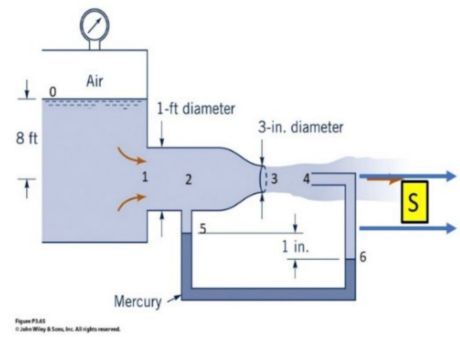
BERNOULLI & UNITS STUDY AID

1. Pressurized water flows steadily from a large, closed tank. Follow the path[1] and determine:
    a. Pressure difference (lbf/ft2)
    b. Volume Flow Rate (ft3/s)
    c. Pressure in the space above the surface of the water in the tank (lbf/ft2)
    d. Pressure at the inlet of the 1" ft Diameter section (lbf/ft2)

$Sg_{HG} = 13.55$



FIRST STEP: LABEL ALL IMPORTANT POINTS

- **Static Pressure Taps**
- **Stagnation Pressure Taps**
- **Free Surfaces**
- **Inlet/Outlets/Free Jets**
- **Change in Diameters**
- **Manometer Fluid Interfaces**
- **Unknown Dimensions**

Next Step: Apply Manometry "IF" Defined

NEED PRESSURE DIFFERENCE

BERRY POINT-to-POINT

Use DUCT Points Aligned With Static/Stagnation TAPS

$$P_2 + \Delta P_{2-4} = P_4$$

$$P_2 + \cancel{\gamma_{H20} S} + \gamma_{Hg}\left(\frac{1}{12}\right) - \gamma_{H20}\left(\frac{1}{12} + \cancel{S}\right) = P_4$$

$$P_2 - P_4 = \frac{1}{12}(\gamma_{H20} - \gamma_{Hg})$$

$$\frac{P_2 - P_4}{\gamma_{H20}} = \frac{1}{12}(1 - Sg_{Hg}) = -1.0458\,ft$$

NO OTHER APPROACH IS ACCEPTABLE

[1] NO NEED TO MOVE FORWARD IF PATH IS NOT FOLLOWED

NEXT STEP:

APPLY BERNOULLI: SAME POINTS

$$\dot{m}_2\left(\frac{P_2}{\gamma_{H20}}+\overbrace{z_2}^{z_2=z_4}+\frac{V_2^2}{2g}\right)=\dot{m}_4\left(\frac{P_4}{\gamma_{H20}}+\overbrace{z_4}^{z_2=z_4}+\overbrace{\frac{V_4^2}{2g}}^{=0:\text{Stagnation}}\right)$$

One Inlet/One Exit

$$\dot{m}_2=\dot{m}_4$$

$$\frac{P_2-P_4}{\gamma_{H20}}=-\frac{V_2^2}{2g}$$

COMBINE WITH MANOMETRY

$$\frac{P_2-P_4}{\gamma_{H20}}=\frac{1}{12}(1-Sg_{Hg})=-1.0458\,ft$$

$$\frac{P_2-P_4}{\gamma_{H20}}=-\frac{V_2^2}{2g}=-1.0458\,ft$$

SOLVE

$$\mathrm{V}_2=\sqrt{(1.0458\,ft)2\bullet 32.2\,ft/s^2}=8.21\,ft/s$$

$$Q=V_2A_2$$

$$Q=8.21\,ft/s\bullet\pi\frac{D_2^2[ft^2]}{4}=6.45\frac{ft^3}{s}$$

PRESSURE ABOVE WATER

Apply Bernoulli 0-3

$$\frac{P_0}{\gamma_{H20}}+\overbrace{z_0}^{=8\,ft}+\overbrace{\frac{V_0^2}{2g}}^{\approx 0}=\overbrace{\frac{P_3}{\gamma_{H20}}}^{=0\,psig:\,free\ jet}+\overbrace{z_3}^{=0\,ft}+\frac{V_3^2}{2g}$$

$$P_0=\left(\frac{V_3^2\left[\frac{ft^2}{s^2}\right]}{2g\left[\frac{ft}{s^2}\right]}-8\,ft\right)\overbrace{\gamma_{H20}}^{62.4}\left[\frac{lbf}{ft^3}\right]=$$

$$P_0=16230.12\frac{lbf}{ft^2}\left[\frac{1\,ft^2}{144in^2}\right]=112.7\,psig$$

$$V_3=\frac{Q}{A_3}=\frac{6.45\frac{ft^3}{s}}{0.04909\,ft^2}=131.4\,ft/s$$

PRESSURE AT INLET

Apply Bernoulli 0-1

$$\overbrace{\frac{P_0}{\gamma_{H20}}}^{known}+\overbrace{z_0}^{=8\,ft}+\overbrace{\frac{V_0^2}{2g}}^{\approx 0}=\overbrace{\frac{P_1}{\gamma_{H20}}}^{unknown}+\overbrace{z_1}^{=0\,ft}+\frac{V_1^2}{2g}$$

$$\frac{P_1}{\gamma_{H20}}=\frac{P_0}{\gamma_{H20}}+z_0-\frac{V_1^2}{2g}$$

$$P_1\left[\frac{lbf}{ft^2}\right]=\left[\frac{P_0}{\gamma_{H20}}[ft]+z_0[ft]-\frac{V_1^2}{2g}[ft]\right]\gamma_{H20}\left[\frac{lbf}{ft^3}\right]$$

2. The temperature distribution in a fluid is given below along with its velocity vector field:

$$T[C](x,y,t)=\left(10[]x^{\frac{1}{3}}+5[]ty^{\frac{-2}{7}}+24[]e^{-\alpha[]t}\right)[C]$$

"10"

$$[C]=[]m^{\frac{1}{3}}\rightarrow[]=\left[\frac{C}{m^{\frac{1}{3}}}\right]$$

"5"

$$[C]=[][s][m]^{-2/7}\rightarrow[]=\frac{C}{s-m^{-2/7}}=\left[\frac{Cm^{2/7}}{s}\right]$$

"24"

$$[C]=[][1]\rightarrow[]=C$$

"α"

$$[1]=[]s\rightarrow[]=\frac{1}{s}$$

$$\vec{V}\left[\frac{m}{s}\right](x,y,t)=u\hat{i}+v\hat{j}$$

$$=\left((20[]xt^{\frac{-3}{14}}+45[]y^{2})\hat{i}-(23[]x^{2}+50[]x^{\frac{6}{7}}yt)\hat{j}\right)\left[\frac{m}{s}\right]$$

χThrough this experience, I began to understand the fundamental concepts of the course, thanks to your focus upon the units of each problem. It sounds like a little thing, but is one that is a very important detail to double check the math through each problem. At times I was frustrated and felt as though I could not understand the material, but with persistence and trying to understand the big picture, I began to make much progress. I will remember this class for how it showed me I can learn a great deal in a short amount of time. I appreciate you providing the external pressure to unlock the better engineer in all of us students.ψ

Former Student, Spring 2022

$$\vec{V}\left[\frac{m}{s}\right](x,y,t) = u\hat{i} + v\hat{j}$$

$$= \left((20[]xt^{\frac{-3}{14}} + 45[]y^{2})\hat{i} - (23[]x^{2} + 50[]x^{\frac{6}{7}}yt)\hat{j} \right)\left[\frac{m}{s}\right]$$

"20"

$$\frac{m}{s} = []ms^{\frac{-3}{14}} \rightarrow [] = \frac{\frac{m}{s}}{ms^{\frac{-3}{14}}} \rightarrow s^{-1}s^{\frac{3}{14}} \rightarrow \left[s^{\frac{-11}{14}} \right]$$

"45"

$$\frac{m}{s} = []m^{2} \rightarrow [] = \frac{\frac{m}{s}}{m^{2}} = \left[\frac{1}{m-s} \right]$$

"23"

$$\frac{m}{s} = []m^{2} \rightarrow [] = \frac{\frac{m}{s}}{m^{2}} = \left[\frac{1}{m-s} \right]$$

"50"

$$\frac{m}{s} = []m^{\frac{6}{7}}ms \rightarrow [] = \frac{\frac{m}{s}}{m^{\frac{6}{7}}ms} = \frac{m}{m^{\frac{13}{7}}s^{2}} = \left[\frac{1}{m^{\frac{6}{7}}s^{2}} \right]$$

where t is in seconds, x and y are in meters, and T is in degrees C. **<u>FOLLOW THE PATH AND:</u>**

a. Determine the correct units for the brackets []?
b. Using the definition of the TOTAL, or MATERIAL DERIVATIVE, determine the "parametric expression" for the time rate of change of temperature $\left(i.e., \frac{DT}{Dt}\left[\frac{C}{s}\right] \right)$ of a fluid traveling with velocity field above $(i.e., \vec{V}\left[\frac{m}{s}\right](x,y,t))$, **AND**, verify units:
c. Determine the spatial derivatives **<u>AND</u>** verify units: $\frac{\partial T}{\partial y}\left[\frac{C}{m}\right] and \frac{\partial u}{\partial x}\left[\frac{m/s}{m}\right]$.

$$T[C](x,y,t)=\left(10[]x^{\frac{1}{3}}+5[]ty^{\frac{-2}{7}}+24[]e^{-\alpha[]t}\right)[C]$$

$$\frac{DT}{Dt}\frac{[C]}{[s]}=\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}$$

$$=\overbrace{-\alpha\left[\frac{1}{s}\right]24[C]e^{-\alpha[]t}}^{\frac{\partial T}{\partial t}}\left\{\frac{[C]}{[s]}\right\}$$

$$+\left\{u\left[\frac{m}{s}\right]\overbrace{10\left[\frac{C}{m^{\frac{1}{3}}}\right]\frac{1}{3}x^{-\frac{2}{3}}\left[m^{-\frac{2}{3}}\right]}^{\frac{\partial T}{\partial x}}\right\}\left\{\frac{[C]}{[s]}\right\}$$

$$+\left\{v\left[\frac{m}{s}\right]5\overbrace{\left[\frac{C-m^{2/7}}{\cancel{s}}\right](-2/7)t[\cancel{s}]y^{-\frac{9}{7}}\left[m^{-\frac{9}{7}}\right]}^{\frac{\partial T}{\partial y}}\right\}\left\{\frac{[C]}{[s]}\right\}$$

"Fluid Mechanics has helped me build a stronger problem-solving skill by helping me not focus on the problem itself, but on the path that must be followed."

Former Student, Winter 2022

$$\frac{\partial T}{\partial x}\left[\frac{C}{m}\right],\frac{\partial T}{\partial y}\left[\frac{C}{m}\right], and\ \frac{\partial u}{\partial x}\left[\frac{m/s}{m}\right]$$

$$T[C](x,y,t)=\left(10[]x^{\frac{1}{3}}+5[]ty^{\frac{-2}{7}}+24[]e^{-\alpha[]t}\right)[C]$$

$$\frac{\partial T}{\partial x}\left[\frac{C}{m}\right]=10\left[\frac{C}{m^{\frac{1}{3}}}\right]\frac{1}{3}x^{-\frac{2}{3}}\left[m^{-\frac{2}{3}}\right]\rightarrow[C]\left[m^{\overbrace{-\frac{2}{3}-\frac{1}{3}}^{\frac{-3}{3}=-1}}\right]\rightarrow\frac{[C]}{[m]}$$

$$\frac{\partial T}{\partial y}\left[\frac{C}{m}\right]=-\frac{10}{7}\left[\frac{Cm^{2/7}}{\cancel{s}}\right]t[\cancel{s}]y^{\frac{-9}{7}}\left[m^{\frac{-9}{7}}\right]\rightarrow[C]\left[m^{\frac{2-9}{7}}\right]\rightarrow\frac{[C]}{[m]}$$

$$u=20\left[s^{\frac{-11}{14}}\right]xt^{\frac{-3}{14}}+45[]y^2$$

$$\frac{\partial u}{\partial x}\left[\frac{m/s}{m}\right]=\frac{-3}{14}20\left[s^{\frac{-11}{14}}\right]t^{\frac{-3}{14}}\left[s^{\frac{-3}{14}}\right]\rightarrow\left[\frac{1}{s}\right]$$