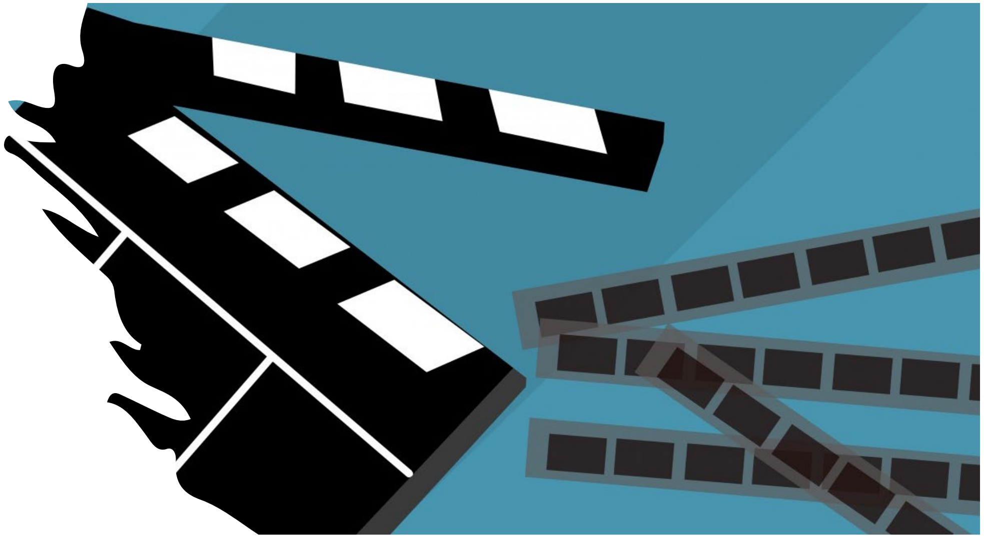
# THERMODYNAMICS & 1st LAW REVIEW

- **CHAPTER 5**
  - **ENERGY CONSERVATION VIDEO**



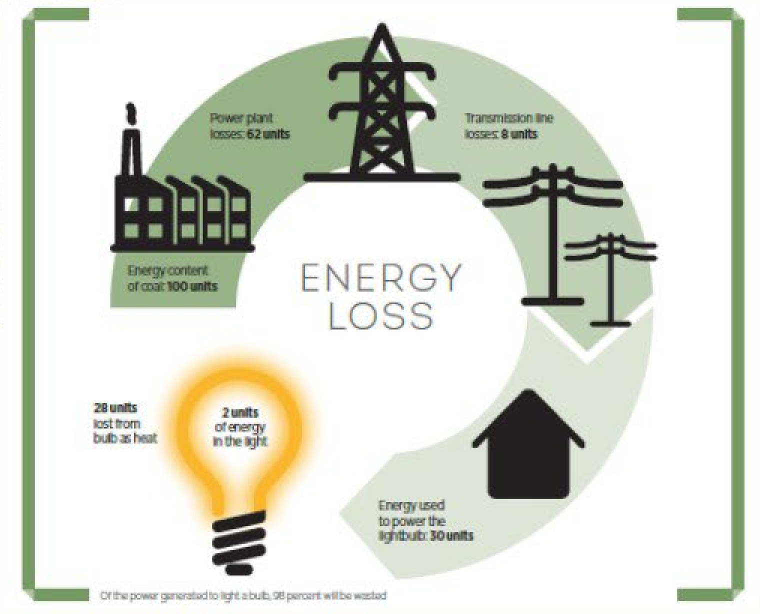
**Dr. K. J. Berry**

# *A FUTURE REALITY*

- *The future world society will represent a standard of living unprecedented in recent history. As any society develops and prospers, **CLEAN WATER** and the efficient generation and use of **CLEAN ENERGY** are both vital elements necessary to ensure economic and social viability. A country's economic strength is dependent upon the ability to provide efficient and affordable electric power for transportation, the conveniences of home and workplace, and therefore helps to build a strong and healthy societal foundation.*



# Primary Grid Power Transmission Losses




70% Energy Loss from Fuel to Community.



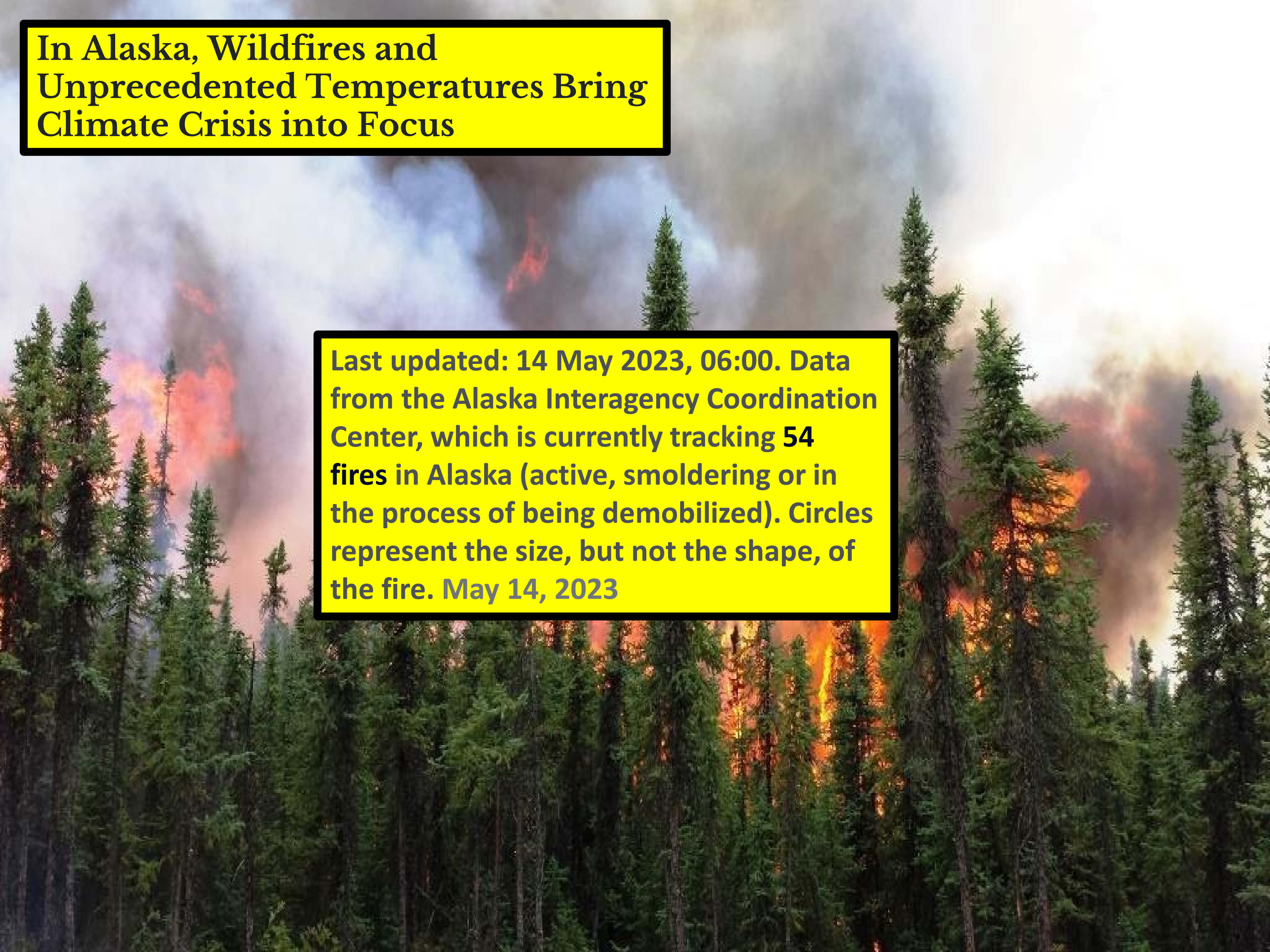
# Renewable Energy

https://www.youtube.com/watch?v=1kUE0BZtTRc&t=3s

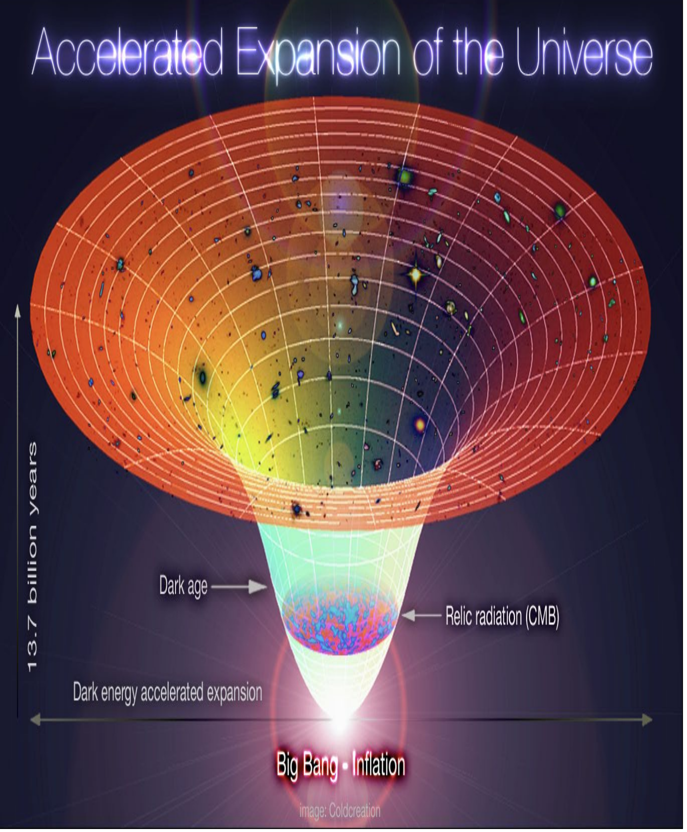

# CONSERVATION LAWS

https://www.youtube.com/watch?v=VxCORJ8dN3Y&list=RDLV_8EEnMwkmZk&index=24

# In Alaska, Wildfires and Unprecedented Temperatures Bring Climate Crisis into Focus



Last updated: 14 May 2023, 06:00. Data from the Alaska Interagency Coordination Center, which is currently tracking **54 fires** in Alaska (active, smoldering or in the process of being demobilized). Circles represent the size, but not the shape, of the fire. May 14, 2023




# Chapter 5

**CONSERVATION OF ENERGY**
**+**
**CONSERVATION OF MOMENTUM**
**+**
**CONSERVATION OF MASS**

- **ENERGY/MOMENTUM/MASS is neither created nor destroyed**
- **Transform from one form to another**

# Conservation of Energy

$$\frac{dM_{sys}}{dt} \rightarrow \rho; \qquad \frac{dMom_{sys}}{dt} \rightarrow \rho\vec{V}$$

$$\frac{dE_{sys}}{dt} \rightarrow \rho e$$

$e =$ Total energy per unit mass $= \frac{E}{M}\left[\frac{J}{kg}\right]$

$\Rightarrow e =$ Internal energy+Potential energy+Kinetic energy

$$\Rightarrow e = \left(u + gz + \frac{1}{2}V^2\right)\left[\frac{J}{kg} = \frac{m^2}{s^2}\right]$$

$h = u + \frac{p}{\rho} = c_p T$ (IDEALGAS)

$p = \rho RT$

# Conservation of Energy

## Recall Mass Conservation:

$$\frac{dM_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho \left(\vec{V} \cdot d\vec{A}\right)$$

Mass is carried away by the fluid particle

## Momentum Conservation:

$$\sum \vec{F} = \frac{d\left(Mom\right)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \vec{V} d\forall + \int_{CS} \rho \vec{V} \left(\vec{V} \cdot d\vec{A}\right)$$

Momentum is carried away by the fluid particle

## Energy Conservation:

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho e \left(\vec{V} \cdot d\vec{A}\right)$$

Energy is carried away by the fluid particle

# First Law of Thermodynamics
# The Energy Equation

- **First Law of Thermodynamics for a System:**

**The *time rate of increase of the total stored energy* of the system is equal to the sum of the net time rate of *energy addition by heat transfer* INTO the system and the net time rate of *energy addition by work OUT or done by system.***

$$\frac{D}{Dt}\int_{sys} \rho e \, \mathrm{d}\forall = \frac{dQ}{dt} - \frac{dW}{dt}, or :$$

$$\dot{Q} = \dot{W} + \frac{D}{Dt}\int_{sys} \rho e \, \mathrm{d}\forall, where :$$

$\dot{Q} \equiv$ HEAT ADDED to system as POSITIVE (+)

$\dot{W} \equiv$ WORK DONE by system as POSITIVE (+)

e $\equiv$ system energy per unit mass (internal, kinetic, potential, chemical,other)

$$e \equiv u + \frac{V^2}{2} + gz, \dot{W} = \dot{W}_{shaft} + \dot{W}_{pressure} + \dot{W}_{viscous}$$

# First Law of Thermodynamics
# The Energy Equation

- **First Law of Thermodynamics for a System:**

**The time rate of increase of the total stored energy of the system is equal to the sum of the net time rate of energy addition by heat transfer into the system and the net time rate of energy addition by work transfer into the system**

$$\dot{Q}_{cs} = \dot{W}_{cs} + \frac{D}{Dt}\int_{sys} \rho e \, d\forall, \; where:$$

$\dot{W} \equiv$ WORK DONE by system as POSITIVE (+)

$$\dot{W} = \dot{W}_{shaft} + \dot{W}_{pressure} + \dot{W}_{viscous}$$

$\dot{W}_{shaft} =$ SHAFT work done by a machine at the CONTROL SURFACE (i.e turbine/pum

$\dot{W}_{pressure} =$ Rate of work done by pressure forces at Control Surface (Pres x Vel)

$$\dot{W}_{pressure} = \int_{cs} P(\vec{V} \bullet n)\, dA$$

$\dot{W}_{viscous} =$ Shear work due to viscous stress at Control Surface

$\dot{W}_{viscous} = -\int_{cs} \vec{\tau} \bullet \vec{V} dA$; Small except for Boundary Layers close to surface

# Conservation of Energy

$$\dot{Q}_{cs} - \dot{W}_{cs} = \frac{dE}{dt} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho e\left(\vec{V}\cdot d\vec{A}\right)$$

**Storage term**

**Transport term**

$$T_{shaft} \times \omega$$

$$\dot{W}_{cs} = \left(\sum \dot{W}_{\substack{shaft\\work}}\right)_{out} + \left(\dot{W}_{\substack{flow\\work}}\right)_{out}$$

$$\left(\dot{W}_{\substack{flow\\work}}\right)_{out} = \delta\vec{F}\cdot\vec{V} = p d\vec{A}\cdot\vec{V} = \int p\left(\vec{V}\cdot d\vec{A}\right)$$

$$\Rightarrow \dot{Q}_{cs} - \dot{W}_{cs} - \int_{cs} p\left(\vec{V}\cdot d\vec{A}\right) = \frac{dE}{dt} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho e\left(\vec{V}\cdot d\vec{A}\right)$$

$$\Rightarrow \dot{Q}_{cs} - \dot{W}_{cs} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho\left(e + \frac{p}{\rho}\right)\left(\vec{V}\cdot d\vec{A}\right)$$

# Conservation of Energy

$$\dot{Q}_{cs} - \dot{W}_{cs} = \frac{d}{dt}\int_{CV} \rho e \, d\forall + \int_{CS} \rho\left(e + \frac{p}{\rho}\right)\left(\vec{V}\cdot d\vec{A}\right)$$

$$\Rightarrow \dot{Q}_{cs} - \dot{W}_{cs} = \frac{d}{dt}\int_{CV} \rho\left(u + gz + \frac{1}{2}V^2\right) d\forall + \int_{CS} \rho\left(\frac{p}{\rho} + u + gz + \frac{1}{2}V^2\right)\left(\vec{V}\cdot d\vec{A}\right)$$

**Net Heat Transfer** — **Shaft work** — **Storage term** — **Transport term**

- a. **Select governing principle**
- b. **Select control volume**
- c. **Assess all the heat transfer and shaft works**
- d. **Assess storage and transport terms**
- e. **Put it altogether**
- f. **Finish calculations for unknown**

# Conservation of Energy

## Things to Keep in Mind with Energy Conservation

- Control Volume Selection is Critical!!!
  - **Cut a surface – account for energy transfer across surfaces**
- **More things to know at inlets and outlets**
  - **Transport terms have: u, ρ, p, z, v**
- **Dot Product – same as before**

$$\vec{V} \cdot d\vec{A} = (sign)\left|V\right| dA$$

**|V| is total velocity – Scalar!!!**

# Different forms of Energy Equations

**"Most basic" Form**

- **Easy to remember**
- **Can always work to simpler forms**

$$\dot{Q}_{cs} - \dot{W}_{shaft} = \left( \frac{d}{dt} \int \rho e \, d\forall \right)_{CV} + \left( \int \rho \left( e + \frac{p}{\rho} \right) \left( \vec{V} \cdot d\vec{A} \right) \right)_{CS}$$

$$\text{STEADY STATE} \rightarrow \frac{d}{dt} \int \rho e \, d\forall \equiv 0$$

$$\text{UNIFORM FLOW} \rightarrow \rho \left( e + \frac{p}{\rho} \right) \neq f(dA)$$

$$and \int \rho \left( \vec{V} \cdot d\vec{A} \right) = \dot{m} \left[ \cos\theta \right]$$

$$\dot{Q}_{cs} - \dot{W}_{shaft} = \sum_{out} \dot{m} \left( \frac{p}{\rho} + u + gz + \frac{1}{2} V^2 \right) - \sum_{in} \dot{m} \left( \frac{p}{\rho} + u + gz + \frac{1}{2} V^2 \right)$$

# ENTHALPY

**Enthalpy, a property of a thermodynamic system, is the sum of the system's internal energy and the product of its pressure and volume. It is a state function used in many measurements in chemical, biological, and physical systems at a constant pressure, which is conveniently provided by the large ambient atmosphere.**

$$h = u + \frac{p}{\rho} = u + pv$$

$$h\left[\frac{J}{kg}\right] = u\left[\frac{J}{kg}\right] + \frac{p\left[\dfrac{N = \frac{J}{m}}{m^2}\right]}{\rho\left[\dfrac{kg}{m^3}\right]} = \overbrace{c_p\left[\frac{J}{kg-K}\right]T[K]}^{\text{IDEAL GAS}};$$

$$q[WATTS] = \dot{m}\left[\frac{kg}{s}\right]c_p\left[\frac{J}{kg-K}\right]\Delta T[K]$$

# Class 12: Different forms of Energy Equations

COMPLETE FORM[WATTS or ft-lbs/s]

$$\dot{Q}_{cs} - \dot{W}_{shaft} = \sum_{out} \dot{m}\left(h + gz + \frac{1}{2}V^2\right) - \sum_{in} \dot{m}\left(h + gz + \frac{1}{2}V^2\right) + \sum \dot{m}_{out}\, g\left(h_{L_{IN-OUT}}\right)$$

HEAD LOSS FORM: SINGLE INPUT/SINGLE OUTPUT[m or ft]

$$\frac{\dot{Q}_{cs}}{\dot{m}g} + \frac{u_{in} - u_{out}}{g} - \frac{\dot{W}_{shaft}}{\dot{m}g} = \frac{p_{out} - p_{in}}{\rho g} + \frac{V_{out}^2 - V_{in}^2}{2g} + z_{out} - z_{in} + \left(h_{L_{IN-OUT}}\right)$$

$$h_l \equiv \text{FLOW HEAD LOSS} = \frac{\dot{Q}_{cs}}{\dot{m}g} + \frac{u_{in} - u_{out}}{g} - \left(h_{L_{IN-OUT}}\right) = \frac{\dot{W}_{shaft}}{\dot{m}g} + \frac{p_{out} - p_{in}}{\rho g} + \frac{V_{out}^2 - V_{in}^2}{2g} + z_{out} - z_{in}$$

$$Efficiency(\eta)_{FLOW} = \frac{\dot{W}_{shaft} - \gamma \overbrace{Q}^{\text{Flow Rate}} \left[\frac{m^3}{s}\right]\left\{h_{L_{FLOW}}[m]\right\}}{\dot{W}_{shaft}}$$

$$h_{L_{FLOW}} \rightarrow [m]$$

GENERAL ENERGY EQUATION--MULTIPLE I/O STREAMS

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in} (\dot{m}g(\frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1)) = \sum_{out} (\dot{m}g(\frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2)) + \sum H_L; W \text{ or } ft-lbf/s;$$

$LET \rightarrow$

UNITS: WATTS or FT-LBF/s

$$\dot{W}_{s_{IDEAL}} = \dot{W}_{Turbine_{IDEAL}} - \dot{W}_{Pump_{IDEAL}};$$

$$H_L[Watts] = \dot{m}_B(kg/s)(g)h_{L_{A-B}}(m) = \dot{m}_B g\left(h_q + h_{\text{minor}} + h_{\text{major}}\right) \rightarrow \text{Total SYSTEM Losses;} \rightarrow \text{OR}$$

$$H_{L_{A-B}}[ft-lbf/\text{sec}] = \dot{m}_B(slugs/s)gh_{A-B}(ft)$$

(one INLET/one EXIT) $\rightarrow$

Energy Equation $\rightarrow$ "m;ft" $\rightarrow (\div \dot{m}g)$

$$\frac{\dot{Q}_{cs}}{\dot{m}g} + \frac{\dot{W}_{Pump_{IDEAL}}}{\dot{m}g} + \frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{\dot{W}_{Turbine_{IDEAL}}}{\dot{m}g} + \frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2 + h_{qL}(m); units = m, \text{or, ft}$$

$$\text{h}_{p\ IDEAL} + h_1 = h_{T_{IDEAL}} + h_2 + h_{qL}$$

UNITS: m or ft

$$h_{\text{minor}}(m) = \sum_i K_i \frac{V_i^2}{2g}; \rightarrow \text{Component Losses}$$

$$h_{\text{major}}(m) = \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g}; \rightarrow \text{Straight Pipe Section Losses}$$

$$h_{qL}(m) = \frac{u_2}{g} - \frac{u_1}{g} - \frac{\dot{Q}_{cs}}{\dot{m}g} + h_{L_{A-B}}; \rightarrow \text{Thermal Losses}$$

# Different forms of Energy Equations

## "SINGLE Input = SINGLE Outputs" Form

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}=\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}+Losses$$

$$Losses\left(h_L\right)=-\overbrace{\left(\frac{\dot{Q}_{cs}}{\dot{m}g}+\frac{u_{in}-u_{out}}{g}\right)}^{BERNOULLI=0}$$

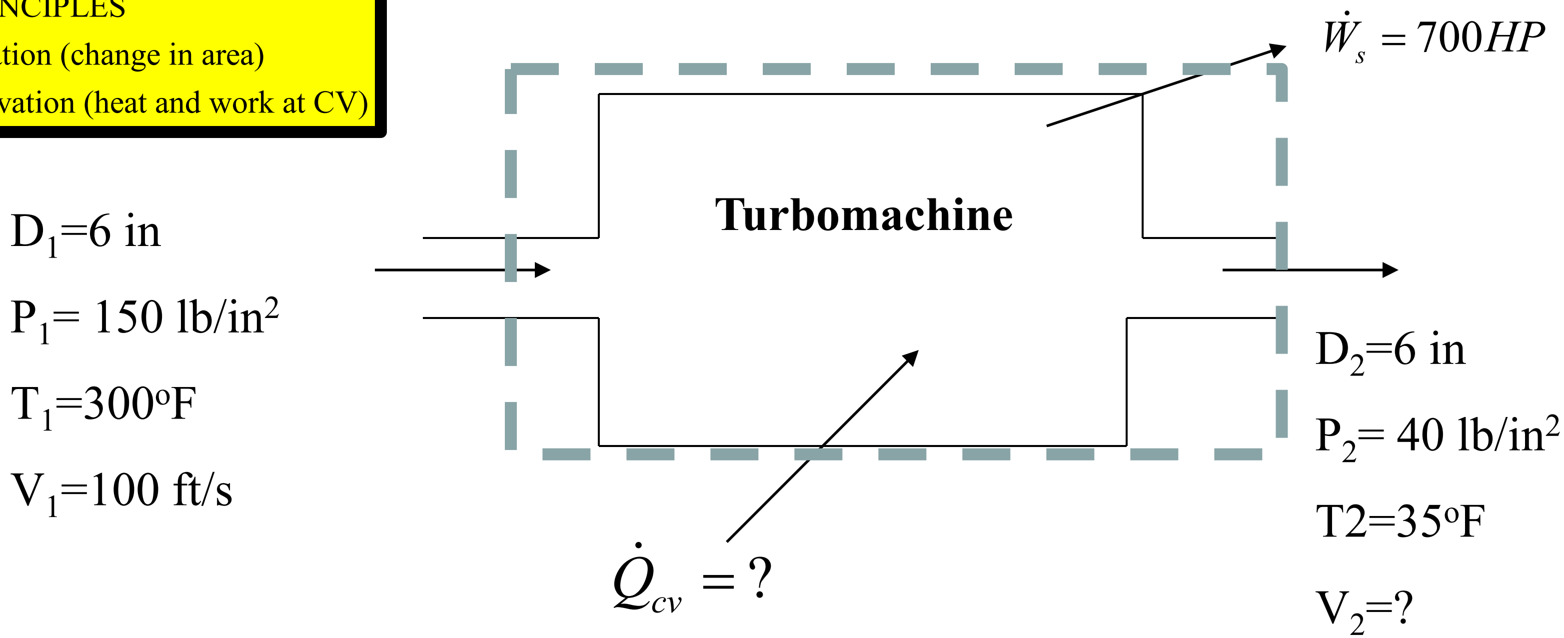
- **It's Bernoulli if losses are ZERO**
- **Units of each term are meters or feet**
- **Fluid losses show up in terms of "heat"**

Head loss represents reversible & irreversible processes.

# Energy Equation Example #1

**Problem #1: Air [R=1716, cp=6003ft.lbf/(slug.°R)] flows steadily, as shown in Figure below, through a turbine that produces 700 hp. For the inlet and exit conditions shown, estimate (a) the exit velocity $V_2$ and (b) the heat transferred $\dot{Q}$ in Btu/h.**

DRIVING PRINCIPLES
Mass Conservation (change in area)
Energy Conservation (heat and work at CV)



# Energy Equation Example

**Solution:** The conservation of energy equation -

$$\dot{Q}_{in} - \dot{W}_{shaft} = \dot{m}\left[\left(\mathrm{u}_2 - \mathrm{u}_1\right) + \frac{1}{2}\left(V_2^2 - V_1^1\right) + g\left(z_2 - z_1\right) + \left(\frac{p_2 - p_1}{\rho}\right)\right]$$

$$\Rightarrow \dot{Q}_{in} - \dot{W}_{shaft} = \dot{m}\left[\left(\mathrm{u}_2 + \frac{p_2}{\rho}\right) + \frac{1}{2}V_2^2 - \left(\mathrm{u}_1 + \frac{p_1}{\rho}\right) - \frac{1}{2}V_1^2\right];\ z_2 = z_1; h = u + \frac{P}{\rho} = c_p T; (IDEAL\ GAS)$$

$$\Rightarrow \dot{Q}_{in} - \dot{W}_{shaft} = \dot{m}\left[c_p T_2 + \frac{1}{2}V_2^2 - c_p T_1 - \frac{1}{2}V_1^2\right]; \text{IDEAL GAS}$$

Now determine the air density at the inlet and at the exit

$$\rho_1 = \frac{P_1}{RT_1} = \frac{\left(150\, lb/in^2\right)\left(144 in^2/1 ft^2\right)}{\left(1716 \frac{ft-lb}{slugs - R^0}\right)\left(460+300\right)^o R} = 0.0166\, slugs/ft^3 \qquad \rho_2 = \frac{P_2}{RT_2} = \frac{\left(40\, lb/in^2\right)\left(144 in^2/1 ft^2\right)}{\left(1716\right)\left(460+35\right)^o R} = 0.00679\, slugs/ft^3$$

Mass flow rate

$$\dot{m}_1 = \rho_1 V_1 A_1 = \left(0.0166\, slugs/ft^3\right)\left(100\, ft/s\right)\left(\frac{\pi}{4}\left(\frac{6in}{12in} ft\right)^2\right) = 0.325\, slugs/s$$

Now $\dot{m}_1 = \dot{m}_2 = \rho_2 V_2 A_2 = \left(0.00679\, slugs/ft^3\right)\left(V_2\, ft/s\right)\left(\frac{\pi}{4}\left(\frac{6in}{12in} ft\right)^2\right) = 0.325\, slugs/s$

$$\Rightarrow V_2 = 244\, ft/s$$

# Energy Equation Example

Now the conservation of energy equation -

$$\dot{Q}_{in} - \dot{W}_{shaft} = \dot{m}\left[c_p T_2 + \frac{1}{2}V_2^2 - c_p T_1 - \frac{1}{2}V_1^2\right];\ h = u + \frac{p}{\rho} = c_p T$$

$$\Rightarrow \dot{Q}_{in} = (700hp)\left(550\frac{ft.lb/s}{hp}\right) + 0.325(slugs/s)\left[\begin{matrix}\left(6003\ ft.lb/\left(slugs.^o R\right)\right)(460+35)^o R + \frac{1}{2}(244\ ft/s)^2 \\ -\left(6003\ ft.lb/\left(slugs.^o R\right)\right)(460+300)^o R - \frac{1}{2}(100\ ft/s)^2\end{matrix}\right]$$

$$\Rightarrow \dot{Q}_{in} = 385{,}000\ ft.lb/s + -1{,}566{,}027\ ft.lb/s$$

$$\dot{Q}_{in} = -1{,}181{,}027\ ft.lb/s = (-1{,}181{,}027\ ft.lb/s)\left(\frac{3600\frac{s}{h}}{778.2(ft.lb/Btu)}\right);(\text{conversion}\ \frac{778.2\ ft.lb}{BTU})$$

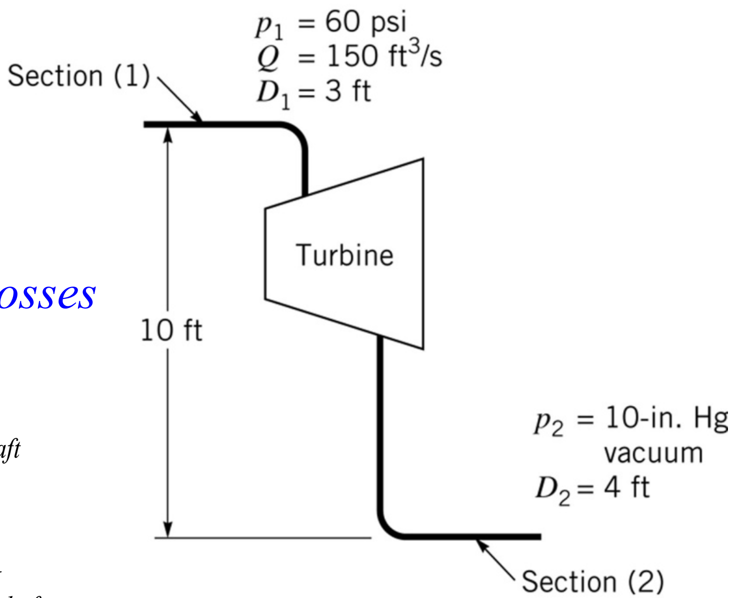
$$\Rightarrow \dot{Q}_{in} = -5{,}463{,}502\ Btu/h,$$

The negative sign indicates that heat transfer is a loss from the control surface.

$$\eta_{thermal} = \frac{W_{net}}{Q_H} = \frac{(700hp)\left(550\frac{ft.lb/s}{hp}\right)\left(\frac{3600\frac{s}{h}}{778.2(ft.lb/Btu)}\right)}{5{,}463{,}502\ Btu/h} = 33\%$$

# Energy Equation Example #2

**Problem 5.113: Water is supplied at 150 ft³/s and 60 psi to a hydraulic turbine through a 3ft inside diameter inlet pipe as shown in the Figure below. The turbine discharge pipe has a 4 ft inside diameter. The static pressure at section (2), 10ft below the turbine inlet, is 10in Hg vacuum. If the turbine develops 2500hp, determine the power loss between section (1) and (2).**




**Solution:** For flow between section (1) and (2), the energy equation can be written in loss form as

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}=\frac{\dot{W}_s}{\dot{m}g}+\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}+Power\ Losses$$

$$-power\ loss=\dot{m}\left[\frac{1}{2}\left(V_2^2-V_1^2\right)+g\left(z_2-z_1\right)+\left(\frac{p_2-p_1}{\rho}\right)\right]+\dot{W}_{shaft}$$

$$=\rho Q\left[\frac{1}{2}\left(V_2^2-V_1^2\right)+g\left(z_2-z_1\right)+\left(\frac{p_2-p_1}{\rho}\right)\right]+\dot{W}_{shaft}$$

**From given data**

$$p_2=-10\ in\ of\ Hg=(\text{-}10in)\frac{14.6558\,psia}{29.9''\,Hg}\frac{144in^2}{ft^2}=-705.83\,lb/ft^2$$

# Energy Equation Example

**Also** $$V_1 = \frac{Q}{A_1} = \frac{Q}{\pi D_1^2/4} = \frac{\left(150\ ft^3/s\right)}{\pi\left(3\ ft\right)^2/4} = 21.22\ ft/s$$

**Now** $$\dot{m}_1 = \dot{m}_2$$

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} = \dot{W}_s + \frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} + Losses$$

$$\Rightarrow Q_1 = Q_2$$

$$\Rightarrow V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\ ft/s\right)\frac{\left(3\ ft\right)^2}{\left(4\ ft\right)^2} = 11.94\ ft/s$$

$$-power\ loss = \rho Q\left[\frac{1}{2}\left(V_2^2 - V_1^2\right) + g\left(z_2 - z_1\right) + \left(\frac{p_2 - p_1}{\rho}\right)\right] + \dot{W}_{shaft}$$

$$= \frac{\left(1.94\,\text{slugs}/\text{ft}^3\right)\left(150\ ft^3/s\right)}{550\ ft.lb/s}\left[\begin{array}{l}\dfrac{11.94^2 - 21.22^2}{2}\dfrac{ft^2}{s^2} \\ -\left(32.2\ ft/s^2\right)\left(10\ ft\right) \\ \dfrac{-\left(705.83\,lb/ft^2\right)\left(-60\,lb/in^2 \bullet 144\,in^2/ft^2\right)}{\left(1.94\,slugs/ft^3\right)}\end{array}\right] + 2500\text{hp}$$

$$\Rightarrow -power\ loss = -301hp$$

$$\Rightarrow \eta = \frac{W_{actual}}{W_{ideal}} = \frac{W_{ideal} - Loss}{W_{ideal}} = \frac{2500 - 301}{2500} = 87\%$$

# Systems w/Turbines and Pumps
# (SINGLE INPUT/OUTPUT)

- When fluid passes through a turbine, the fluid system is DOING WORK (SHAFT) on the surroundings.
- When fluid passes through a pump, the pump DOES WORK on the fluid.

$$\frac{\dot{Q}_{cs}}{\dot{m}g}+\frac{u_{in}-u_{out}}{g}-\frac{\dot{W}_{shaft_{IDEAL}}}{\dot{m}g}=\frac{p_{out}-p_{in}}{\rho g}+\frac{V_{out}^2-V_{in}^2}{2g}+z_{out}-z_{in}+\frac{H_l}{\dot{m}g}$$

$$\frac{\dot{W}_{shaft_{IDEAL}}}{\dot{m}g}=\frac{\dot{W}_{turbine_{IDEAL}}-\dot{W}_{pump_{IDEAL}}}{\dot{m}g},\ h_l[m]=\frac{H_l[power]}{\dot{m}g}$$

$$\frac{\dot{Q}_{cs}}{\dot{m}g}+\frac{u_{in}-u_{out}}{g}+\frac{\dot{W}_{Pump_{IDEAL}}}{\dot{m}g}=\frac{\dot{W}_{Turbine_{IDEAL}}}{\dot{m}g}+\frac{p_{out}-p_{in}}{\rho g}+\frac{V_{out}^2-V_{in}^2}{2g}+z_{out}-z_{in}+h_l$$

$$h_{P_{IDEAL}}=h_{T\,IDEAL}+h_{out}-h_{in}+h_{L_{FLOW}}\quad(head-ft:m)$$

$$\dot{W}_{Pump_{IDEAL}}[W]=h_{P_{IDEAL}}[m]\dot{m}g,\ \dot{W}_{Turbine_{IDEAL}}[W]=h_{T\,IDEAL}[m]\dot{m}g$$

$$h_{out}=\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}$$

$$h_{L_{FLOW}}[m]=-\left[\frac{\dot{Q}_{cs}}{\dot{m}g}+\frac{u_{in}-u_{out}}{g}\right]+h_l\qquad h_{in}=\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}$$

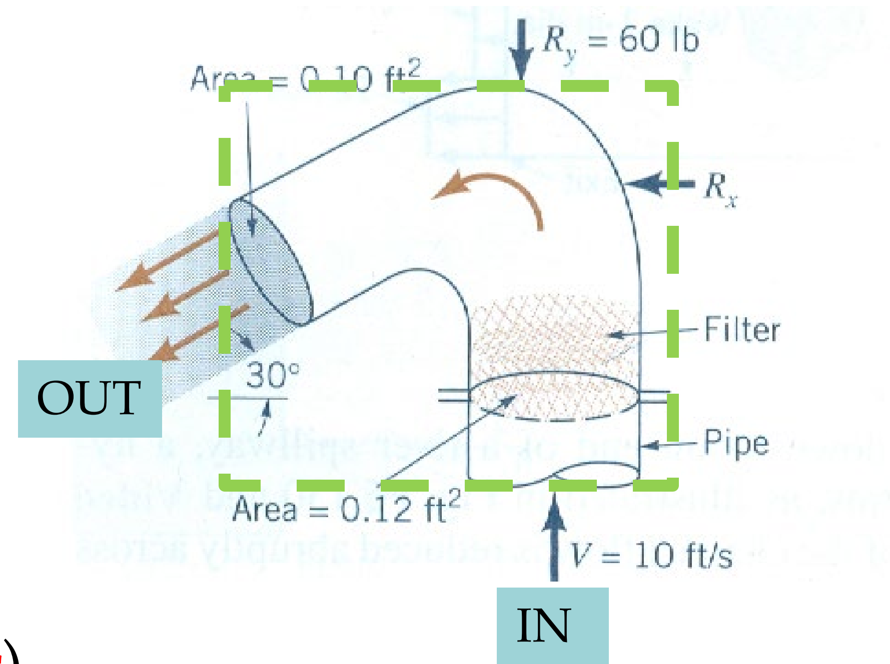
# Isentropic Efficiency ($2^{nd}$ Law)

$$\eta_{Pump} = \frac{\dot{W}_{p_{IDEAL-IN}}}{\dot{W}_{p_{ACTUAL-IN}}} = \frac{\dot{W}_{p_{ACTUEL-IN}} - \dot{W}_{LOSS}}{\dot{W}_{p_{ACTUAL-IN}}}$$

$$\eta_{Turbine} = \frac{\dot{W}_{Turbine_{ACTUAL-OUT}}}{\dot{W}_{Turbine_{IDEAL-OUT}}} = \frac{\dot{W}_{Turbine_{ACTUAL-OUT}}}{\dot{W}_{Turbine_{ACTUAL-OUT}} + \dot{W}_{LOSS}}$$

# Mass, Momentum and Energy

- Water flows steadily through an end cap and exits as a free jet and contains a filter. The axial force to hold end cap stationary is 60lb.
  - *Pipe Weight =200lb*
  - *Flow Volume = 20ft3*




Determine the head loss.

- Mass (change in diameter)
- Energy (flow loss--filter)
- Momentum (external forces)

# ENERGY/MOMENTUM/MASS

$$\frac{p_{in}}{\gamma}+\frac{V_{in}^2}{2g}+z_{in}=\frac{p_{out}}{\gamma}+\frac{V_{out}^2}{2g}+z_{out}+Losses$$

$$h_l=\frac{p_{in}-p_{out}}{\rho g}+\frac{V_{in}^2-V_{out}^2}{2g};\ p_{out}=0;\ Free\ \text{Jet}$$

$$=\frac{p_{in}}{\rho g}+\frac{V_{in}^2-V_{out}^2}{2g}$$

*Momentum*

$$\sum F_y=\cancel{\frac{dM_{CV}}{dt}}+\sum_{out}(v_{out}\pm)\dot{m}-\sum_{in}(v_{in}\pm)\dot{m}$$

$$\uparrow\sum F_y=-Ry+p_{in}A_{in}+p_{out}A_{out}\sin\Theta-W=0+\left(-V_{out}\sin\Theta-V_{in}\right)\dot{m}$$

$W$ = Weight of pipe and weight of fluid

$$\text{W}=200lbf+20\,ft^3\bullet\gamma_f\frac{lbf}{ft^3}=200+20*62.4=1448lbf$$

Mass Cons

$$V_{out}=\frac{A_{in}}{A_{out}}V_{in}=\frac{0.12\,ft^2}{0.10\,ft^2}10\frac{ft}{s}=12\frac{ft}{s}$$

# ENERGY/MOMENTUM/MASS

$$h_l = \frac{p_{in}}{\rho g} + \frac{V_{in}^2 - V_{out}^2}{2g}$$

$$\uparrow \sum F_y = -Ry + p_{in} A_{in} - W = 0 + \left(-V_{out} \sin\Theta - V_{in}\right)\dot{m}$$

$$p_{in} = \frac{\left(-V_{out} \sin\Theta - V_{in}\right)\dot{m} + Ry + W}{A_{in}}$$

$$V_{out} = \frac{A_{in}}{A_{out}} V_{in} \rightarrow \text{MASS CONS}$$

$$p_{in} = \frac{\left(-\frac{A_{in}}{A_{out}} V_{in} \sin\Theta - V_{in}\right)\rho A_{in} V_{in} + Ry + W}{A_{in}}$$

$$= \frac{Ry + W - \rho A_{in} V_{in}^2 \left(\frac{A_{in}}{A_{out}} \sin\Theta + 1\right)}{A_{in}}$$

$$\frac{60[lbf] + W[lbf] - 1.94\frac{slugs}{ft^3} 0.12 ft^2 100\frac{ft^2}{s^2}\left(\frac{0.12}{0.10}\sin 30 + 1\right)\left[\frac{slugs - ft}{s^2} = \frac{\frac{lbf - s^2}{ft} - ft}{s^2}\right]}{0.12\left[ft^2\right]}$$

$$p_{in} = 189.6\frac{lb_f}{ft^2} + \frac{W lb_f}{0.12 ft^2}$$

$$p_{in} = 189.6\frac{lb_f}{ft^2} + \frac{W lb_f}{0.12\, ft^2}$$

$$h_l = \frac{p_{in}}{\rho g} + \frac{V_{in}^2 - V_{out}^2}{2g}$$

$$= \frac{189.6\frac{lb_f}{ft^2} + \frac{W lb_f}{0.12\, ft^2}}{62.4\frac{lb_f}{ft^3}} + \frac{V_{in}^2\left(1-\left(\frac{A_{in}}{A_{out}}\right)^2\right)}{2x32.2\frac{ft}{s^2}}$$

$$= 2.35\, ft + \frac{\frac{W lb_f}{0.12\, ft^2}}{62.4\frac{lb_f}{ft^3}}; \text{ENERGY LOSS}$$

FLOW POWER LOST DUE TO "$h_L$"

$$\dot{P}_{LOST}[hp] = \dot{m}\left[\frac{slugs}{s}\right] g\left[\frac{ft}{s^2}\right] h_L\,[ft]$$

$$= \dot{m}\left[\frac{slugs}{s} = \frac{\frac{lbf - s^2}{ft}}{s}\right] g\left[\frac{ft}{s^2}\right] h_L\,[ft]$$

$$= \dot{m}\left[\frac{slugs}{s}\right] g\left[\frac{ft}{s^2}\right] h_L\,[ft] \rightarrow [lbf - ft/s] \bullet \frac{1hp}{550}\, lbf - ft/s$$

$$\dot{P}_{LOST}[W] = \dot{m}\left[\frac{kg}{s}\right] g\left[\frac{m}{s^2}\right] h_L\,[m]$$

$$\frac{kg - m}{s^2} \equiv N$$

$$\dot{P}_{LOST}[W] = \dot{m}\left[\frac{kg}{s}\right] g\left[\frac{m}{s^2}\right] h_L\,[m] \rightarrow \frac{N - m}{s}$$

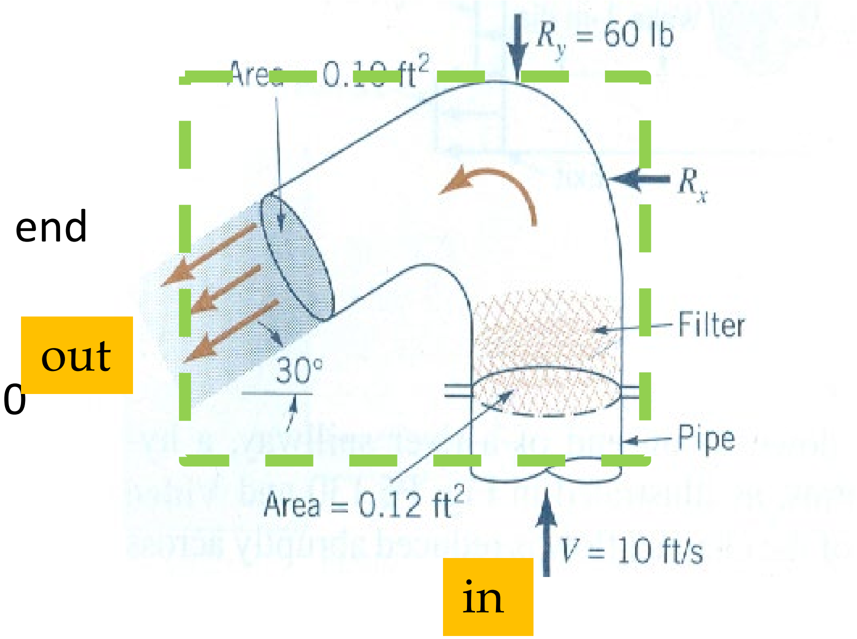
$$N - m \equiv J$$

$$J/s \equiv W$$

# PARAMETRIC MODELLING
## *"The Language of Engineers"*

Water, flows steadily in a pipe and exits through an end cap that contains a filter as shown.

The pipe weight is 200 $lb_f$ and the flow volume is 20 $ft^3$. The axial component, Ry, is 60lb, and the flow head loss is 15 ft-$lb_f$/slug

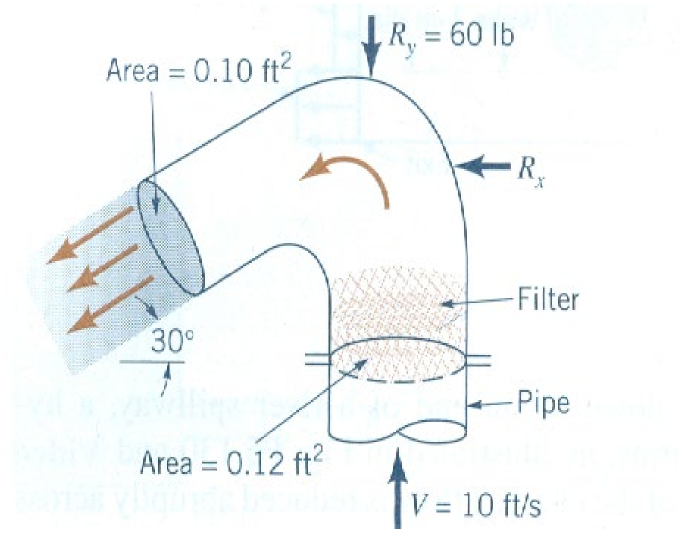



$$(1\ slug = \frac{lb_f - s^2}{ft})$$

- What is the magnitude and direction of the total anchoring forces (lbf).
- Provide a plot with derivation for Rx vs Theta, for theta going from $30^0$ to $90^0$ in increments of 5 degrees.

FOUR Equations, FOUR Unknows, $P_{in}$, $P_{out}$, $R_x$, $V_{out}$.
LAWS:
Momentum X
Momentum Y
Mass Conservation
Energy Conservation

# ENERGY




$Energy \rightarrow EQUATION\ 1$

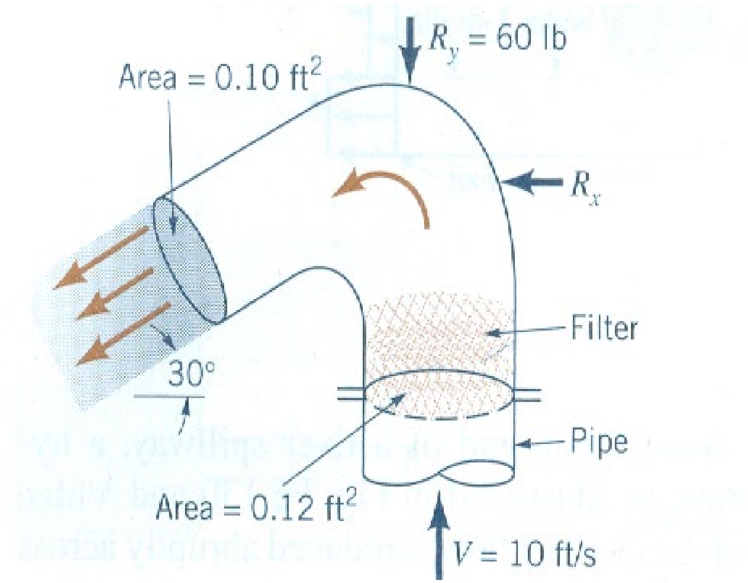
$$h_l = 15\frac{ft-lbf}{slug} = \frac{15\,\frac{ft-lbf}{slug}}{32.2\,ft/s^2} = \frac{15\,\frac{\frac{ft-lbf}{lbf-\sec^2}}{ft}}{32.2\,ft/s^2} = 0.46\,ft$$

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} = \frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} + Losses$$

$$h_l = \frac{p_{in} - p_{out}}{\rho g} + \frac{V_{in}^2 - V_{out}^2}{2g};\ p_{out} \neq Free\ \text{Jet}$$

$$\frac{p_{in}}{\gamma} - \frac{p_{out}}{\gamma} = \left[-\left(\frac{V_{in}^2 - V_{out}^2}{2g}\right) + h_l\right]$$

# MOMENTUM Y/MASS




*Momentum → EQUATION 2*

$$\sum F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out} \pm)\dot{m} - \sum_{in}(v_{in} \pm)\dot{m}$$

$$\uparrow \sum F_y = -Ry + p_{in}A_{in} + p_{out}A_{out}\sin\Theta - W = \left(-V_{out}\sin\Theta - V_{in}\right)\dot{m}$$

$W$ = Weight of pipe and weight of fluid

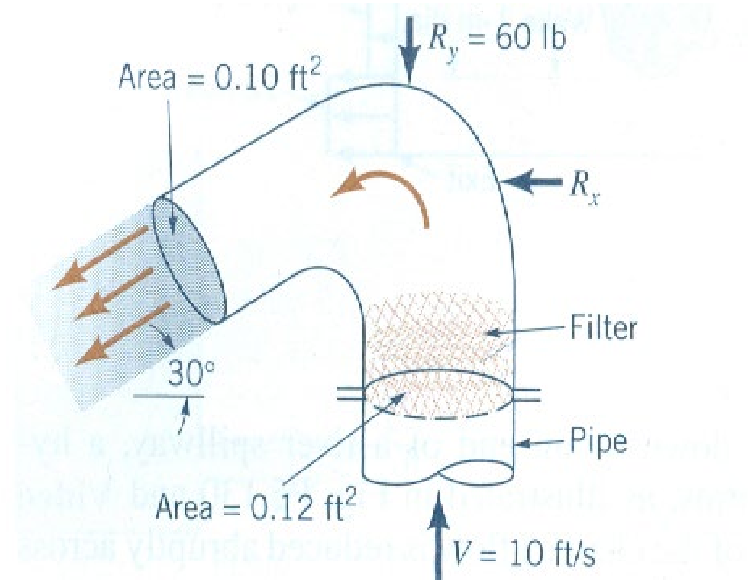
$$W = 200lbf + 20\,ft^3 \bullet \gamma_f \frac{lbf}{ft^3} = 200 + 20 * 62.4 = 1448lbf$$

$$\dot{m} = \rho Q$$

*MASS →*

$$V_{out} = \frac{A_{in}}{A_{out}} V_{in} = \frac{0.12\,ft^2}{0.10\,ft^2} 10\frac{ft}{s} = 12\frac{ft}{s}$$

# MOMENTUM X

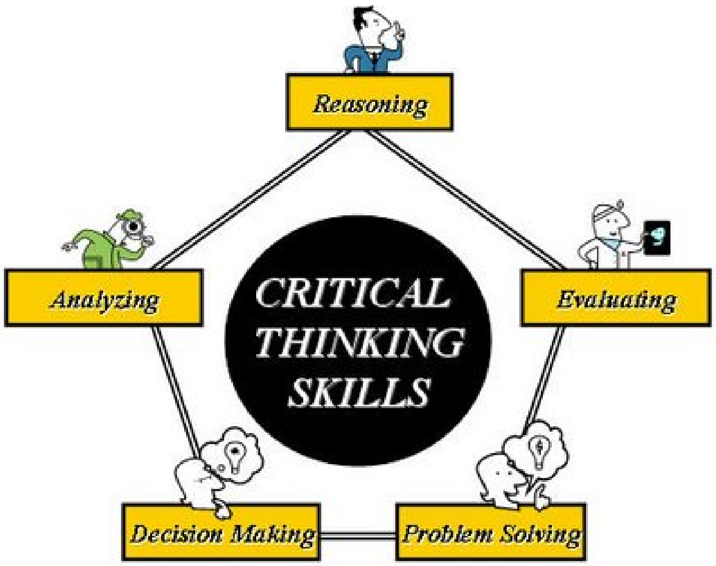


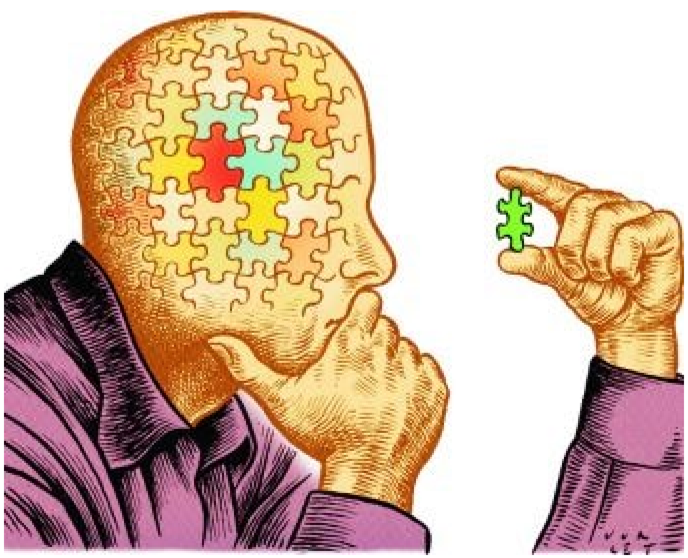

$$Momentum \rightarrow EQUATION\ 3$$

$$\sum F_x = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(u_{out} \pm)\dot{m} - \sum_{in}(u_{in} \pm)\dot{m}$$

$$\rightarrow \sum F_x = -Rx + p_{out} A_{out} \cos\Theta = \left(-V_{out} \cos\Theta\right)\dot{m} + 0$$

Three Equations, Three Unknows, Pin, Pout, Rx.



*Energy → EQUATION 1*

$$h_l = 15\frac{ft-lbf}{slug} = \frac{15\frac{ft-lbf}{slug}}{32.2\,ft/s^2} = \frac{15\frac{\frac{ft-lbf}{lbf-\sec^2}}{ft}}{32.2\,ft/s^2} = 0.46\,ft$$

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$$h_l = \frac{p_{in}-p_{out}}{\rho g}+\frac{V_{in}^2-V_{out}^2}{2g};\ p_{out} \neq Free\ \text{Jet}$$

$$\frac{\mathrm{p}_{in}}{\gamma}-\frac{p_{out}}{\gamma} = \left[-\left(\frac{V_{in}^2-V_{out}^2}{2g}\right)+h_l\right]$$

*Momentum → EQUATION 2*

$$\sum F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out}\pm)\dot{m} - \sum_{in}(v_{in}\pm)\dot{m}$$

$$\uparrow \sum F_y = -Ry + p_{in}A_{in} + p_{out}A_{out}\sin\Theta - W = \left(-V_{out}\sin\Theta - V_{in}\right)\dot{m}$$

$W$ = Weight of pipe and weight of fluid

$$\mathrm{W}=200lbf + 20\,ft^3 \bullet \gamma_f \frac{lbf}{ft^3} = 200+20*62.4 = 1448lbf$$

$$\dot{m} = \rho Q$$

*Mass*

$$V_{out} = \frac{A_{in}}{A_{out}}V_{in} = \frac{0.12\,ft^2}{0.10\,ft^2}10\frac{ft}{s} = 12\frac{ft}{s}$$

*Momentum → EQUATION 3*

$$\sum F_x = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(u_{out}\pm)\dot{m} - \sum_{in}(u_{in}\pm)\dot{m}$$

$$\rightarrow \sum F_x = -Rx + p_{out}A_{out}\cos\Theta = \left(-V_{out}\cos\Theta\right)\dot{m} + 0$$



Three Equations, Three Unknows, Pin, Pout, Rx.

# EQUATION SUMMARY

$$Energy \rightarrow EQUATION\ 1$$

$$\frac{p_{in}}{\gamma} - \frac{p_{out}}{\gamma} = \left[ -\left( \frac{V_{in}^2 - V_{out}^2}{2g} \right) + h_l \right]$$

$$Momentum \rightarrow EQUATION\ 2$$

$$p_{in} A_{in} + p_{out} A_{out} \sin\Theta - W = Ry + \left( -V_{out} \sin\Theta - V_{in} \right) \dot{m}$$

$$Momentum \rightarrow EQUATION\ 3$$

$$-Rx + p_{out} A_{out} \cos\Theta = \left( -V_{out} \cos\Theta \right) \dot{m} + 0$$

Three Equations, Three Unknows, Pin, Pout, Rx

MATRIX FORMAT

$$\begin{bmatrix} a_{11}dof_1 & a_{12}dof_2 & a_{13}dof_3 \\ a_{21}dof_1 & a_{22}dof_2 & a_{23}dof_3 \\ a_{31}dof_1 & a_{32}dof_2 & a_{33}dof_3 \end{bmatrix} \begin{Bmatrix} dof_1 \\ dof_2 \\ dof_3 \end{Bmatrix} = \begin{Bmatrix} Force_1 \\ Force_2 \\ Force_3 \end{Bmatrix}$$

→ EQUATION 1 EXAMPLE

$$\frac{p_{in}}{\gamma} - \frac{p_{out}}{\gamma} + 0 = h_l - \frac{V_{in}^2 - V_{out}^2}{2g}$$

$$\begin{bmatrix} \frac{1}{\gamma} & -\frac{1}{\gamma} & 0 \end{bmatrix} \begin{Bmatrix} p_{in} \\ p_{out} \\ R_x \end{Bmatrix} = \left\{ Force_1 = h_l - \frac{V_{in}^2 - V_{out}^2}{2g} \right\}$$

$$[A]\{x\} = \{b\}$$

$$\{x\} = [A]^{-1}\{b\}$$

# NUMERICAL METHODS

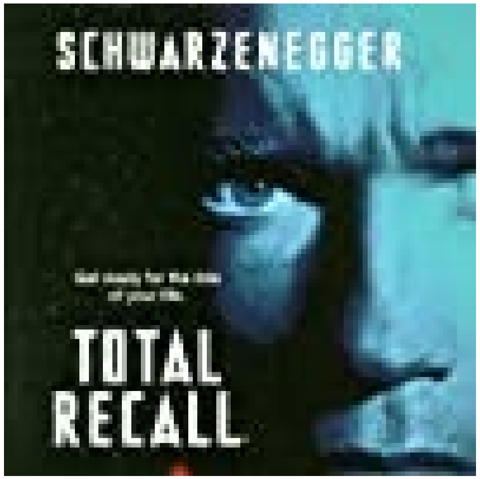

ELEGANT SOLUTION

MATRIX SOLUTION → NUMERICAL METHODS

$$[A]\{x\} = \{b\}$$

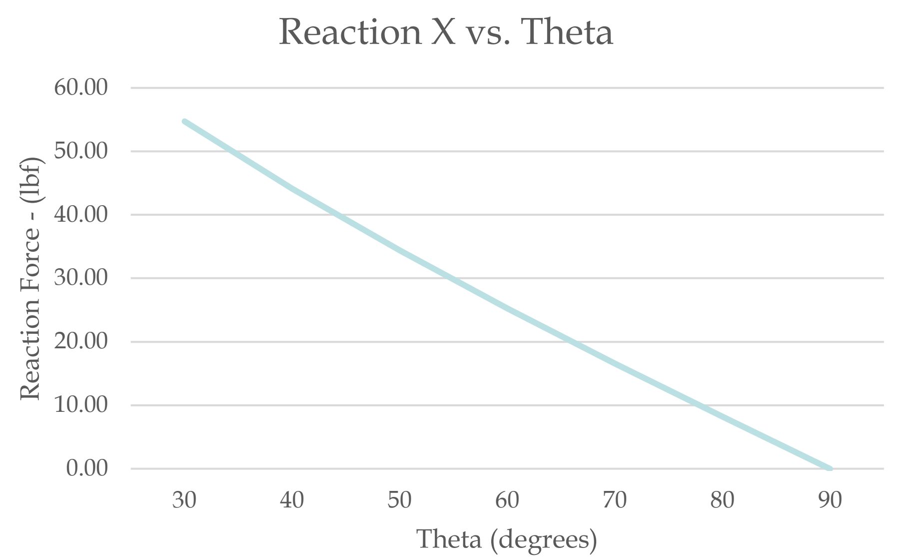
$$\{x\} = [A]^{-1}\{b\}$$

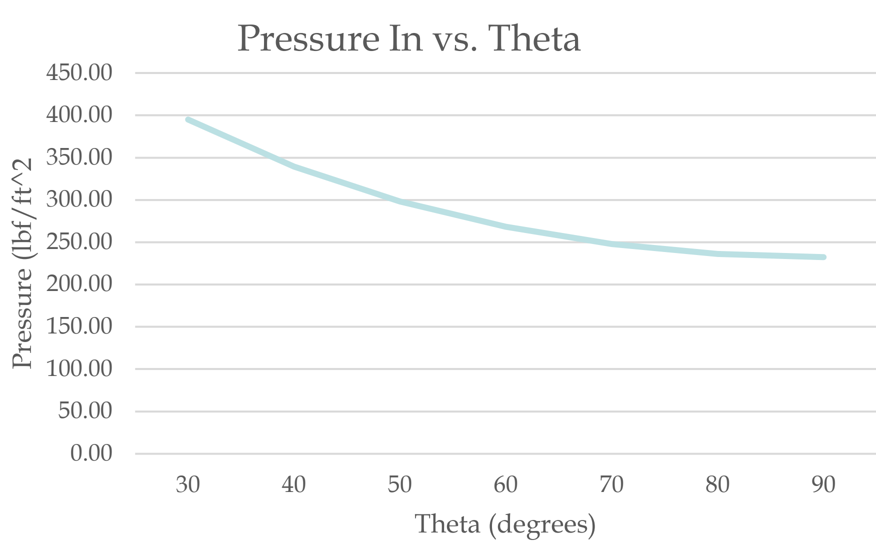
$$\rightarrow \begin{Bmatrix} EQUATION\ 1 \\ EQUATION\ 2 \\ EQUATION\ 3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{\gamma} & \frac{-1}{\gamma} & 0 \\ A_{in} & A_{out}\sin\Theta & 0 \\ 0 & A_{out}\cos\Theta & -1 \end{bmatrix} \begin{Bmatrix} P_{in} \\ P_{out} \\ R_x \end{Bmatrix} = \begin{Bmatrix} h_l + \left(\frac{-V_{in}^2 + V_{out}^2}{2g}\right) \\ R_y + W - \dot{m}(V_{out}\sin\Theta + V_{in}) \\ -\dot{m}(V_{out}\cos\Theta) \end{Bmatrix}$$

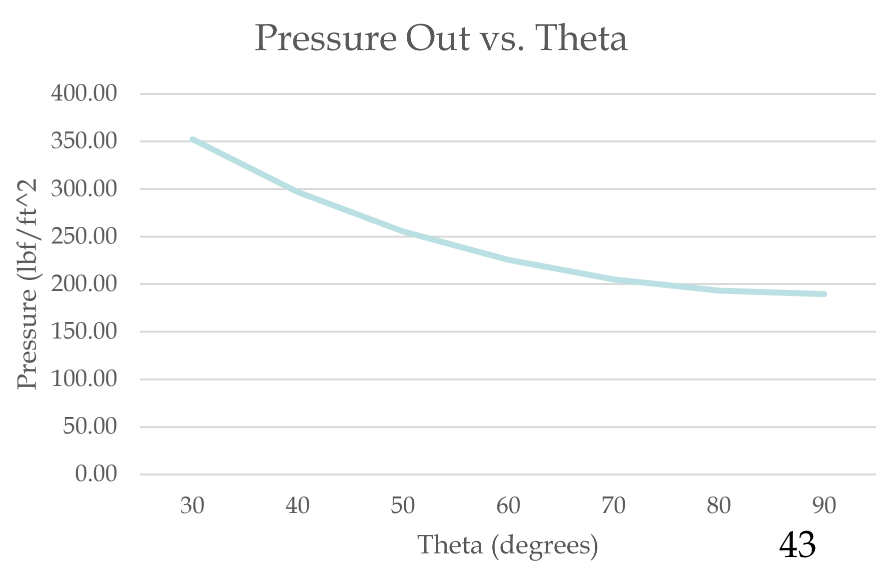



| Angle (degrees) | Angle (radians) | Pin (lbf/ft^2) | Pout (lbf/ft^2) | Rx (lb) |
|---|---|---|---|---|
| 30 | 0.52 | 395.12 | 352.49 | 54.69 |
| 40 | 0.70 | 339.58 | 296.94 | 44.12 |
| 50 | 0.87 | 298.12 | 255.49 | 34.36 |
| 60 | 1.05 | 268.13 | 225.49 | 25.23 |
| 70 | 1.22 | 247.82 | 205.19 | 16.56 |
| 80 | 1.40 | 236.06 | 193.43 | 8.20 |
| 90 | 1.57 | 232.21 | 189.57 | 0.00 |

| Given Values | | |
|---|---|---|
| Vin | 10 | f/s |
| Vout | 12 | f/s |
| Ain | 0.12 | ft^2 |
| Aout | 0.1 | ft^2 |
| Ry | 60 | lb |
| γf | 62.4 | lbf/ft^3 |
| Weight | 200 | lb |
| ∀ | 20 | ft^3 |
| g | 32.2 | ft/s^2 |

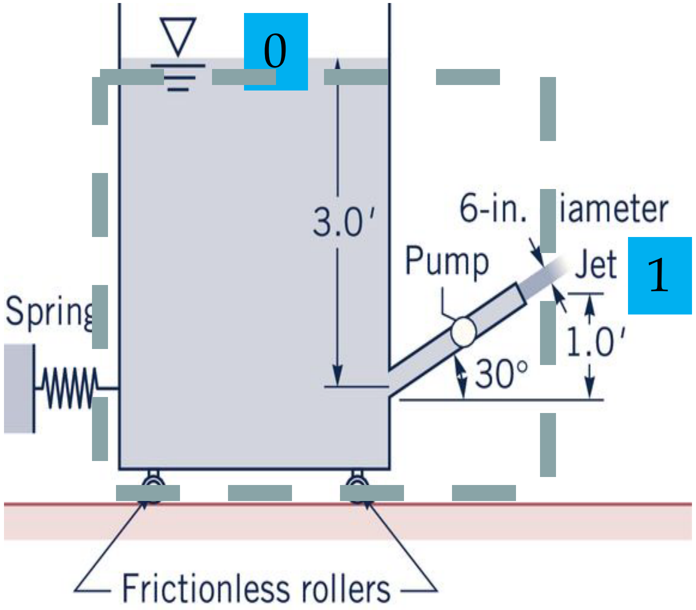


# Energy/Momentum/Mass

- The large water tank is evacuated by a pump with a volume flow rate of 5 ft3/sec with a pump efficiency of 85%.
  - Flow Head Loss$=3.0\frac{V_{jet}^2}{2g}$
  - Find pump work and spring deflection with linear spring deflection K=200lbf/ft.




$$\dot{W}_{p_{IDEAL}} = \dot{m}gh_{p_{IDEAL}} = \rho\left(AV\right)gh_{p_{IDEAL}} = \gamma Qh_{p_{IDEAL}}$$

$$\eta_p = \frac{W_{p_{IDEAL}}}{W_{P_{ACTUAL}}}$$

$$W_{P_{ACTUAL}} = \frac{W_{p_{IDEAL}}}{\eta_p} = \frac{\gamma Qh_{p_{IDEAL}}}{\eta_p}$$

# Solution

*Mass*

$$\frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} + h_{pump_{IDEAL}} = h_{turbine_{IDEAL}} + \frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} + Losses$$

$$V_{jet} = \frac{Q}{A} = 25.5\frac{ft}{s}$$

*Energy*

$$p_1 = p_2 = V_1 = 0, z_1 = 3, z_2 = 1;$$

$$h_{p_{IDEAL}} = \frac{V_{jet}^2}{2g} - 2 + h_l = \frac{V_{jet}^2}{2g}(1+3) - 2 = 38.4 ft$$

$$W_{p_{ACTUAL}} = \frac{\gamma Q h_{p_{IDEAL}}}{\eta_p} = 62.4\frac{lbf}{ft^3} \bullet 5\frac{ft^3}{s} \bullet 38.4 ft \bullet \frac{1HP}{550\frac{ft-lbf}{s}}\frac{1}{\eta_p} = 25.6HP$$

*Momemntum*

$$\sum \vec{F}_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out} \pm)\dot{m} - \sum_{in}(u_{in} \pm)\dot{m}$$

$$\sum \vec{F}_x = K\Delta = 0 + V_{jet}\cos\theta\dot{m} - 0$$

$$\Delta = \frac{V_{jet}\cos\theta\dot{m}}{K} = \frac{\rho Q^2\cos\theta}{A_{jet}K} = 1.07 ft$$

How to include static friction between wheels and ground?

i.e.

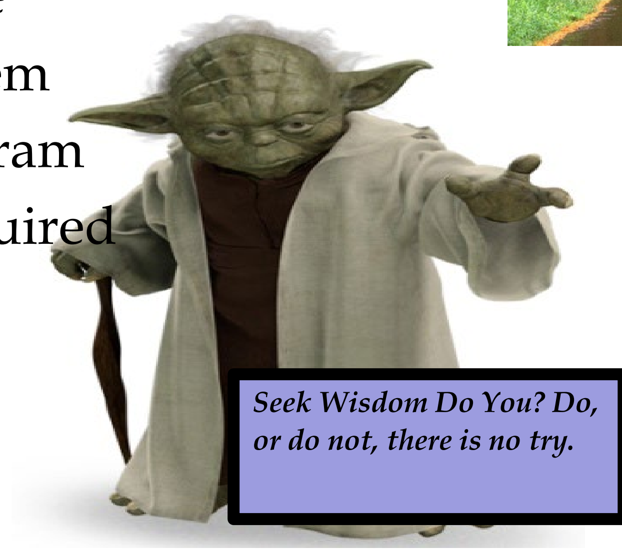
$F_n = \mu_s N \rightarrow$ always opposite to motion

N= Normal Force

# EXTENDED MODEL w/FRICTION

# Road Map

- Identify Points Along Streamline
- Manometry
- Conservation of Energy + Mass
- Conservation of Momentum
- Friction Loss
- Control Volume
- Coordinate Sytem
- Free Body Diagram
- Combine if Required



# Build Parametric Model to Vary Angle and Friction Coefficient

*A large water tank of diameter* $D = 10'$ *and height* $H = 5'$ *is evacuated by a pump (efficiency of* $\eta_p = 80\%$*). The flow head loss through the pipe jet at the exit is, the spring force is* 300 *lbf, and the coefficient of friction between the wheels and the surface is* 0.3. Jet flow loss is $5.0\dfrac{V_j^2}{2g}$

**Apply Conservation of Energy**

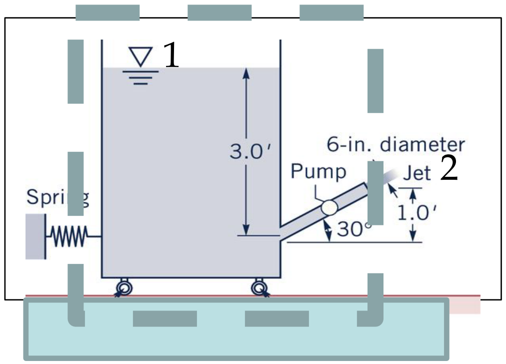
$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_{p_{IDEAL}} = \cancel{h_{T_{IDEAL}}}+\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

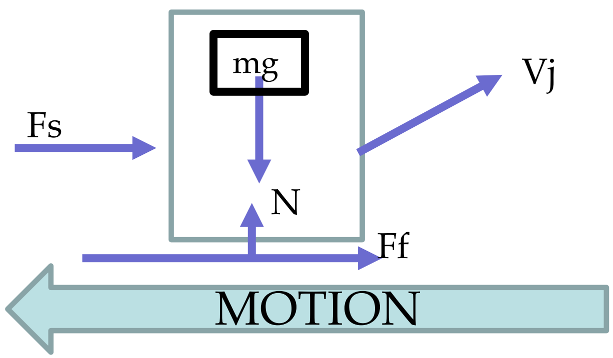
$$P_1 = P_2 = V_1 = 0$$

$$h_{p_{IDEAL}} = \left(\frac{V_2^2}{2g}+z_2-z_1+h_L\right)$$

$$h_{p_{IDEAL}} = \left(\frac{V_2^2}{2g}-2.0+5.0\frac{V_2^2}{2g}\right) = \left(6.0\frac{V_2^2}{2g}-2.0\right)$$

$$\dot{W}_{p_{ACTUAL}} = \frac{\gamma_f Q h_{p_{IDEAL}}}{\eta_p} = \frac{\gamma_f (V_2 A) h_{p_{IDEAL}}}{\eta_p}$$



# Apply Momentum

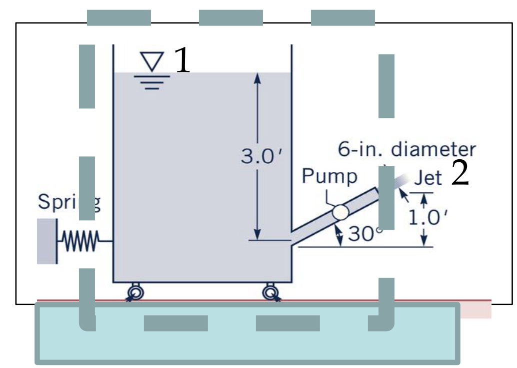
$$\sum \overrightarrow{F_x} = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(u_{out}\pm)\dot{m}_{out} - \sum_{in}(u_{in}\pm)\dot{m}_{in}$$

$$\sum F_x = F_s + F_f = 0 + (V_2\cos\theta +)\dot{m}_2 - 0$$

$$\dot{m}_2 = \rho A_2 V_2$$

$$\sum F_x = F_s + F_f = \rho A V_2^2 \cos\theta$$

$$\sum F_x = F_s + \mu_s N = \rho A V_2^2 \cos\theta$$

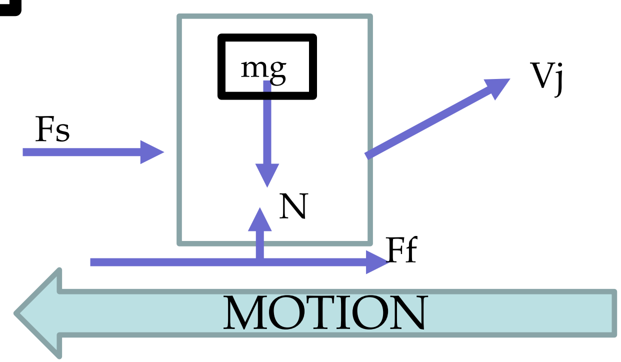



$$\uparrow \sum F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out}\pm)\dot{m}_{out} - \sum_{in}(v_{in}\pm)\dot{m}_{in}$$

$$\uparrow \sum F_y = N - mg = 0 + (V_2\sin\theta +)\dot{m}_2 - 0$$

$$\uparrow \sum F_y = N - mg = \rho A V_2^2 \sin\theta$$

$$N = mg + \rho A V_2^2 \sin\theta$$



# COMBINE

50

$$\rightarrow \sum F_x = F_s + \mu_s N = \rho A V_2^2 \cos\theta : (1)$$

$$\uparrow \sum F_y = N - mg = \rho A V_2^2 \sin\theta : (2)$$

$$N = mg + \rho A V_2^2 \sin\theta \rightarrow (1)$$

$$F_s + \mu_s (mg + \rho A V_2^2 \sin\theta) = \rho A V_2^2 \cos\theta$$

$$F_s + \mu_s mg = \rho A V_2^2 (\cos\theta - \mu_s \sin\theta)$$

Solve for $V_2$

$$V_2 = \left[ \frac{F_s + \mu_s (mg)}{\rho A (\cos\theta - \mu_s \sin\theta)} \right]^{1/2}$$

$$V_2 = \left[ \frac{F_s + \mu_s (\gamma_f \forall_{vol})}{\rho A (\cos\theta - \mu_s \sin\theta)} \right]^{1/2}$$

| MU | | | | | |
|---|---|---|---|---|---|
| 0.3 | | | | | |
| | | | | | |
| | ft/s | ft3/s | ft | ft-lbf/s | HP |
| Theta | V2 | Q | hp | Wp | Wp |
| 30 | 167.49 | 32.89 | 2611.62 | 6,699,183 | 12,180 |
| 35 | 176.19 | 34.59 | 2890.10 | 7,798,486 | 14,179 |
| 40 | 187.20 | 36.76 | 3262.81 | 9,354,315 | 17,008 |
| 45 | 201.45 | 39.55 | 3778.83 | 11,658,484 | 21,197 |
| 50 | 220.54 | 43.30 | 4529.55 | 15,299,241 | 27,817 |
| 55 | 247.53 | 48.60 | 5706.47 | 21,633,101 | 39,333 |
| 60 | 289.18 | 56.78 | 7789.30 | 34,498,043 | 62,724 |
| 65 | 365.05 | 71.68 | 12413.94 | 69,405,046 | 126,191 |
| 70 | 578.05 | 113.50 | 31129.43 | 275,589,099 | 501,071 |

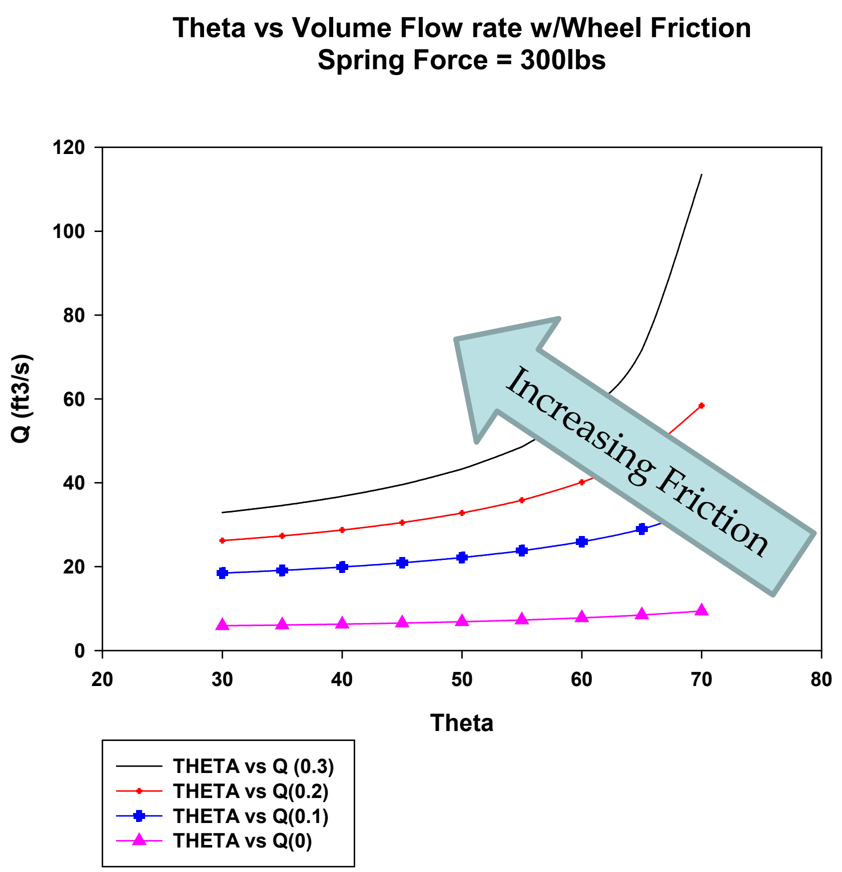
*Analysis:*

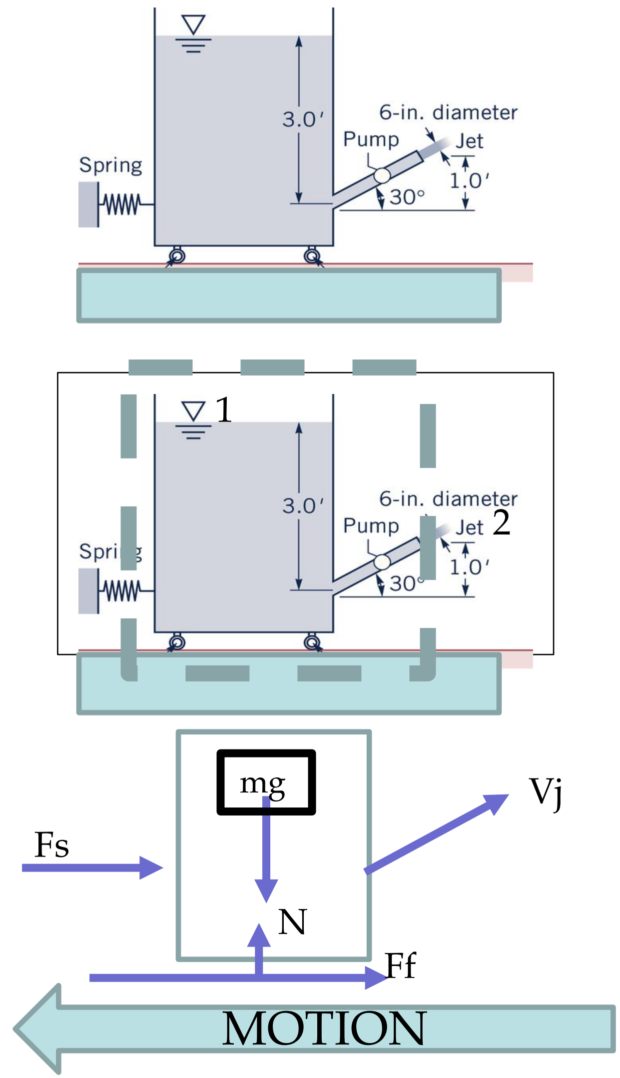
*Large pumping power is required due to large external reaction force of 300lbs combined with the wheel friction force which is a function of coefficient of friction and water volume.*

*The X momentum jet exit force must balance the X spring and friction forces. As theta increases, the X momentum force must decrease (via $\cos\theta$ ), as such the jet velocity must increase to compensate (via $V^2$). And therefore, the pumping power must also increase with theta.*

*Learning points from parametric analysis and thought.*

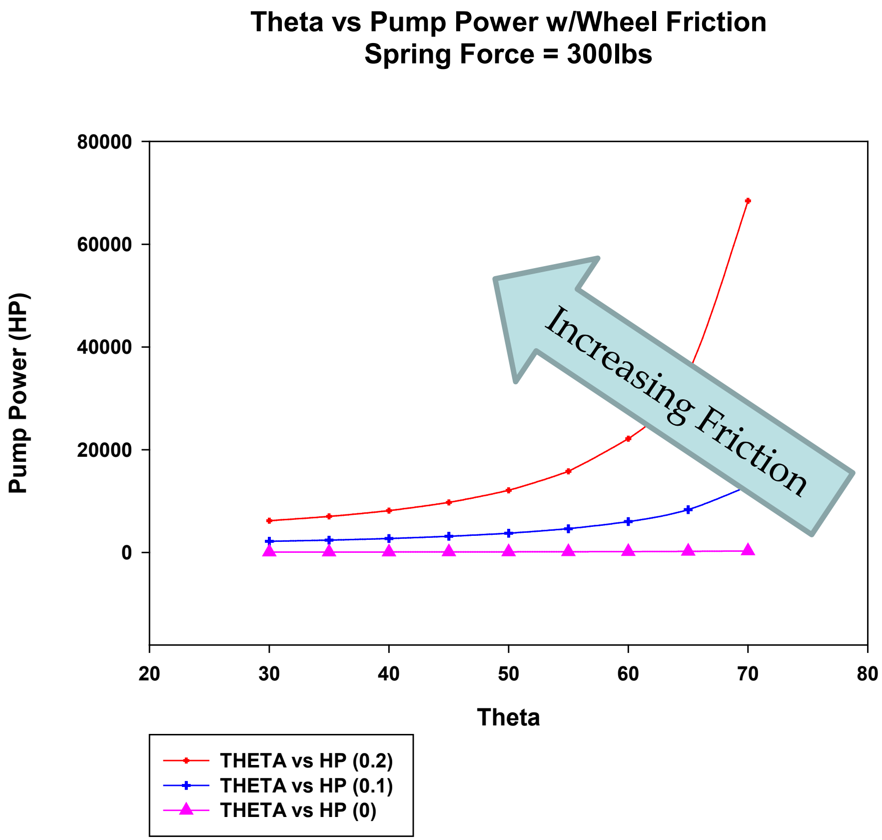
# Parametric Model: Tank Jet



# Parametric Model: Tank Jet

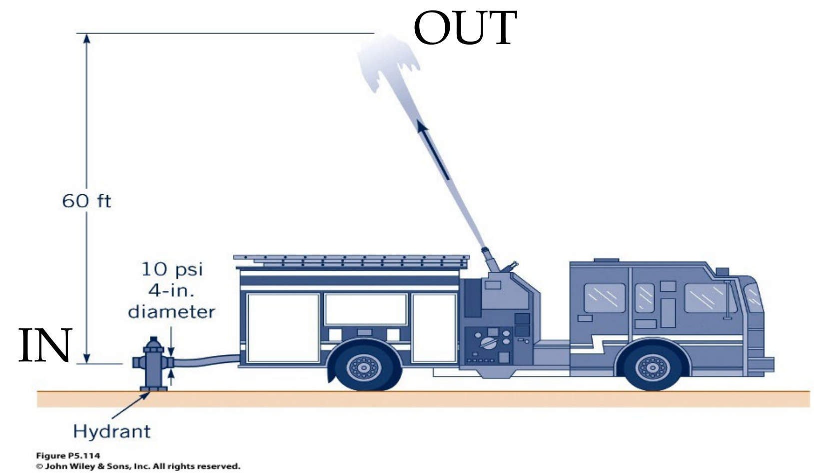





| | | | | | |
|---|---|---|---|---|---|
| MU | | | | | |
| 0.2 | | | | | |
| | | | | | |
| | ft/s | ft3/s | ft | ft-lbf/s | HP |
| Theta | V2 | Q | hp | Wp | Wp |
| 30 | 133.51 | 26.21 | 1658.61 | 3,391,326 | 6,166 |
| 35 | 139.22 | 27.34 | 1803.80 | 3,846,036 | 6,993 |
| 40 | 146.35 | 28.74 | 1993.44 | 4,468,019 | 8,124 |
| 45 | 155.36 | 30.50 | 2246.72 | 5,345,747 | 9,720 |
| 50 | 167.00 | 32.79 | 2596.29 | 6,640,317 | 12,073 |
| 55 | 182.54 | 35.84 | 3102.53 | 8,673,707 | 15,770 |
| 60 | 204.40 | 40.13 | 3890.55 | 12,179,237 | 22,144 |
| 65 | 237.84 | 46.70 | 5268.48 | 19,191,171 | 34,893 |
| 70 | 297.68 | 58.45 | 8253.76 | 37,628,946 | 68,416 |

| | | | | | |
|---|---|---|---|---|---|
| MU | | | | | |
| 0 | | | | | |
| | | | | | |
| | ft/s | ft3/s | ft | ft-lbf/s | HP |
| Theta | V2 | Q | hp | Wp | Wp |
| 30 | 30.16 | 5.92 | 82.73 | 38,207.97 | 69.47 |
| 35 | 31.01 | 6.09 | 87.58 | 41,588.28 | 75.62 |
| 40 | 32.06 | 6.30 | 93.79 | 46,055.26 | 83.74 |
| 45 | 33.37 | 6.55 | 101.77 | 52,016.90 | 94.58 |
| 50 | 35.00 | 6.87 | 112.15 | 60,123.78 | 109.32 |
| 55 | 37.06 | 7.28 | 125.93 | 71,465.01 | 129.94 |
| 60 | 39.69 | 7.79 | 144.75 | 87,985.09 | 159.97 |
| 65 | 43.17 | 8.48 | 171.62 | 113,466.85 | 206.30 |
| 70 | 47.99 | 9.42 | 212.54 | 156,198.72 | 284.00 |

Truck delivers Q=1.5cfs to an elevation of 60ft above hydrant. The pressure in 4″ Diameter outlet is 10 psi. If flow losses are small, find pump power input that must be added to the water if pump eff = 80%.




Fluid Fundamentals

Mass Conservation: Q=VA

Must apply energy equation due to work/power along streamline.

Energy Equation (Single Input/Output)

$$h_{P_{IDEAL}} = \cancel{h_{T\,IDEAL}} + h_{out} - h_{in} + \cancel{h_{L_{FLOW}}} \quad (head - ft:m)$$

$$\dot{W}_{Pump_{IDEAL}} = h_{P_{IDEAL}}\dot{m}g, \dot{W}_{Turbine_{IDEAL}} = h_{T\,IDEAL}\dot{m}g$$

$$h_{out} = \cancel{\frac{p_{out}}{\gamma}} + \cancel{\frac{V_{out}^2}{2g}} + z_{out}$$

$$h_{in} = \frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in}$$

$$h_{P_{IDEAL}} = z_{out} - z_{in} - \frac{p_{in}}{\gamma} - \frac{V_{in}^2}{2g}$$

$$= 60\,ft - \frac{10\,psi\,\frac{144\,ft^2}{in^2}}{62.4\,\frac{lbf}{ft^3}} - \frac{\left(\frac{Q}{A_{in}}\right)^2}{2g} = 32.3\,ft \rightarrow \text{input pump work}$$

$$\eta_{Pump} = \frac{\dot{W}_{P_{IDEAL-IN}}}{\dot{W}_{P_{ACTUAL-IN}}}$$

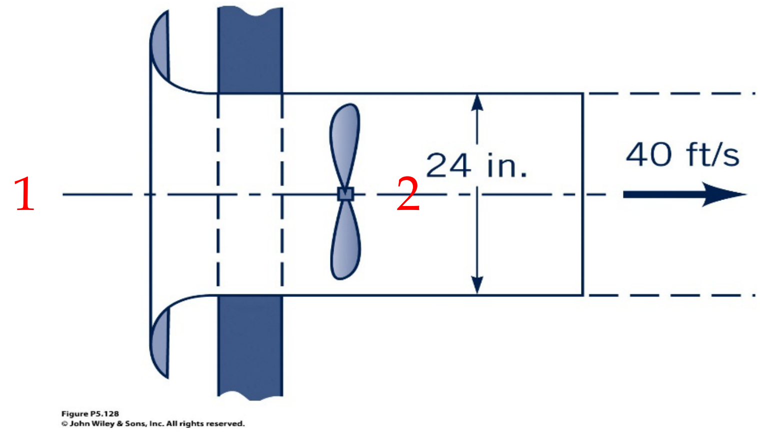
$$\dot{W}_{P_{ACTUAL-IN}} = \frac{\dot{W}_{P_{IDEAL-IN}}}{\eta_{Pump}} = \frac{\dot{m}gh_{P_{IDEAL}} = \gamma_{H20}Qh_{P_{IDEAL}}}{\eta_{Pump}}$$

$$= \frac{62.4\frac{lbf}{ft^3}1.5\frac{ft^3}{s}32.3\,ft}{0.80} = 3779\frac{ft-lbf}{s}$$

$$= 3779\frac{ft-lbf}{s}\frac{1HP}{550\frac{ft-lbf}{s}} = 6.48HP$$

# Energy Conservation + Momentum

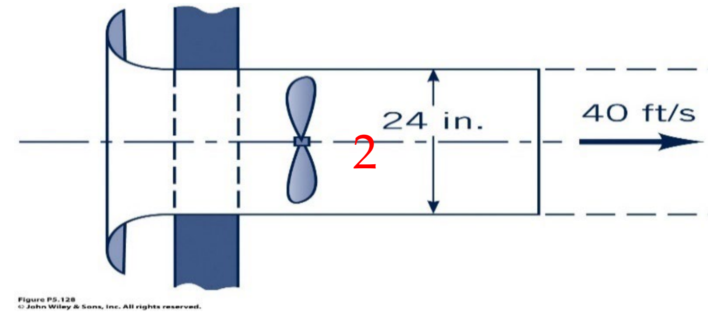
- Ventilation air at 530R and 14.7 psia uses a 3/4hp fan to produce steady air velocity of 40ft/s in a 24″ duct, find maximum fan efficiency and thrust of air on supporting infrastructure.




Fluid Type – IDEAL GAS AIR

Mass: NO
Manometer: NO
Energy: YES
Momentum: YES

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+\cancel{z_1}+h_{p_{IDEAL}}=\cancel{h_{T_{IDEAL}}}+\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+\cancel{z_2}+h_l$$



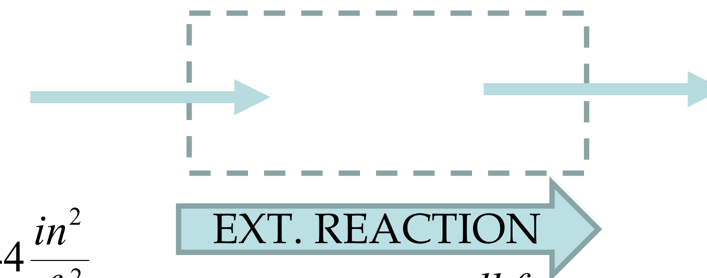

Maximum Fan Efficiency; $h_l = 0$

$$P_1 \approx P_2, V_1 \approx 0$$

$$h_{p_{IDEAL}} = \frac{V_2^2}{2g} \rightarrow \text{Fan work is converted to air kinetic energy}$$

$$=24.8\text{ft}$$

$$\dot{W}_{Ideal} = \gamma Q h_{p_{IDEAL}}; \eta_{fan} = \frac{\dot{W}_{ideal}}{\dot{W}_{actual}}$$




$$\gamma = \rho g = \frac{P_{abs}}{R_{air}T_{abs}} g = \frac{14.7\,psia \bullet 144\frac{in^2}{ft^2}}{1716.49\frac{ft-lbf}{slug-R}\bullet 530R} 32.2\,ft/s^2 = 0.075\frac{lbf}{ft^3}$$

$$Q = AV = 126\frac{ft^3}{s}, A = \frac{\pi D^2}{4}$$

$$\dot{W}_{Ideal} = \gamma Q h_{p_{IDEAL}} \frac{hp}{550\frac{ft-lbf}{s}} = 0.426hp; \eta_{fan} = \frac{\dot{W}_{ideal}}{\dot{W}_{actual}}$$

$$\eta_{fan} = \frac{\dot{W}_{ideal}}{\dot{W}_{actual}} = \frac{0.426}{0.75} = 0.56 \rightarrow 56\%$$

**MOMENTUM**

$$\rightarrow + \sum_x F_x = V_2 \dot{m} = V_2 \rho Q$$

GENERAL ENERGY EQUATION--MULTIPLE I/O STREAMS

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in} (\dot{m}g(\frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1)) = \sum_{out} (\dot{m}g(\frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2)) + \sum h_L [m] \dot{m}_{out} g; W \text{ or } ft - lbf / s;$$

$LET \rightarrow$

UNITS: WATTS or FT-LBF/s

$$\dot{W}_{s_{IDEAL}} = \dot{W}_{Turbine_{IDEAL}} - \dot{W}_{Pump_{IDEAL}};$$

$H_L [Watts] = \dot{m}_B (kg / s)(g) h_{L_{A-B}} (m) = \dot{m}_B g \left(h_q + h_{\text{minor}} + h_{\text{major}}\right) \rightarrow$ Total SYSTEM Losses; $\rightarrow$ OR

$H_{L_{A-B}} [ft - lbf / \sec] = \dot{m}_B (slugs / s) g h_{A-B} (ft)$

(one INLET/one EXIT) $\rightarrow$

Energy Equation $\rightarrow$ "m;ft" $\rightarrow (\div \dot{m}g)$

$$\frac{\dot{Q}_{cs}}{\dot{m}g} + \frac{\dot{W}_{Pump_{IDEAL}}}{\dot{m}g} + \frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{\dot{W}_{Turbine_{IDEAL}}}{\dot{m}g} + \frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2 + h_q(m); units = m, \text{or, ft}$$

$$\text{h}_p + h_1 = h_T + h_2 + h_q$$

UNITS: m or ft

$h_{\text{minor}}(m) = \sum_i K_i \frac{V_i^2}{2g}; \rightarrow$ Component Losses

$h_{\text{major}}(m) = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g}; \rightarrow$ Straight Pipe Section Losses

$h_q(m) = \frac{u_2}{g} - \frac{u_1}{g} - \frac{\dot{Q}_{cs}}{\dot{m}g} + h_{L_{A-B}}; \rightarrow$ Thermal Losses

# Where are we in the term?