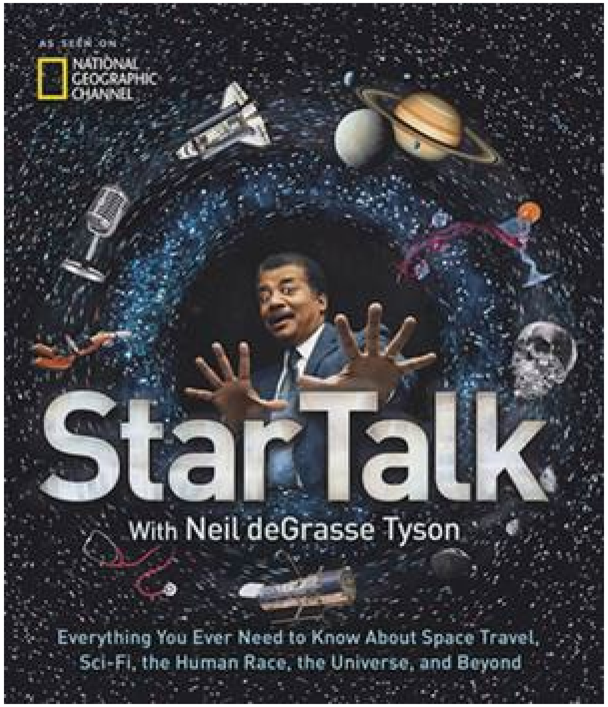


Chapter 4
Fluid Kinematics

KNOWING "***<u>HOW</u>***" TO THINK, EMPOWERS YOU FAR BEYOND THOSE WHO ONLY KNOW WHAT TO THINK!!

1

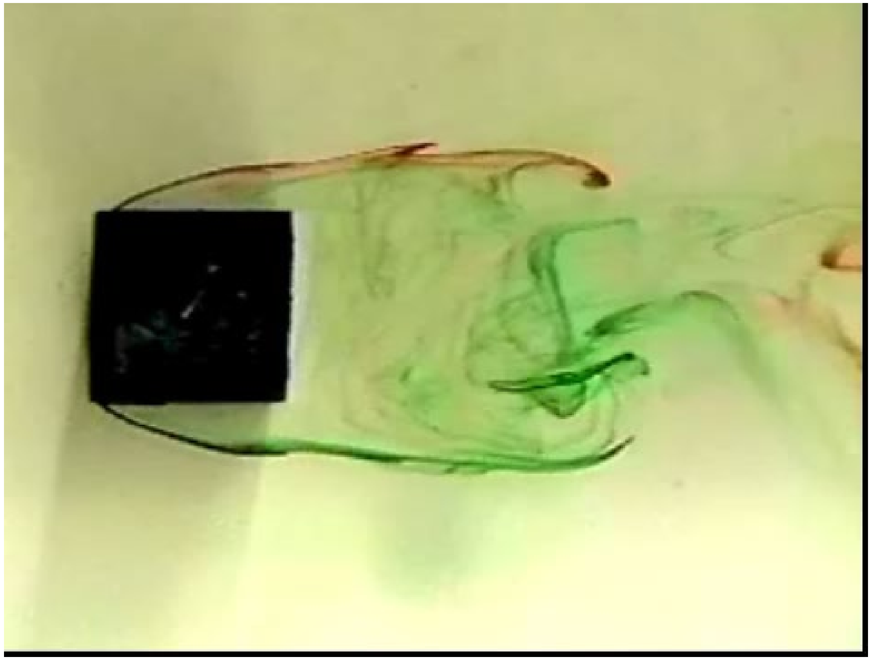
Educational Objectives

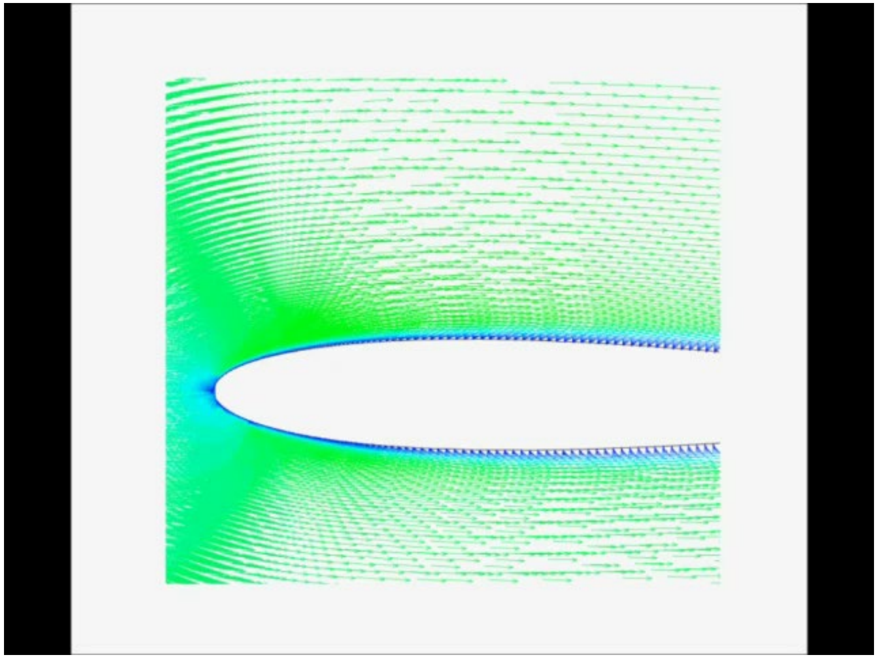
Understand velocity and acceleration of particles along streamlines.

Be able to compute and explain **LOCAL** and **CONVECTIVE** acceleration for a **SCALAR** variable.

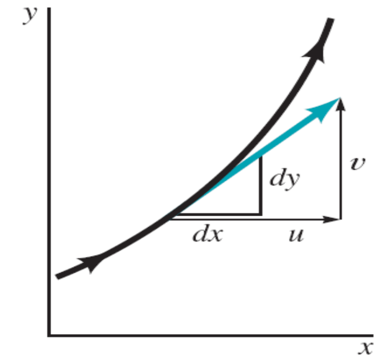
Flow Types Steady and Unsteady Flow

- **Fluid Kinematics**



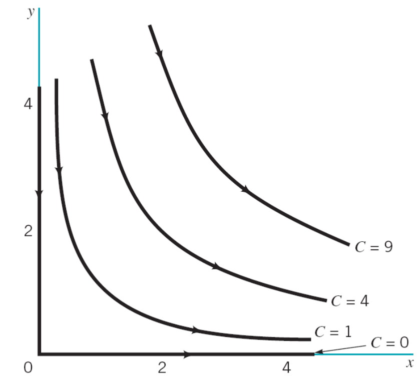


Class 09: Fluid Kinematics



Streamlines, Streaklines and Pathlines

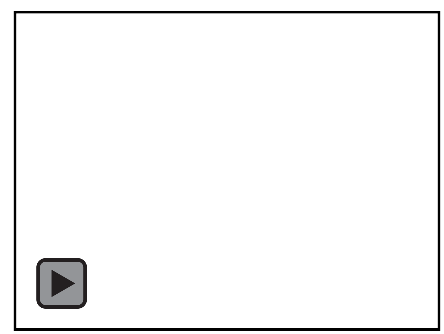
- **To visualize and anlyze the flow fields, streamlines, streaklines and pathlines are often used**



- **A streamline is a line that is everywhere tangent to the velocity field**

- **For *unsteady* flows the streamlines may change shape with time**

- **A pathline is a line traced out by a given particle as it flows from one point to another (Lagrangian concept)**



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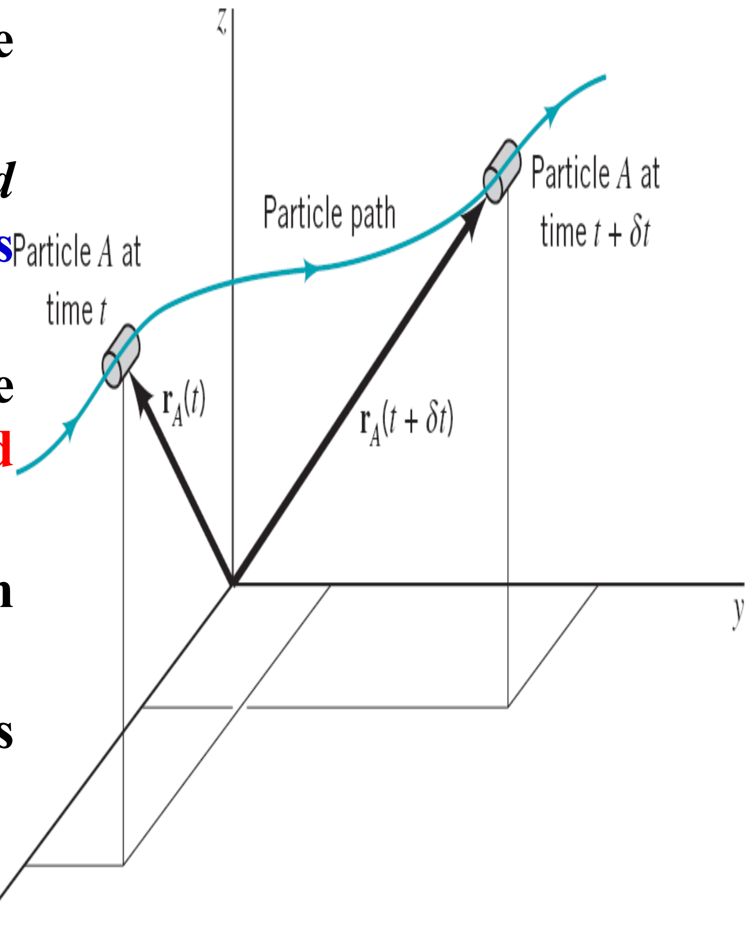
- Fluids have a well-known tendency to move or flow
- Fluid kinematics considers the ***velocity and acceleration*** of the fluid **instead of forces that causes fluid motion**
- The **motion of fluid particles** can be described **in terms of velocity and acceleration**
- Fluid flow must be described as a function of spatial coordinates (x,y,z) and time (t)
- One of the most important fluid variables is the velocity field

  $V = u(x,y,z,t)i + v(x,y,z,t)j + w(x,y,z,t)k$

- *The position vector r_A is a function of time,* $V_A = dr_A/dt$



Velocity Vector: $\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k} = u\vec{i} + v\vec{j} + w\vec{k}$

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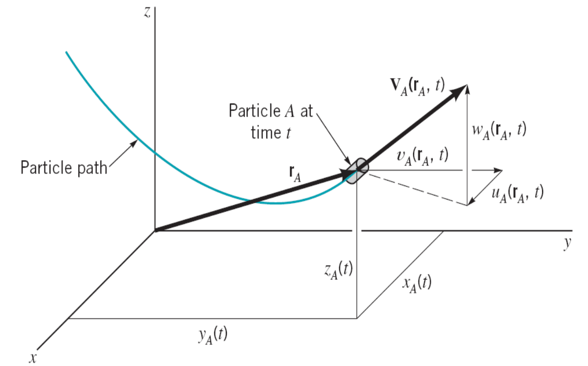

The Material Derivative

- **Consider a fluid particle moving along a pathline. The particle's velocity** $V_A = V_A\,(r_A, t)$
- **Using the CHAIN RULE OF DIFFERENTIATION the acceleration of particle A,** a_A **can be derived**

$$\vec{V} = \vec{V}\left[t, x(t), y(t), z(t)\right]$$

$$\Rightarrow \vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz}{dt}$$

$$\Rightarrow \vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

$$\Rightarrow \vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

$$\frac{D(\beta)}{Dt} = \frac{\partial(\beta)}{\partial t} + u\frac{\partial(\beta)}{\partial x} + v\frac{\partial(\beta)}{\partial y} + w\frac{\partial(\beta)}{\partial z}$$

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$u = \frac{\partial x}{\partial t}$$

$$v = \frac{\partial y}{\partial t}$$

$$w = \frac{\partial z}{\partial t}$$

Material Derivative

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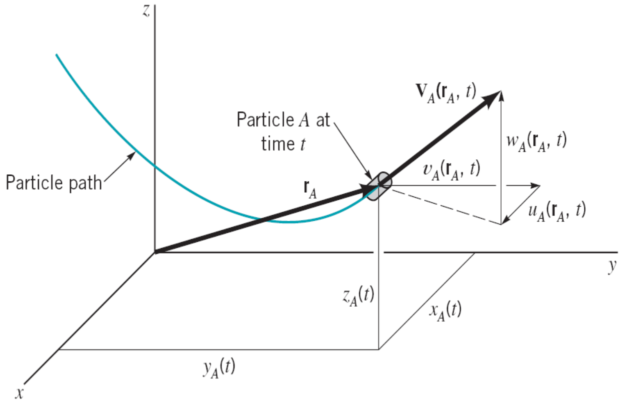
X-, Y-, and Z-component of acceleration



$$a_x(x,y,,z,t) = \frac{Du}{Dt} = \text{TOTAL ACCELERATION} = \underbrace{\frac{\partial u}{\partial t}}_{\text{Local Acceleration}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective Acceleration}}$$

$$a_x(x,y,z,t) = \frac{Du}{Dt} \rightarrow \text{Time Rate of Change of "U-VELOCITY"}$$

$$a_y(x,y,z,t) = \frac{Dv}{Dt} = \underbrace{\frac{\partial v}{\partial t}}_{\text{Local acceleration}} + \underbrace{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}}_{\text{Convective acceleration}} \rightarrow \text{Time Rate of Change of "V-VELOCITY"}$$

$$a_z(x,y,z,t) = \frac{Dw}{Dt} = \underbrace{\frac{\partial w}{\partial t}}_{\text{Local acceleration}} + \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{Convective acceleration}} \rightarrow \text{Time Rate of Change of "W-VELOCITY"}$$

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Example: $\vec{V}=\left(t^2+Cx^2\right)\hat{i}+\left(2t+Cxy^2\right)\hat{j}$; C=constant; *Find* a_x & a_y.

Solution: $u=t^2+Cx^2;\quad v=2t+Cxy^2$

$$a_x=\frac{Du}{Dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y};\rightarrow \textit{Valid for all x,y,z,t}$$

Time Rate of Change of "u" velocity

$$\Rightarrow a_x(x,y,z,t)=2t+\left(t^2+Cx^2\right)\left(2Cx\right)+\left(2t+Cxy^2\right)\left(0\right)$$

$$\Rightarrow a_x(x,y,z,t)=2\left(t+Ct^2x+C^2x^3\right)$$

$$a_y=\frac{Dv}{Dt}=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y};\rightarrow \textit{Valid for all x,y,z,t}$$

Time Rate of Change of "v" velocity

$$\Rightarrow a_y(x,y,z,t)=2+\left(t^2+Cx^2\right)\left(Cy^2\right)+\left(2t+Cxy^2\right)\left(2Cxy\right)$$

$$\Rightarrow a_y(x,y,z,t)=2\left(1+2Ctxy+C^2x^2y^3\right)+\left(t^2+Cx^2\right)Cy^2$$

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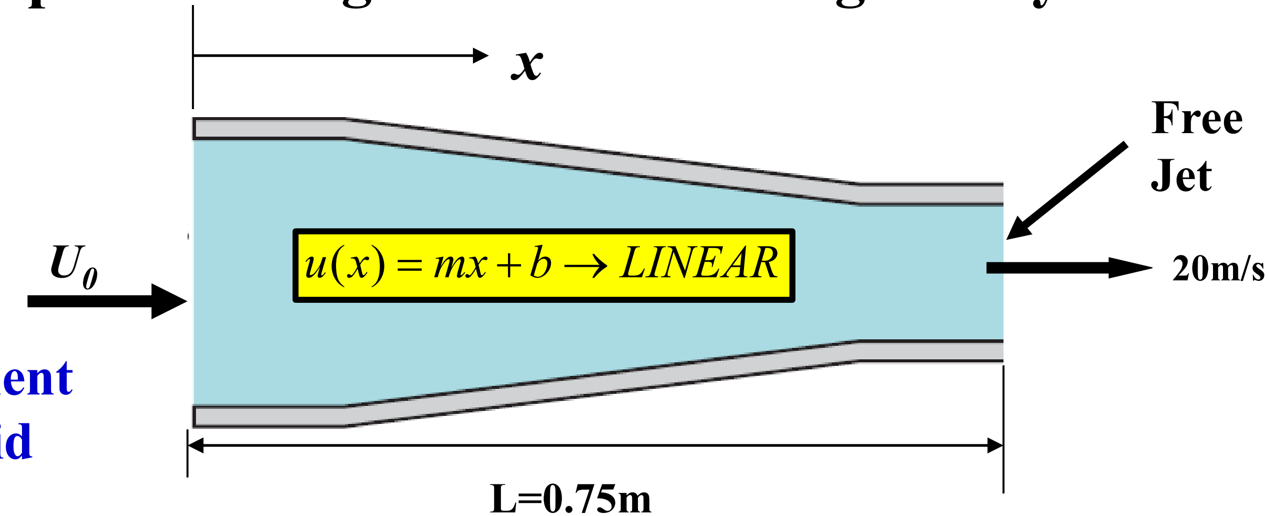
Problem: **Water flows through a 0.75m long nozzle. The flow enters with a uniform velocity $U_0 = 5m/s$ and increases to a speed of 20m/s where it exits as a free jet. The flow can be considered inviscid. The velocity profile along the centerline is given by**

$$u(x) = \frac{U_0}{1 - \frac{0.5x}{L}}$$



Determine the x-component of acceleration of the fluid along the centerline.

Solution:

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t}$$

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$$u(x)=\frac{U_0}{1-\frac{0.5x}{L}}$$

For our case:

$$a_x=\frac{\partial u}{\partial x}\frac{\partial x}{\partial t}=u\frac{\partial u}{\partial x} \qquad \frac{\partial u}{\partial x}=\left(\frac{0.5U_0}{L}\right)\left(\frac{1}{\left(1-0.5x/L\right)^2}\right)$$

$$\Rightarrow a_x=\left(\frac{U_0}{\left(1-0.5x/L\right)}\right)\left(\frac{0.5U_0}{L}\right)\left(\frac{1}{\left(1-0.5x/L\right)^2}\right)$$

$$\Rightarrow a_x(x,y,z,t)=\left(\frac{0.5U_0^2/L}{\left(1-0.5x/L\right)^3}\right);\rightarrow 0\le x\le L$$

@ x=0

$$a_x\left(x=0\right)=0.5U_0^2/L=16.67\,m/s^2$$ ← **Acceleration minimum**

@ x=L

$$a_x\left(x=L\right)=4U_0^2/L=133.33\,m/s^2$$ ← **Acceleration maximum**

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A 3D velocity vector is given by u=2x, v=-y, and w=z, determine the acceleration vector.

Solution:

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w$$

$$a_x = \frac{Du}{Dt} = 0 + (2x)(2) + (0)(0) + (0)(0) = 4x$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w$$

$$a_y = \frac{Dv}{Dt} = 0 + 0(0) + (-1)(-y) + (0)(0) = y$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w$$

$$a_z = \frac{Dw}{Dt} = 0 + 0(0) + 0(0) + (1)z = z$$

$$\vec{a}(x,y,z,t) = \left(4x\hat{i} + y\hat{j} + z\hat{k}\right)\frac{m/s}{s} \longrightarrow valid\ for\ all\ x,y,z,t$$

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Water flows through a constant diameter pipe with a uniform velocity by u(x,y,z,t) = 0, V(x,y,z,t) = (8/t + 5)m/s, , w(x,y,z,t) = 0. where t is in seconds. Find the acceleration at time = 1 and 10 sec.

$$a_y(x,y,z,t) = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w$$

$$= \frac{\partial v}{\partial t}\hat{j} = -\frac{8}{t^2}\hat{j}\frac{m}{s^2} \rightarrow \textit{Time rate of change of "v" velocity} \rightarrow \frac{m/s}{s}$$

$$a_y(x,y,z,t=1s) = -\frac{8}{1^2}\hat{j}\frac{m}{s^2}$$

$$a_y(x,y,z,t=10s) = -\frac{8}{10^2}\hat{j}\frac{m}{s^2} = -0.08\hat{j}\frac{m}{s^2}$$

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Example:

Consider Velocity Field

$$\vec{V}(x,y,z,t)=\left[\left(1t^2\left\{\frac{m}{s^3}\right\}+1x^2\left\{\frac{1}{m-s}\right\}\right)\hat{i}+\left(2t+xy^2\right)\hat{j}\right]\frac{m}{s}=u(x,y,z,t)\hat{i}+v(x,y,z,t)\hat{j}$$

Consider Temperature Field

$$\mathrm{T(x,t)}=\left(34\left\{\frac{K}{s^2}\right\}\mathrm{t}^2+5\left\{\frac{K}{m}\right\}x\right)K$$

$FIND$: Time Rate of Change of Temperature due to convective fluid motion: $\frac{K}{s}$

$$\frac{DT}{Dt}(x,y,z,t)=\frac{\partial T}{\partial t}+\frac{\partial T}{\partial x}u+\frac{\partial T}{\partial y}v+\frac{\partial T}{\partial z}w$$

$$=68t+(5)(\left(t^2+x^2\right))+0+0$$

$$\frac{DT}{Dt}(x,y,z,t)\frac{K}{s}=68\left\{\frac{K}{s^2}\right\}t\{s\}+5\left\{\frac{K}{m}\right\}(t^2\left\{\frac{m}{s^3}\right\}+x^2\left\{\frac{1}{m-s}\right\})\frac{K}{s}$$

HOMEWORK

4-3,25,26,29,34

End of
Chapter 4

任何人都可以记住事情，但重要的是要了解它。