P_2 = \gamma_{oil}(3 + h) - \gamma_m 3'$$

$$\frac{P_1 - P_2}{\gamma_{oil}} = (3 + h) - 3\frac{\gamma_m}{\gamma_{oil}} = 3(1 - \frac{\gamma_m}{\gamma_{oil}}) + h$$



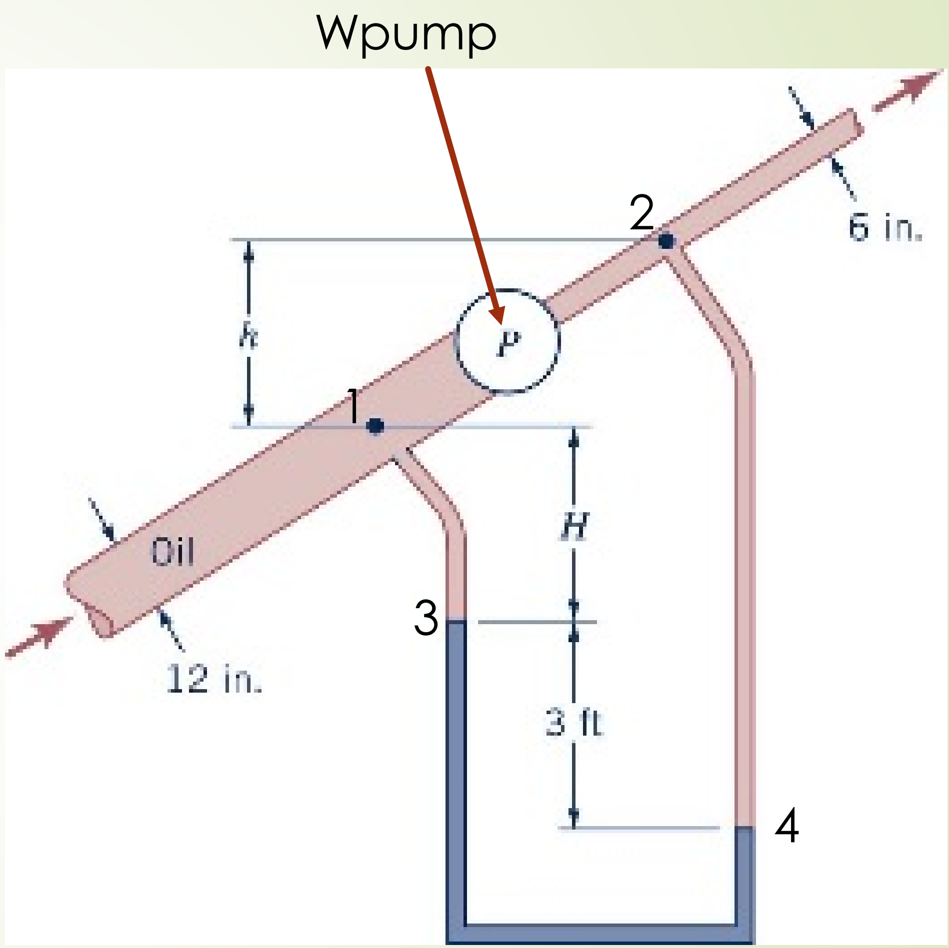
Step 3:

- Mass Conservation (Q Given)
  - Apply control volume around pump

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1}$$

$$V_2 = \frac{Q}{A_2}$$



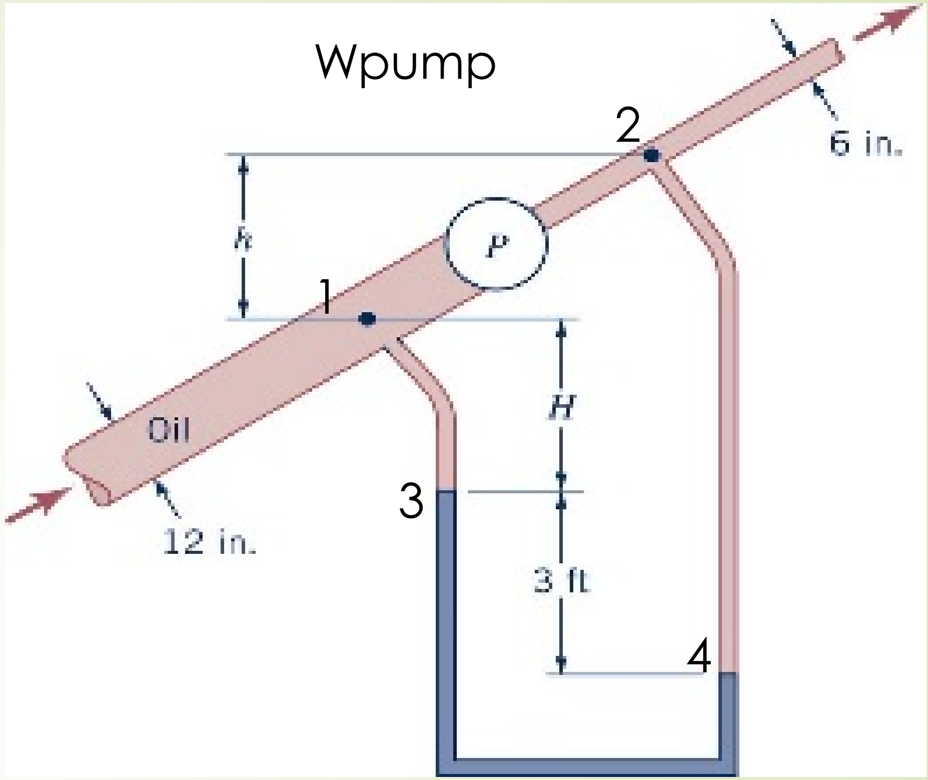
Step 4:

- Energy Conservation
  - Apply control volume around pump

$$\frac{P_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_{p_{IDEAL}} = \cancel{h_T} + \frac{P_2}{\gamma_{oil}} + \frac{V_2^2}{2g} + z_2 + h_l$$

$$h_{pIDEAL} = \frac{P_2}{\gamma_{oil}} - \frac{P_1}{\gamma_{oil}} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + (z_2 - z_1) + h_l$$

Combine with mass and energy conservation and Manometry



$$h_{pIDEAL} = 3\left(\frac{\gamma_m}{\gamma_{oil}} - 1\right) - h + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + (z_2 - z_1) + \frac{h_l \dfrac{ft-lb}{slug\left(\dfrac{lb-s^2}{ft}\right)}}{g\,\dfrac{ft}{s^2}}$$

$$h_{p\,IDEAL} = 3\left(\frac{\gamma_m}{\gamma_{oil}} - 1\right) + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + \frac{h_l}{g} \rightarrow ft$$

$$\dot{W}_{p_{IDEAL}} = \dot{m}\left[\frac{slugs}{s}\right] g\left[\frac{ft}{s^2}\right] h_{pIDEAL}[ft] \rightarrow \frac{ft-lb}{s} \rightarrow \text{MINIMUM WORK IN}$$

Final: Actual Pump Work

- Isentropic Work

$$\eta_p = \frac{W_{p_{\text{IDEAL IN}}}}{W_{p_{\text{ACTUAL IN}}}}, \gamma_m = 13.55\gamma_{H_2O}, \gamma_{oil} = 0.88\gamma_{H_2O}$$

$$W_{p_{\text{ACTUAL IN}}} = \frac{W_{p_{\text{IDEAL IN}}}}{\eta_p} = \frac{\dot{m}gh_{pIDEAL}[ft]}{\eta_p}$$

$$W_{p_{\text{ACTUAL IN}}} = \frac{\dot{m}g}{\eta_p}\left\{3\left(\frac{\gamma_m}{\gamma_{oil}} - 1\right) + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + \frac{h_l}{g}\right\}$$

$$\dot{m}g = \rho(AV)g = \gamma Q$$

$$W_{p_{\text{ACTUAL IN}}}(Q,\eta_p) = \frac{\gamma_{oil} Q}{\eta_p}\left\{3\left(\frac{\gamma_m}{\gamma_{oil}} - 1\right) + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + \frac{h_l}{g}\right\} \rightarrow f(Q^3)$$

D1 (")	A1 (ft2)	D2 (")	A2 (ft2)	hl(ft-lbf/slug)	g(ft/s2)	Gm/Goil	Eff	GM (lbf/ft3)
8	0.3491	6	0.1963	12	32.2	15.40	0.8	62.4
Q (ft3/s)	Work(ft-lbf/s)	Work (HP)						
1	2,745.80	4.99						
1.5	4,158.96	7.56						
2	5,620.45	10.22						
2.5	7,146.37	12.99						
3	8,752.81	15.91						
3.5	10,455.90	19.01						
4	12,271.74	22.31						
4.5	14,216.43	25.85						
5	16,306.08	29.65						
5.5	18,556.80	33.74						
6	20,984.69	38.15						
6.5	23,605.87	42.92						
7	26,436.43	48.07						
7.5	29,492.49	53.62						
8	32,790.15	59.62						
8.5	36,345.52	66.08						
9	40,174.71	73.04						
9.5	44,293.81	80.53						
10	48,718.95	88.58						
10.5	53,466.22	97.21						
11	58,551.74	106.46						
11.5	63,991.60	116.35						
12	69,801.92	126.91						
12.5	75,998.81	138.18						
13	82,598.36	150.18						
13.5	89,616.69	162.94						
14	97,069.90	176.49						
14.5	104,974.10	190.86						
15	113,345.40	206.08						
15.5	122,199.91	222.18						

PUMP ENERGY MODEL
5-113

In reality, flow head loss is a function of Q, i.e. $h_l \rightarrow f(Q)$