# MULTI STREAM MOMENTUM ENERGY STUDY AID

MECH-322 Fluid Mechanics

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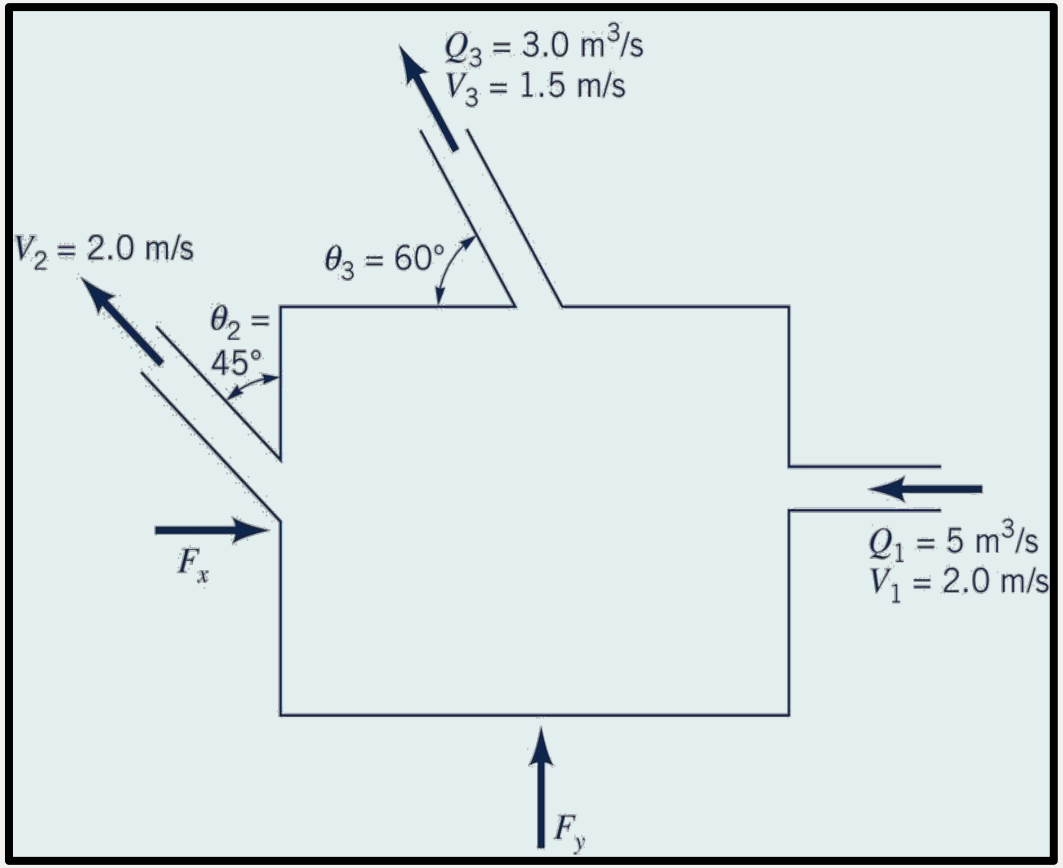
# Problem Geometry

**Water flows in the plane steadily through a chamber (neglect chamber weight) with a volume of 0.20 m3 with a gravitational vector defined as:**

$$\vec{g} = (20.6\hat{i} + 32.2\hat{j})\frac{m}{s^2}$$



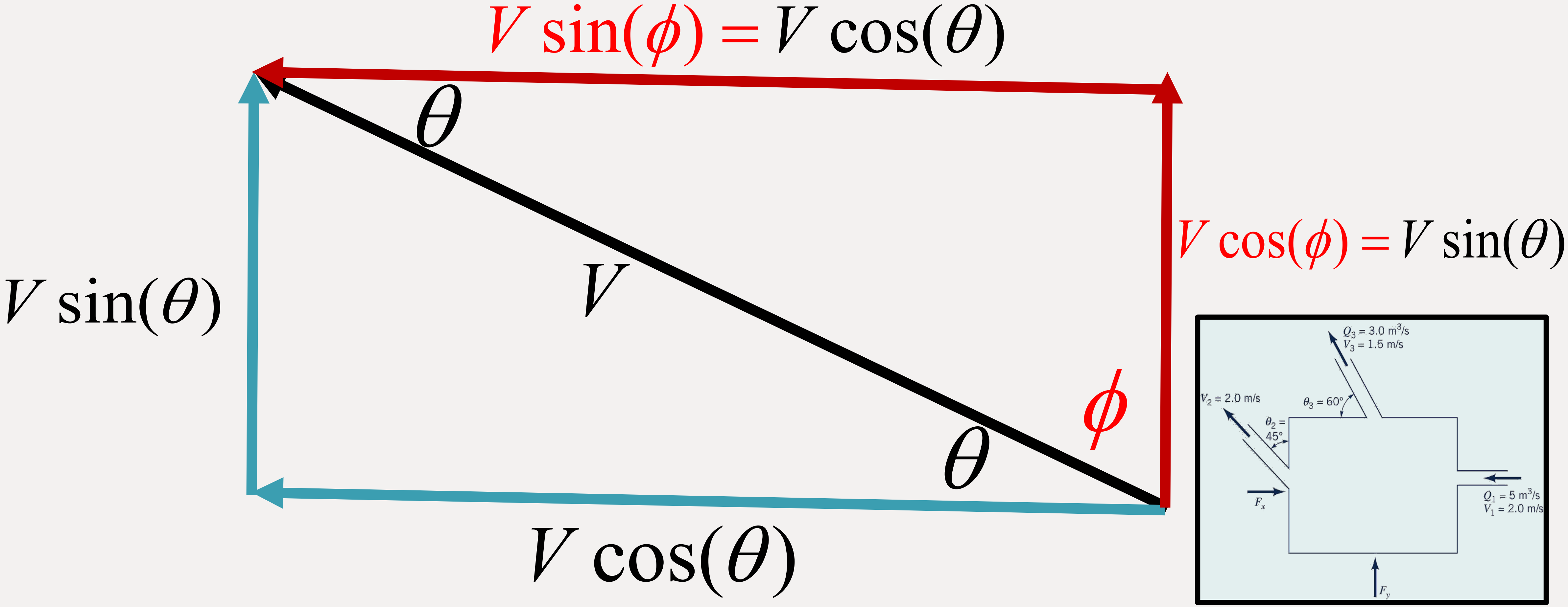

$$h_{L_{1-3}} = 150J/kg, h_{L_{1-2}} = 20J/kg \rightarrow \text{ENERGY LOSS}$$

$$h_{L_{1-3}}[m] = \frac{h_{L_{1-3}}[(J = N - m)/kg]}{g[\frac{m}{s^2}]}$$

$$= \frac{\frac{kg - \frac{m}{s^2}}{kg}}{\frac{m}{s^2}} = [m]$$

# VECTOR ALGEBRA

$$\phi = 90 - \theta$$



# CONSERVATION OF ENERGY

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in}(\dot{m}g(\frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1)) =$$

$$\sum_{out}(\dot{m}g(\frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2)) + \sum H_L(= \dot{m}g(h_l[m; ft])); W \text{ or } ft - lbf / s;$$

## CONSERVATION OF MATH

$$\frac{dM_{ass}}{dt} = \sum \dot{m}_{out} - \sum \dot{m}_{in}$$

# CONSERVATION OF MOMENTUM

$$\sum \vec{F}_{CV} = \frac{d\overrightarrow{M_{CV}}}{dt} + \sum_{OUT_{CS}} \dot{m}\vec{V}(\pm) - \sum_{IN_{CS}} \dot{m}\vec{V}(\pm)$$

$$\dot{m}[\frac{kg}{s};\frac{slugs}{s}] \equiv \text{mass flow rate} = \rho[\frac{kg}{m^3};\frac{slugs}{ft^3}]\mathrm{A}_n[m^2, ft^2]V[\frac{m}{s};\frac{ft}{s}]$$

# Parametric Goals

Develop parametric simulation matrix in terms of known & unknown variables as:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{Bmatrix} p_2 \\ p_3 \\ F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix}$$

Problem Variables

$$\dot{m}\left[\frac{kg}{s}\right], \rho\left[\frac{kg}{m^3}\right], P\left[\frac{N}{m^2}\right], V\left[\frac{m}{s}\right], Q\left[\frac{m^3}{s}\right], A[m^2], \theta, F[N], h_L\left[\frac{J}{kg}\right]$$

ENERGY 1 – 2 → EQUATION 1

$$\text{let } \mathrm{H}_L[W] = \dot{m}\left[\frac{kg}{s}\right] g \left[\frac{m}{s^2}\right]\left(\frac{u_2}{g} - \frac{u_1}{g} - \frac{h_l\left[\frac{J}{kg}\right]}{g}\right)[m] = \left[\frac{N-m}{s}\right] = \left[\frac{J}{s}\right] = [W]$$

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in}(\dot{m}_1 g(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1)) = \sum_{out}(\dot{m}_2 g(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2)) + \sum H_L; W \text{ or } ft - lbf/s;$$

SOLVE FOR UNKNOWN $\mathrm{P}_2$

$$\left\{\frac{\dot{m}_1 g\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g}\right)}{\dot{m}_2 g}[m] - \frac{V_2^2}{2g}[m] - \frac{H_{L_{1-2}}[W]}{\dot{m}_2 g\left[\frac{W}{m}\right]}[m]\right\}\gamma[\frac{N}{m^3}] = p_2[\frac{N}{m^2}]$$

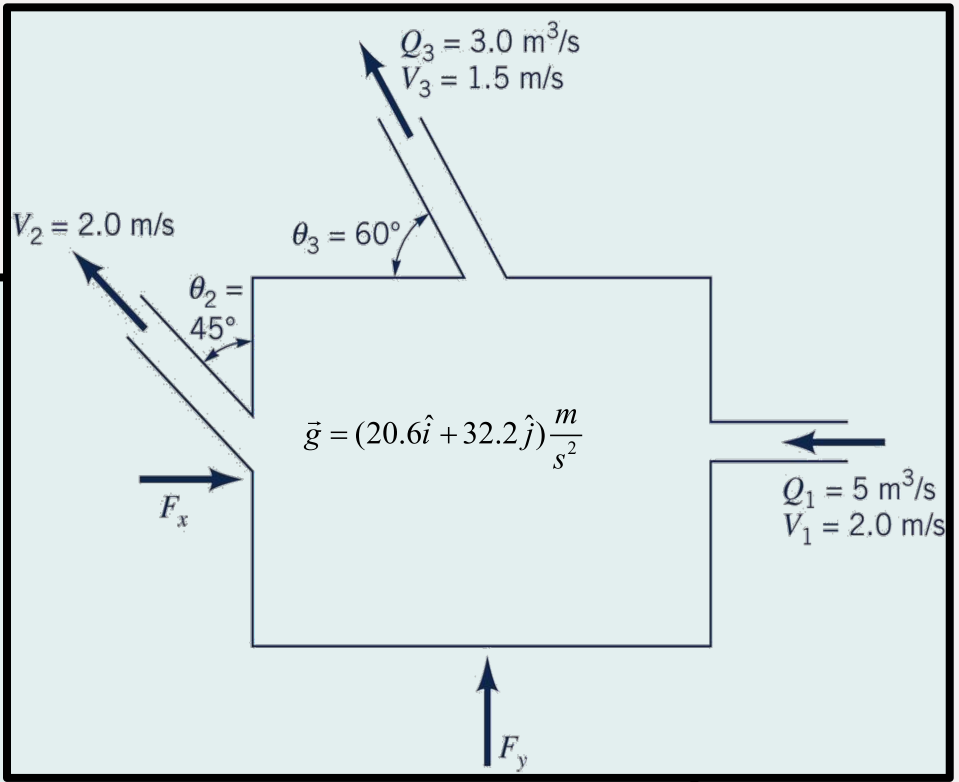
ENERGY $1-2 \rightarrow$ EQUATION 2

$$\text{let } \mathrm{H}_L[W] = \dot{m}\left[\frac{kg}{s}\right] g\left[\frac{m}{s^2}\right]\left(\frac{u_2}{g} - \frac{u_1}{g} - h_l\right)[m] = \left[\frac{N-m}{s}\right] = \left[\frac{J}{s}\right] = [W]$$

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in}(\dot{m}_1 g(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1)) = \sum_{out}(\dot{m}_3 g(\frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3)) + \sum H_L; W \text{ or } ft-lbf/s;$$

SOLVE FOR UNKNOWN $\mathrm{P}_3$

$$\left\{\frac{\dot{m}_1 g\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g}\right)}{\dot{m}_3 g}[m] - \frac{V_3^2}{2g}[m] - \frac{H_{L_{1-3}}[W]}{\dot{m}_3 g\left[\frac{W}{m}\right]}[m]\right\}\gamma[\frac{N}{m^3}] = p_3[\frac{N}{m^2}]$$




MOMENTUM

$\rightarrow + \uparrow +$

$$\vec{V}_3 = -V_3\cos(\theta_3)\hat{i} + V_3\sin(\theta_3)\hat{j}$$

$$\vec{V}_2 = -V_2\sin(\theta_2)\hat{i} + V_2\cos(\theta_2)\hat{j}$$

*X --EQUATION 3*

$$F_x + P_2A_2\sin(\theta_2) + P_3A_3\cos(\theta_3) + g_x\rho_f\forall - P_1A_1 = 0 \overbrace{-(V_3\cos(\theta_3)\dot{m}_3 + V_2\sin(\theta_2)\dot{m}_2)}^{MOM\ \text{OUT}} \overbrace{-V_1(-)\dot{m}_1}^{MOM\ \text{IN}}$$

*Y --EQUATION 4*

$$F_y - P_2A_2\cos(\theta_2) - P_3A_3\sin(\theta_3) + g_y\rho_f\forall = 0 + \overbrace{(V_3\sin(\theta_3)\dot{m}_3 + V_2\cos(\theta_2)\dot{m}_2)}^{MOM\ \text{OUT}}$$

$$[A]\{x\} = \{B\}$$

$$|\vec{g}| = \sqrt{g_x^2 + g_y^2}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{Bmatrix} p_2 \\ p_3 \\ F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix}$$

MASS CONSERVATION

$$\dot{m} = \rho Q$$

$$\sum_{in} \dot{m} - \sum_{out} \dot{m} = 0$$

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3$$

$$\dot{m}_2 = \rho Q_1 - \rho Q_3 = \rho Q_2 = \rho A_2 V_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ A_2 \sin(\theta_2) & A_3 \cos(\theta_3) & 1 & 0 \\ -A_2 \cos(\theta_2) & -A_3 \sin(\theta_3) & 0 & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ p_3 \\ F_x \\ F_y \end{Bmatrix} = \begin{Bmatrix} \left\{ \dfrac{\dot{m}_1 g \left( \dfrac{p_1}{\gamma} + \dfrac{V_1^2}{2g} \right)}{\dot{m}_2 g} [m] - \dfrac{V_2^2}{2g} [m] - \dfrac{H_{L_{1-2}} [W]}{\dot{m}_2 g \left[ \dfrac{W}{m} \right]} [m] \right\} \gamma [\dfrac{N}{m^3} \\ \left\{ \dfrac{\dot{m}_1 g \left( \dfrac{p_1}{\gamma} + \dfrac{V_1^2}{2g} \right)}{\dot{m}_3 g} [m] - \dfrac{V_3^2}{2g} [m] - \dfrac{H_{L_{1-3}} [W]}{\dot{m}_3 g \left[ \dfrac{W}{m} \right]} [m] \right\} \gamma [\dfrac{N}{m^3}] \\ -g_x \rho_f \forall + P_1 A_1 - (V_3 \cos(\theta_3) \dot{m}_3 + V_2 \sin(\theta_2) \dot{m}_2) + V_1 \dot{m}_1 \\ -g_y \rho_f \forall + (V_3 \sin(\theta_3) \dot{m}_3 + V_2 \cos(\theta_2) \dot{m}_2) \end{Bmatrix}$$