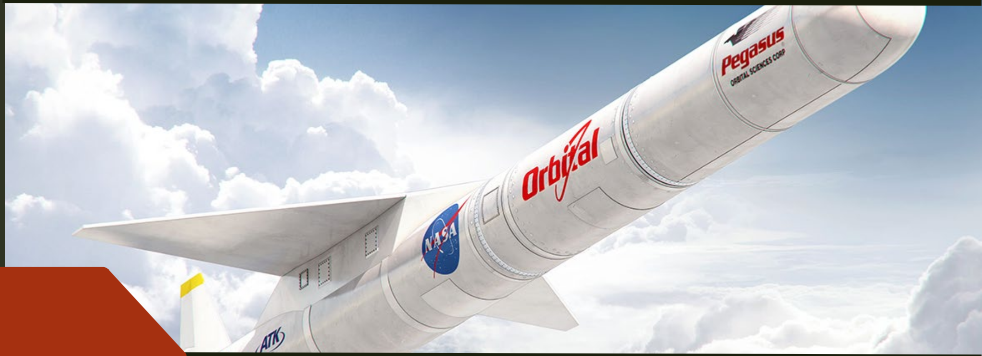
FINS: EXTENDED SURFACES (CLICK)

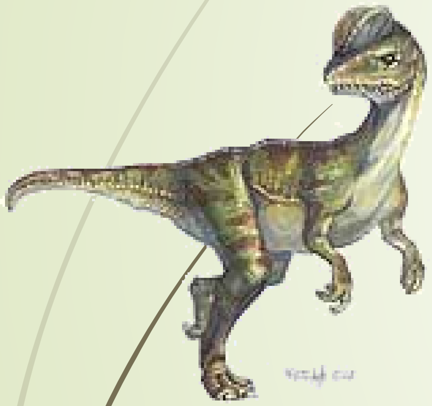
MECH-420 Heat Transfer

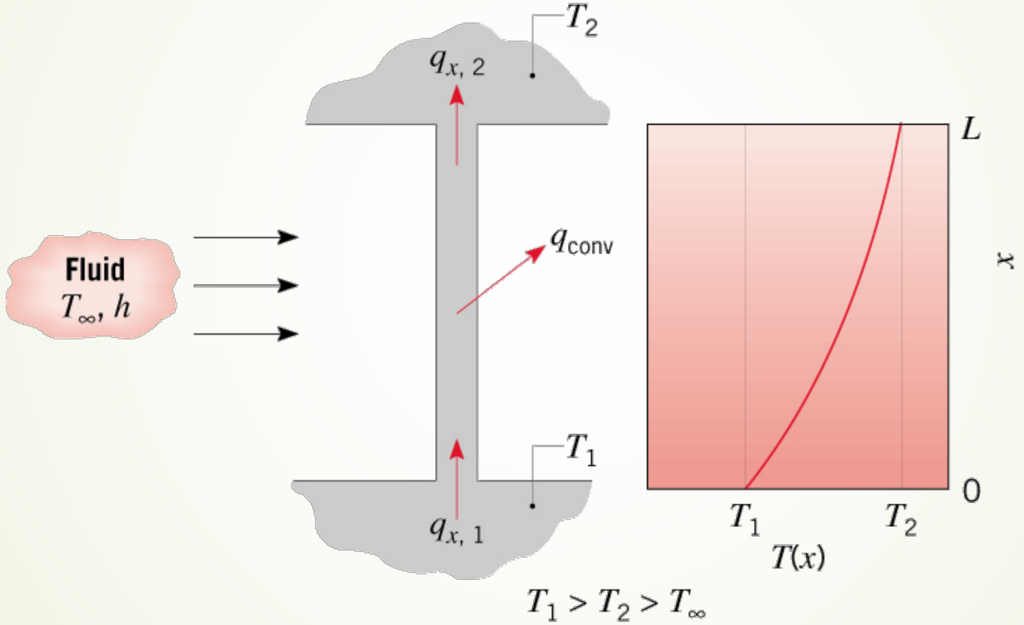
Section 3.6



NATURE AND RATIONALE OF EXTENDED SURFACES

- An extended surface (also know as a **combined conduction-convection system or a FIN**) is a solid within which heat transfer by conduction is *assumed* to be **one dimensional**, while heat is also transferred by **CONVECTION** (and/or radiation) from the surface in a direction **TRANSVERSE** to that of **CONDUCTION**.



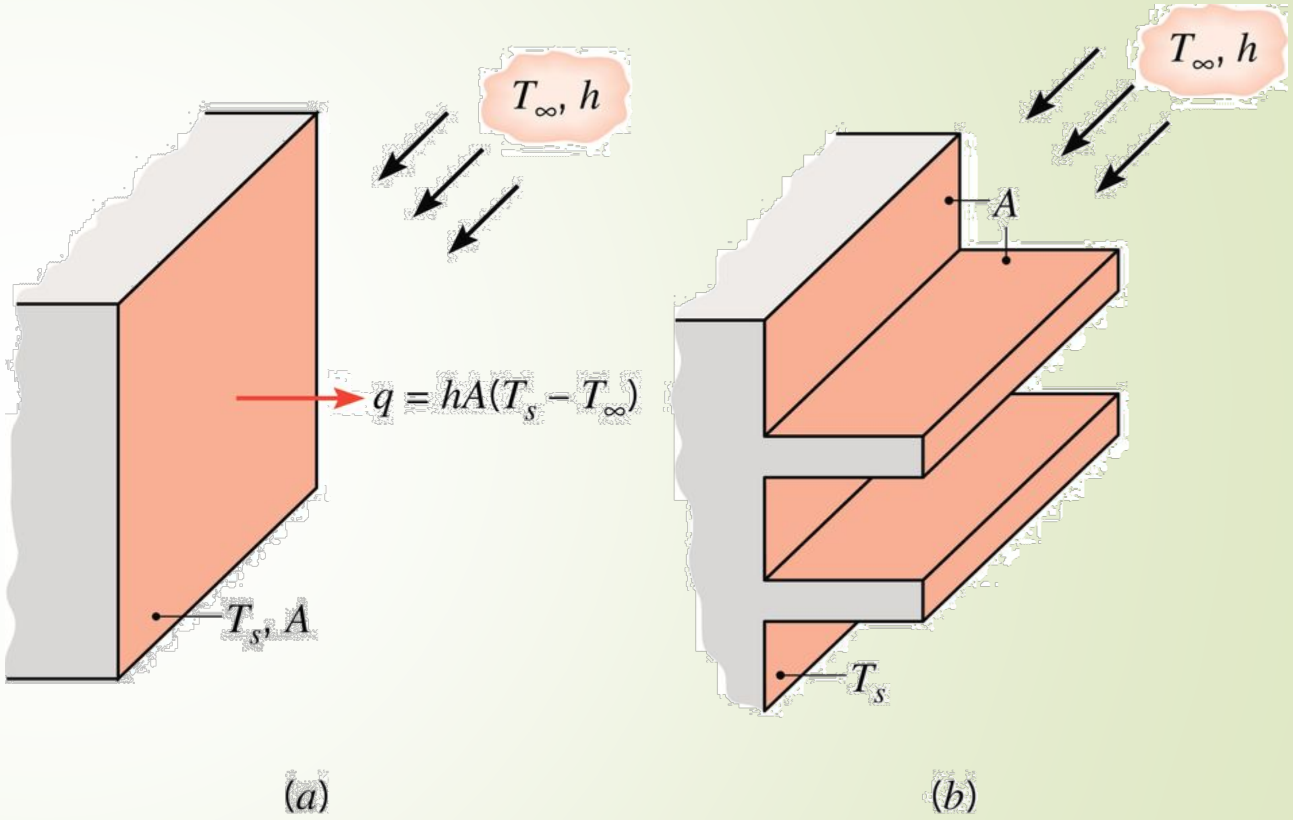


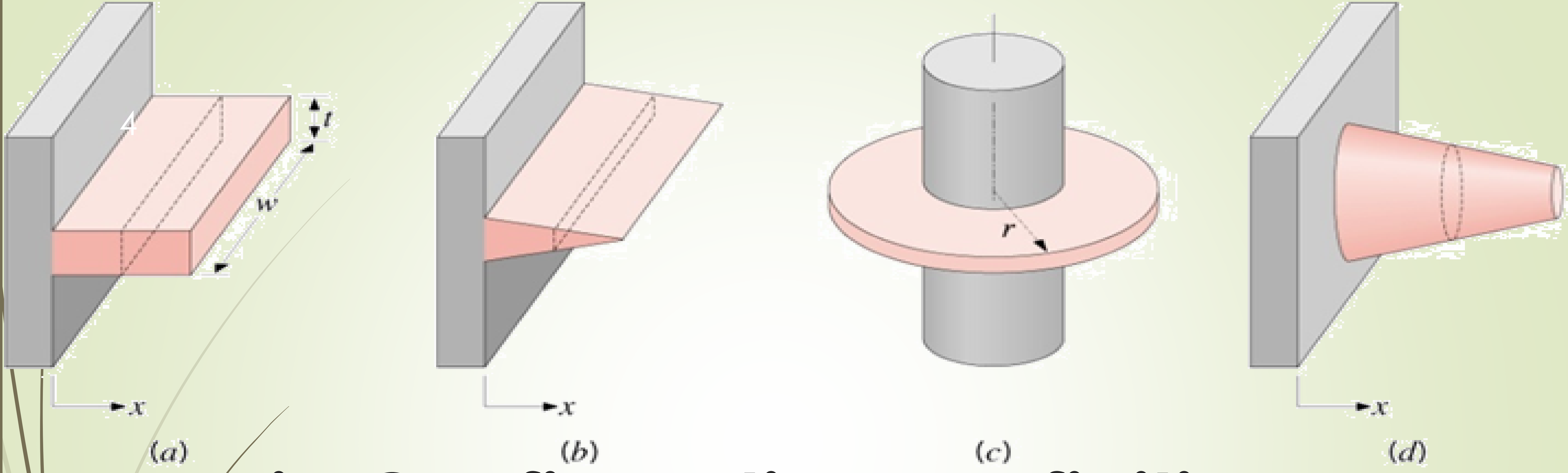
  – If heat is transferred from the surface to the fluid **by convection**, what surface condition is dictated by the **conservation of energy** requirement?

  – Why is heat transfer by conduction in the x-direction not, in fact, one-dimensional?

DEFINITION

- Extended surfaces enhance heat transfer by increasing the surface **AREA** available or **CONVECTION** (and/or radiation).
- Convection requires higher "h" and maybe a larger fan or pump, and thus more external work.





Fin Configurations Definitions

a) STRAIGHT fin of UNIFORM cross section.
b) STRAIGHT fin of NON-UNIFORM cross section.
c) ANNULAR Fin of UNIFORM cross section.
d) PIN Fin of NON-UNIFORM cross section.

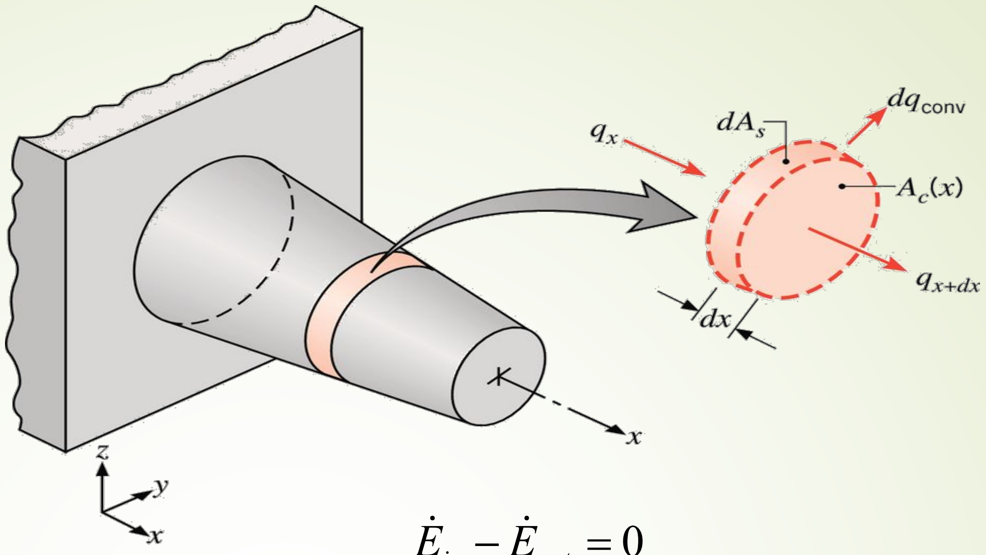
Very good for small “h” with a gas or Free Natural Convection

ANALYSIS OBJECTIVES

- **What is the actual functional dependence of the temperature distribution in the solid?**
- **If the temperature distribution is assumed to be one-dimensional, that is, $T=T(x)$, how should the value of T be interpreted for any x location?**
- **How does HEAT TRANSFER vary with x ?**
- **When may the assumption of one-dimensional conduction be viewed as an excellent approximation? i.e. THE THIN FIN APPROXIMATION.**

6

STEADY STATE ANALYSIS



$$q'' = -k\frac{dT}{dx}$$

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{E}_{in} = q''_x A_x, \dot{E}_{out} = q''_x A_x + \frac{d}{dx}(q''_x A_x)dx + hdA_s(T(x) - T_\infty)$$

$$\dot{E}_{in} - \dot{E}_{out} \rightarrow$$

$$-\frac{d}{dx}(q''_x A_x)dx - hdA_s(T(x) - T_\infty) = 0, dA_s = Pdx$$

For Fins of CONSTANT CROSS SECTION, Ax ≠ F(x)

$$\frac{d^2T}{dx^2} - \frac{hP}{k_x A_c}(T(x) - T_\infty) = 0; P \equiv \text{Perimeter}$$

let θ(x)= $T(x) - T_\infty, m^2 = \frac{hP}{k_x A_x}$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \rightarrow \text{THIN FIN EQUATION}$$

Straight and PIN Fins of UNIFORM CROSS SECTION

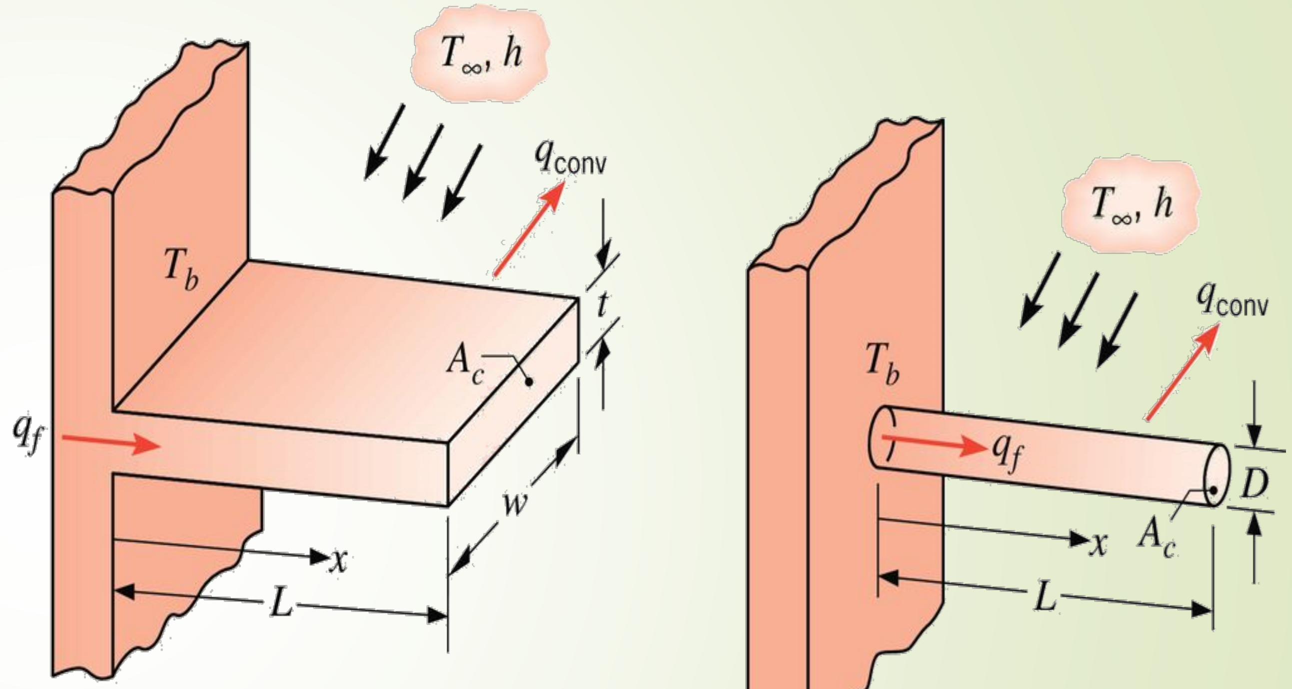
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \rightarrow \text{THIN FIN EQUATION}$$

$$m^2 = \frac{hP}{k_x A_c}, M = \left(\sqrt{hPkA_c}\right)\theta_B; \theta_B = T_B - T_\infty$$

SOLUTION

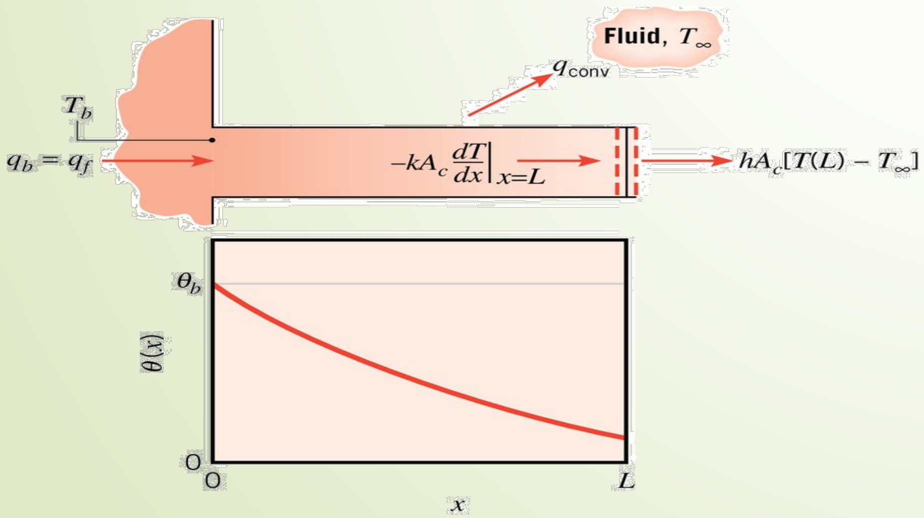
$$\theta(x) = T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

$$C_1, C_2 \rightarrow \text{BOUNDARY CONDITIONS}$$

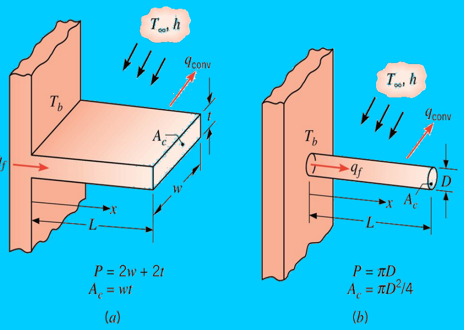


$P = 2w + 2t$
$A_c = wt$
(a)

$P = \pi D$
$A_c = \pi D^2/4$
(b)



FIN SOLUTIONS: STRAIGHT FIN OF UNIFORM CROSS SECTION



CASE	TIP	$\dfrac{\theta(x)}{\theta_B} = \dfrac{T(x)-T_\infty}{T_B - T_\infty}$	$q_f = -k_f A_c \dfrac{dT}{dx}$
A	$h\theta(L) = -k\dfrac{d\theta}{dx}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	$\dfrac{d\theta}{dx} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	$\theta(L) = \theta_L$	$\dfrac{\left(\dfrac{\theta_L}{\theta_B}\right)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{\left(\cosh mL - \dfrac{\theta_L}{\theta_B}\right)}{\sinh mL}$
D	$@ L \to \infty, \theta(L) \to 0$	e^{-mx}	M

$$L > 2.65\left(\frac{kA_c}{hP}\right)^{1/2}$$

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$M = \left(\sqrt{hPkA_c}\right)\theta_B$$

FIN PERFORMANCE #1 EFFECTIVENESS

Fins are used to INCREASE the heat transfer from a surface by increasing the effective surface area.

But fins do represent a "RESISTANCE" to heat transfer from the original surface.

An assessment is made from the DEFINITION of the FIN EFFECTIVENESS:

$$\varepsilon_{fin} = \frac{\text{wall heat transfer WITH fin}}{\text{wall heat transfer WITHOUT fin}} = \frac{q_f}{hA_{c,b}\theta_b} \rightarrow \text{Desire} > 2$$

$A_{c,b} =$ fin cross section area at FIN BASE

$\theta_b = T_b - T_\infty$

FIN PERFORMANCE
EFFECTIVENESS

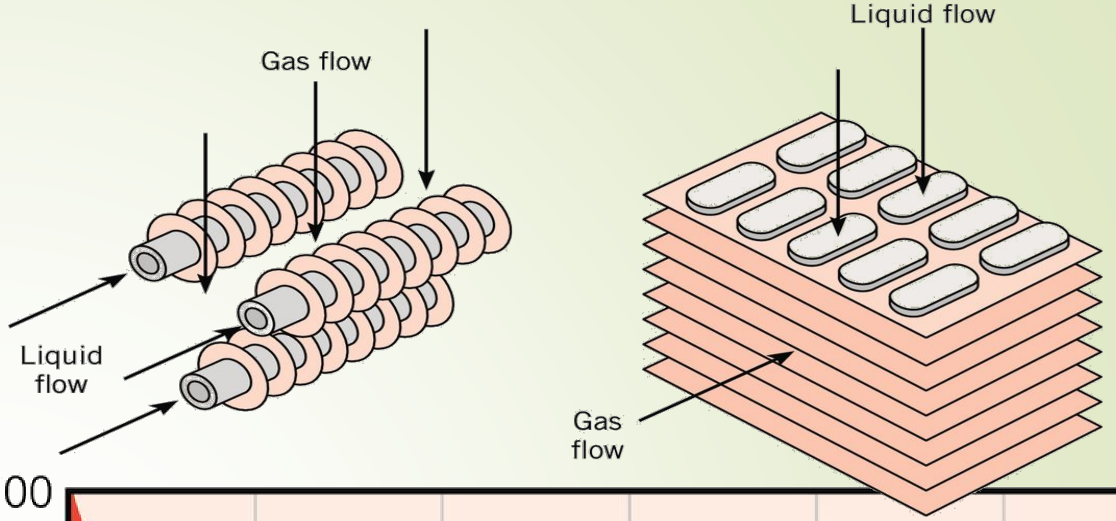
$$\varepsilon_{fin} = \frac{q_f}{hA_{c,b}\theta_b}, \text{LONG FIN, } q_f = M$$

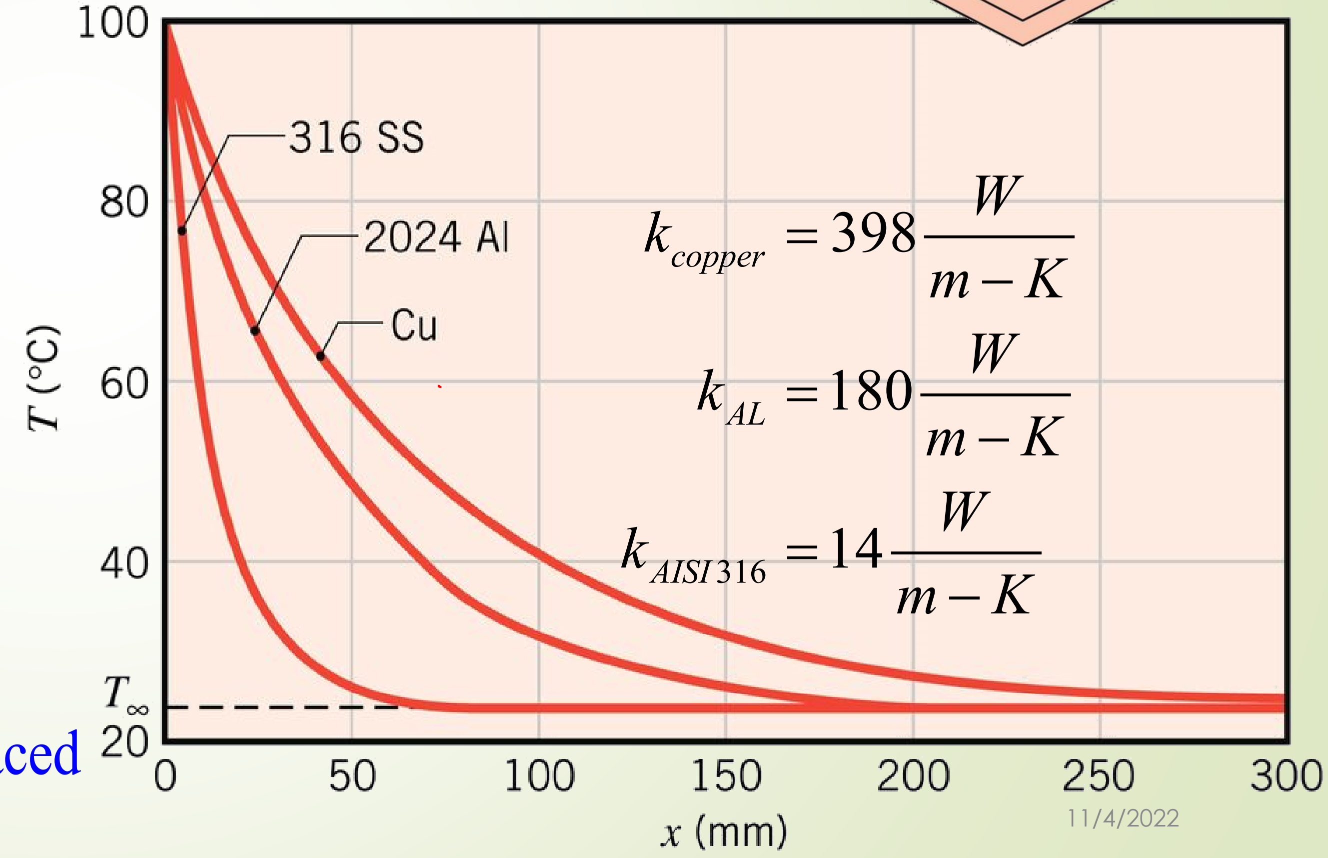
Consider LONG FIN, $q_f = \sqrt{hPk_f A_c}\,\theta_b$

$$\varepsilon_{fin} = \frac{q_f}{hA_{c,b}\theta_b} = \frac{\sqrt{hPk_f A_c}\,\theta_b}{hA_{c,b}\theta_b} = \left(\frac{kP}{hA_c}\right)^{1/2}$$

MAXIMIZE FIN PERFORMANCE

1. High Thermal Conductivity
2. Low Heat Transfer Coefficienct
3. $\frac{Perimeter}{A_c}$ is high $\rightarrow$ "thin" and closely spaced





$$k_{copper} = 398 \frac{W}{m-K}$$

$$k_{AL} = 180 \frac{W}{m-K}$$

$$k_{AISI\,316} = 14 \frac{W}{m-K}$$

FIN PERFORMANCE #2

EFFICIENCY

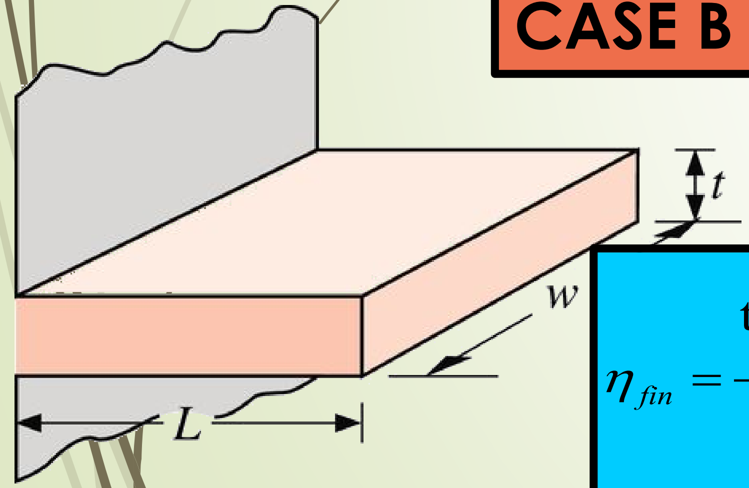
$$q_{fin} = \eta_{fin} \bullet q_{\max} = \eta_{fin} \bullet h \bullet A_{fin} \bullet \theta_b$$

$$\eta_{fin} = \frac{\text{Fin Heat Transfer Rate}}{\text{MAX HEAT TRANSFER POSSIBLE}} = \frac{q_{fin}}{q_{\max}} = \frac{q_{fin}}{hA_{FIN\ TOTAL}\theta_b} \rightarrow 0-1$$

$A_{FIN\ TOTAL}$ = TOTAL FIN AREA EXPOSED TO FLUID

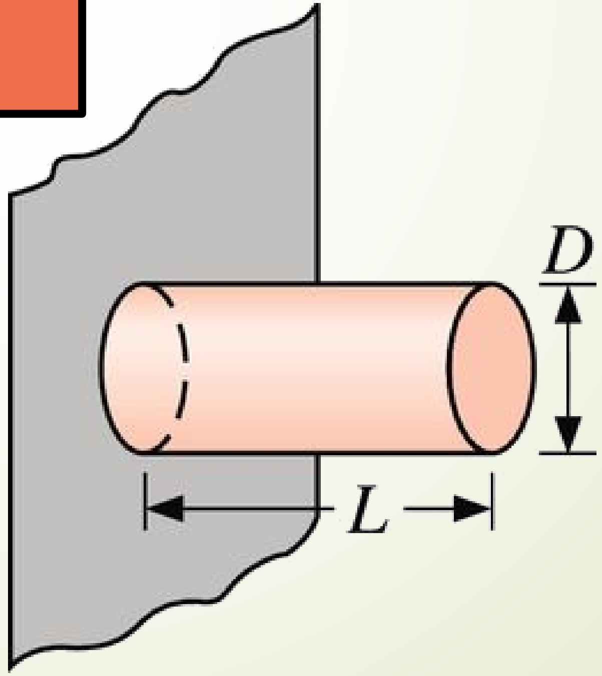
$\theta_b = T_b - T_\infty$

INSULATED TIP ONLY
CASE B (SLIDE 8)



$$\eta_{fin} = \frac{\tanh\left[m(L+\frac{t}{2})\right]}{m(L+\frac{t}{2})}$$

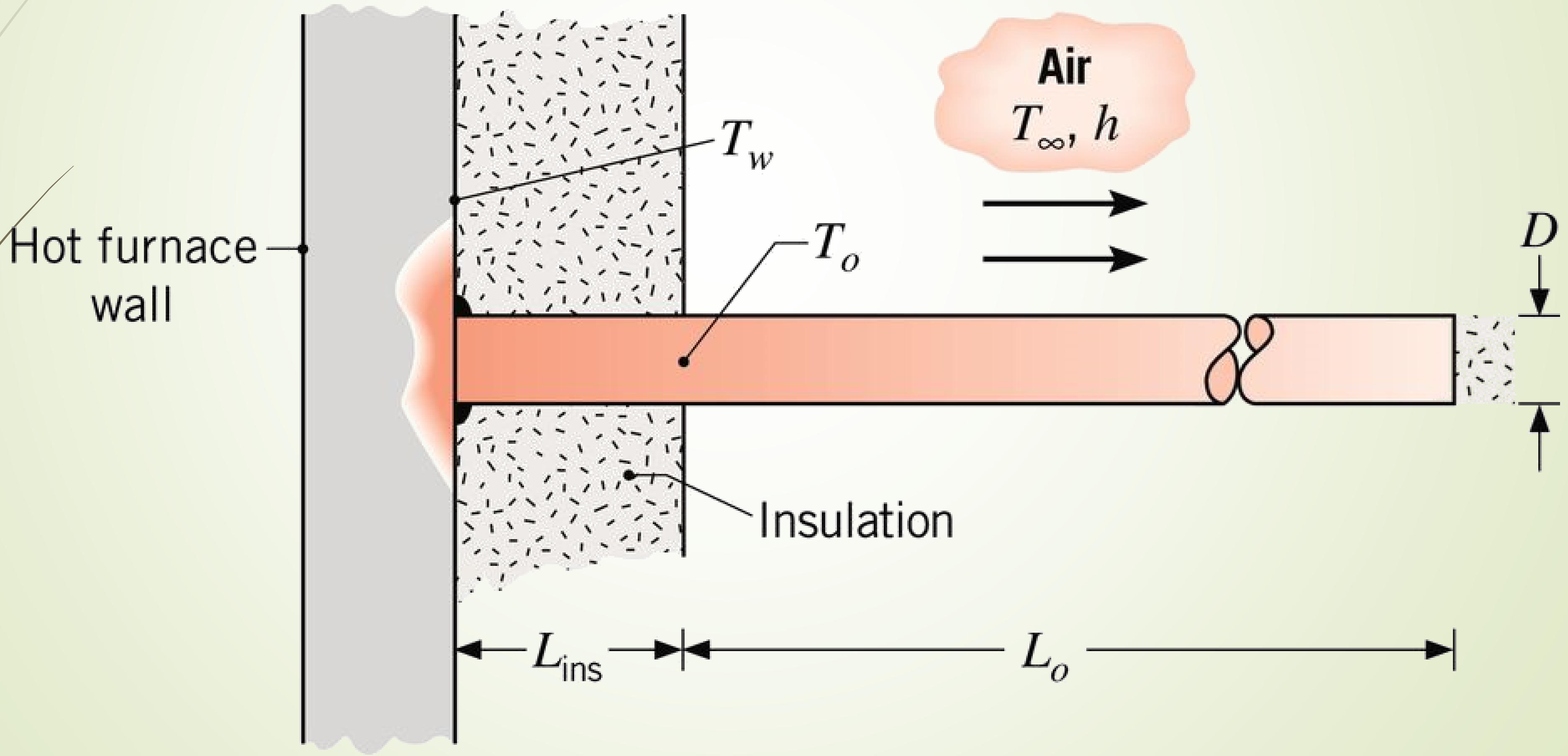
$$A_{fin} = 2w\left(L+\frac{t}{2}\right)$$



$$\eta_{fin} = \frac{\tanh\left[m(L+\frac{D}{4})\right]}{m(L+\frac{D}{4})}$$

$$A_{fin} = \pi D(L+\frac{D}{4})$$

A rod of diameter D = 25mm and k=60 W/m-K protrudes from furnace wall at T_w =200C and is covered with insulation thickness L_{ins} =200mm. The rod at its exposed surface, T_0, must be maintained below $T_{max} = 100C$ with air temp $T_\infty = 25C$ and h=15W/m-K.



Derive T(x) exact solution in terms of TW and L0 for the rod immersed within furnace wall. Fin tip is insulated.

$$HDE(0 \le x \le L_{ins})$$

$$\frac{d^2T}{dx^2} = 0; T(x) = C_1 x + C_2$$

$$BC\#1 :, T(x=0) = T_w, C_2 = T_w$$

$$T(x) = C_1 x + T_w$$

$$BC\#2$$

$$-k\frac{dT}{dx}_{x=L_{ins}} = q_{fin} / A_c$$

$$-kC_1 = M \tanh(mL_0)$$

$$C_1 = \frac{-M \tanh(mL_0)}{kA_c}$$

$$T(x) = T_w - \frac{M \tanh(mL_0)}{kA_c} x$$

$$T(x) = T_w - \frac{M \tanh mL_0}{kA_c} x$$

$$M = \sqrt{hPkA_c}\,\theta_b$$

$$T(x) = T_w - \frac{\sqrt{hPkA_c}\,\theta_b \tanh mL_0}{kA_c} x$$

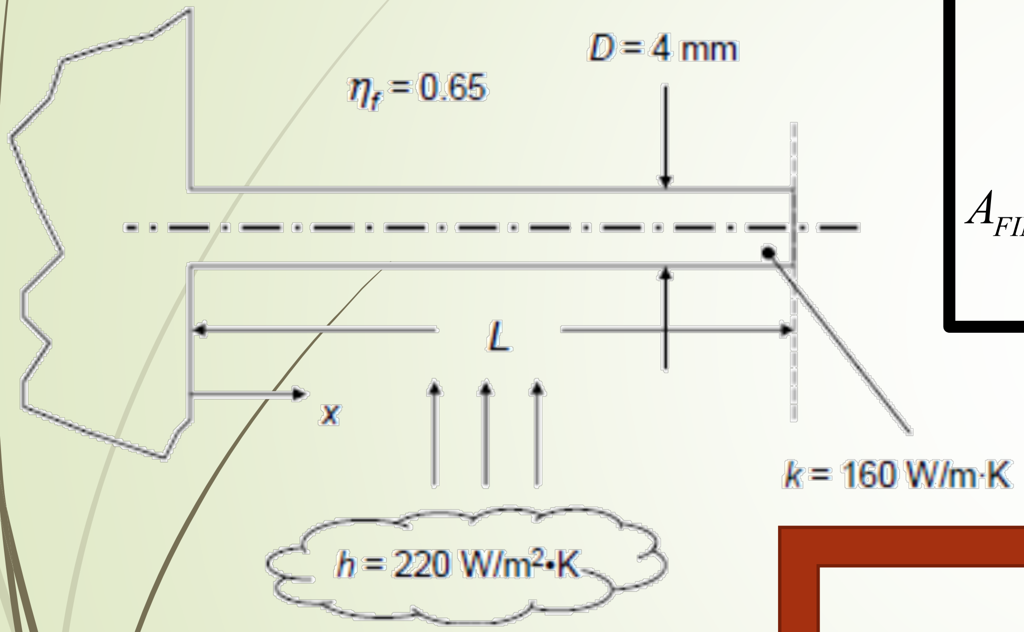
at $x=L_{ins}$

$$T_0 = T_w - \frac{\sqrt{hPkA_c}(T_0 - T_\infty)\tanh mL_0}{kA_c} L_{ins}$$

$$T_0\left(\frac{kA_c}{L_{ins} \tanh mL_0 \sqrt{hPkA_c}} + 1\right) = T_w \frac{kA_c}{L_{ins} \tanh mL_0 \sqrt{hPkA_c}} + T_\infty$$

$$T_0 = \frac{T_w \dfrac{kA_c}{L_{ins} \tanh mL_0 \sqrt{hPkA_c}} + T_\infty}{\left(\dfrac{kA_c}{L_{ins} \tanh mL_0 \sqrt{hPkA_c}} + 1\right)}$$

A PIN fin of **uniform cross section** (TYPE A) with k and h defined. Find fin heat rate and **EFFECTIVENESS** if efficiency is 65% with L=25mm, wall temperature = 200C and fluid temperature = 25C



$$\eta_{fin} = \frac{\text{Fin Heat Transfer Rate}}{\text{MAX HEAT TRANSFER POSSIBLE}} = \frac{q_{fin}}{q_{max}} = \frac{q_{fin}}{hA_{FIN\ TOTAL}\theta_b}$$

$$= 0.65$$

$$A_{FIN\ TOTAL} = \text{TOTAL FIN AREA EXPOSED TO FLUID}$$

$$\theta_b = T_b - T_\infty$$

$$\eta_{fin} = 0.65 = \frac{q_{fin}}{q_{max}} \rightarrow q_{fin} = 0.65 q_{max}$$

$$q_{fin} = 0.65 \bullet h \bullet A_f \bullet \theta_b$$

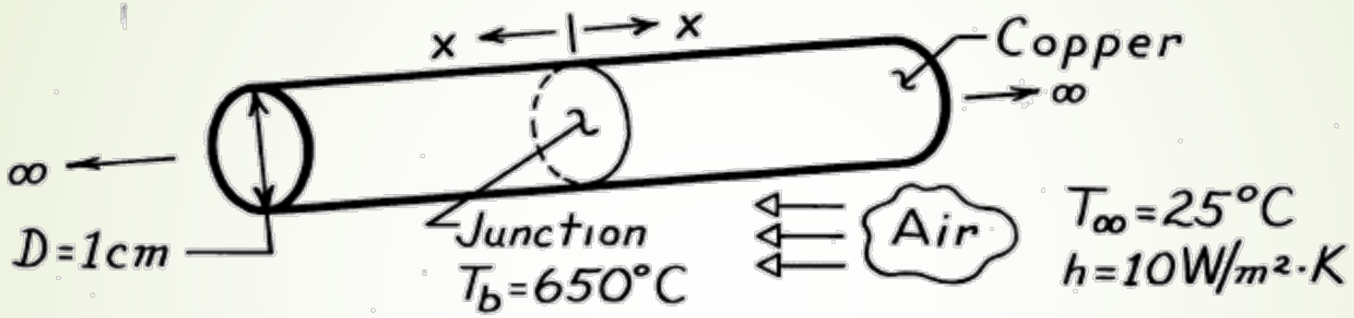
$$A_f = \pi DL = \pi \bullet \frac{4}{1000} \bullet \frac{100}{1000} = 1.3x10^{-3} m^2$$

$$q_{fin} = 0.65 \bullet h \bullet \pi DL \bullet (T_b - T_\infty) = 7.85W$$

$$\varepsilon_{fin} = \frac{q_f}{hA_{c,b}\theta_b} = \frac{q_f}{h\left[\frac{\pi D^2}{4}\right](200-25)} = \frac{7.85W}{0.4838W}$$

$$= 16.2$$

Two **long** copper rods are soldered together with melting point of 650C as shown, what is minimum power input needed to effect soldering. Neglect radiation.



$$k = 379W/m-K \quad \left(@\bar{T}[K] = \frac{650+25}{2} + 273 \approx 600K \right)$$

"*Assume* Long Fin"

$$q_{\min} = 2 \bullet M = 2\sqrt{hPkA_c}\,\theta_b = 120.9W \rightarrow \text{minimum neglecting radiation}$$

$$P = \pi D, A_c = \frac{\pi D^2}{4}, D = \frac{1}{100}m$$

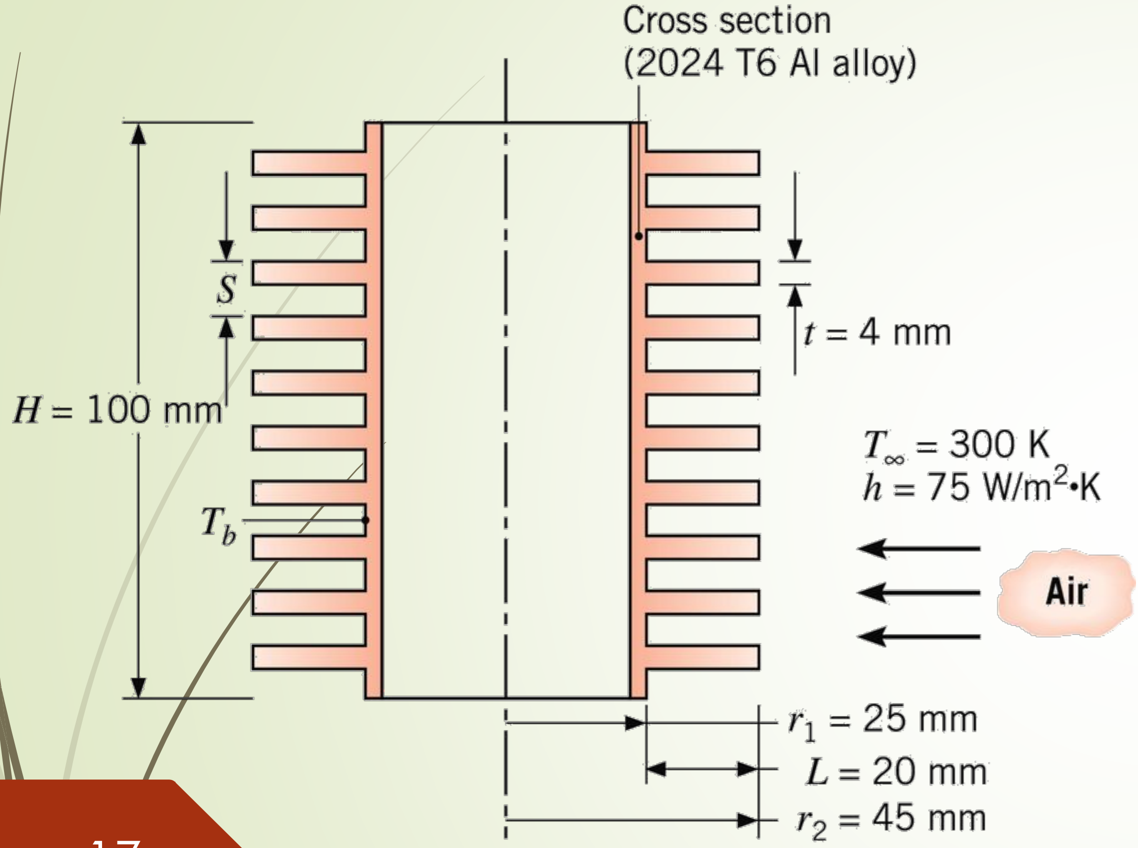
$$\theta_b = 650C - 25C$$

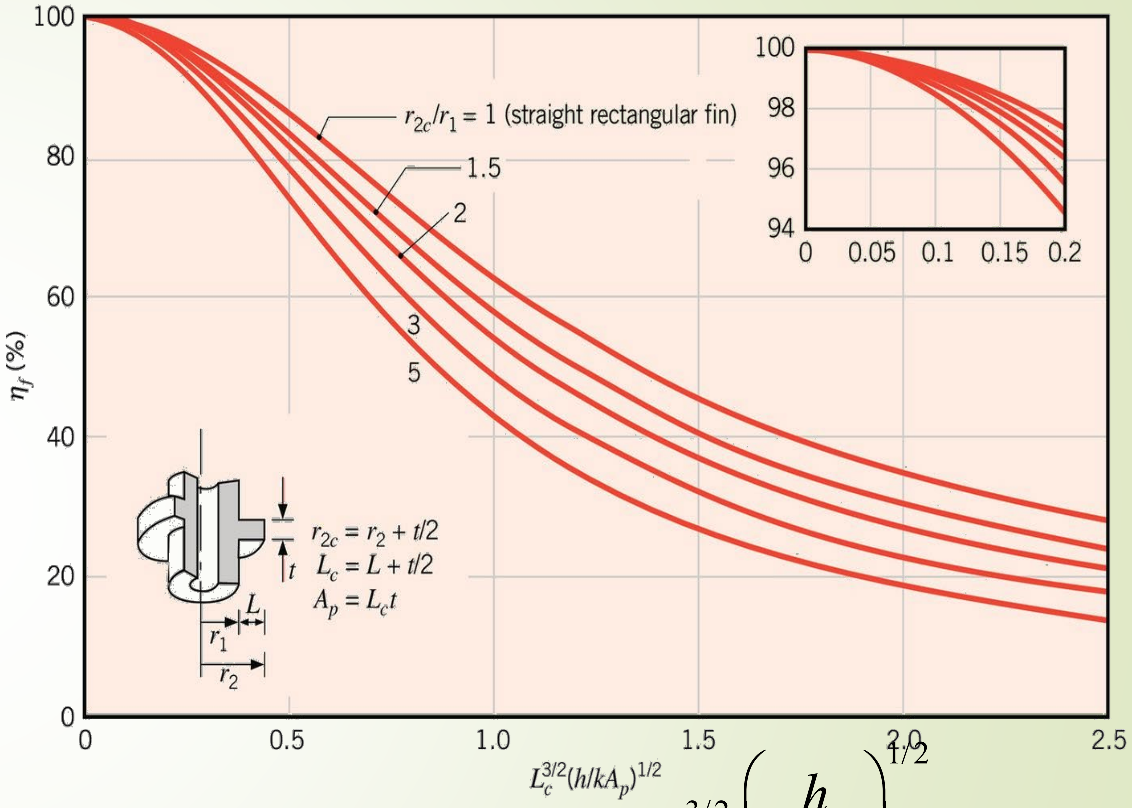
TYPE B

Rectangular Fins of ANNULAR CROSS SECTION

ONLY OPTION

$$q_{fin} = \eta_{fin} \bullet q_{max} = \eta_{fin} \bullet h \bullet A_{fin} \bullet \theta_b$$

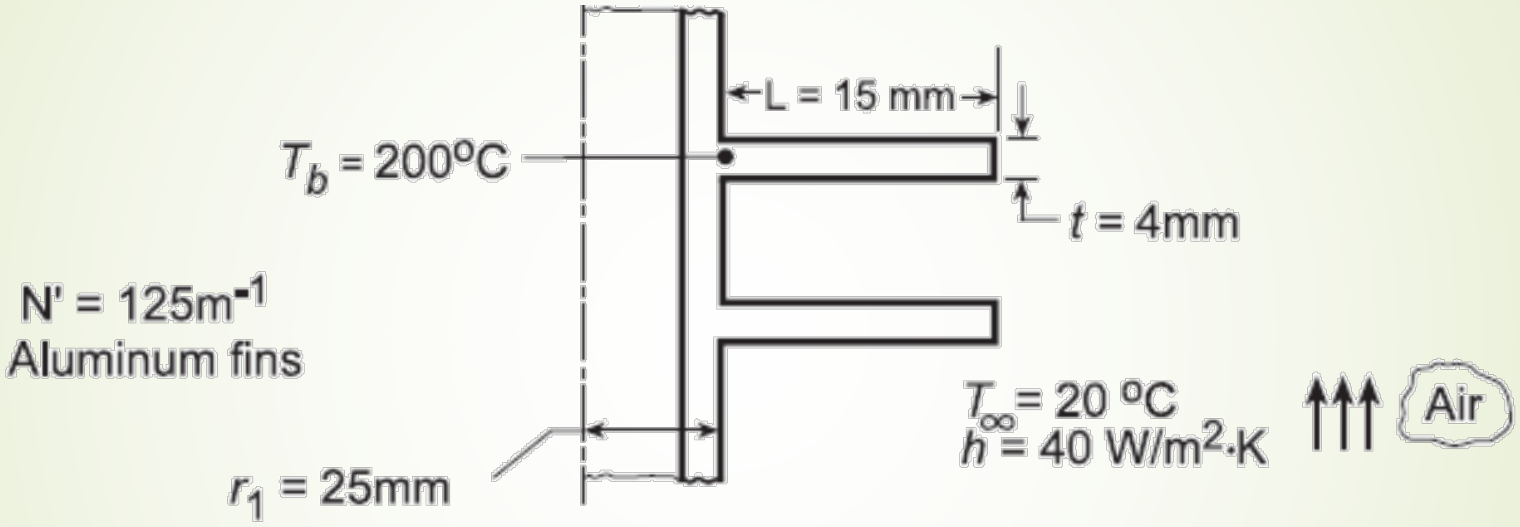




$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2}$$

ANNULAR FINS of REGTANGULAR PROFILE-EFFICIENCY

An ANNULAR AL fin of rectangular profile (TYPE B) is attached to a tube as shown. What is fin efficiency and effectiveness?



$$\eta_{fin} = \frac{\text{Fin Heat Transfer Rate}}{\text{MAX HEAT TRANSFER POSSIBLE}} = \frac{q_{fin}}{q_{max}} = \frac{q_{fin}}{hA_{FIN\ TOTAL}\theta_b}$$

$A_{FIN\ TOTAL}$ = TOTAL FIN AREA EXPOSED TO FLUID

$$\theta_b = T_b - T_\infty$$

$$\varepsilon_{fin} = \frac{q_f}{hA_{c,b}\theta_b}$$

FIN EFFICIENCY

Table A-1, Aluminum, T ≈ 400K:k=240W/m-K

FIGURE 3.20

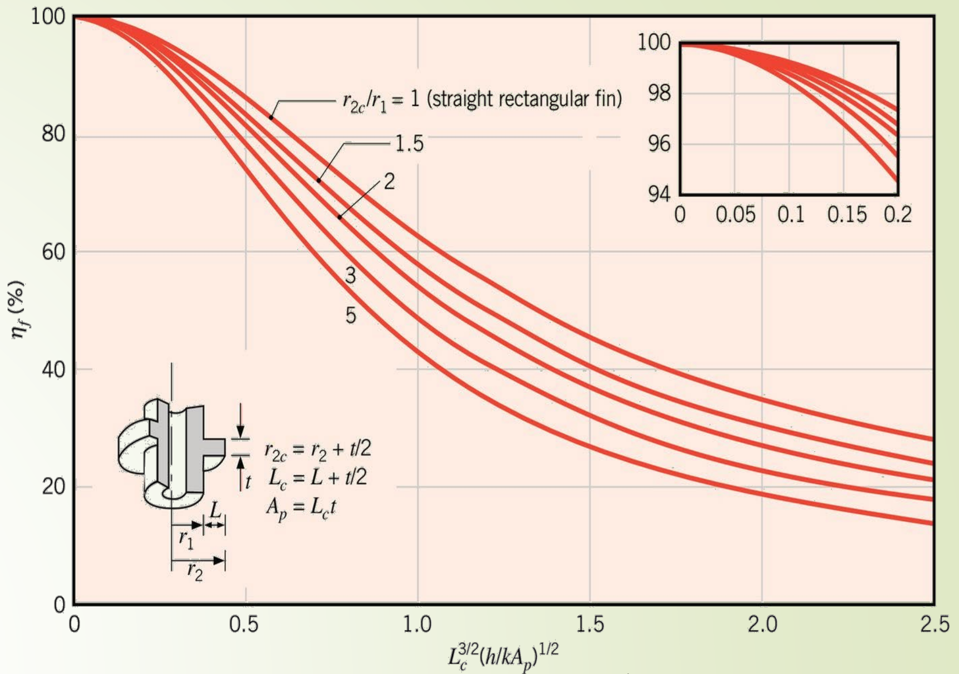
$$r_{2c} = r_2 + \frac{t}{2} = 40mm + 2mm = 0.042m,\ L_c = L + \frac{t}{2} = 15mm + 2mm = 0.017m$$

$$\frac{r_{2c}}{r_1} = 0.042m / 0.025m = 1.68$$

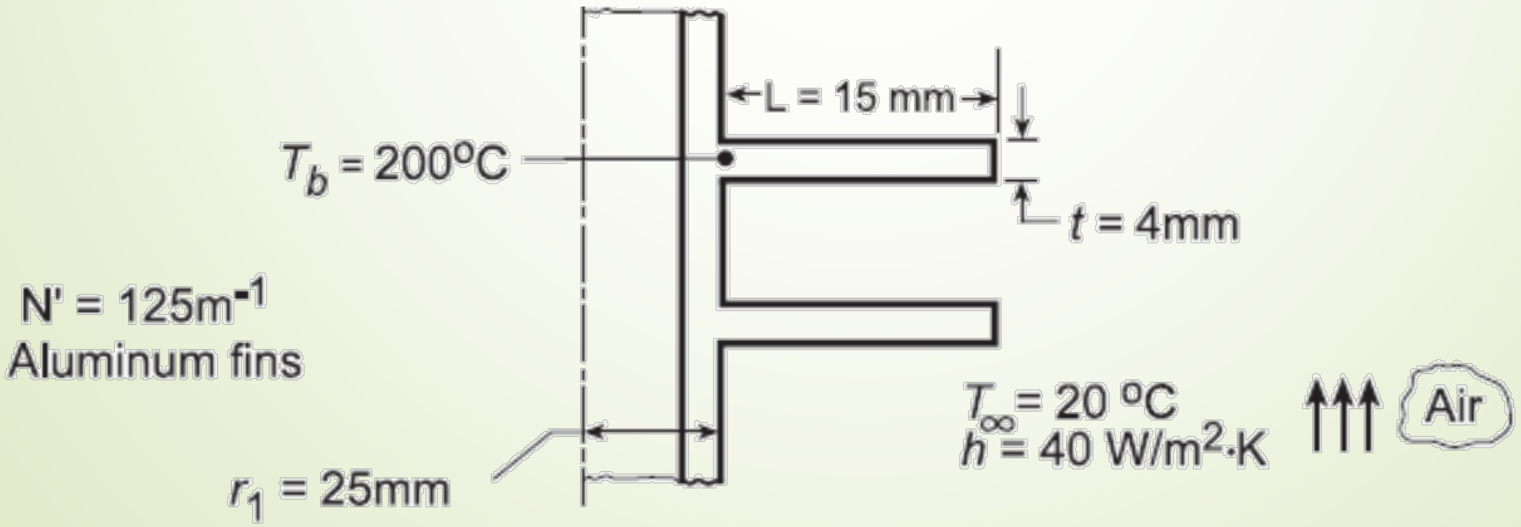
$$A_p = L_c t = 0.017m \bullet 0.004m = 6.85x10^{-5} m^2$$

$$L_c^{3/2}\sqrt{\frac{h}{kA_p}} = 0.017^{3/2}\sqrt{\frac{40\frac{W}{m^2-K}}{240\frac{W}{m-K}\bullet 6.85x10^{-5}m^2}} = 0.11$$

$\eta_f \approx 0.97 \rightarrow$ FIN EFFICIENCY



$$L_C^{3/2}\left[\frac{h}{kA_p}\right]^{1/2}$$



SINGLE FIN HEAT TRANSFER RATE

$$\eta_{fin} = \text{Fin Efficiency} = \frac{q_f}{q_{max}}$$

$$q_f = \eta_{fin} \bullet q_{max}$$

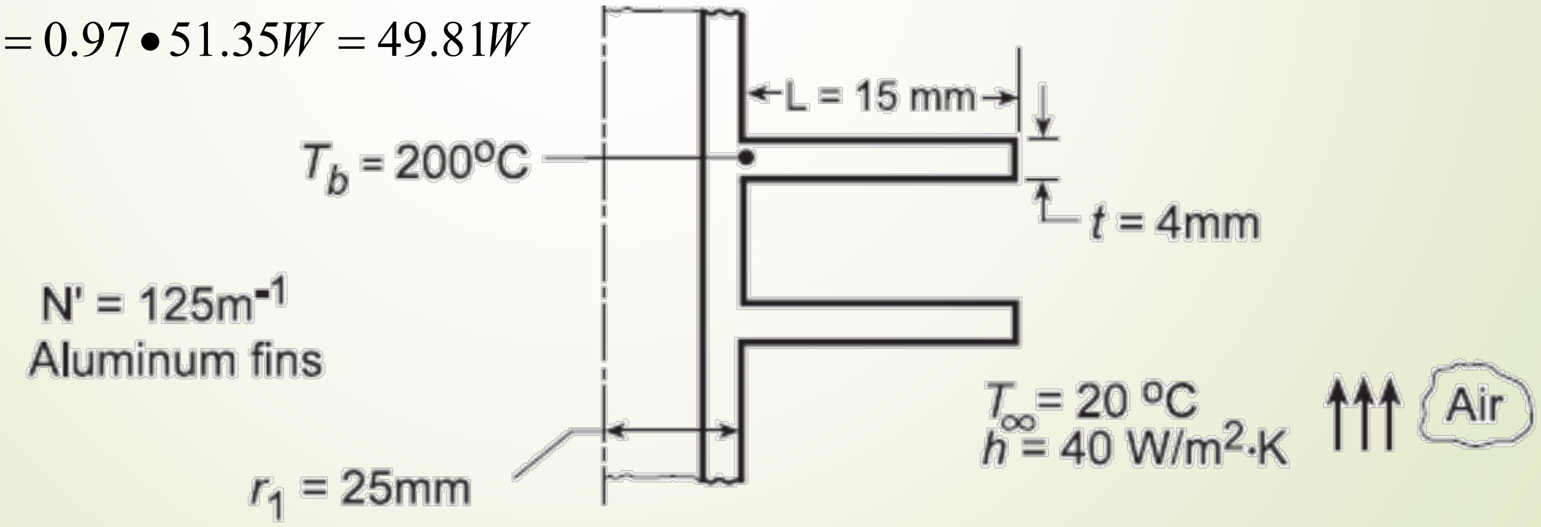
$$q_{max} = h \bullet A_{\text{FIN TOTAL}} \bullet \theta_b,$$

$$A_{FIN} = \left[2\pi(r_2^2 - r_1^2) + 2\pi t(r_2)\right] = 2\pi(40^2 - 25^2) + 2\pi \bullet 4 \bullet (40)$$

$$= 2\pi(975 + 160) \bullet \frac{1m^2}{1000^2 mm} = 7.1314x10^{-3} m^2$$

$$q_{max} = 40\frac{W}{m^2 - K} \bullet A_{\text{FIN TOTAL}} \bullet (200 - 20) = 51.35W$$

$$q_f = 0.97 \bullet 51.35W = 49.81W$$

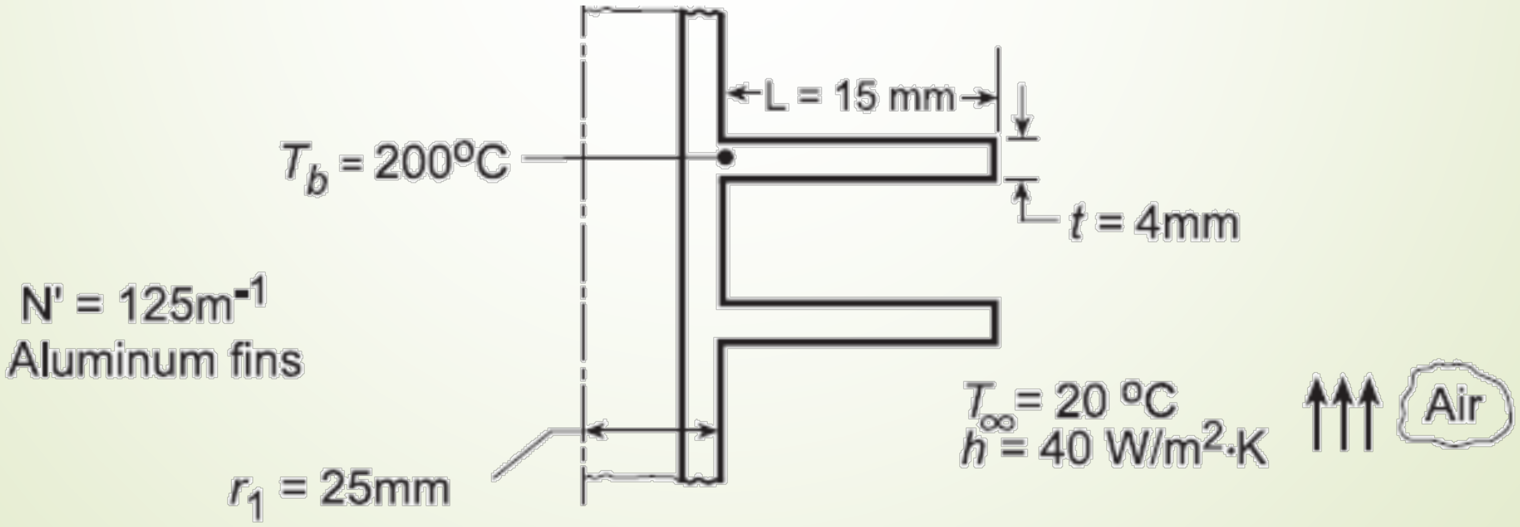


FIN EFFECTIVENESS

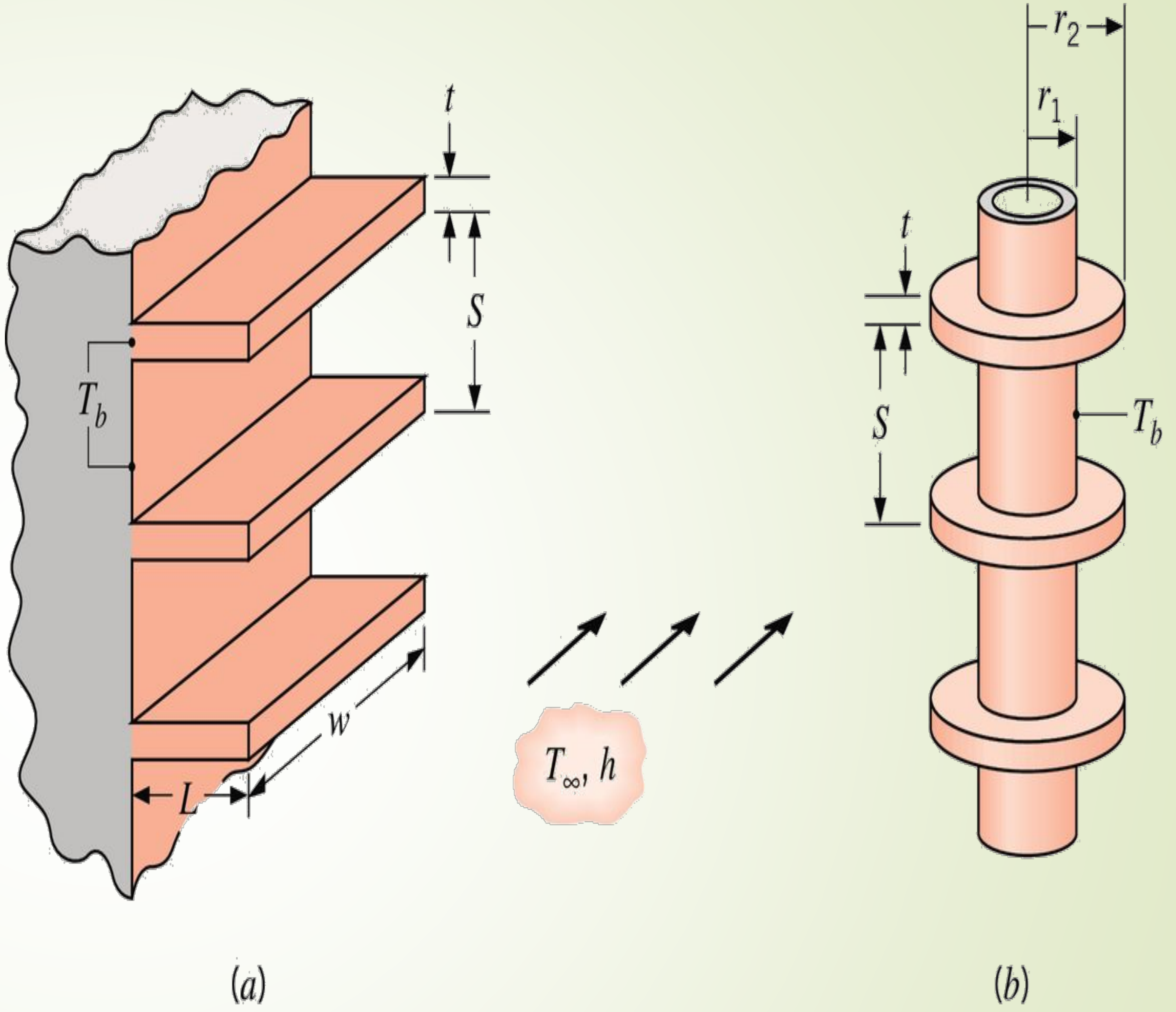
$$\varepsilon_{fin} = \frac{q_f}{hA_{c,b}\theta_b}$$

$$A_{c,b} = 2\pi r_1 t = 6.283x10^{-4}\,m^2$$

$$\varepsilon_{fin} = \frac{q_f = 49.81W}{40\dfrac{W}{m^2-K}A_{c,b}(200-20)K} = 11.01$$



FIN ARRAY SYSTEMS



TOTAL FIN ARRAY HEAT TRANSFER

$$q_{\text{TOTAL}}[W] = q_{fin} + q_{wall-exposed}$$

q_{fin} = Heat transfer for all fins

$q_{wall-exposed}$ = Heat transfer for exposed wall with NO Fins

$$q_{\text{TOTAL}}[W] = N\eta_{fin} q_{MAXIMUM} + hA_0\theta_b$$

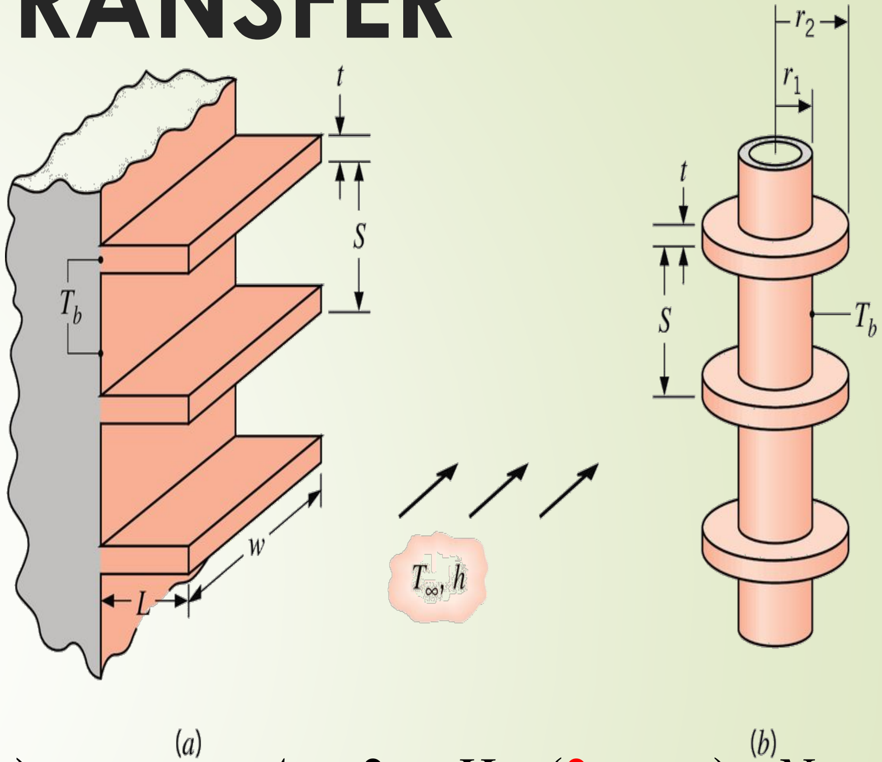
A_0 = Wall exposed surface area

N= Number of Fins

$$\theta_b = T_b - T_\infty$$

$$\eta_{fin} = \text{Fin Efficiency} = \frac{q_f}{q_{\max}}$$

$$q_{\max} = h \bullet A_{\text{FIN TOTAL}} \bullet \theta_b$$



$$A_0 = HW - (W \bullet t) \bullet N$$

H = Wall Height

$$A_{FIN_{TOTAL}} = 2L(W + t)$$

(Table 3.4)

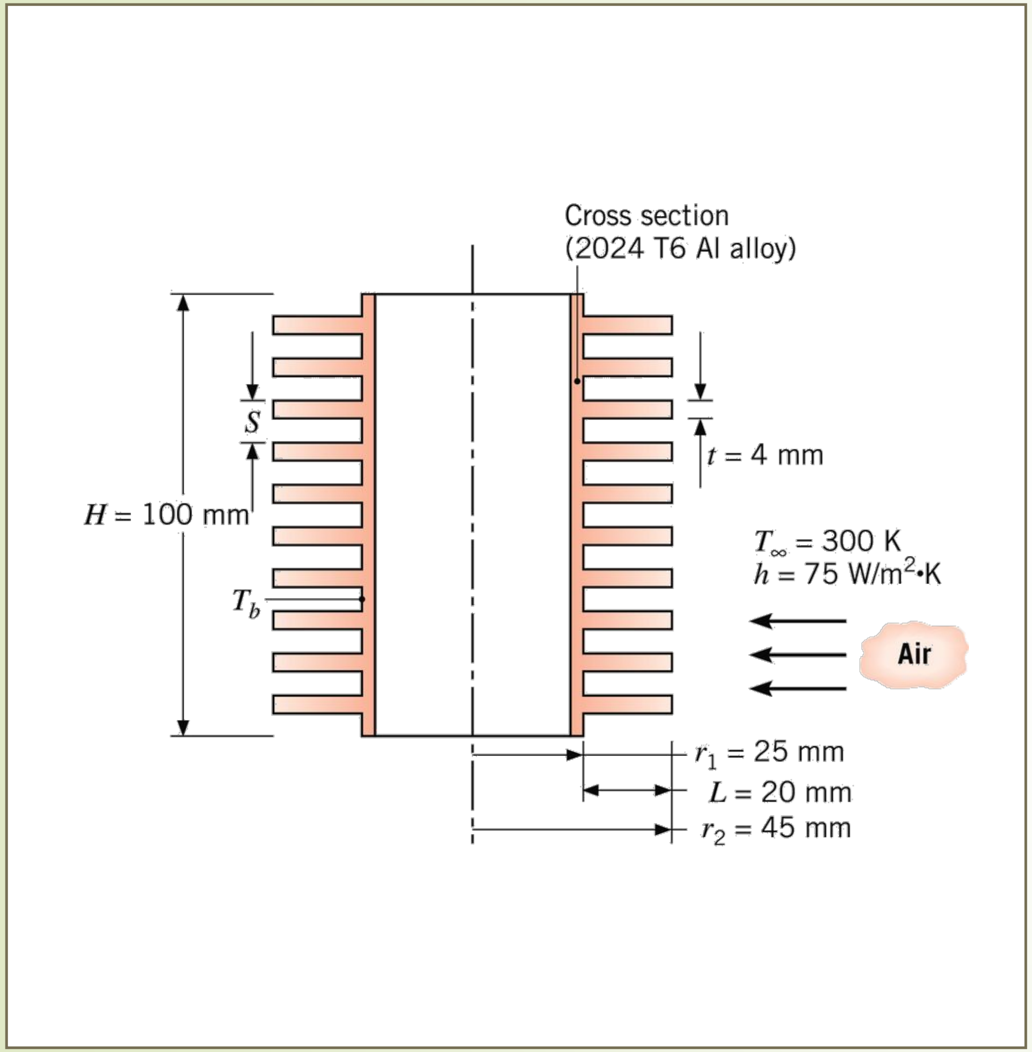
$$A_0 = 2\pi r_1 H - (2\pi r_1 \bullet t) \bullet N$$

H = *Tube* Height

$$A_{FIN_{TOTAL}} = \left[2\pi(r_2^2 - r_1^2) + 2\pi t(r_2 - r_1)\right]$$

(Figure 3.20)

PROBLEM



Find the HEAT TRANSFER RATE PER UNIT LENGTH (W/m) if the number of fins per unit length is N=400 Fins/m.

k = 177 W/m-K
Tb = 450K

$$q_{\text{TOTAL}}[W] = N\eta_{fin}q_{MAXIMUM} + hA_0\theta_b$$

$$A_0 = 2\pi r_1 H - 2\pi r_1 \bullet t \bullet N$$

$$H = Tube\ \text{Height}$$

$$q_{\max} = h \bullet A_{\text{FIN TOTAL}} \bullet \theta_b$$

$$\text{A}_{FIN} = \left[2\pi(r_2^2 - r_1^2) + 2\pi t(r_2 - r_1)\right]$$

$$q_{\text{TOTAL}}[W] = N\eta_{fin}q_{MAXIMUM} + h\theta_b\left[2\pi r_1 H - 2\pi r_1 \bullet t \bullet N\right]$$

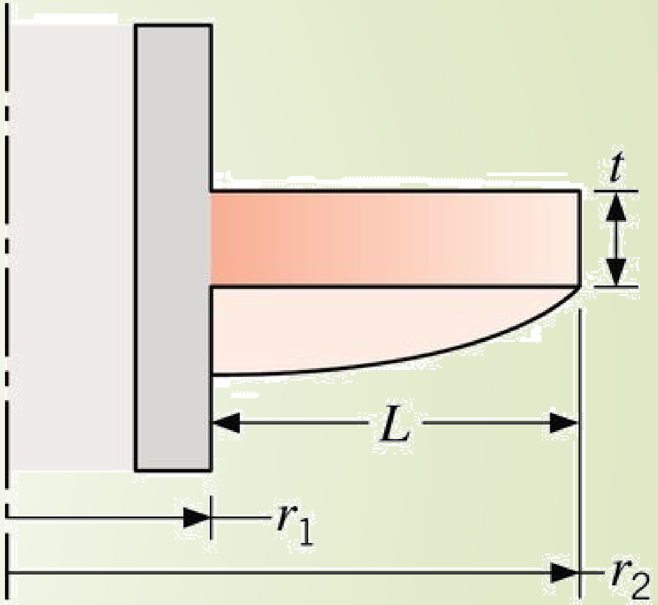
$$= N\eta_{fin}q_{MAXIMUM} + h\theta_b\left[2\pi r_1 H - 2\pi r_1 \bullet t \bullet N\right]$$

$$\div H$$

$$\frac{q_{\text{TOTAL}}[W]}{H} = \frac{N}{H}\eta_{fin}q_{MAXIMUM} + h\theta_b\left[2\pi r_1\frac{H}{H} - [2\pi r_1 \bullet t] \bullet \frac{N}{H}\right]$$

$$q'\left[\frac{W}{L}\right] = N'\eta_{fin}q_{MAXIMUM} + \left[2\pi r_1 - [2\pi r_1 \bullet t] \bullet N'\right]$$

$$q'\left[\frac{W}{L}\right] = N'(\eta_{fin}q_{MAXIMUM} - [2\pi r_1 \bullet t]h\theta_b) + h\theta_b 2\pi r_1$$



$$q'\left[\frac{W}{L}\right] = N'(\eta_{fin}q_{MAXIMUM} - [2\pi r_1 \bullet t]h\theta_b) + h\theta_b 2\pi r_1$$

$$N' = \frac{q'\left[\frac{W}{L}\right] - h\theta_b 2\pi r_1}{(\eta_{fin}q_{MAXIMUM} - [2\pi r_1 \bullet t])}$$