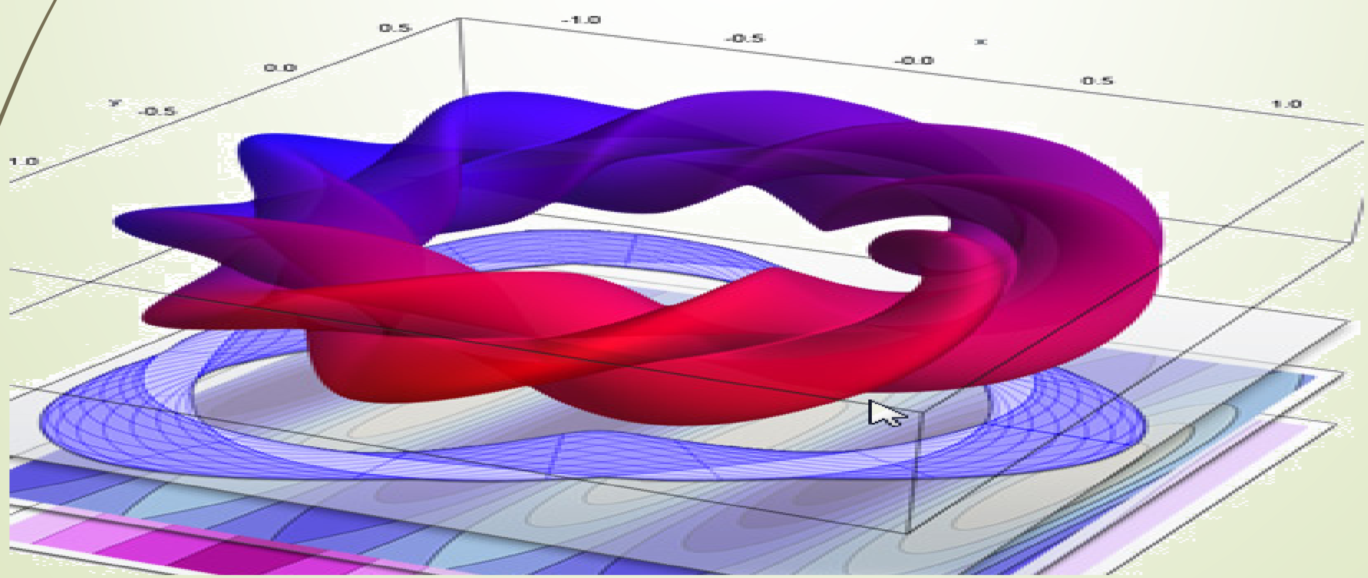
Fluid Mechanics
Introduction

What is Mechanical Engineering ?

- Oldest and most diverse and versatile engineering fields.
- ***APPLIED PHYSICS*** *used to study objects and systems that move and have motion; and is the study of the* ***Conversion of Energy*** *from one form to another to do useful* ***Work*****.**
- As such, the field of mechanical engineering touches virtually every aspect of modern life, including diverse systems from space flight to automotive to the human body.

FLUIDS STUDENT SPRING 2018

I believe MECH-322 has enhanced my abilities as an engineer. Parametric thinking is a useful tool that allows the mind to make connections from one variable to multiple variables. The parametric thinking taught in this course will definitely stay with me to later courses and will allow me to think more-abstractly about problems.



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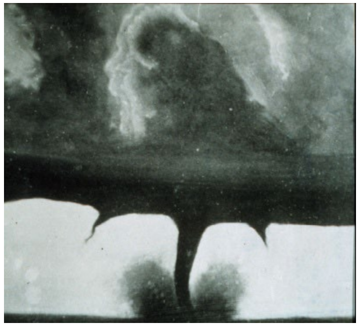
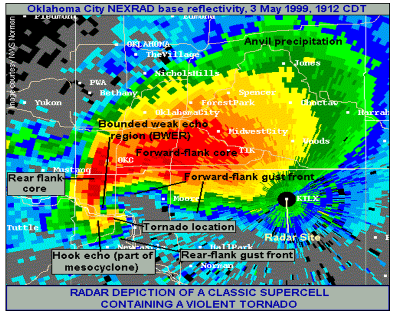
Fluid Definition

- Fluid Mechanics deals with the **behavior** of fluids at **rest** and in **motion**.
- The study of fluids is important since they are an integral part of the environment in which we live, etc;
  - Air
  - Water
  - Blood
  - Work and Power

Application of Fluid Mechanics



Tornado



Weather/Climate

Bridge & Sky Scraper

Space Research

Aerospace & Aeronautics

Auto Industry

Submarine & Submersibles

Biofluids

During your career as an engineer you will be involved in various systems design and analysis works those require a good understanding of Fluid Mechanics.

2023: WARMEST YEAR IN KNOWN HISTORY

Fluids Characteristics: DEFINITIONS

Solid (Steel, Concrete, etc.): it has densely spaced molecules with large intermolecular cohesive forces that allow the solid to maintain its shape, and to not be easily deformed. The movement of the molecules are restricted. They are arranged in a lattice formation .

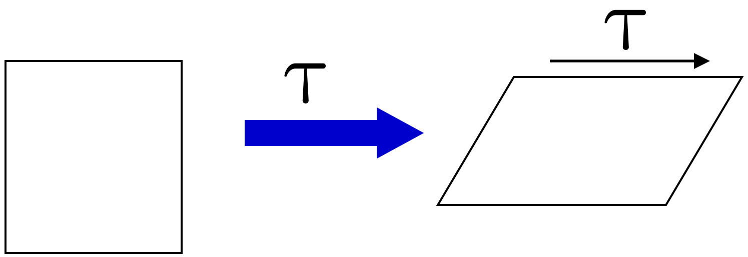
Liquid (water, oil, etc.): the molecules are spaced farther apart, the intermolecular forces are smaller than for solids and the molecules have more freedom of movement. Thus, liquids can be easily deformed (but not easily compressed) and can be poured into containers or forced through a tube.

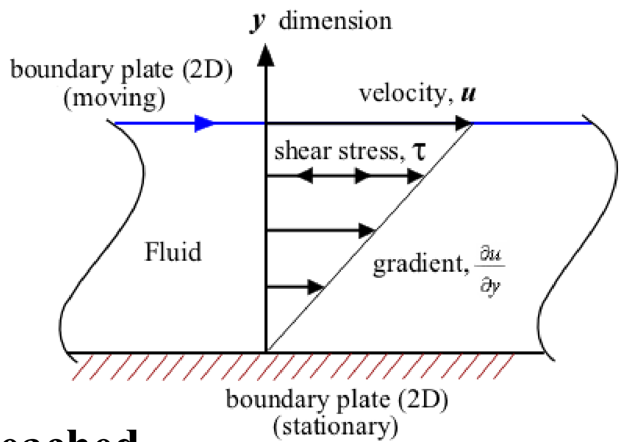
Gas (air, oxygen, etc.): it has even greater molecular spacing and freedom of movement with negligible cohesive intermolecular forces and as a consequence can be easily deformed and compressed, and will completely fill the volume of any container in which they are placed.

Fluids: It is defined as a substance that deforms continuously (a continuum) when acted on by a shearing stress (force per unit area) of any magnitude.

Analysis of Fluid Behaviors

Shear Stress and Shear Strain:





- **Materials deform until equilibrium state is reached.**

Shear stress ∞ Strain $\Rightarrow \tau \infty \dot{\gamma} \Rightarrow \tau = G\,\dot{\gamma}$

In a viscous fluid, the shear stress is proportional to the time rate of strain, i.e.

Example: water, air, oil, etc.

$$\frac{\partial u}{\partial n} \rightarrow \text{TIME RATE OF STRAIN}$$

SHEARING STRESS UNITS

In a viscous fluid, the shear stress is proportional to the time rate of strain, i.e.

$$\tau \propto \dot{\gamma} \Rightarrow \tau = \mu \frac{\partial u}{\partial y}$$

$$\tau \propto \dot{\gamma} \Rightarrow \tau[\frac{F}{Area}] = \mu[\frac{F-s}{Area}]\left\{ \frac{\partial u \left[\frac{m}{s} \right]}{\partial y [m]} [\frac{1}{s}] \right\}$$

$$\mu \equiv \text{Viscosity}[\frac{F-s}{Area}] \rightarrow \frac{N-s}{m^2} \rightarrow \frac{lbf-s}{ft^2}$$

$$\frac{du}{dy} \equiv \text{Time Rate of Strain or Velocity Gradient}[\frac{1}{s}]$$

Stress Field - Normal & Shear stresses

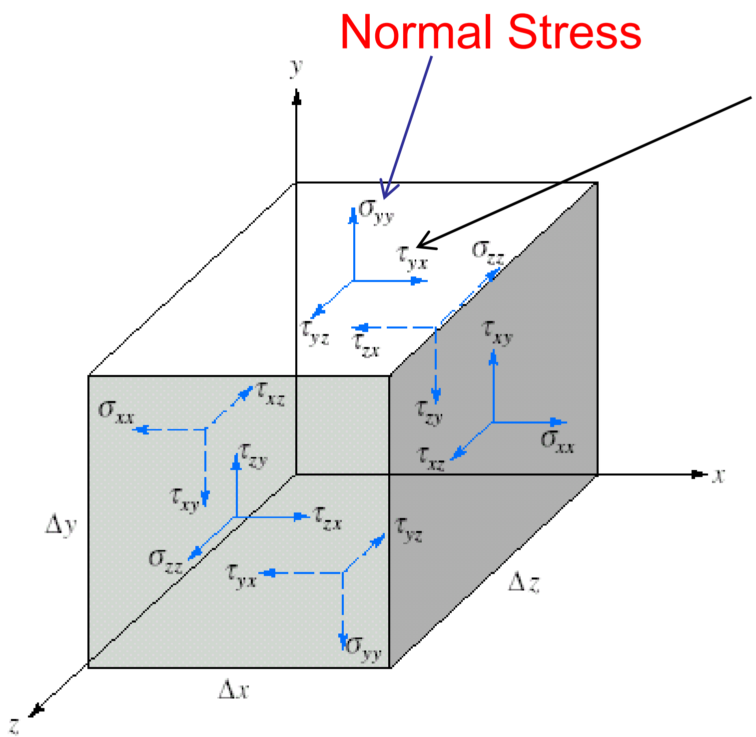


Fig. 2.6 Notation for stress.

Newtonian Fluids

$$\tau_{yx} \propto \frac{du}{dy}$$

$$\tau_{yx} = \mu \frac{du}{dy}$$

Example: water, air, oil, etc.

Shear stress is proportional to shear strain

Non-Newtonian Fluids

$$\tau \neq \mu \frac{\partial u}{\partial y}$$

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n$$

$$\tau_{yx} = k\left|\frac{du}{dy}\right|^{n-1} \frac{du}{dy} = \eta \frac{du}{dy}$$

Example: Blood, Tar, Slurries, etc.

Newtonian/Non-Newtonian Fluid Shear Stress

$$\tau \neq \mu \frac{\partial u}{\partial y}$$

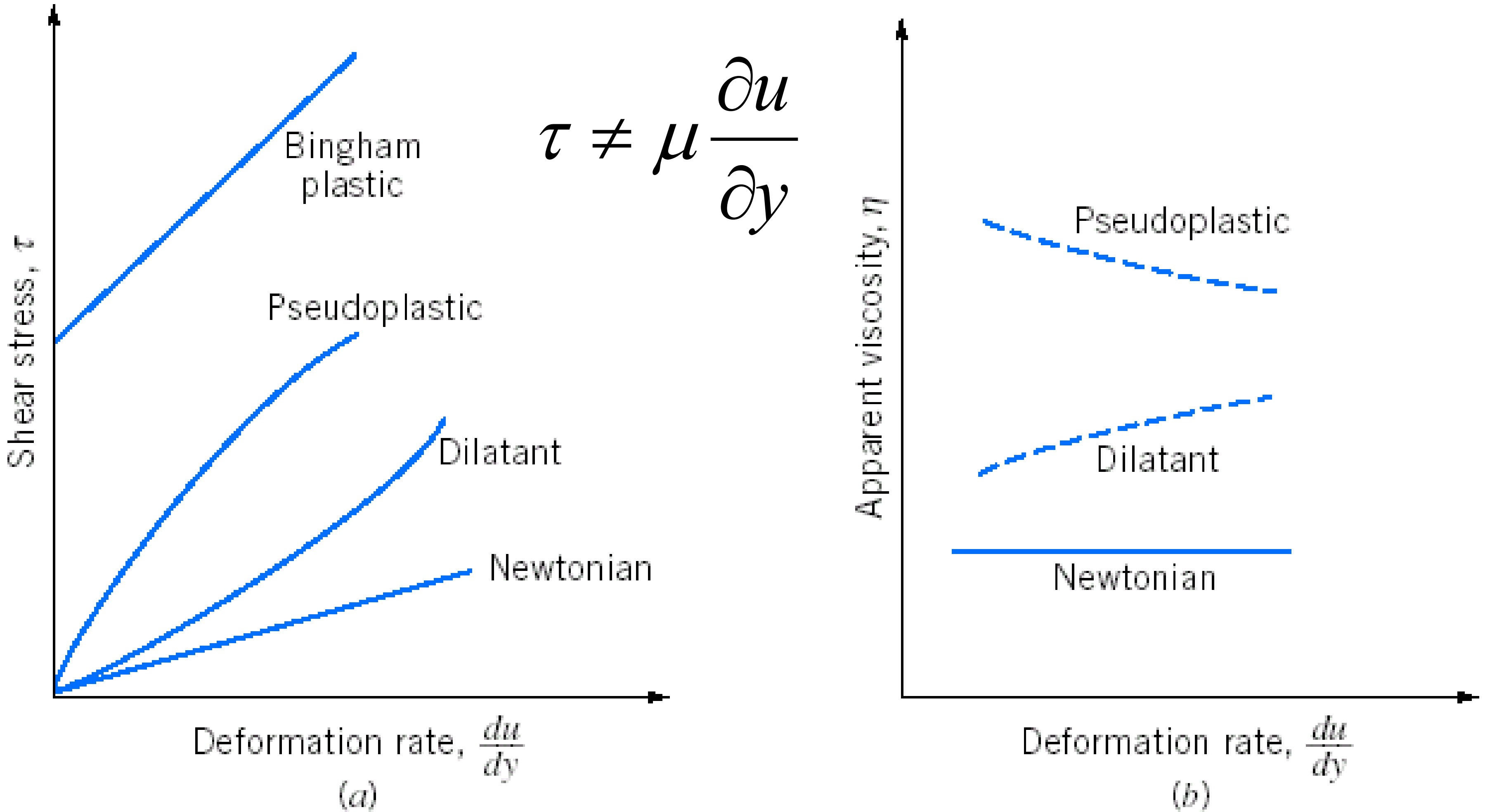


Fig. 2.8 (a) Shear stress, τ, and (b) apparent viscosity, η, as a function of deformation rate for one-dimensional flow of various non-Newtonian fluids.

SHEAR STRESS and SHEAR FORCE

- **SHEAR STRESS** is primarily caused by friction between fluid particles, due to fluid **VISCOSITY**.

- Fluids at rest cannot resist a shear stress; in other words, when a **SHEAR STRESS** is applied to a fluid at rest, the fluid will not remain at rest, but will move because of the **SHEAR STRESS**.

- **SHEAR FORCE** is the summation of the effect of shear stress over a **SURFACE AREA**, and often results in shear strain.

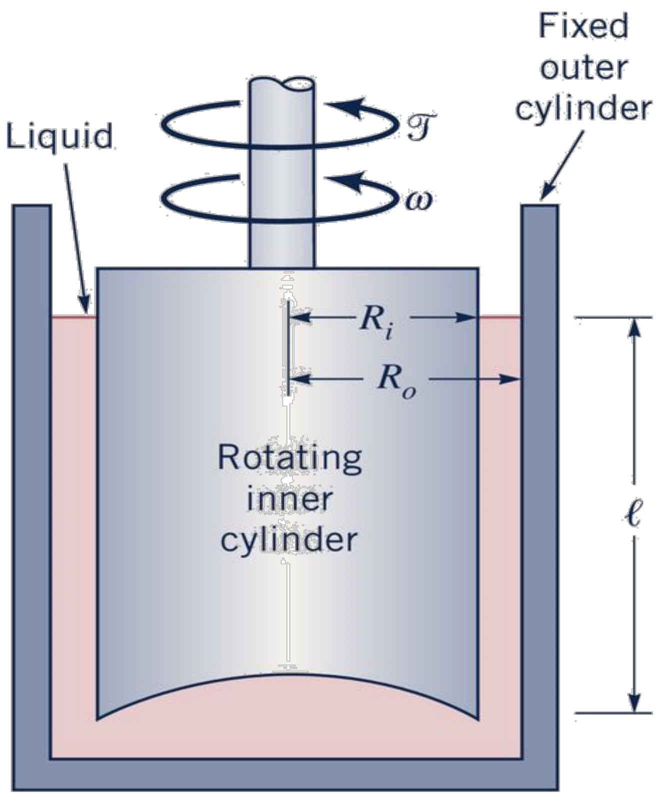
$$\tau\left[\frac{FORCE}{AREA}\right] \equiv \text{SHEAR STRESS} = \mu\frac{dV}{dy} \rightarrow \text{Viscosity}\left[\frac{Force-\cancel{s}}{AREA}\right] \bullet \text{Time Rate of Strain}\left[\frac{1}{\cancel{s}}\right]$$

$$F_s[FORCE] \equiv \text{SHEAR FORCE} = \text{SHEAR STRESS}\left[\frac{FORCE}{\cancel{AREA}}\right] \bullet A_c[\cancel{AREA}]$$

$A_c \equiv$ AREA IN CONTACT BETWEEN FIXED SURFACE and FLOWING FLUID

SHEAR AREA (examples)

EVERY GEOMETRY IS DIFFERENT!!



$$A_c = 2\pi(R_0 - R_i)L$$

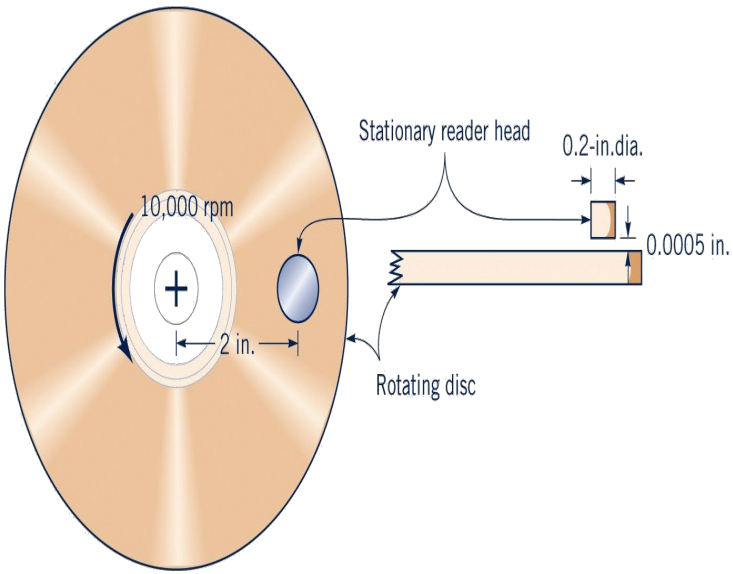


Figure P1.84

$$A_c = \frac{\pi D^2}{4}$$

WORLD-RENOWNED ASTROPHYSICIST: NEIL DE GRASSE TYSON

"Those who know **HOW** to think will surpass those who only know **WHAT** to think"?

The 'PATH' to success in Fluid Mechanics or ANY engineering course is paved with the understanding of "UNITS" and "DEFINITIONS" that drives the APPLICATION of fluid mechanics engineering equations as a part of a process.

The 'PATH' to problem solving success is NOT driven by merely applying arbitrary engineering equations without a knowledge of "what" are the variables, "what" are the units of BOTH the equation and the units, "what" is the expected outcome of the equation, and "when" is it appropriate to apply the equation?

UNITS

UNIT CONVERSIONS			
	SI	BKS	Conversion
Force	N	lbf	1N = 0.224809 lbf
Pressure	PA=N/m2	PSF=lbf/ft2	1PA=0.0208855 lbf/ft2
Mass	kg	slug	1kg = 0.0685 slug $\left(\frac{lb_f - s^2}{ft}\right)$
Length	m	ft	1m = 3.28084 ft
Volume	m3	ft3	1m3 = 35.3147 ft3
Velocity	m/s	ft/s	1m/s=3.28084 ft/s
Energy	J	BTU	1BTU = 1,055 J
Power	W (J/s)	ft-lbs/s	1W=0.74 ft-lbf/s=0.00134 hp=3.41BTU/h
Temperature	C (K)	F (R)	C=(F-32)/1.8, K=C+273, R=F+460
Time	s	s	

Engineering Analysis W/O Proper Units Receives 0 Credits

PROPERTIES

FLUID PROPERTIES			
Pressure	N/m2=Pa=F/A	lbf/ft2	1 Pa=0.021 psf
Dynamic Viscosity	N-s/m2=Pa-s	lbf-s/ft2	1Pa-s = 0.02089 lbf-s/ft2
Kinematic Viscosity	m2/s	ft2/s	1m2/s = 10.75381 ft2/s
Density	kg/m3	slugs/ft3	1kg/m3 = 0.00194 slug/ft3
Specific Weight	N/m3=F/V	lbf/ft3	1N/m3 = 0.00637 lbf/ft3
Shear Stress	Pa	lbf/ft2	1 Pa = 0.021 psf

In SI Units:

$$W = 1\,\mathrm{N} = 1\,\mathrm{kg} \bullet 1\frac{m}{s^2}$$

BKS

$$W = 1lb_f = \overbrace{32.174lbm}^{SLUG} \bullet 1\frac{ft}{s^2}$$

$$W = 1lb_f = 1slug[\frac{lb_f - s^2}{ft}] \bullet 1\frac{ft}{s^2}$$

$$1slug = 32.174[lbm]$$

Fluid Properties

- Fluid Mechanics can be subdivided into
  - Fluid Statics
    - (Fluids at REST)
  - Fluid Dynamics
    - (Fluid is ACCELERATING)

Analysis of Fluid Behaviors

♦ In fluid mechanics, same fundamental laws are applicable as of physics and other mechanics courses (such as Newton's laws of motion, conservation of mass and momentum, the first and second laws of thermodynamics, etc.).

Density of a fluid: defined as its mass (kg) per unit volume (m3). Density is typically used to characterize the mass of a fluid system. It is denoted by ρ.

Specific volume: the volume per unit mass and is therefore the reciprocal of the density i.e. $v = 1/\rho$.

Analysis of Fluid Behaviors

Specific Weight of a fluid: defined as its weight per unit volume. Specific weight, γ, is related to density through the equation $\gamma = \rho g = Force/Volume$, g is the local acceleration of gravity.

Specific Gravity: defined as the ratio of the density of the fluid to the density of water at some specific temperature (usually 4°C (39.2°F) – at this temperature the density of water is ***1000kg/m³ or 1.94slug/ft³***).

$$S = \frac{\rho_{fluid}}{\rho_{water}} = \frac{\gamma_{fluid}}{\gamma_{water}}$$

YOU MUST KNOW WATER PROPERTIES

Incompressible flow: $\rho = \text{CONSTANT}$

Compressible flow: A flow is considered a compressible flow if the change in density of the flow with respect to pressure is non-zero.

Analysis of Fluid Behaviors

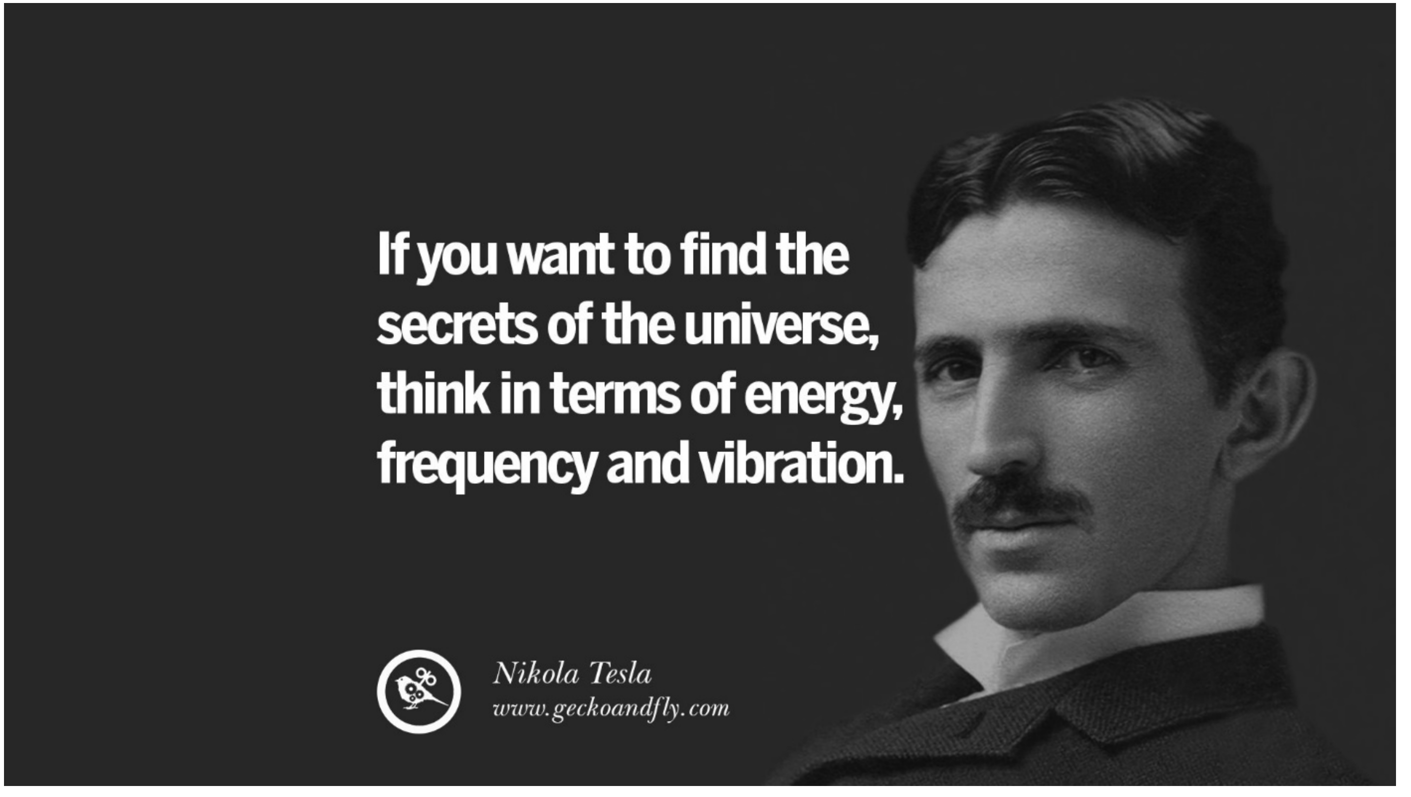
Ideal/Perfect Gas Law (equation of state for an ideal gas): Gases are highly compressible in comparison to liquids, with changes in gas density directly related to changes in pressure and temperature through the equation $p = \rho R_{gas} T$, *p* is the pressure **ABSOLUTE**, ρ is the density, *T* is the ***ABSOLUTE*** temperature (K or R) and *Rgas* is the gas constant.

Pressure (when fluid is at rest): defined as the normal force per unit area (p=F/A) exerted on a plane surface immersed in a fluid. In ideal gas law, pressure must be expressed as an absolute pressure denoted by (abs).

- In engineering, it is common practice to measure pressure relative to the local atmospheric pressure, and when measure in this fashion it is called gage pressure. Sea-level atmospheric pressure is 14.7 psi (abs) or 101 kPa.

Absolute pressure = Atmospheric pressure + gage pressure

Engineering answers WITHOUT UNITS have NO meaning or value!!!



Analysis of Fluid Behaviors

Viscosity: *Viscosity* is an internal property of a fluid that offers resistance to flow. It is a measure of the resistance of a fluid to deform under shear stress. It is denoted by μ. Also called as dynamic viscosity or absolute viscosity. $UNITS: \frac{N-s}{m^2}(Pa-s); \frac{lbf-s}{ft^2}$

Kinematic Viscosity: Often viscosity appears in fluid flow problems combined with the density in the form

$$\nu(T) = \frac{\mu(T)}{\rho}$$

This ratio is called **KINEMATIC VISCOSITY**.

- Viscosity is very sensitive to temperature and varies from fluid to fluid. As temperature increases, intermolecular cohesive forces in fluids (liquids and gases) are reduced which causes variations in fluid viscosity.

KINEMATIC VISCOSITY

$$\nu = \frac{\mu}{\rho} \rightarrow \frac{\text{Dynamic Viscosity}}{\textit{Density}}$$

$$\nu\left[\frac{ft^2}{s}\right] = \frac{\mu\left[\frac{lbf-s}{ft^2}\right]}{\rho[?]}$$

$$UNITS: \frac{\frac{lbf-s}{ft^2}}{\frac{lbm}{ft^3}} = ??? = \left[\frac{ft^2}{s}\right]$$

KINEMATIC VISCOSITY

$$\nu = \frac{\mu}{\rho} \rightarrow \frac{\text{Dynamic Viscosity}}{\textit{Density}}$$

$$UNITS: \frac{\frac{N-s}{m^2}}{\frac{kg}{m^3}} = \frac{\frac{kg-m/s^2}{m^2}s}{\frac{kg}{m^3}} = \frac{m^2}{s}$$

$$UNITS: \frac{\frac{lbf-s}{ft^2}}{\frac{slug}{ft^3}} = \frac{\frac{\cancel{lbf}-\cancel{s}}{ft^2}}{\frac{\frac{\cancel{lbf}-s^{\cancel{2}}}{ft}}{ft^3}} = \frac{ft^3}{ft-s} = \frac{ft^2}{s}$$

Viscosity of a Fluid

Which Oil Do You Use? 0W30? 5W30? 10W30?

Let's compare the kinematic viscosity of the three AMSOIL lubricants

AMSOIL 0W-30 is 57.3 cST @ 40 deg. C, & 11.3 cST @ 100 deg. C

AMSOIL 5W-30 is 59.5 cST @ 40 deg. C, & 11.7 cST @ 100 deg. C

AMSOIL 10W-30 is 66.1 cST @ 40 deg. C, & 11.7 cST @ 100 deg. C.

Centistoke is a Kinematic Viscosity measurement unit.

$$1cST = \frac{mm^2}{s}$$

Analysis of Fluid Behaviors

- The effect of temperature on fluids viscosity can be closely approximated using two empirical equations.

For Liquids, the equation is known as Andrade's equation defined by

$$\mu = De^{B/T}$$

where D and B are constants and T is absolute temperature.

For Gases, the equation is known as Sutherland equation defined by

$$\mu = \frac{CT^{3/2}}{T + S}$$

where C and S are constants and T is absolute temperature.

If the viscosity is known at two temperatures, then two constants can be determined for each cases.

BABY MATH REVIEW

$$Ln(AB) = Ln(A) + Ln(b)$$

$$Ln(\frac{A}{B}) = Ln(A) - Ln(b)$$

$$Ln(B) = A(\text{no units}) \rightarrow B = e^{A}$$

$$Ln(e^{a}) = a \rightarrow \text{"}a\text{" has NO UNITS}$$

GENERAL SOLUTION

$\mu(T) = De^{\frac{B}{T}}; \rightarrow$

$\ln(\mu) = \ln(D) + \ln(e^{\frac{B}{T}}) \rightarrow \ln(\mu) = \ln(D) + \frac{B}{T}$

$$\ln(\mu_1) = \ln(D) + \ln\left(e^{\frac{B}{T_1}}\right); \rightarrow \text{Eqn. 1}$$

$$\ln(\mu_2) = \ln(D) + \ln\left(e^{\frac{B}{T_2}}\right); \rightarrow \text{Eqn. 2}$$

SOLUTION

Eqn. 1 – Eqn. 2

$$\ln(\mu_1) - \ln(\mu_2) = \frac{B}{T_1} - \frac{B}{T_2} = B\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

Two Equations and Two Unknowns

$$B = \frac{\ln(\mu_1) - \ln(\mu_2)}{\left(\frac{1}{T_1[K]} - \frac{1}{T_2[K]}\right)}$$

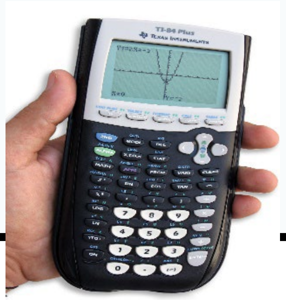
AND from Equation 1

$$\ln(D) = \left\{\ln(\mu_1) - \ln\left(e^{\frac{B}{T_1}}\right)\right\} = \beta$$

$D = e^{\beta}$

FINAL

$\mu(T) = De^{B/T}$



Given: $\mu_1 = 1 \times 10^{-3}\ N \cdot s/m^2$ @ 20^oC; AND, $\mu_2 = 6.53 \times 10^{-4}\ N \cdot s/m^2$ @ 40^oC

Find viscosity of $H2O$ @ 30^0C.

FIND B?

$$B = \frac{\ln(\mu_1) - \ln(\mu_2)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = \frac{\ln\left(\frac{\mu_1}{\mu_2}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} [K]$$

$$B = \frac{\ln(1 \times 10^{-3}\ N \cdot s/m^2) - \ln(6.53 \times 10^{-4}\ N \cdot s/m^2)}{\frac{1}{(20+273)K} - \frac{1}{(40+273)K}}$$

$B = 1954.22^0 K$; NOTE UNITS

FIND D?

$$B = 1954.22K$$

$$\ln(D) = \ln(\mu_1) - \ln\left(e^{\frac{B[K]}{T_1}}\right) = \ln\left(\frac{\mu_1}{e^{\frac{B[K]}{T_1}}}\right)$$

$$\ln(D) = \ln\left(\frac{1\times10^{-3}\ N\cdot s/m^2}{e^{\frac{B[K]}{20+273}}}\right) = -13.577$$

$$D = e^{-13.5169} = 1.2688x10^{-6}\ \frac{N-s}{m^2}$$; NOTE UNITS

Find viscosity of H2O @ 30^0C.

$$\mu(T=30C)=De^{B/T}$$

$$=1.2688x10^{-6}\frac{N-s}{m^2}\bullet e^{\frac{1954.22K}{30+273}}$$

$$=8.0242x10^{-4}\frac{N-s}{m^2}(Pa-s)$$

NOTE: VISCOSITY vs TEMP NOT LINEAR

I.E.: CAN NOT DO LINEAR INTERPOLATION

Fluid Properties - Viscosity of a Fluid

- Viscosity of Fluids in laminar Flow

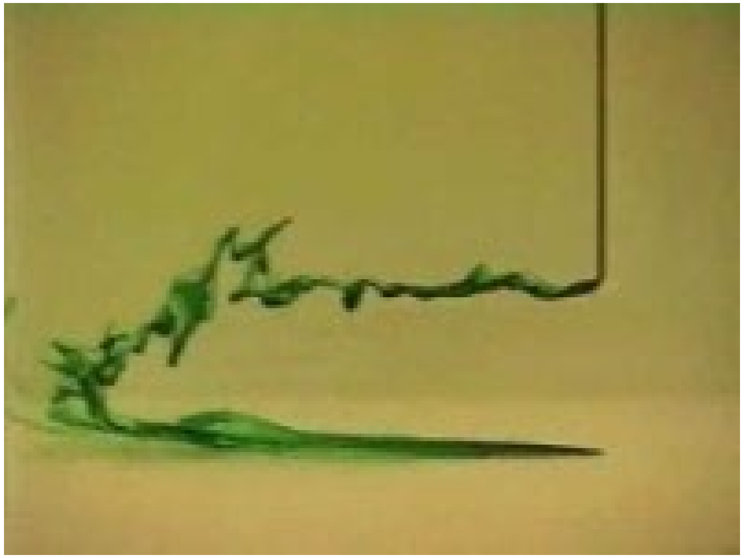
LAMINAR FLOW MIXING (CLICK)


THE HUNT IS ON.
SEAN CONNERY
THE HUNT FOR RED OCTOBER
ALEC BALDWIN

Fluid Properties – Viscous effect

No-slip: The experimental observation reveals that the fluid 'sticks' to the solid boundaries which is very important issue in fluid mechanics and is usually referred to as the no-slip condition. All fluids satisfy this condition.

No-slip: Flow Visualization



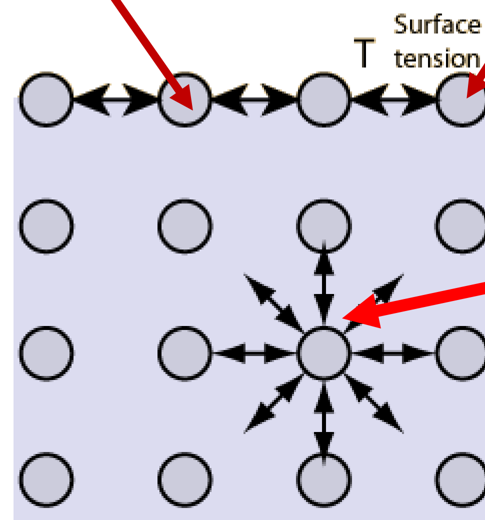
NO SLIP VIDEO (CLICK)

Fluid Behaviors – Surface Tension

The cohesive forces between molecules within a liquid beneath surface are shared with all neighboring atoms. Those **on the surface** have no neighboring atoms above, and exhibit stronger attractive forces upon their nearest neighbors on the surface. This enhancement of the intermolecular attractive forces at the surface is called surface tension.

Surface Tension σ (sigma)

Unbalanced Forces



Balanced Forces

Question: Can you walk on water?

You can't,……., But Water Striders Can……….

WHY?????

CLICK

Surface tension effects play a role in many fluid mechanics problems associated with liquid-gas, liquid-liquid, or liquid-gas-solid interfaces.

$$\tau(y)\left[\frac{N}{m^2};\frac{lbf}{ft^2}\right]=\mu\left[\frac{N-s}{m^2};\frac{lbf-s}{ft^2}\right]\frac{\partial u(y)}{\partial y}\left[\frac{m/s}{m};\frac{ft/s}{ft}\right]$$

$\rightarrow$ SHEAR STRESS FUNCTION

$u(y)\rightarrow$ Velocity Distribution Function

$\frac{\partial u(y)}{\partial y}\rightarrow$ Velocity Gradient (or Shear Strain)

$\mu\rightarrow$ Fluid Viscosity

A velocity difference between two surfaces separated by a fluid with viscosity μ experienecs a SHEAR STRESS $\tau[\frac{Force}{Area}]$ due to NO SLIP condition at the surface.

The shear stress causs a DRAG FORCE $\rightarrow$ $F_D = \tau[\frac{Force}{Area}] \bullet A_{contact}$

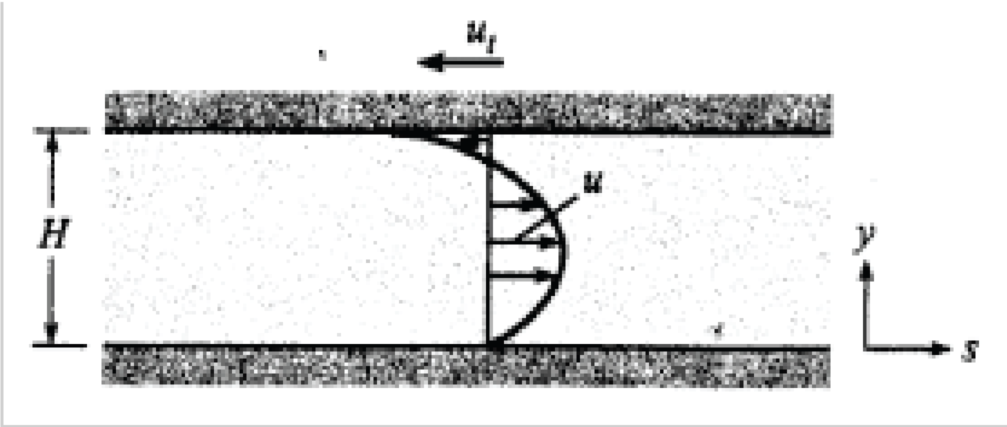
The POWER [Watts] to overcome drag is:

$P[J/s=W]=F_D[N]\bullet Velocity[m/s]$

$P[ft\text{-}lbf/s]=F_D[lbf]\bullet Velocity[ft/s]$

Class Example 2 – GIVEN VELOCITY PROFILE

Glycerin at 20°C $\left(\mu = 1.5\,\text{N.s/m}^2\right)$ flows through two narrowly separated parallel plates. The plates are separated by only 5 cm and the **pressure drops (dP/ds)** from left to right (see picture) at a rate of ***1.6 kN/m² per meter (kPa/m)***. Top plate is moving to the left at 5 cm/s (-s direction).



$$u(y) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t\frac{y}{H}$$

$$\frac{du}{dy} = \frac{-1}{2\mu}\frac{dP}{ds}(H - 2y) + \frac{u_t}{H}$$

$$0 \le y \le H$$

GIVEN:
Every Problem is different with a different velocity distribution between the two surfaces

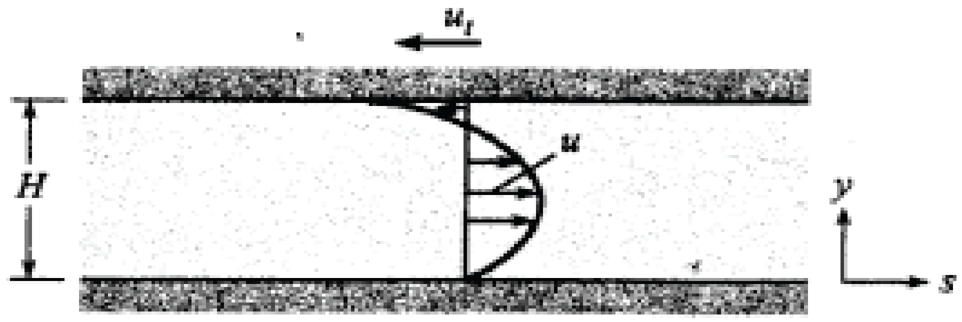
Class Example – GIVEN VELOCITY PROFILE

(a) Determine the velocity and shear stress at a height of 12 mm from the lower (stationary) surface. (b) Where is the location of maximum velocity?

$$\tau(x, y) = \mu \frac{du(x, y)}{dy} \rightarrow \text{SHEAR STRESS FUNCTION}$$

Class Example

Solution: (a) Given $\mu = 1.5\, N.s/m^2$ $H = 5cm = 5\times 10^{-2}\, m$

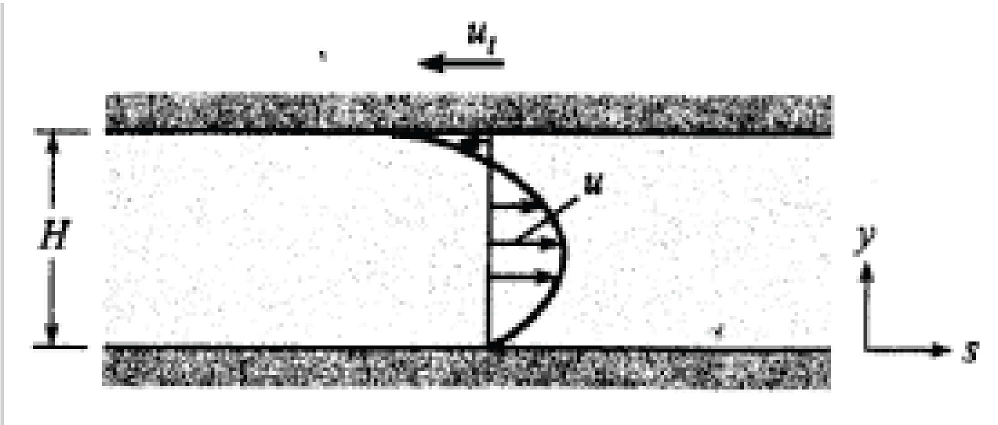


VELOCITY PARAMETRIC FUNCTION

$$u(\mu, y, H, u_t, \frac{dP}{ds}) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t\frac{y}{H};\ 0 \le y \le H;$$

$$u_t = -5\, cm/s = -5\times 10^{-2}\, m/s;$$

$$\frac{\partial p}{\partial s} = \frac{p_f - p_i}{1m} = -\frac{1.6\, kN/m^2}{1m} = -1.6\times 10^3\, N/m^3\ (\frac{N/m^2}{m} = \frac{Pa}{m})$$



$$u(y) = -\frac{1}{2\left(1.5\ N.s/m^2\right)}\left(-1.6\times10^3\ N/m^3\right)\left[\left(5\times10^{-2}\,m\right)y - y^2\right] + \left[\frac{-5\times10^{-2}\ m/s}{5\times10^{-2}\,m}\right]y$$

$$u(y) = \left[\left(533.331/(m-s)\right)\left[\left(5\times10^{-2}\,m\right)y - y^2\right]\right](m/s) + \left[\left[\frac{-5\times10^{-2}\ m/s}{5\times10^{-2}\,m}\right]y(m)\right](m/s)$$

$$@\,y = 12mm = 12\times10^{-3}\,m$$

$$u\left(y = 0.012m\right) = 0.2312\,m/s$$

EXCEL DATA

$$u(\mu, y, H, u_t, \frac{dP}{ds}) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t\frac{y}{H}$$

viscosity	P- Grad	Width	Top Plate
N-s/m2	N/m2/m	M	m/s
1.5	-1.60E+03	5.00E-02	-5.00E-02
y	u(y)	Shear	
m	m/s	N-s/m2=Pa-s	
0	0.00E+00	3.85E+01	
0.001	0.05780	3.69E+01	
0.002	0.11320	3.53E+01	
0.003	0.16620	3.37E+01	
0.004	0.21680	3.21E+01	
0.005	0.26500	3.05E+01	
0.006	0.31080	2.89E+01	
0.007	0.35420	2.73E+01	MAX VELOCITY →
0.008	0.39520	2.57E+01	
0.009	0.43380	2.41E+01	
0.01	0.47000	2.25E+01	
0.011	0.50380	2.09E+01	
0.012	0.53520	1.93E+01	

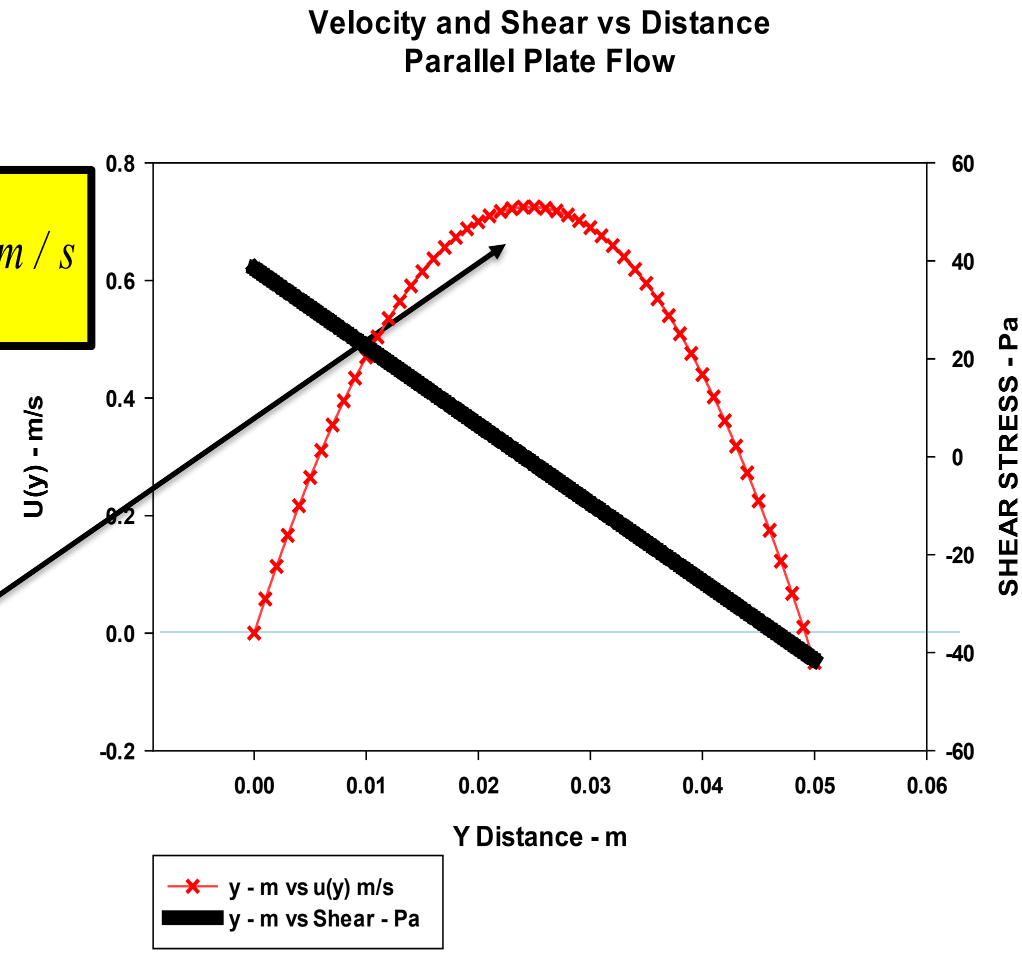
0.015	0.61500	1.45E+01
0.016	0.63680	1.29E+01
0.017	0.65620	1.13E+01
0.018	0.67320	9.70E+00
0.019	0.68780	8.10E+00
0.02	0.70000	6.50E+00
0.021	0.70980	4.90E+00
0.022	0.71720	3.30E+00
0.023	0.72220	1.70E+00
0.024	0.72480	1.00E-01
0.025	0.72500	-1.50E+00
0.026	0.72280	-3.10E+00
0.027	0.71820	-4.70E+00
0.028	0.71120	-6.30E+00
0.029	0.70180	-7.90E+00
0.03	0.69000	-9.50E+00

- FULL PLOT OVER CHANNEL
- Velocity is Parabolic
- Shear is Linear

$$u(\mu, y, H, u_t, \frac{dP}{ds}) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t\frac{y}{H}; m/s$$

$$\tau(y) = \mu\frac{\partial u}{\partial y}$$

$$\frac{-1}{2}\frac{dP}{ds}(H - 2y) + (\mu)\frac{u_t}{H}; \frac{N-s}{m^2}$$



MAX VELOCITY
Y=0.025m
U=0.7250 m/s

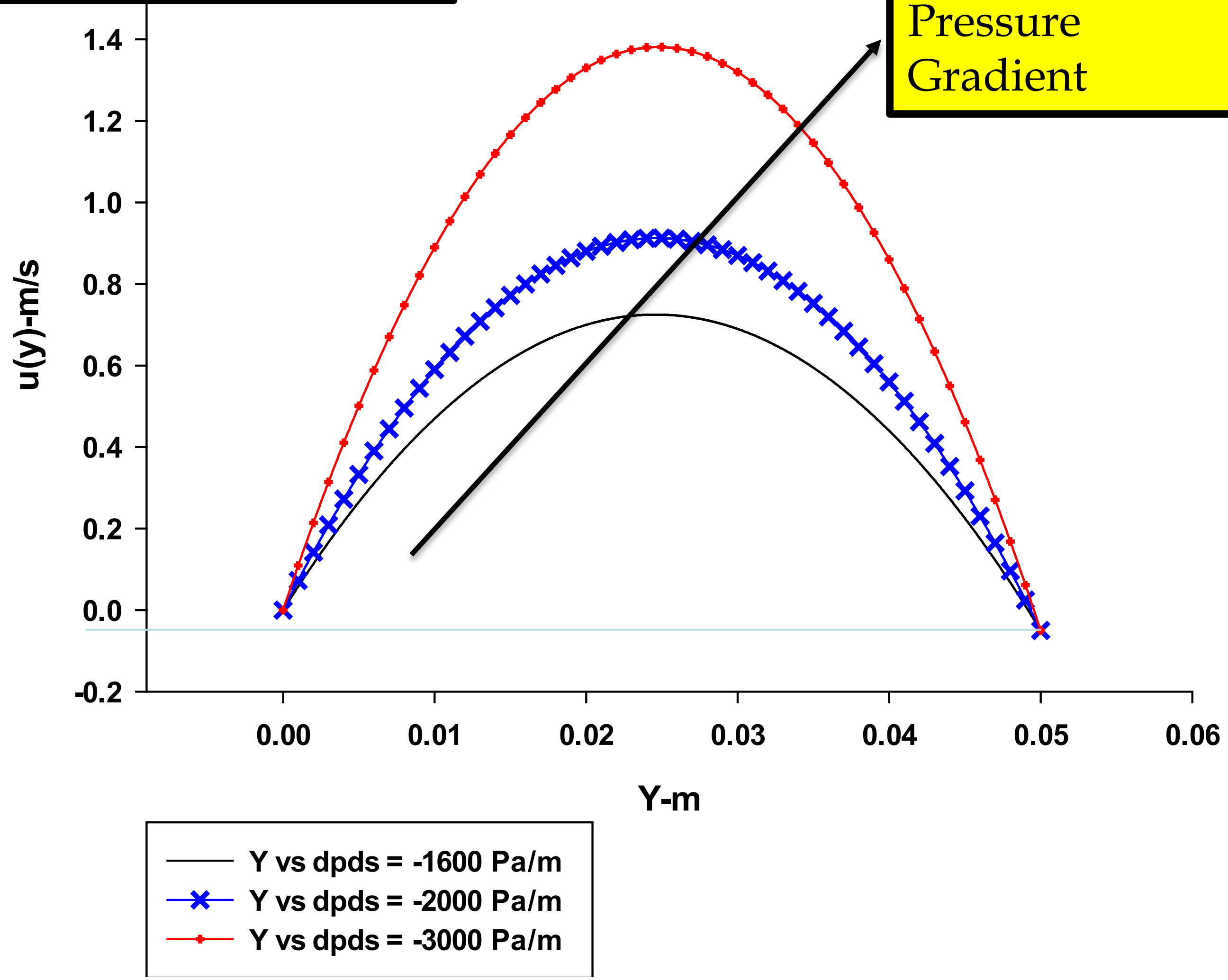
Use calculator to verify EXCEL velocity and shear output for three (3) points.

Distance vs Velocity
Variable Pressure Gradient

$$u(\mu, y, H, u_t, \frac{dP}{ds}) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t\frac{y}{H}; m/s$$

Increased Pressure Gradient Drives Velocity;

Does Y Location of MAX Velocity Change ?



Class Example

Solution: (C) To find the location where the velocity is maximum; put

$$u(\mu, y, H, u_t, \frac{dP}{ds}) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t\frac{y}{H}; m/s^2$$

$$\frac{\partial u}{\partial y} = \left[\frac{-1}{2\mu}\frac{dP}{ds}(H - 2y_{max}) + \frac{u_t}{H}\right] = 0$$

$$y_{max} = \frac{-\frac{u_t}{H} + \frac{1}{2\mu}\frac{dP}{ds}H}{2\frac{1}{2\mu}\frac{dP}{ds}} \rightarrow \text{Parametric Equation for Location of Max Velocity}$$

$$= \frac{-\left(-0.05/0.05\right) + \frac{0.5}{1.5}(-1600) * 0.05}{-\frac{1}{1.5} * 1600} = 0.02406m \rightarrow \text{Max Velocity Location}$$

NOTE: as ratio of Ut/H decrease, YMAX becomes independent of pressure gradient.

The maximum velocity occurs 24.06 mm above the bottom plate

$$u_{\max} = u(y = y_{\max})$$

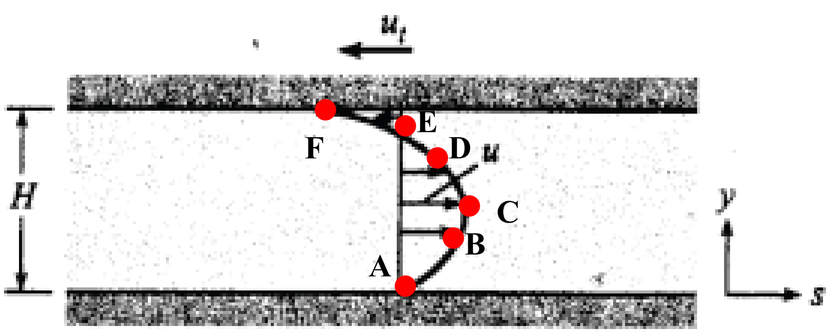
Class Example

The figure below shows a typical velocity profile that is produced when a fluid is driven through a parallel plate system and where the top plate is moving to the left at a velocity u_t

- Where in this velocity profile is the shear stress the greatest?
- Where is it the least?

$$u(y) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t \frac{y}{H}$$

$$0 \le y \le H$$

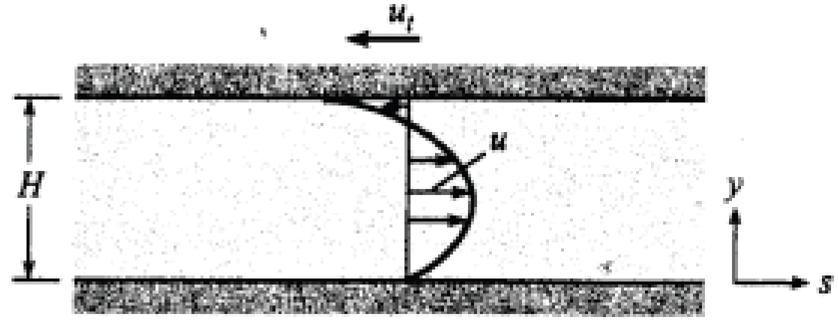


Class Example

Problem:

$$u(y) = \frac{-1}{2\mu}\frac{dP}{ds}(Hy - y^2) + u_t \frac{y}{H}$$

$$0 \le y \le H$$



$$\frac{dp}{ds} = -20\, N/m^3\ ;\ H{=}3.5cm;\ \mu{=}2.4\times 10^{-4}\ N.S/m^2\ ;\ u_t = -3.8\, m/s$$

Determine the maximum value of the shear stress (magnitude only).

Solution:

$$\tau = \mu\frac{\partial u}{\partial y} = \mu\left[\frac{-1}{2\mu}\frac{dp}{ds}(H-2y) + \frac{u_t}{H}\right]$$

$$\Rightarrow \tau = \frac{-1}{2}\frac{dp}{ds}(H-2y) + \mu\frac{u_t}{H} = 10\frac{N}{m^3}(0.035m - 2y) - 0.02605\frac{N}{m^2}$$

$$\tau(0) = 0.324\frac{N}{m^2}$$

$$\tau(y{=}H{=}0.035m) = -0.376\frac{N}{m^2}$$

1. A 12” Dia plate is placed over a fixed bottom plate with a $\Delta = 0.1$ in gap. The fluid between the plates is glycerin $\mu = 3.13E-2\frac{lbf-s}{ft^2}$ and the plate rotates with an angular velocity of $\omega = 2$ rpm. Assume the velocity in the gap is linear (straight line) and neglect shear stress on the edge of the rotating plate. An electric motor rotates the plate with TORQUE T_q.

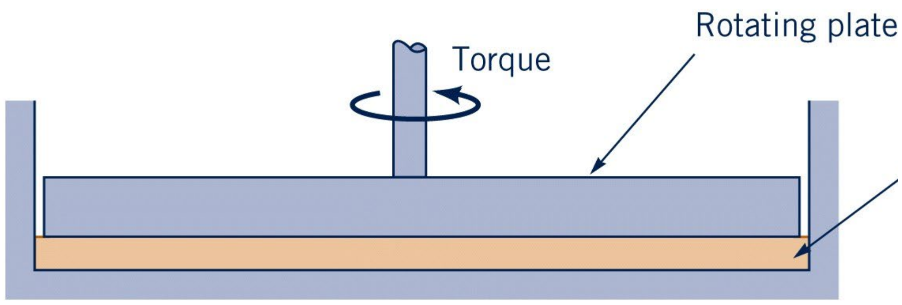


Figure P1.89

PROBLEM 1.89

a. In words, explain what fluid principle governs the magnitude of the Torque?

b. What is the parametric expression of the linear velocity profile in the gap, i.e., $V(y, \Delta, D, \omega_0)\left[\frac{ft}{s}\right]$

c. What is parametric expression for SHEAR FORCE [lbf], $F_D(y, \Delta, D, \omega_0\left[\frac{rad}{s}\right], \mu)[lbf]$, (also known as drag force) and verify units?

d. What is parametric expression for TORQUE [ft-lbf] and verify units.

e. What is parametric expression for POWER [ft-lbs/sec] and verify units.

a. In words, explain what fluid principle governs the magnitude of the Torque?

i. *The shear stress between the rotating disk and the stationary surface results in a SHEAR or DRAG FORCE. The shaft TORQUE (N-m) or energy must oppose the DRAG FORCE acting at a distance. This TORQUE also creates an internal material shear stress within the solid rotating shaft which is a function of the shaft diameter and material type.*

ii. *Torque (N-m) must be greater than the opposing shear Drag Force x Moment Arm (N-m).*

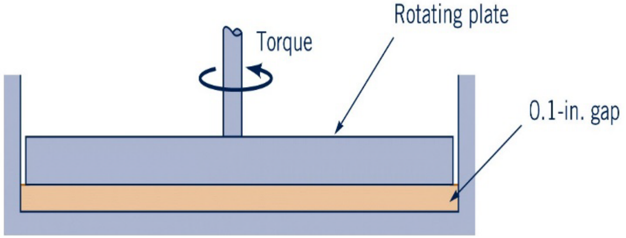


Figure P1.89

b. What is the parametric expression of the **LINEAR VELOCITY PROFILE** in the gap, i.e.,

$$V(y,\Delta,D,\omega_0)\left[\frac{ft}{s}\right]=\frac{\frac{D[ft]}{2}\omega_0\left[\frac{rad}{s}\right]}{\Delta[ft]}y[ft];\quad 0\le y\le\Delta$$

$$\omega_0\left[\frac{rad}{s}\right]=\omega\frac{REV}{MIN}\bullet\frac{2\pi Radians}{REV}\bullet\frac{1MIN}{60SEC}$$

c. What is parametric expression for SHEAR FORCE [lbf] (also known as drag force) and verify units?

$$V(y,\Delta,D,\omega_0)\left[\frac{ft}{s}\right] = \frac{\frac{D[ft]}{2}\omega_0\left[\frac{rad}{s}\right]}{\Delta[ft]}y[ft]$$

$F_D \equiv$ Shear Force or Shear Drag

$=$ Shear Stress x $A_{contact}$

$$= \mu\frac{\partial V}{\partial y} \bullet A_{contact}$$

$$= \mu[]\overbrace{\frac{\frac{D[]}{2}\omega_0[rad/s]}{\Delta[]}}^{\frac{\partial V}{\partial y}}\overbrace{\left(\frac{\pi D^2[]}{4}\right)}^{A_{contact}} \rightarrow f(D^3)$$

$$= \left[\frac{lbf - \cancel{s}}{\cancel{ft^2}}\right]\left[\cancel{ft^3}\right]\left[\frac{1}{\cancel{s}}\right]\left[\frac{1}{\cancel{f}t}\right]$$

$$= lbf$$

$$F_D[lbf] = \mu[\frac{lbf-s}{ft^2}]\frac{\frac{D[ft]}{2}\omega_0[rad/s]}{\Delta[ft]}\left(\frac{\pi D^2[ft^2]}{4}\right)$$

$T_q[lbf-ft] = F_D[lbf]\bullet$ Moment Arm $\rightarrow$ TORQUE

$$=\mu[\frac{lbf-s}{ft^2}]\frac{\frac{D[ft]}{2}\omega_0[rad/s]}{\Delta[ft]}\left(\frac{\pi D^2[ft^2]}{4}\right)\bullet\frac{D}{2}$$

$$=\frac{\mu D^4\omega_0\pi}{16\Delta} \rightarrow f(D^4)$$

$$P[\frac{lbf-ft}{s}] = F_D[lbf] \bullet \text{Velocity}\left[\frac{ft}{s}\right] \rightarrow \text{POWER}$$

$$= \mu[\frac{lbf-s}{ft^2}] \frac{\frac{D[ft]}{2}\omega_0[rad/s]}{\Delta[ft]} \left(\frac{\pi D^2[ft^2]}{4}\right) \bullet \overbrace{\frac{D[ft]}{2}\omega_0\left[\frac{rad}{s}\right]}^{\text{Velocity}}$$

$$= \frac{\mu D^4 \pi \omega_0^2}{16\Delta}[\frac{lbf-ft}{s}]$$

$$= \underbrace{T_q[lbf-ft] \bullet \omega_0\left[\frac{rad}{s}\right]}_{\frac{ft-lbs}{s}} \rightarrow \text{POWER}$$

FOLLOW THE PATH

As you work to "re-work" the problems from the notes and the homework you should always recall the following basic engineering solution pathways:

1. If you don't understand the definitions used within the problem statement, how can one possibly have any chance of obtaining a valid solution.

2. Never put any number on paper unless one also state the units. (unless it is for certain that is indeed has no units, and can be proven).

3. Always ask? What are the fluid fundamentals driving the solution path?

4. Always form solution is terms of problem "variables" with "units" expressed, and not with numbers.

5. Always check units of final solution to ensure "form" of result is correct with proper units as expected, BEFORE moving to final stage of just plugging a number into a calculator.