MECH-420
SPECIAL TOPICS HW #1

TYPED INDIVIDUAL WORK

SUBMIT TO BLACKBOARD W/

SOLUTION AND SPREADSHEET/MATHLAB

A spherical shell of inner radius "r_1" and outer radius "r_2" serves as radiation containment vessel is exposed to a convective fluid, $T_\infty(r_2) = 300K$, and convective heat transfer coeff. "h=25W/m^2-K".

The internal material has a volumetric heat generation rate defined as:

$$\dot{q}(r) = \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0\left[\frac{W}{m^3}\right](2.0 + \frac{r^4}{r_1^4}); 0 \le r \le r_1, r_1 = 2.5m, r_2 = 5m, S_0 = 20kW/m^3.$$

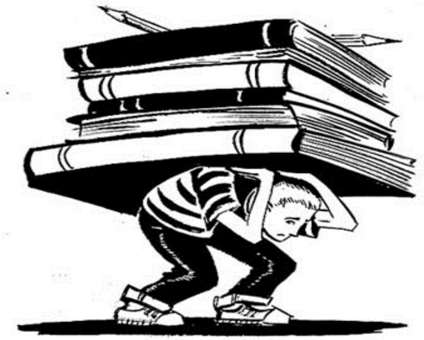
If $V_{sphere} = \frac{4}{3}\pi r^3$, find:

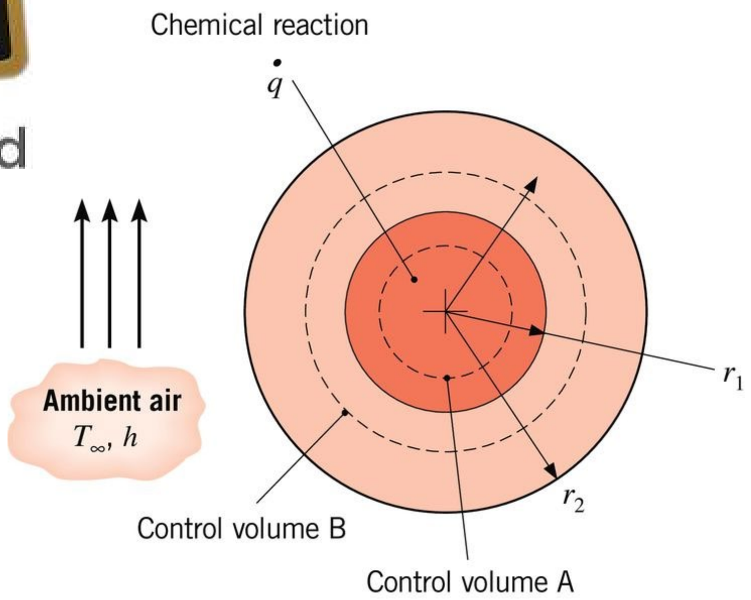
a) Total heat generated [W],

b) steady state surface temperature $T_s(r_1)$

c) Plot $T_s(r_1)$ vs r_1 as $0 \le r_1 \le 4.5m$

(submit typed PDF solution and spreadsheet/MATLAB)





TOTAL ENERGY GENERATED

$$\dot{q}(r) = \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0\left[\frac{W}{m^3}\right](2.0 + \frac{r^4}{r_1^4});\ 0 \le r \le r_1$$

$$\dot{E}_{gen} = \int_V \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right]dV, V_{sphere} = \frac{4}{3}\pi r^3, \frac{dV}{dr} = 4\pi r^2, dV = 4\pi r^2 dr$$

$$\dot{E}_{gen} = \int_0^{r_1}(2.0 + \frac{r^4}{r_1^4})4\pi r^2 dr$$

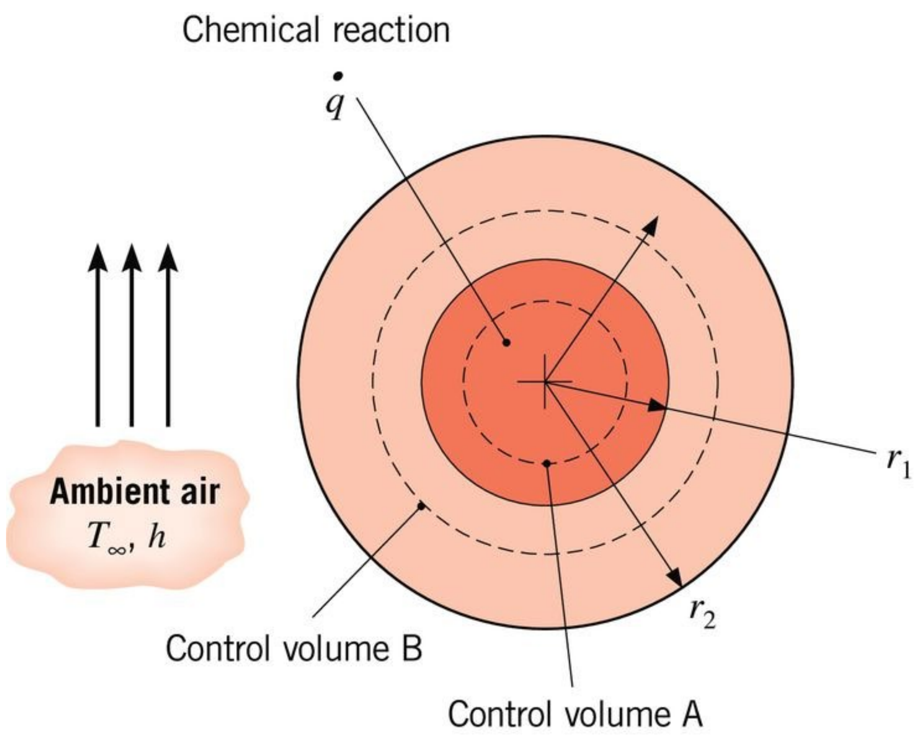
$$= 4\pi S_0\left[\frac{W}{m^3}\right]\int_0^{r_1}(2.0 + \frac{r^4}{r_1^4})r^2 dr = 4\pi S_0\left[\frac{W}{m^3}\right]\int_0^{r_1}(2.0r^2 + \frac{r^6}{r_1^4})dr$$

$$= 4\pi S_0\left[\frac{W}{m^3}\right]\left[\frac{2r^3}{3} + \frac{r^7}{7r_1^4}\right]_{0-r_1} = 4\pi S_0\left[\frac{W}{m^3}\right]\left[\frac{2r_1^3}{3} + \frac{r_1^7}{7r_1^4}\right]$$

$$= 4\pi S_0\left[\frac{W}{m^3}\right]r_1^3\left[\frac{2}{3} + \frac{1}{7}\right] = 4\pi S_0\left[\frac{W}{m^3}\right]r_1^3\left[\frac{14}{21} + \frac{3}{21}\right]$$

$$= 4\pi S_0\left[\frac{W}{m^3}\right]r_1^3\frac{17}{21}$$

$$\dot{E}_{gen} = \frac{68\pi}{21}S_0\left[\frac{W}{m^3}\right]r_1^3 \equiv W$$



Surface Temperature, Ts(r1) @ r2

Parametric Road Map

$$\dot{E}_{gen} = \frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3 \equiv W$$

$$\overset{+}{\dot{E}}_{in} - \overset{+}{\dot{E}}_{out} + \overset{\pm}{\dot{E}}_{g} = \frac{dE_{st}}{dt} \equiv \rho \forall c_p \frac{dT}{dt}$$

$$\overset{+}{\dot{E}}_{out} = \overset{\pm}{\dot{E}}_{g}$$

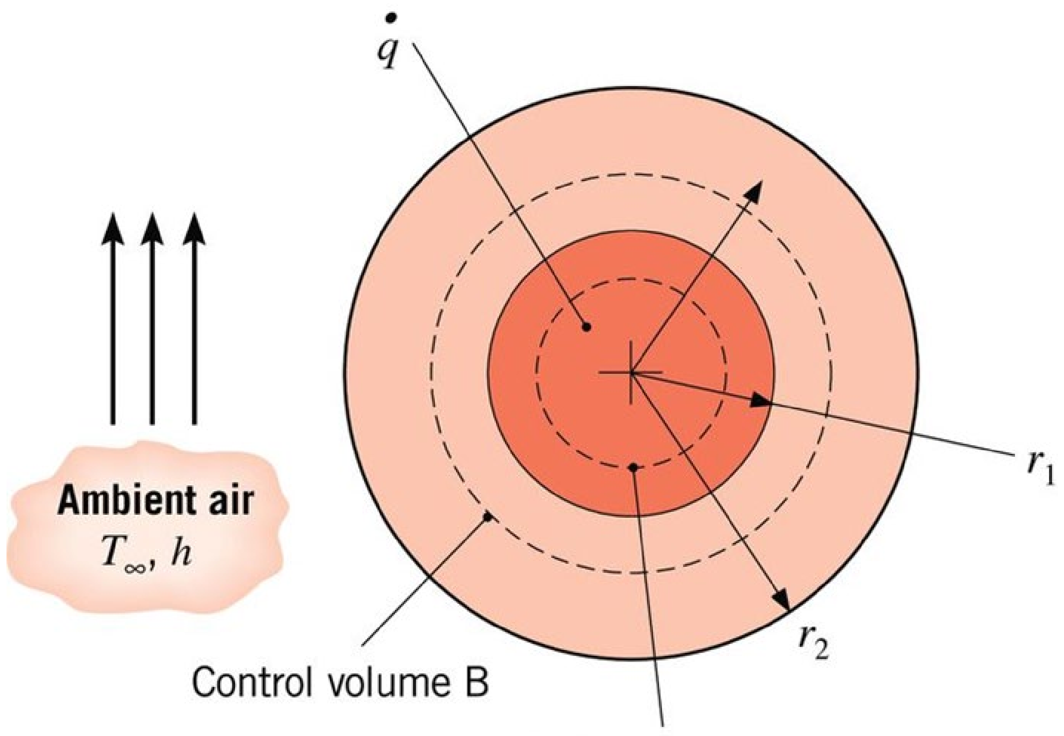
$$\bar{h} A_0 (T_s - T_\infty) = \frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3$$

$$T_s(r_1^3) = T_\infty + \frac{\frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3}{\bar{h} A_0}$$

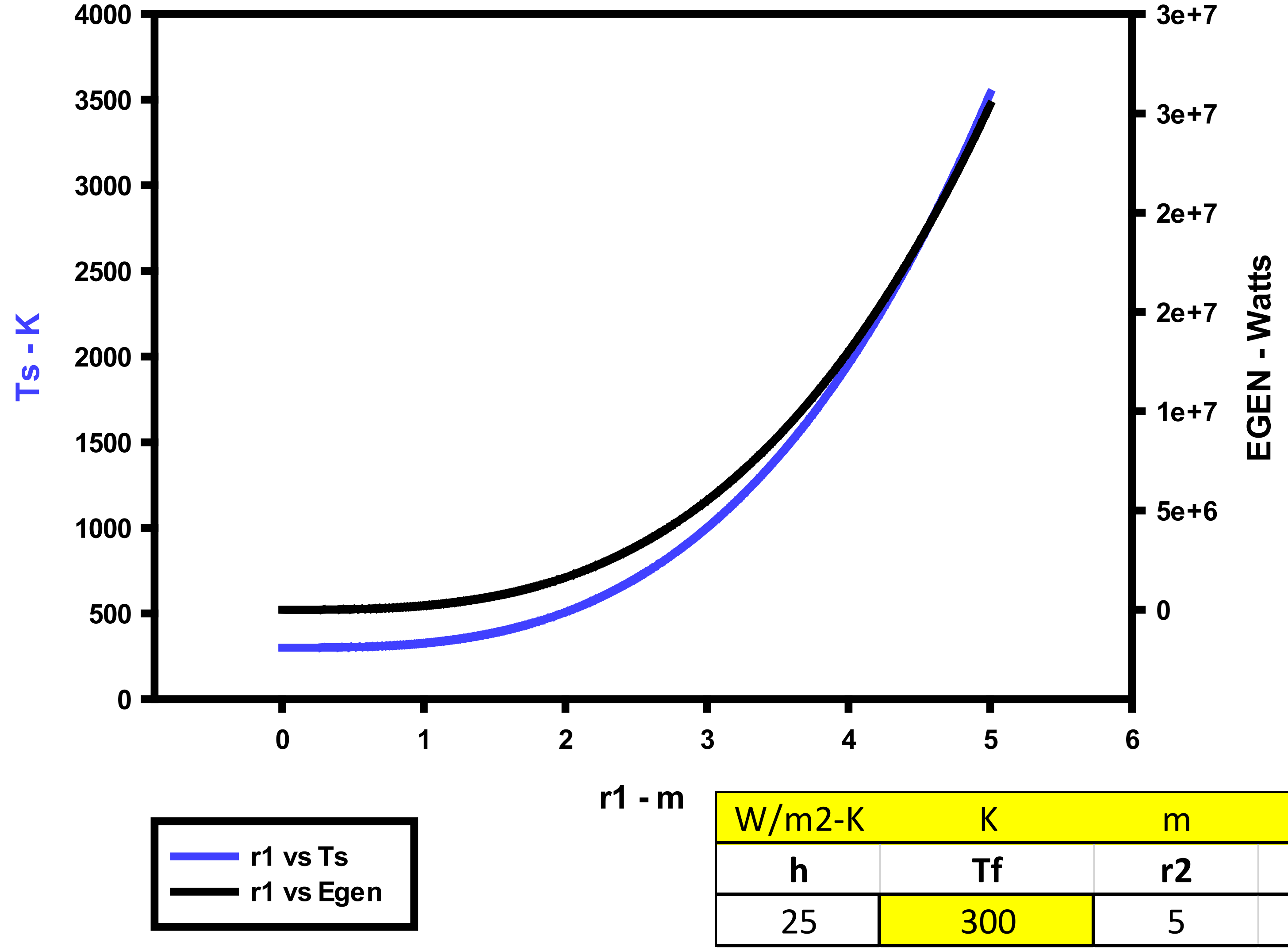
$$T_{s_{r=r_2}}(r_1) = T_\infty[K] + \frac{\frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3}{\bar{h} \frac{W}{m^2 - K} (4\pi r_2^2)} = [K]$$

$$T_{s_{r=r_2}}(r_1) = T_\infty[C] + \frac{\frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3}{\bar{h} \frac{W}{m^2 - K} (4\pi r_2^2)} = [C]$$

Chemical reaction

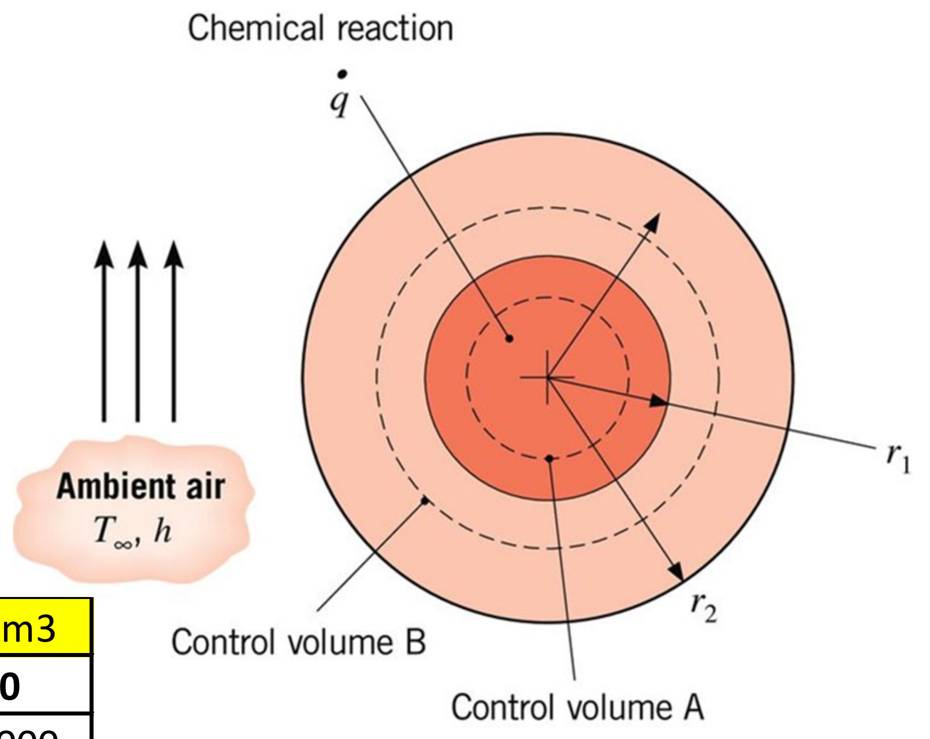


HW #1 - Problem 2

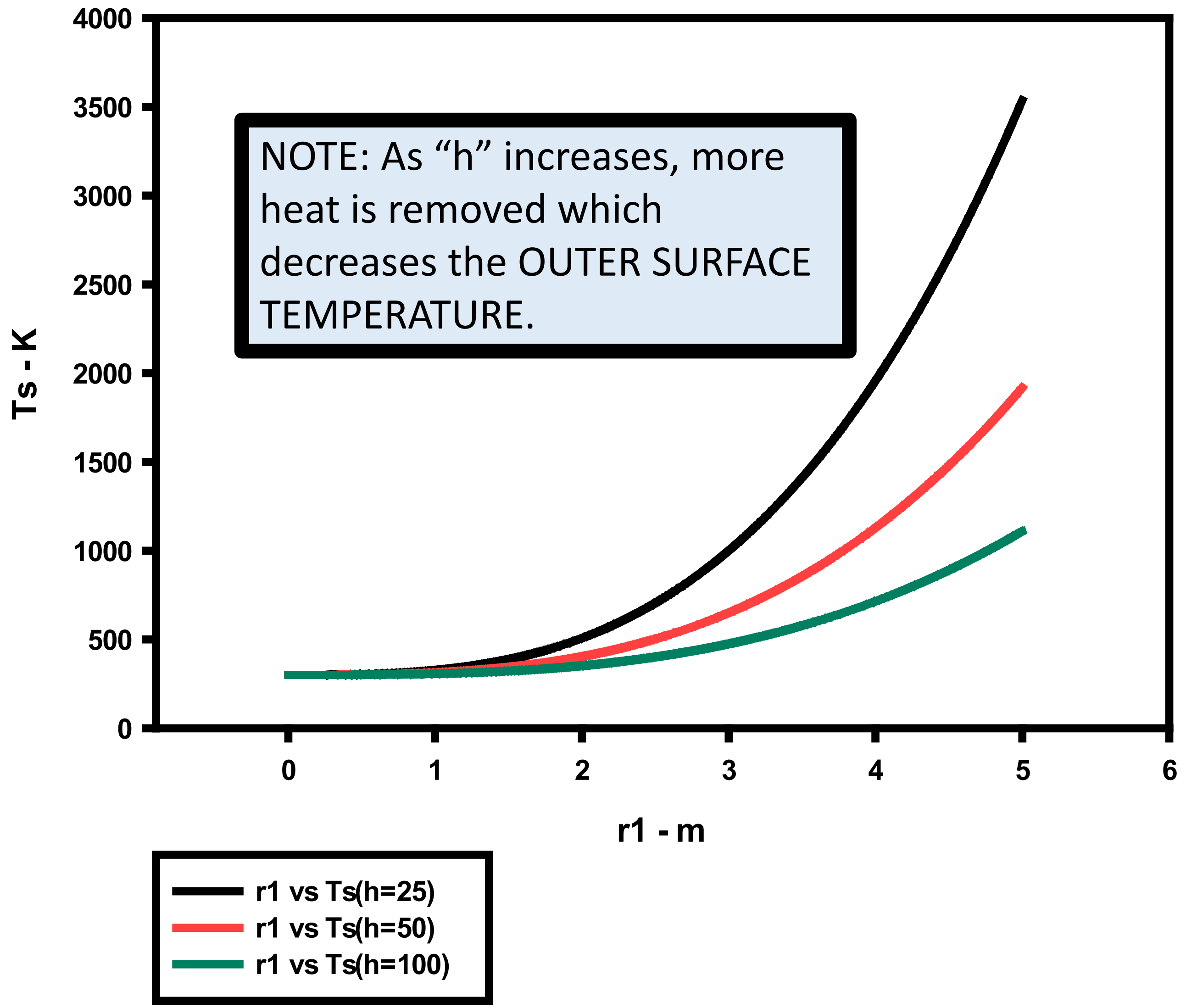


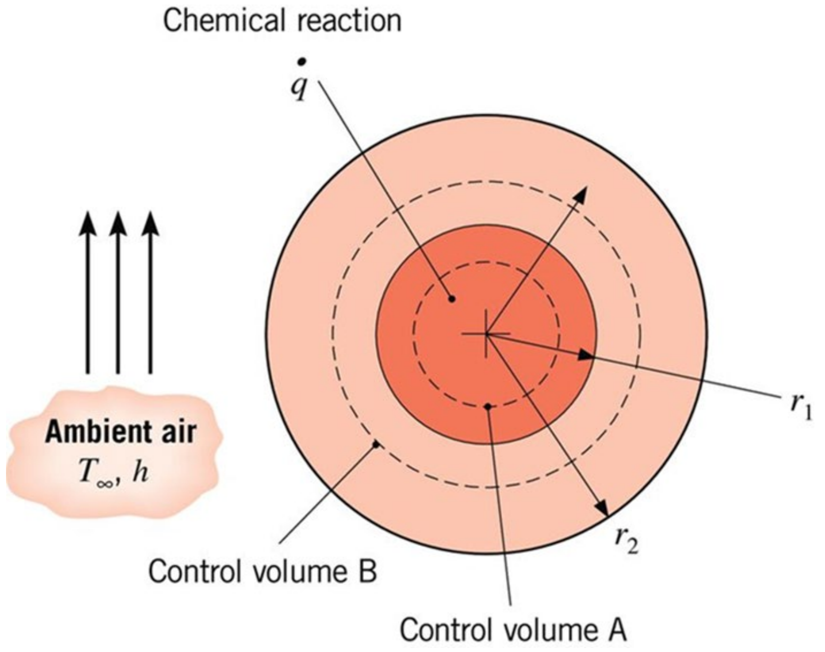
$$T_{s_{r=r_2}}(r_1) = T_\infty[C] + \frac{\frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3}{\bar{h}\,\frac{W}{m^2-K}(4\pi r_2^2)} = [C]$$

W/m2-K	K	m	W/m3
h	Tf	r2	S0
25	300	5	20,000



HW #1 - Problem 2
r1 vs Ts vs HT Coeff





$$T_{s_{r=r_2}}(r_1) = T_\infty[C] + \frac{\frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3}{\bar{h}\,\frac{W}{m^2-K}(4\pi r_2^2)} = [C]$$

CORE LEARNING OBJECTIVES

1. Internal heat generation rate $\dot{S}_{gen}(r)$ for this problem is only within internal sphere: $0 \le r \le r_1$

$$i.e., \dot{E}_{gen} = \int_0^{r_1} \dot{S}_{gen}(r)dV = \int_0^{r_1} S_0 \left[\frac{W}{m^3}\right](2.0 + \frac{r^4}{r_1^4})dV = \frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3 \equiv W$$

2. The convective heat transfer coefficient is only applied at the OUTER boundary exposed to fluid.

$$\text{i.e.,q[W]} = \bar{h}A_0(T_s(r = r_2) - T_\infty), \; A_0 = 4\pi r_2^2$$

3. An over all energy balance for the entire canister yields for outer surface temperature:

$$T_{s_{r=r_2}}(r_1) = T_\infty[K] + \frac{\frac{68\pi}{21} S_0 \left[\frac{W}{m^3}\right] r_1^3}{\bar{h}\frac{W}{m^2 - K}(4\pi r_2^2)} = [K] \sim r_1^3$$

which is a function of inner sphere radius (r_1) with internal heat generation rate.

As r_1 increases, the more HEAT is generated, and the more heat must be transfeered to the outer suface to be carried away by the external convective fluid.