MECH-420 STUDY AID

Energy Conservation

Heat Diffusion Equation

Thermal Circuits

Dr. K. J. Berry

PROBLEM DEFINITION

A solid spherical galactic probe exploring the dead moon Ulysses circling the exploding star DELTA 45XC1, is of radius r0 = 0.3m with a thermal conductivity 250 W/m-K and has a volumetric internal heat generation of

.

$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0\left(1.0 + \left[\frac{7r}{2r_0}\right]^{\frac{1}{3}}\right); S_0 = 30,000\left[\frac{W}{m^3}\right]$$

The probe has an Iridium (k = 47 W/m-K) shielding to protect it from the NEUTRON STAR J-CLASS Radiation of thickness 0.02m. The outer Iridium surface emissivity is 0.8 and the temperature of the surroundings is 20K.

- What is the value of the MINIMUM and MAXIMUM Temperature?
- What is the value and location of the MINIMUM and MAXIMUM heat transfer rate?

ANALYSIS

Since the surface emissivity and the surrounding temperature is given, the probe is experiencing radiation heat transfer at the surface with the large surroundings.

Since we are asked to find surface temperature and the HEAT FLUX IS KNOWN AT ALL BOUNDARIES then we can apply an overall control volume and use energy conservation to find temperature.

Since we are transferring heat by radiation at the surface, the internal heat generation must be dissipated to the surface by conduction and transferred to the surroundings by radiation.

Since energy is leaving the probe at the surface the MAXIMUM temperature must be at the probe center, and the MINIMUM must be at the surface.

MINIMUM TEMPERATURE

$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0(1.0+\left[\frac{7r}{2r_0}\right]^{\frac{1}{3}}); S_0 = 30,000\left[\frac{W}{m^3}\right]$$

OVERALL CONTROL VOLUME

$$\cancel{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_{gen} = \cancel{\dot{E}_{st}}$$

$$\dot{E}_{gen} = \dot{E}_{out}$$

$$\dot{E}_{gen} = \int_0^{r_i} \dot{S}_{gen}(r)dV; \rightarrow V = \frac{4}{3}\pi r^3, dV = 4\pi r^2 dr$$

$$\dot{E}_{gen} = 4\pi S_0 \int_0^{r_0} (1.0+\left[\frac{7r}{2r_i}\right]^{\frac{1}{3}})r^2 dr = 4\pi S_0\left[\frac{r^3}{3} + \frac{7^{1/3} r^{10/3}}{(10/3)(2r_i)^{1/3}}\right]_{0-r_i}$$

$$\dot{E}_{gen} = 4\pi S_0\left[\frac{W}{m^3}\right] r_i^3 \left[m^3\right](\frac{1}{3} + \frac{7^{1/3}}{\frac{10}{3}2^{1/3}}), r_i = 0.3m \text{ (important)}$$

$$\dot{E}_{gen} = 8,030W$$

$$\dot{E}_{gen} = 8,030W$$

$$\dot{E}_{gen} = \dot{E}_{out} = q_{radiation} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4)$$

$$T_s[K] = \left[\frac{\dot{E}_{gen}}{\varepsilon\sigma A_s} + T_{surr}^4[K]\right]^{\frac{1}{4}}, r_0 = 0.3m + 0.02 = 0.32m \text{ (important)}$$

$$A_s = 4\pi r_0^2 = 1.2868m^2, \varepsilon = 0.8, \sigma = 5.67x10^{-8}\frac{W}{m^2 - K^4}$$

$$T_{surr} = 20K$$

$$T_s[K] = 609K = 335.9C$$

INTERFACE TEMPERATURE (IRIDIUM)

$$\dot{E}_{gen} = q = 8,030W$$

THERMAL CIRCUITS (1D,SS,Homogenous,Sgen = 0)

$$q = \frac{T_i - T_s}{R_t}$$

$$T_i = q \bullet R_t + T_s = q \bullet \frac{\frac{1}{r_i} - \frac{1}{r_0}}{4\pi k_{Iridium}} + T_s$$

$$T_i = 8,030\,\not{W} \bullet \frac{\left(\frac{1}{0.30} - \frac{1}{0.32}\right)\frac{1}{\not{m}}}{(4\pi)47\frac{\not{W}}{\not{m} - K}} + 609K$$

$$T_i = 611.83K$$

CENTER TEMPERATURE

$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0(1.0 + \left[\frac{7r}{2r_i}\right]^{\frac{1}{3}}); S_0 = 30,000\left[\frac{W}{m^3}\right]$$

HDE −−SPHERE

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dT}{dr}) = -\frac{\dot{S}_{gen}(r)}{k} = \frac{-S_0}{k}(1.0 + \left[\frac{7r}{2r_i}\right]^{\frac{1}{3}})$$

$0 \le r \le r_i = 0.3m$

multiple by r^2

$$\frac{d}{dr}(r^2\frac{dT}{dr}) = \frac{-S_0}{k}(r^2 + \left[\frac{7r}{2r_i}\right]^{\frac{1}{3}} r^2)$$

INTEGRATE

$$r^2\frac{dT}{dr} = \frac{-S_0}{k}\left[\frac{r^3}{3} + \left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r^{10/3}}{10/3}\right] + C_1$$

$\div r^2$

$$\frac{dT}{dr} = \frac{-S_0}{k}\left[\frac{r^1}{3} + \left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r^{4/3}}{10/3}\right] + \frac{C_1}{r^2}$$

INTEGRATE

$$T(r) = \frac{-S_0}{k}\left[\frac{r^2}{6} + \left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r^{7/3}}{10/3 \bullet 7/3}\right] - \frac{C_1}{r} + C_2$$

BOUNDARY CONDITIONS

$$T(r)=\frac{-S_0}{k}\left[\frac{r^2}{6}+\left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r^{7/3}}{10/3\bullet 7/3}\right]-\frac{C_1}{r}+C_2$$

Boundary Conditions

@r=0, T(r=0) = FINITE $\rightarrow C_1=0$

$@r=r_i, T(r=r_i)=T_i$

$$T_i=\frac{-S_0}{k}\left[\frac{r_i^2}{6}+\left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r_i^{7/3}}{10/3\bullet 7/3}\right]+C_2$$

$$T_i+\frac{S_0}{k}\left[\frac{r_i^2}{6}+\left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r_i^{7/3}}{10/3\bullet 7/3}\right]=C_2$$

$$T_i+\frac{S_0 r_i^2}{k}\left[\frac{1}{6}+\left(\frac{7}{2}\right)^{\frac{1}{3}}\bullet\frac{9}{70}\right]=C_2$$

EXACT SOLUTION

$$T(r)=\frac{-S_0}{k}\left[\frac{r^2}{6}+\left(\frac{7}{2}\right)^{\frac{1}{3}}\frac{1}{r_i^{\frac{1}{3}}}\frac{r^{7/3}}{10/3\bullet 7/3}\right]+C_2$$

$0\leq r\leq r_i=0.3m$

MAX TEMP = T(r=0)

$$T_{max}=C_2=T_i+\frac{S_0 r_i^2}{k_{probe}}\left[\frac{1}{6}+\left(\frac{7}{2}\right)^{\frac{1}{3}}\bullet\frac{9}{70}\right]=615.7K$$

$$k_{probe}=250\frac{W}{m-K}, r_i=0.30m, T_i=611.83K$$

MINIMUM/MAXIMUM HEAT RATE

MINIMUM HEAT RATE

$$q=-k_{probe} A \frac{dT}{dr} = -kA \left\{ \frac{-S_0}{k} \left[\frac{r^1}{3} + \left(\frac{7}{2}\right)^{\frac{1}{3}} \frac{1}{r_i^{\frac{1}{3}}} \frac{r^{4/3}}{10/3} \right] \right\}$$

$$A = 4\pi r^2$$

$$q_{\min}(r=0) \rightarrow \left.\frac{dT}{dr}\right|_{r=0} = 0$$

MAXIMUM HEAT RATE

$$q_{\max} \rightarrow \text{ outer surface} \rightarrow q_{radiation} = 8,030W$$