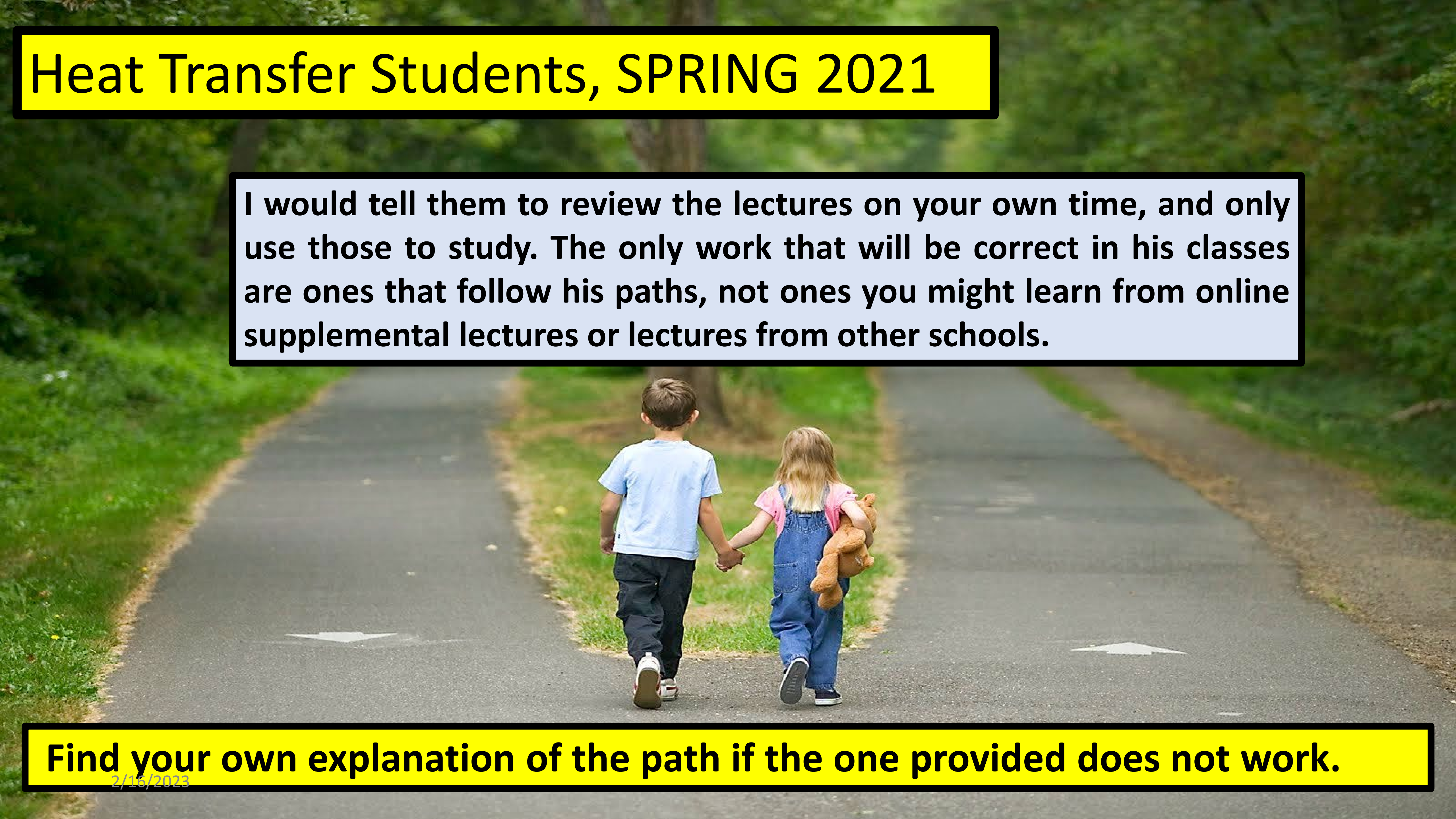
# Conservation of Mass

# The Continuity Equation

MECH-322 FLUID MECHANICS

# Heat Transfer Students, SPRING 2021

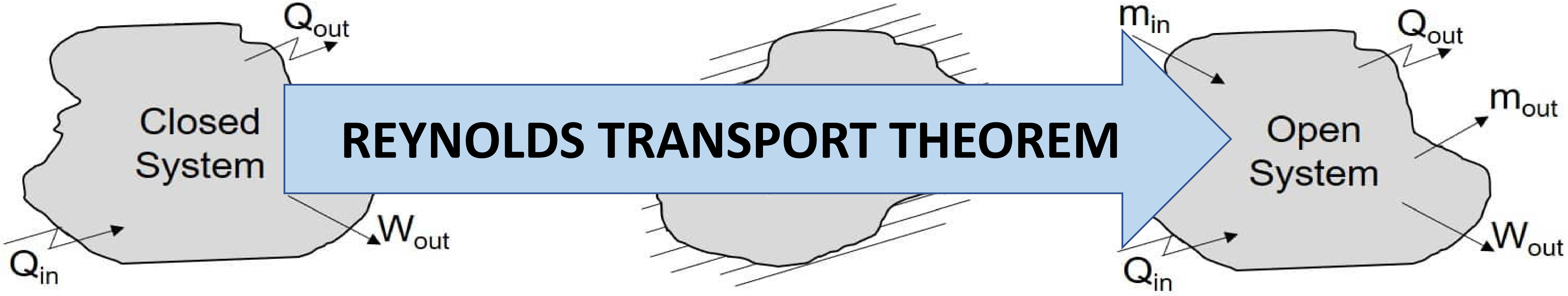


**I would tell them to review the lectures on your own time, and only use those to study. The only work that will be correct in his classes are ones that follow his paths, not ones you might learn from online supplemental lectures or lectures from other schools.**

**Find your own explanation of the path if the one provided does not work.**

A thermodynamic system is defined as **a quantity of matter or a region in space that is of interest**. The mass or region outside the system is called the surroundings, and the surface that separates the system and the surroundings is called the boundary.

- Thermodynamic System
  - Control mass system
    - Closed system
    - Isolated system
  - Control volume system
    - Open system



# REYNOLDS TRANSPORT THEOREM

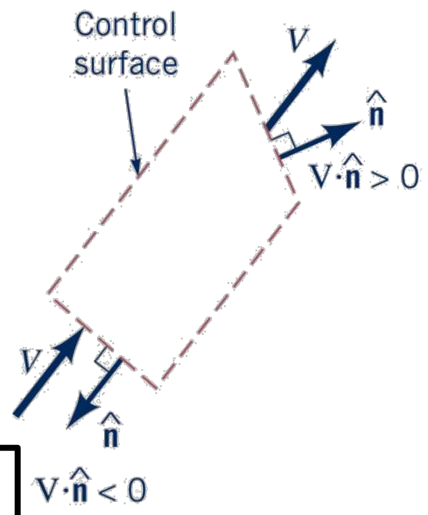
$$\frac{DB}{Dt}_{SYSTEM} \equiv \frac{\partial}{\partial t}\left[\int_{CV}\left(\rho\left[\frac{kg}{vol}\right]b\left[\frac{prop}{kg}\right]\right)d\forall\right] + \int_{CS}\rho b\left(\vec{V}\bullet\hat{n}\right)dA$$

$\frac{DB}{Dt}_{SYSTEM} \equiv$ *SYSTEM* Time rate of change of extensive parameter (e.g. B)

$\frac{\partial}{\partial t}\left[\int_{CV}\left(\rho\left[\frac{kg}{vol}\right]b\left[\frac{prop}{kg}\right]\right)d\forall\right] \equiv$ Rate of storage of B within CONTROL VOLUME

$\int_{CS}\rho b\left(\vec{V}\bullet\hat{n}\right)dA \equiv$ *NET* TRANSPORT of B accross the entire control surface

# CONTINUITY




- Mass Conservation: "TIME RATE OF CHANGE OF SYSTEM MASS = 0"

$$\frac{DM}{Dt}_{SYSTEM} = 0;$$
$$M = system\ \mathrm{MASS} = DENSITY(kg/m3)\ x\ VOLUME(m3)$$
$$M_{sys} = \int_{sys} \rho d\forall$$

- REYNOLDS TRANSPORT THM.

$$\frac{DM}{Dt}_{SYSTEM} \equiv 0 = \frac{\partial}{\partial t}\left[\int_{CV}\left(\rho\left[\frac{kg}{vol}\right]\right)d\forall\right] + \int_{CS}\rho\left(\vec{V}\bullet\hat{n}\right)dA$$

- Constant Properties & Flow

$\dot{m} = \rho A V_n = \rho Q$ : MASS FLOW RATE (mass/time)

$V_n$ = NORMAL VELOCITY @ SURFACE

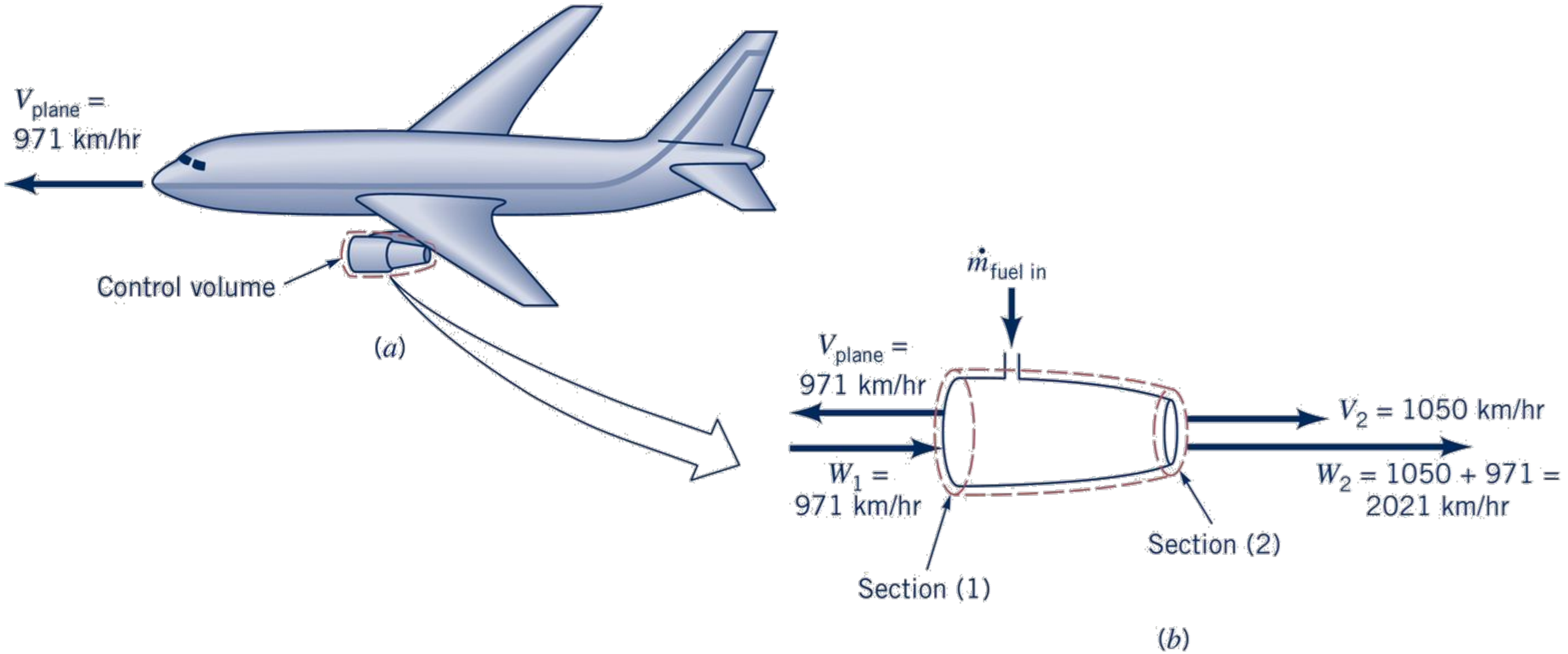
$$0 = \frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in}$$

# Example

- Find mass flow rate of fuel into engine- Steady Flow.

$$Area-Inlet = 0.80m^2, \rho_{air} = 0.736kg/m^3,$$

$$Area-Exit = 0.558m^2, \rho_{exhaust} = 0.515kg/m^3$$



# SOLUTION

Intake Velocity, W1 $\equiv$ relative to moving control volume

Exhaust Vedlocity, W2 $\equiv$ relative to moving control volume

$$\frac{D}{Dt}\int_{sys}\rho d\forall = \frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall = 0 \rightarrow STEADY$$

$$(\rho A W)_2 - (\rho A W)_1 = 0$$

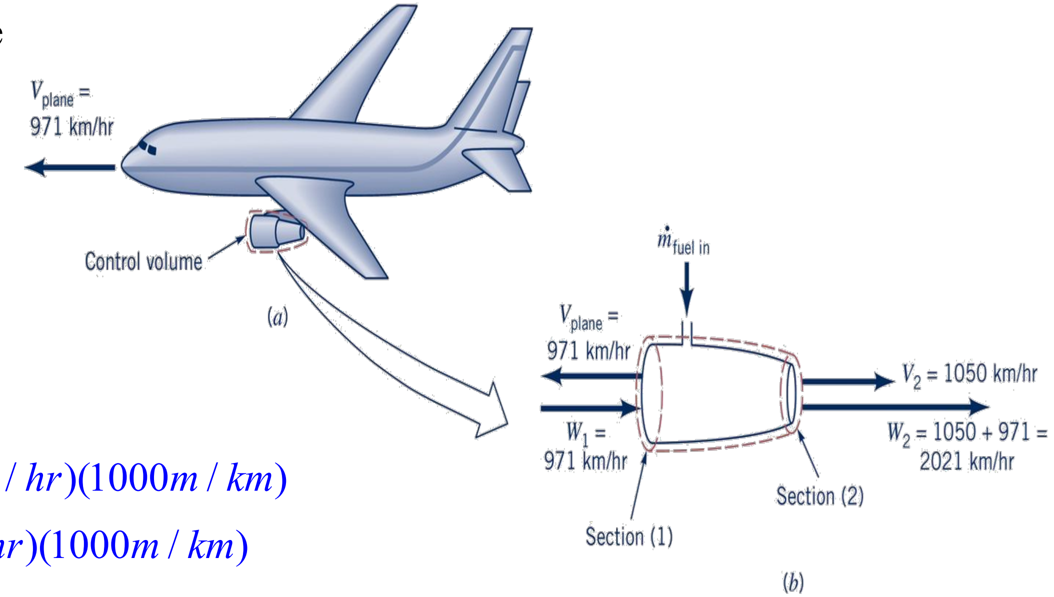
$$\rho_2 A_2 W_2 - \rho_1 A_1 W_1 - \dot{m}_{fuel} = 0$$

$$\dot{m}_{fuel} = (0.515 kg/m3)(0.558 m2)(2021 km/hr)(1000 m/km)$$

$$-(0.736 kg/m3)(0.80 m2)(971 km/hr)(1000 m/km)$$

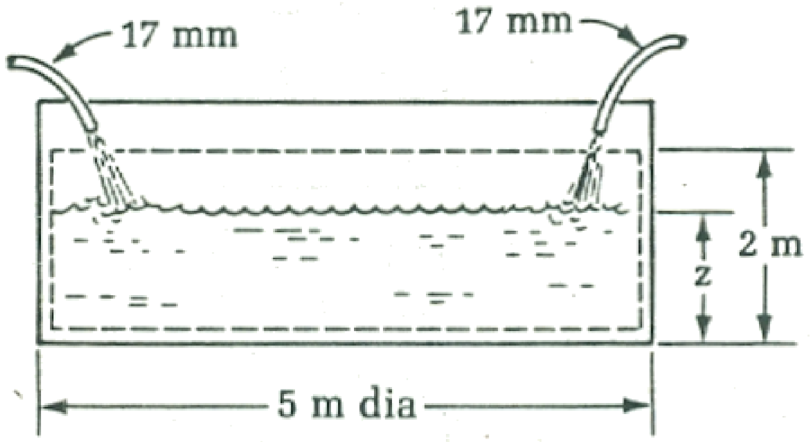
$$= (580{,}800 - 517{,}700) kg/hr$$

$$= 9100 kg/hr$$



# Example

- Circular Pool with two hoses with fill rates of 2 m/s & 1.5 m/s.
- Find time required to fill pool to a depth of 2 m.
- Solution: **Note mass of control volume is changing with time.**



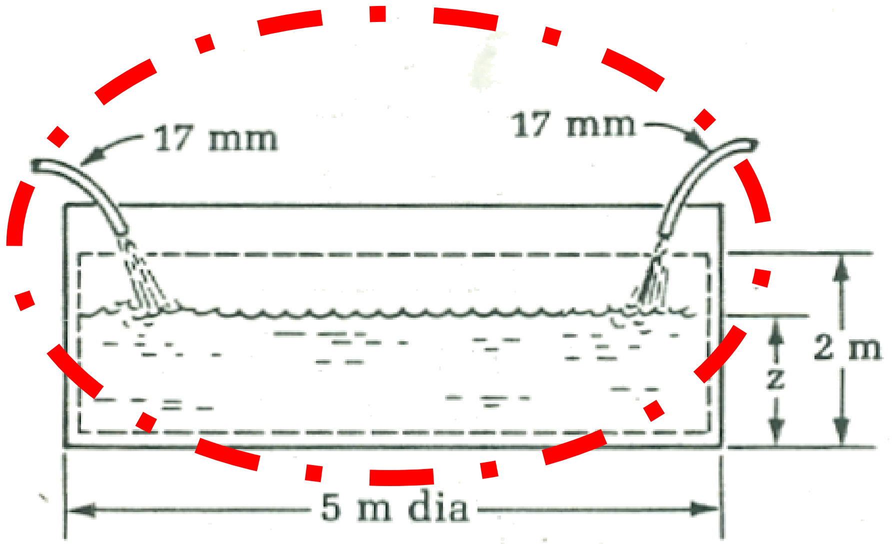
# SOLUTION

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

$$m(z(t)) = mass = \rho(kg/m3)\forall(m3) =$$

$$= 1000 kg/m^3 \left(\frac{\pi D^2}{4} m^2\right) \bullet z(t) m$$

$m(z(t)) = 19{,}600 \bullet z(t) kg$, constant density




$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall = \frac{d}{dt}(\rho V) = \frac{d}{dt}(19{,}600z) = 19{,}600\frac{dz}{dt}$$

$$\sum \dot{m}_{out} = 0$$

$$\sum \dot{m}_{in}(t) = \dot{m}_1(t) + \dot{m}_2(t) = \rho(A_1V_1(t) + A_2V_2(t))$$

$$19{,}600\frac{dz}{dt} = \sum \dot{m}_{in}(t)$$

Separate Variables and Integrate

First Order O.D.E.

$$a\frac{dy}{dt} = b(t)$$

$$19{,}600\int_0^{z^*} dz = \int_0^{t^*}\left(\sum \dot{m}_{in}\right)dt = \int_0^{t^*}\rho(A_1V_1(t) + A_2V_2(t))dt$$

$$19{,}600(z^*) = \rho(A_1V_1 + A_2V_2)t^*$$

so for a known "$z^*$", we can find the time as:

$$t^* = \frac{19{,}600(z^*)}{\rho(A_1V_1 + A_2V_2)}, z^* = 2m$$

*SOLVE*

$$t^* = 49{,}300s$$

NOTE: Velocity may be a function of time.

i.e. $V(t) = a_0 e^{-\alpha t}$

i.e. $V(t) = c_0 \sin(\omega t)$

If so, would need to integrate velocity function over time interval.

# Examples of Time Varying Velocity Fields

i.e. $\mathrm{V(t)=a}_0 e^{-\alpha t}$

$$\int_{t_1}^{t_2} \mathrm{a}_0 e^{-\alpha t} dt = \frac{\mathrm{a}_0}{-\alpha}\left[e^{-\alpha t}\right]_{t_1 \to t_2} = \frac{\mathrm{a}_0}{-\alpha}\left[e^{-\alpha t_2} - e^{-\alpha t_1}\right]$$

$i.e.\mathrm{V(t)=c}_0 \sin(\omega t)$

$$\int_{t_1}^{t_2} \mathrm{c}_0 \sin(\omega t) dt = -\frac{\mathrm{c}_0}{\omega}\left[\cos(\omega t)\right]_{t_1 \to t_2} = -\frac{\mathrm{c}_0}{\omega}[\cos(\omega t_2) - \cos(\omega t_1)]$$

# Example

Oxygen is supplied via tanks 30 cm diameter and 1.3m tall at 13,800 kPa. The exhaust valve is 12.5 mm in diameter and the exit velocity is 1.5 m/s at a constant temperature of 25C. FIND TANK PRESSURE AFTER 60S.

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

$$\sum \dot{m}_{in} = 0$$

$IDEAL - GAS$

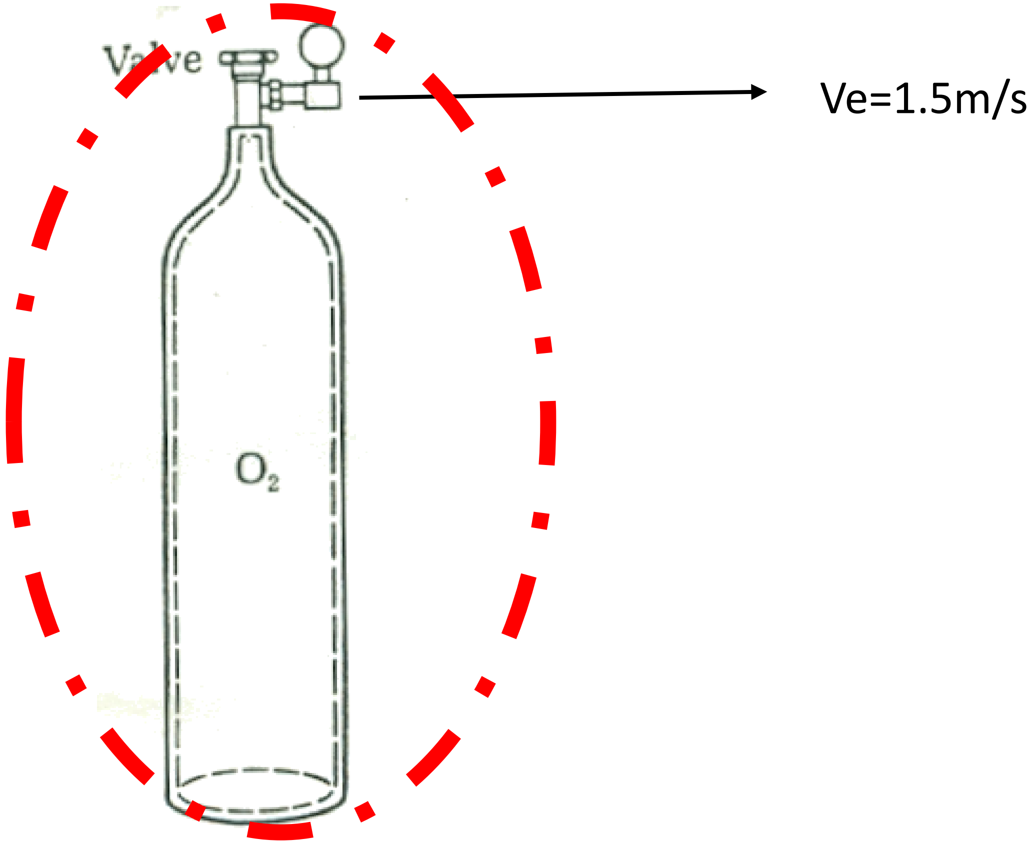
$$\rho(t) = \frac{P(t)}{RT}$$

$$m = \rho \bullet \forall$$

$$\forall = Area\ x\ Height$$

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall = \frac{D}{Dt}(mass = \rho\forall) = \frac{\forall}{RT}\frac{DP}{Dt}$$

$$\frac{\forall}{RT}\frac{DP}{Dt} = -\sum \dot{m}_{out} = -\rho A_e V_e = -A_e V_e \frac{P(t)}{RT}$$



# FINAL SOLUTION

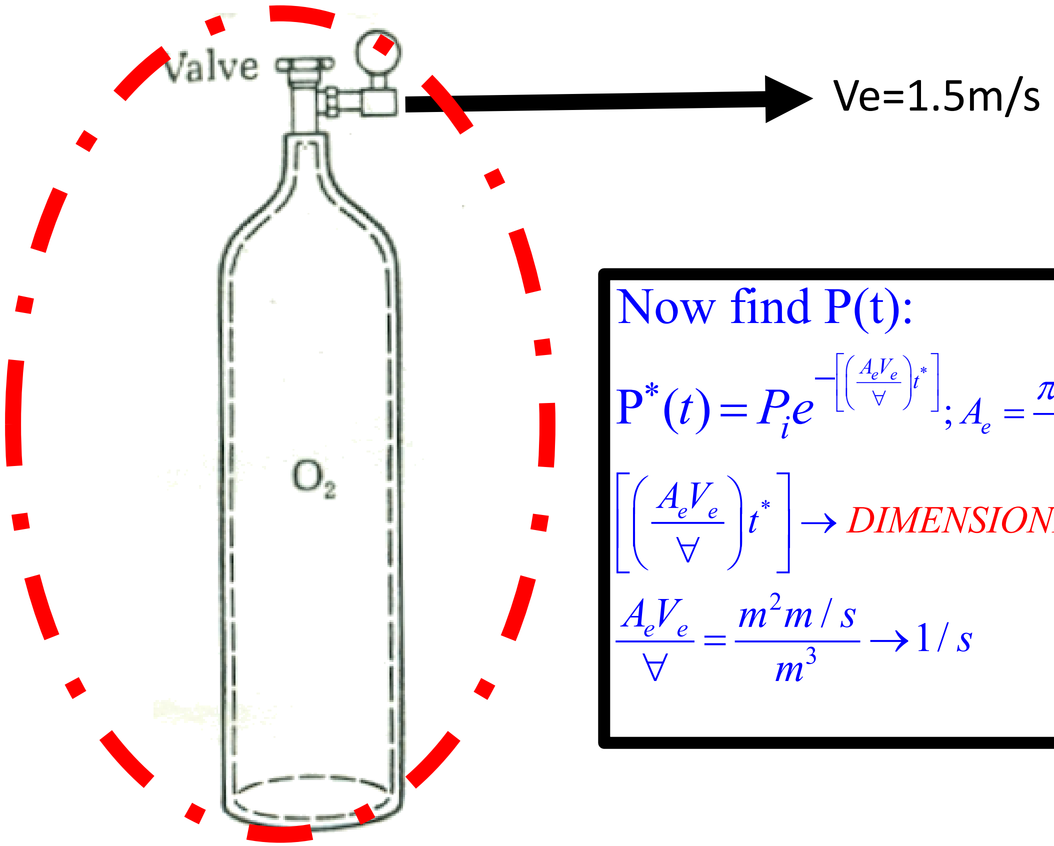
$$\frac{\forall}{RT}\frac{dP}{dt} = -\sum \dot{m}_{out} = -\rho A_e V_e = -A_e V_e \frac{P(t)}{RT}$$

Cancel, Separate Variables, Integrate

$$\int_{P_i}^{P^*(t)} \frac{dP}{P} = \frac{-A_e V_e}{\forall} \int_0^{t^*} dt$$

$$\ln\left(\frac{P^*(t)}{P_i}\right) = \frac{-A_e V_e}{\forall} t^*$$

$$-\ln\left[\frac{P^*(t)}{P_i}\right]\frac{\forall}{A_e V_e} = t^*(P^*);\text{(Is time negative ?)}$$




Now find P(t):

$$P^*(t) = P_i e^{-\left[\left(\frac{A_e V_e}{\forall}\right)t^*\right]};\ A_e = \frac{\pi D_e^2}{4}$$

$$\left[\left(\frac{A_e V_e}{\forall}\right)t^*\right] \rightarrow \textit{DIMENSIONLESS}\ \text{TIME CONSTANT}$$

$$\frac{A_e V_e}{\forall} = \frac{m^2 m/s}{m^3} \rightarrow 1/s$$

Air at STP enters the compressor as shown. Determine a: Rate (slugs/s) at which the mass of air in the tank is increasing/decreasing; b: Average time rate of change of air density within the tank.

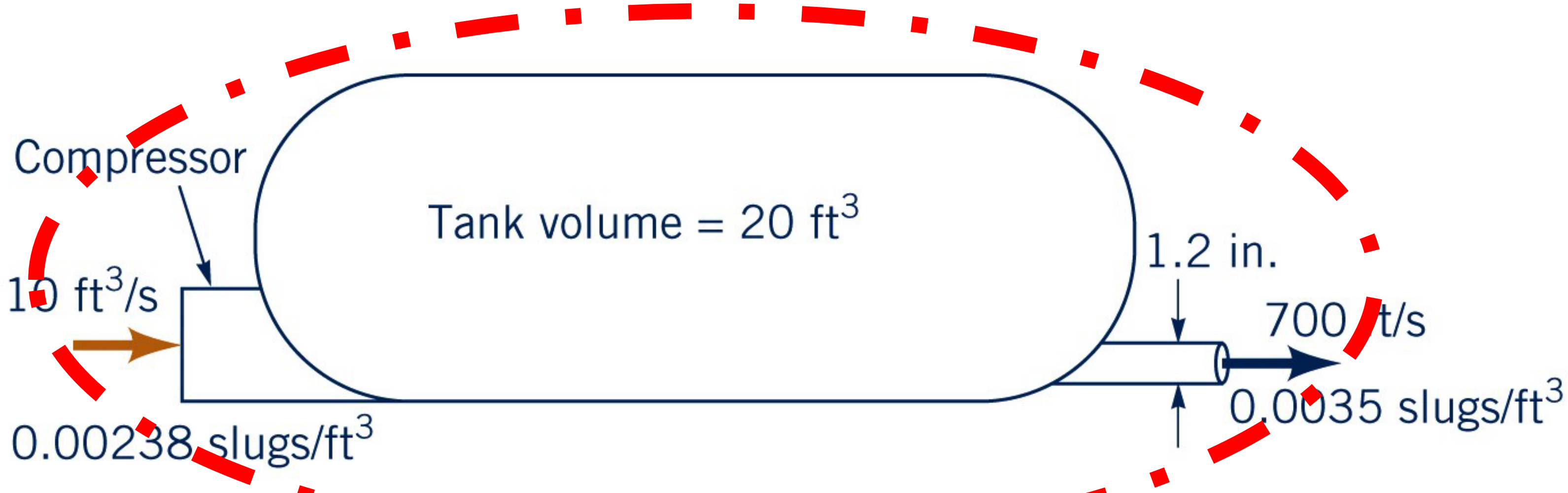


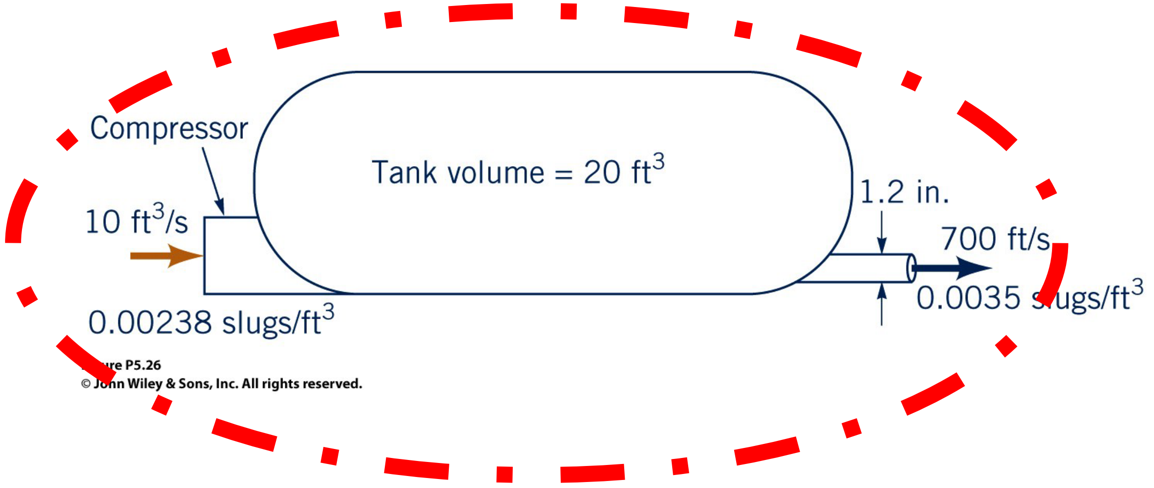

Figure P5.26



$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

# Determine a: Rate (slugs/s) at which the mass of air in the tank is increasing/decreasing;

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$




$$\frac{D}{Dt}(\rho\forall) = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\frac{D}{Dt}(mass) = \rho_{in}Q_{in} - \rho_{out}Q_{out}$$

$$= 0.00238\frac{slugs}{ft^3}10\frac{ft^3}{s} - 0.0035\frac{slugs}{ft^3}(V_{out}A_{out})$$

$$= 0.00238\frac{slugs}{ft^3}10\frac{ft^3}{s} - 0.0035\frac{slugs}{ft^3}(700\frac{ft}{s}\pi\frac{D_{out}^2}{4})$$

$$= 0.00456\frac{slugs}{s} \uparrow \text{(increasing)}$$

# b: Average time rate of change of air density within the tank

$$\frac{\partial}{\partial t}\int_{cv} \rho d\forall = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

Water is Incompressible

$$\frac{D}{Dt}(\rho V_{sys}) = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$V_{sys}\frac{D}{Dt}(\rho) = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\frac{D}{Dt}(\rho) = \frac{\sum \dot{m}_{in} - \sum \dot{m}_{out}}{V_{sys}}$$

$$= \frac{0.00456\,\frac{slugs}{s}}{20\,ft^3}$$

$$= 2.28x10^{-4}\,\frac{slugs/ft^3}{s}$$

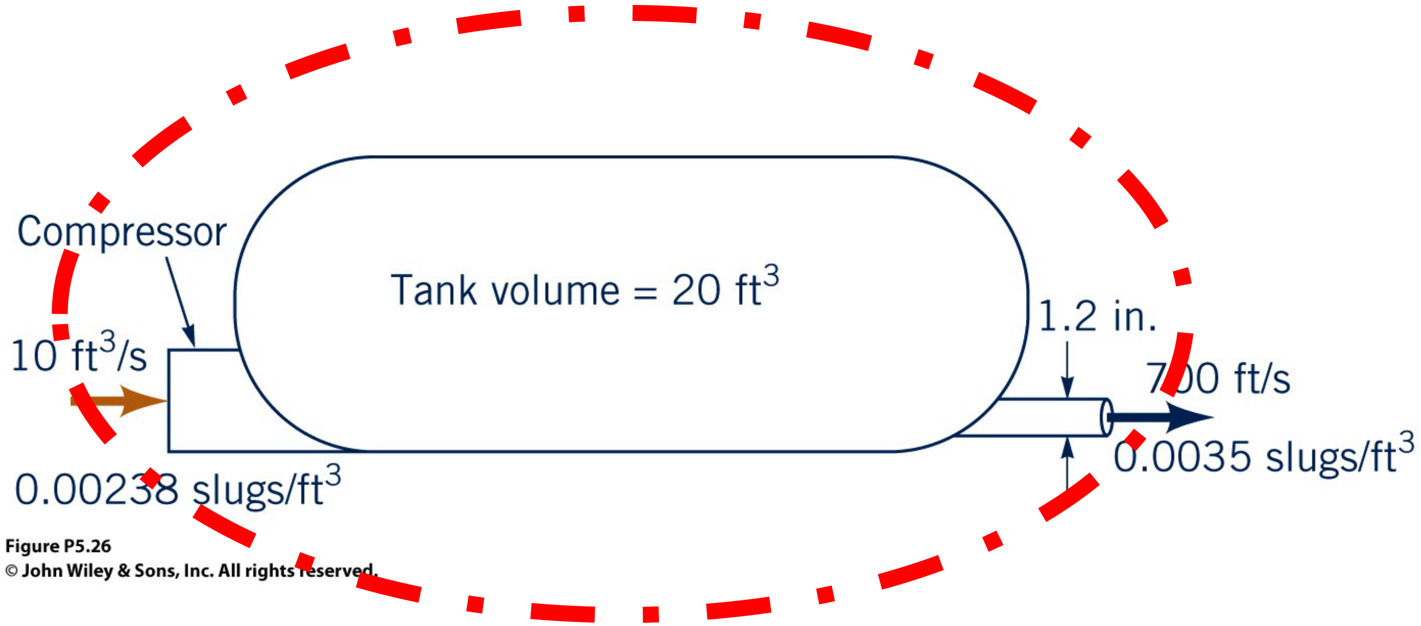



Figure P5.26

Water enters a rigid, sealed, cylindrical tank at steady rate of 100 liters/hr and forces gasoline (SG=0.68) out. What is the time rate of change of mass of gas in the tank?

Gas, which floats on the water without mixing is forced out and both are incompressible.

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

$$\frac{D}{Dt}(\rho\forall) + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

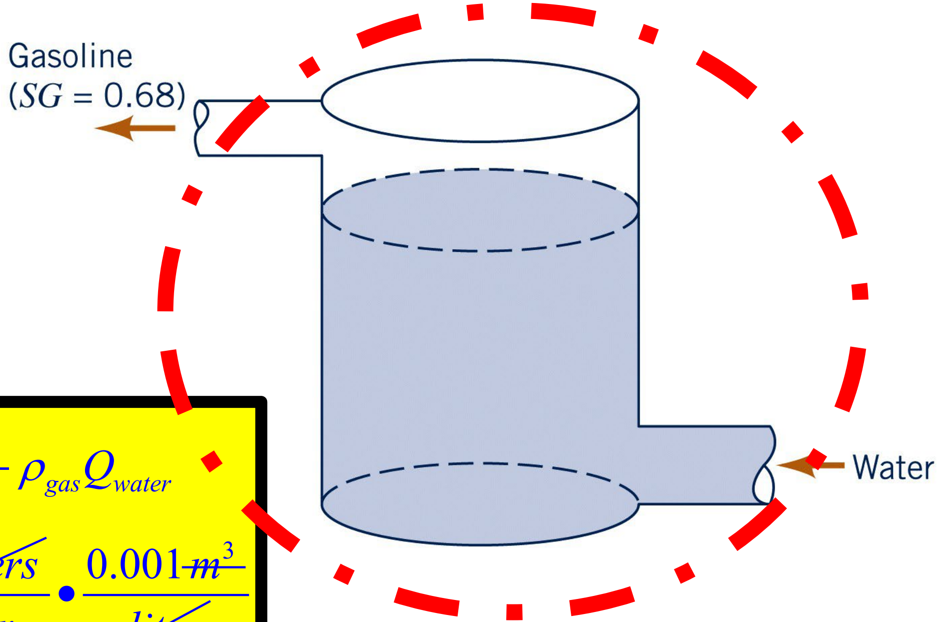
$$\frac{D}{Dt}(m_{gas}) + \frac{D}{Dt}(m_{water}) + \rho_{out}Q_{out} - \rho_{in}Q_{in} = 0$$

$$\frac{D}{Dt}(m_{gas}) + \frac{D}{Dt}(\rho_{water}\forall) + \rho_{out}Q_{out} - \rho_{in}Q_{in} = 0$$

$$\frac{D}{Dt}(m_{gas}) + \rho_{water}\frac{D}{Dt}(\forall) + \rho_{gas}Q_{gas} - \rho_{water}Q_{water} = 0$$

$$\frac{D}{Dt}(m_{gas}) + \cancel{\rho_{water}Q_{water}} + \rho_{gas}Q_{gas} - \cancel{\rho_{water}Q_{water}} = 0$$

$$\frac{D}{Dt}(m_{gas}) = -\rho_{gas}Q_{gas} = --\rho_{gas}Q_{water}$$




$$\frac{D}{Dt}(m_{gas}) = -\rho_{gas}Q_{gas} = --\rho_{gas}Q_{water}$$

$$= -0.68 \bullet 9800\frac{kg}{\cancel{m^3}}100\frac{\cancel{liters}}{hr} \bullet \frac{0.001\cancel{m^3}}{\cancel{liter}}$$

$$= -666.40\frac{kg}{hr}$$

# HOMEWORK

**4.2,3,23,25,26,29,34**