Chapter 2

Fluid Statics

FOLLOW THE PATH

As you work to "re-work" the problems from the notes and the homework you should always recall the following basic engineering solution pathways:

1. If you don't understand the definitions used within the problem statement, how can one possibly have any chance of obtaining a valid solution.

2. Never put any number on paper unless one also state the units. (unless it is for certain that is indeed has no units, and can be proven).

3. Always ask? What are the fluid fundamentals driving the solution path?

4. Always form solution is terms of problem "variables" with "units" expressed, and not with numbers.

5. Always check units of final solution to ensure "form" of result is correct with proper units as expected, BEFORE moving to final stage of just plugging a number into a calculator.

Class 03: Fluid Statics - Pressure

- Pressure = force (normal to surface) per unit area

$$P = \lim_{\delta A \to \delta A'} \frac{\delta F}{\delta A} \text{(scalar)}$$

- S. I. Units = N/m^2 = 1 Pa (Pascal)
- British Gravitational Units = lbf/ft^2

$$1\frac{lbf}{in^2}(PSI) = \frac{144 lbf}{ft^2}(PSF)$$

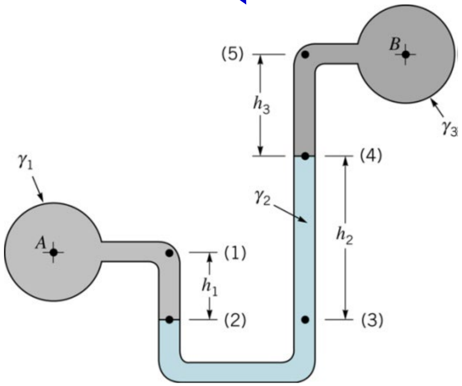
- 1 bar = 10^5 Pa = 100kPa = 0.1MPa
- 1 atm = 101.325 kPa = 14.696 psi=**2116.2 lbf/ft2** = 29.9 in Hg

Going from p to F --> force always acts normal to the surface

Class 03: Fluid Statics - Pressure

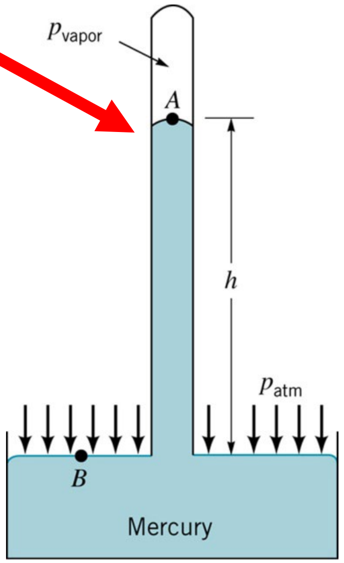
$$\Delta H_2 0_{FEET} = \Delta Hg_{Mercury} \bullet \frac{\gamma_{hg}}{\gamma_{H_2 0}} = Hg_{Mercury} \bullet 13.55$$

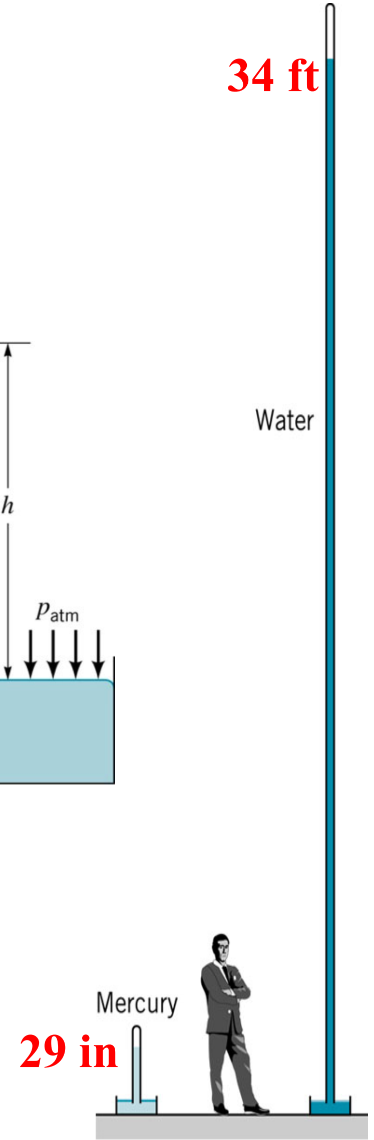
- **Manometer (in H_2O)**



Measuring Pressure

- **Barometer (in Hg)**

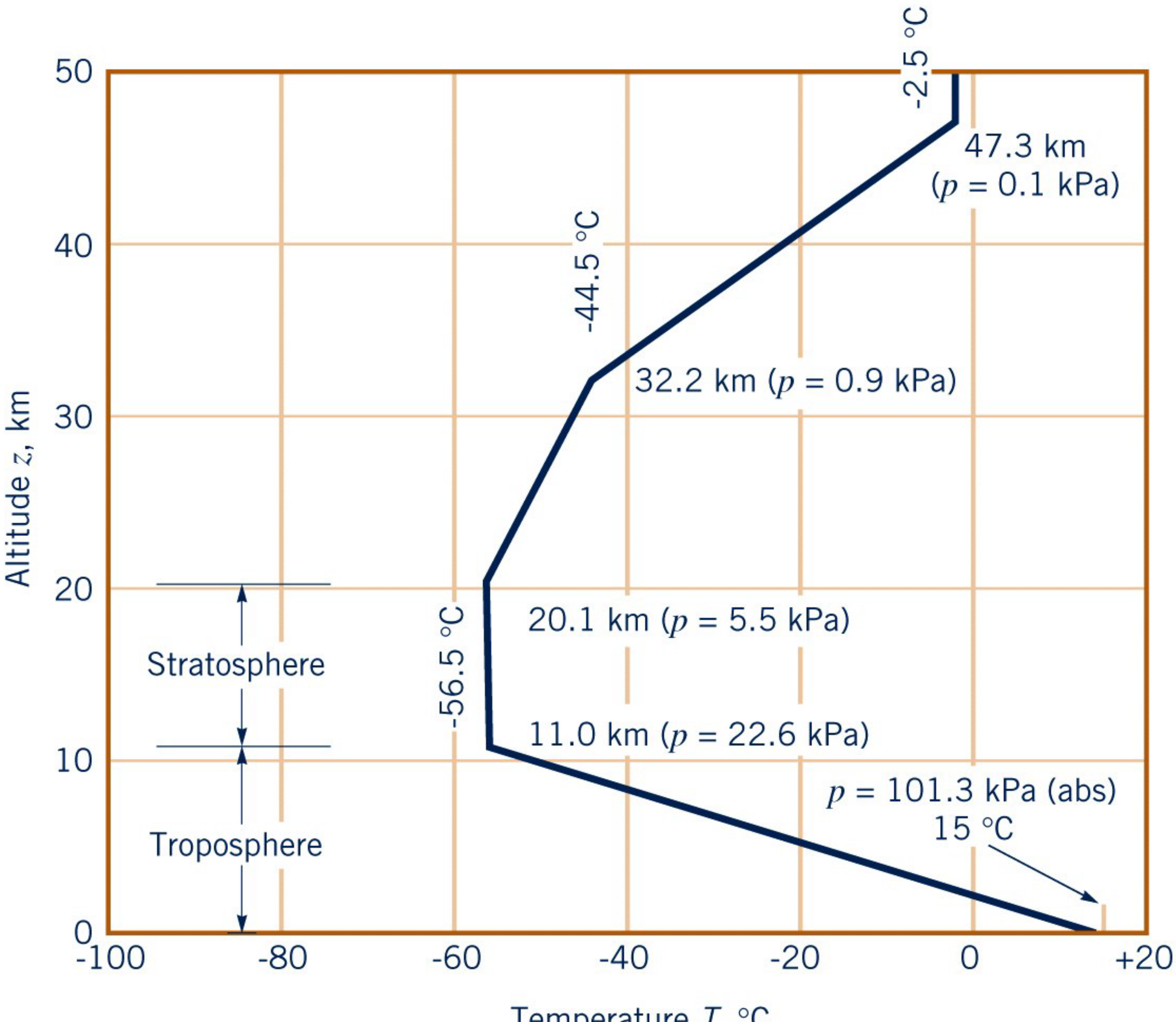




Used to Measure Pressure Difference

https://www.youtube.com/watch?v=EkDhlzA-lwI

Pressure and Temperature vs. Altitude



Class 03: Absolute, Gage, Vacuum Pressure

In engineering, we measure pressure relative to the local atmospheric pressure, and it is called gage pressure. Sea-level atmospheric pressure is 14.7 psi (abs) or 101 kPa.

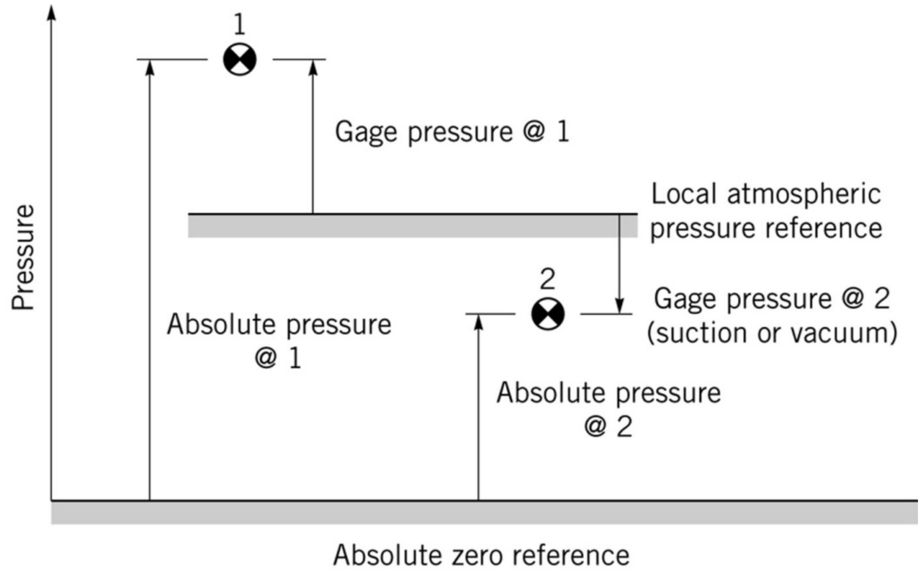
Gage pressure (psig), Absolute pressure (psia)

Absolute pressure =

Atmospheric pressure

+

gage pressure



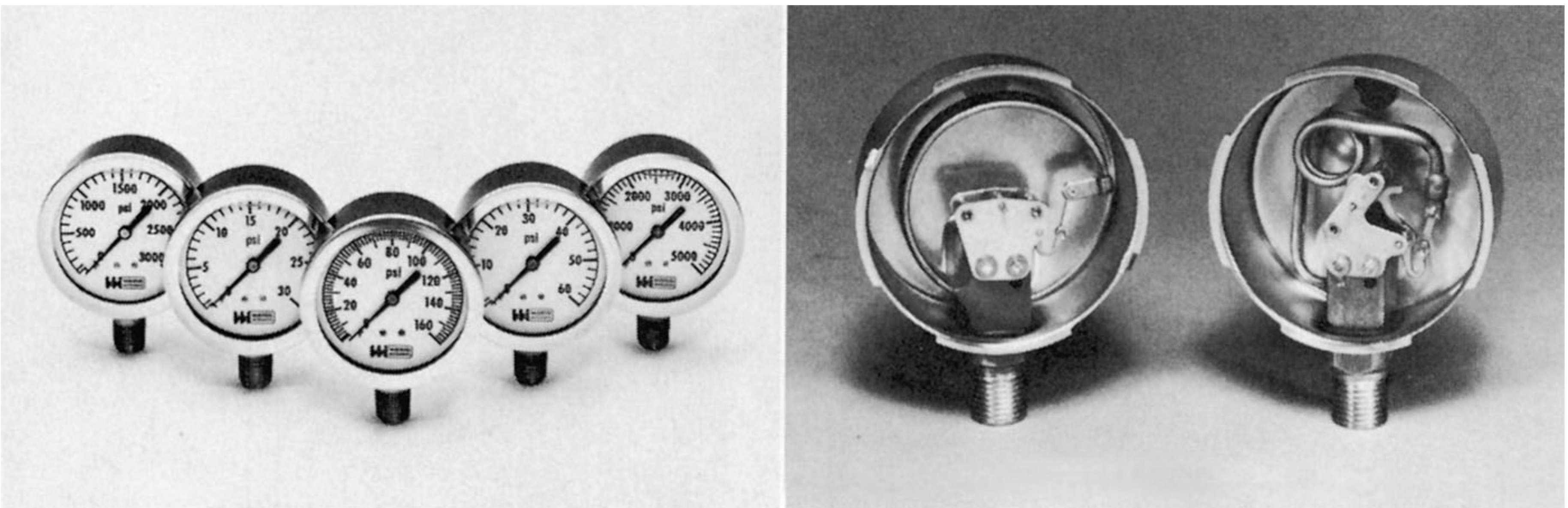
Vacuum Pressure

Ex: 32 psi car's tire pressure

Total pressure =(32+14.7) = 46.7 psi

Class 03: Pressure Measuring Devices

Commercial Product: Bourdon Pressure Gage



(*a*) (*b*)

Liquid-filled Bourdon Pressure gages for various pressure ranges.

Internal elements of Bourdon Pressure Gages.

DIFFERENTIAL PRESSURE ELEMENT

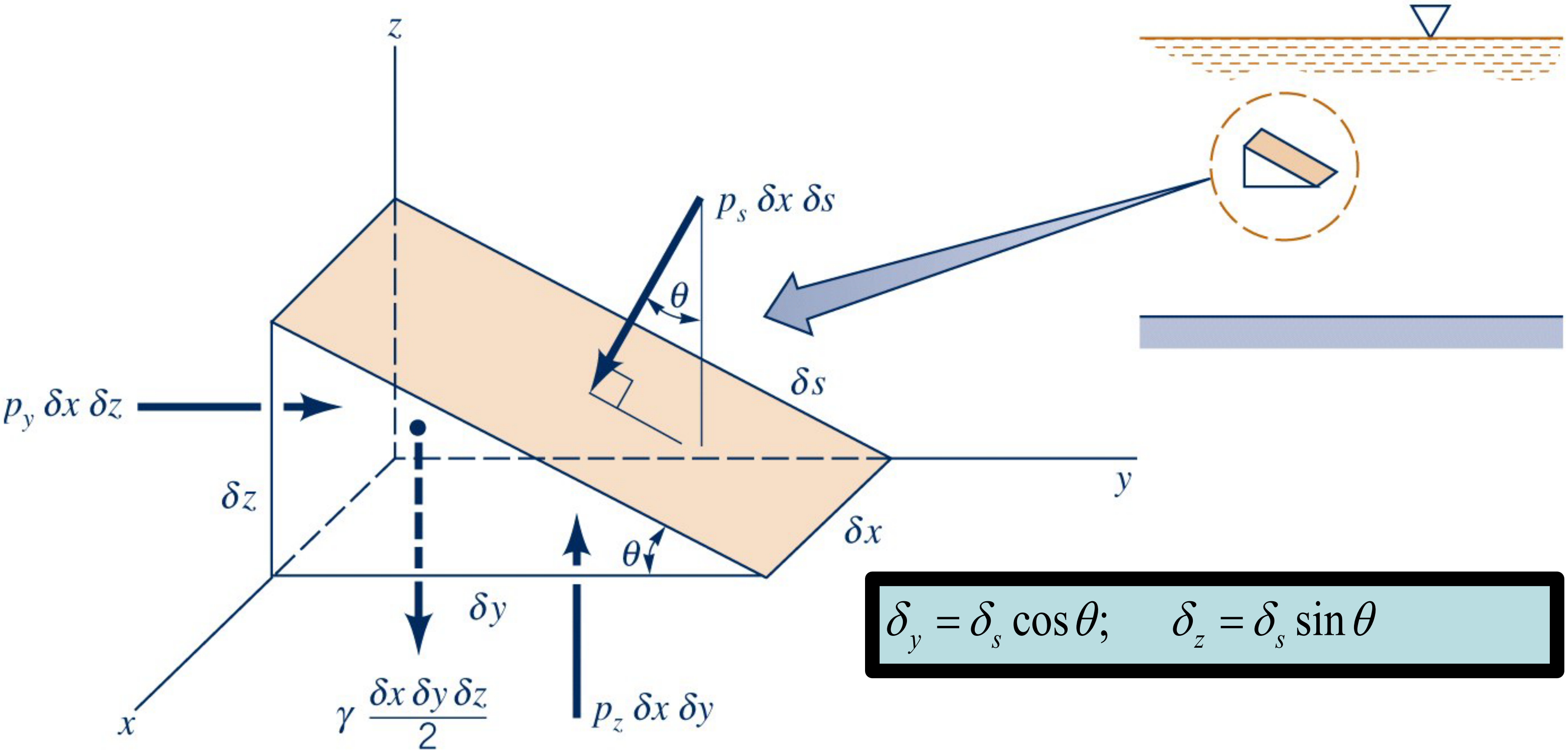


Figure 2.1

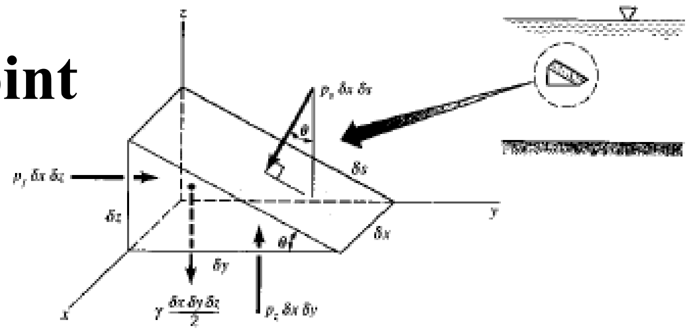
$$\delta_y = \delta_s \cos\theta; \qquad \delta_z = \delta_s \sin\theta$$

Pressure = force (normal to surface) per unit area

Class 03: Fluid Statics

Question: How does the pressure in a fluid vary from point to point at same elevation, assuming there are no shearing stresses?

Case I: Pressure at a same elevation point



Newton's Second Law: F=ma

The equation of motion, F = ma, in the **y-direction** is

$$\sum F_y = p_y \delta_x \delta_z - p_s \delta_x \delta_s \sin\theta = ma_y = \rho \frac{\delta_x \delta_y \delta_z}{2} a_y \quad (1)$$

γ and ρ specific weight and density of the fluid

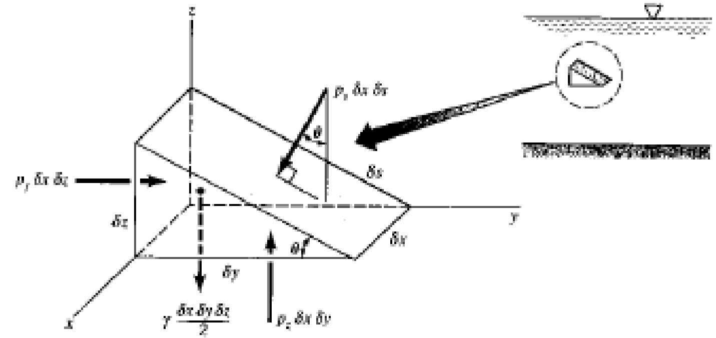
The equation of motion, F = ma, in the **z-direction** is

$$\sum F_z = p_z \delta_x \delta_y - p_s \delta_x \delta_s \cos\theta - \gamma \frac{\delta_x \delta_y \delta_z}{2} = ma_z = \rho \frac{\delta_x \delta_y \delta_z}{2} a_z \quad (2)$$

Pressure must be multiplied by an appropriate area to obtain the force generated by the pressure

$$\delta_y = \delta_s \cos\theta; \qquad \delta_z = \delta_s \sin\theta \qquad (3)$$

From Eq. (1) & (3), we get

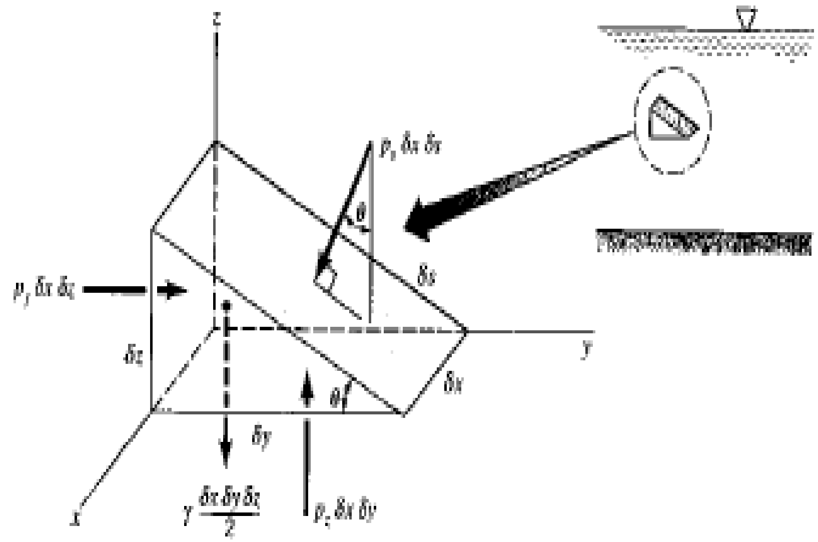


$$p_y \cancel{\delta_x}(\delta_s \sin\theta) - p_s \cancel{\delta_x}\delta_s \sin\theta = ma_y = \rho \frac{\cancel{\delta_x}\delta_y\delta_z}{2}a_y$$

$$\rho\frac{\delta_y\delta_z}{2}a_y \approx 0, \delta_y\delta_z \rightarrow \text{Higher Order Terms-Small}$$

$$\Rightarrow p_y - p_s = \rho\frac{\delta_y\delta_z}{2}a_y \approx 0 \;(4);$$

From Eq. (2) & (3), we get



$$p_z \delta_x (\delta_s \cos\theta) - p_s \delta_x (\delta_s \cos\theta) - \gamma \frac{\delta_x \delta_y \delta_z}{2} = m a_z = \rho \frac{\delta_x \delta_y \delta_z}{2} a_z$$

$$\Rightarrow p_z - p_s = \left(\rho a_z + \gamma\right) \frac{\delta_z \delta_y}{2} \approx 0 \qquad (5)$$

Class 03: Fluid Statics- Pressure at a Point

What is happening at a point, we take the limit as δ_x, δ_y, and δ_z approach zero (while maintaining the angle θ

It follows that $p_y = p_s; \qquad p_z = p_s$

Similarly, for x-direction $p_x = p_s$

$$\Rightarrow \; p_s = p_x = p_y = p_z$$

Observation: The pressure at a point in a fluid at rest or in motion, is independent of direction as long as there are no shearing stresses present.

PASCAL's LAW

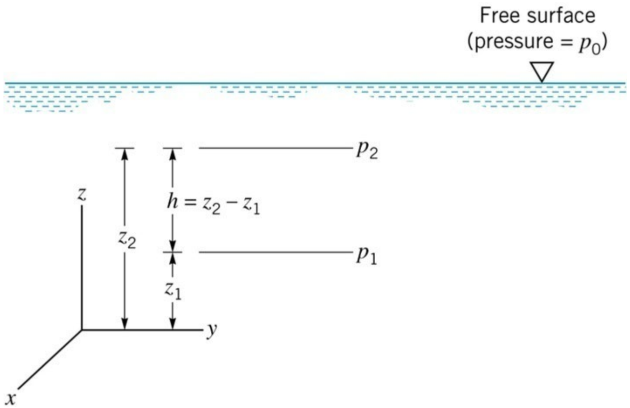
Case III: Variation with Elevation - Incompressible fluids

Since, specific weight $\gamma = \rho g$; changes in γ are caused either by a change in ρ or g .

ρ constant, incompressible fluids.

$$\frac{\partial p}{\partial x} = 0; \quad \frac{\partial p}{\partial y} = 0; \quad \frac{\partial p}{\partial z} = -\gamma$$

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz \Rightarrow p_2 - p_1 = -\gamma\left(z_2 - z_1\right) = -\gamma h$$



$$\Rightarrow p_1 - p_2 = \gamma\left(z_2 - z_1\right) \Rightarrow p_1 = p_2 + \gamma\left(z_2 - z_1\right) = p_2 + \gamma h$$

Pressure head

$$h = \frac{p_1 - p_2}{\gamma}$$

$$\Rightarrow p_1 = p_2 + \gamma h$$

Valid for same fluid only!!

Observation: Fluid at rest, pressure will increase as we move downward and will decrease as we move upward.

Derivations: Variation with Elevation

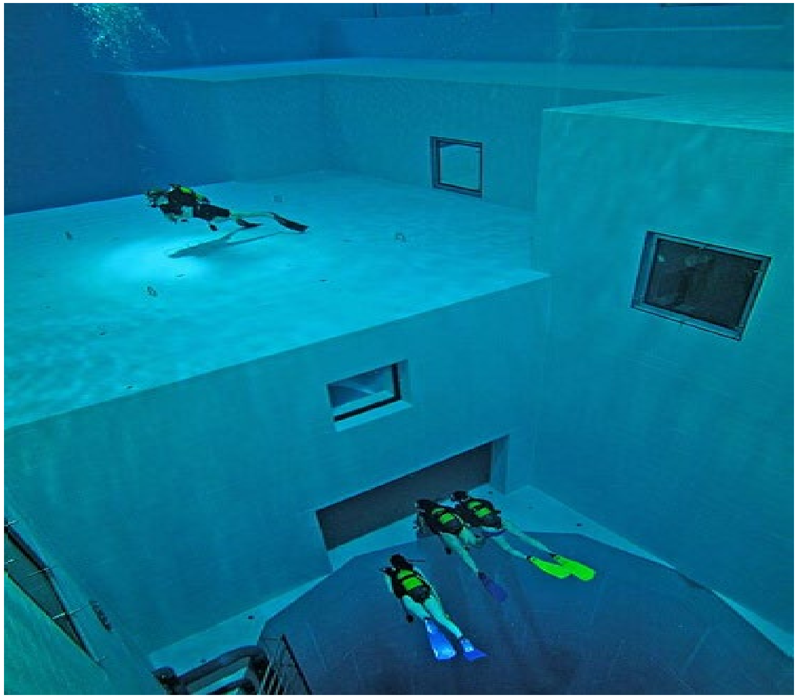
Question:

Picture a tub of water the size of the ocean …

No waves - no wind - calm, cool water

If you descend to the bottom of the tub, will the pressure stay the same? change? increase? decrease?

if you stay at the same depth and move from side to side, will the pressure stay the same? change? increase? decrease?



FLUID STATICS-LAW OF HYDROSTATICS

$$\frac{dP}{dz} = +\gamma \quad g \downarrow z \downarrow$$

$$\frac{dP}{dz} = -\gamma \quad g \downarrow z \uparrow$$

Incompressible + No Shear Stress

POINT-TO-POINT METHOD

$$\mathrm{P}_A + \Delta P_{A-B} = P_B$$

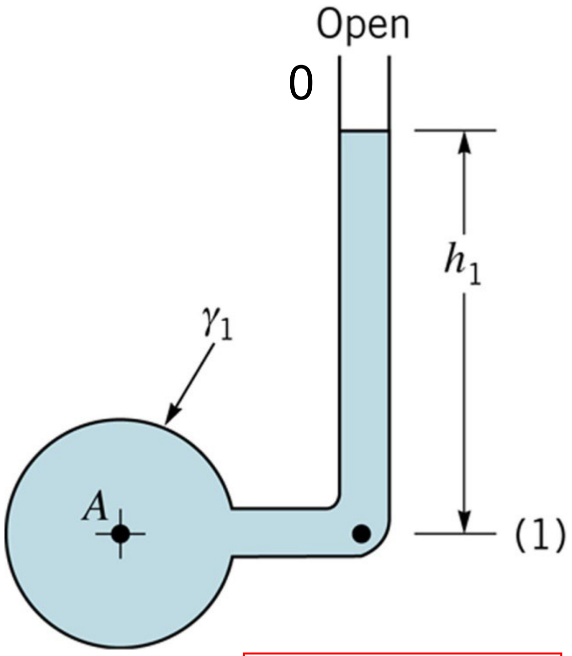
FOLLOW THE PATH

Measurement of pressure – Laboratory Experiment/Research

A standard technique measuring pressure involves the use of liquid columns in vertical or inclined tubes. This technique is called Manometry.

Measuring pressure intensity:

- Piezometers



Manometers are often used to measure the difference in pressure in two points

It's very simple & easy !

$$p_0 + \gamma h_1 = p_1$$

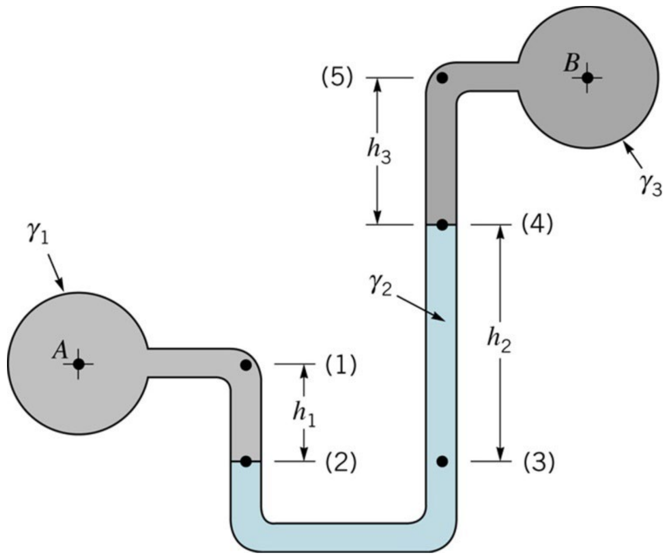
Law of Hydrostatics

PA=P1 (Pascal's Law)

Class 03: Differential Manometers

Measuring pressure intensity:

- **Differential Manometers**



Slightly more difficult !!

The difference in pressure between A and B can be found by starting one end of the system and working around to the other end.

Example: At A, the pressure is p_A, which is equal to p_1. As we move to pt 2, the pressure increases by $\gamma_1 h_1$. The pressure at p2 is equal to p3, and as we move upward to pt 4, the pressure decreases by $\gamma_2 h_2$. Similarly, continue to move upward from pt 4 to pt 5, the pressure decreases by $\gamma_3 h_3$. Finally, $p_5 = p_B$.

Thus $$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$\Rightarrow p_A - p_B \text{ (pressure difference)} = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Use consistent system of units

GENERAL POINT-to-POINT ROAD MAP

Identify and label elevation levels associated with different fluids using Pascal's Law.

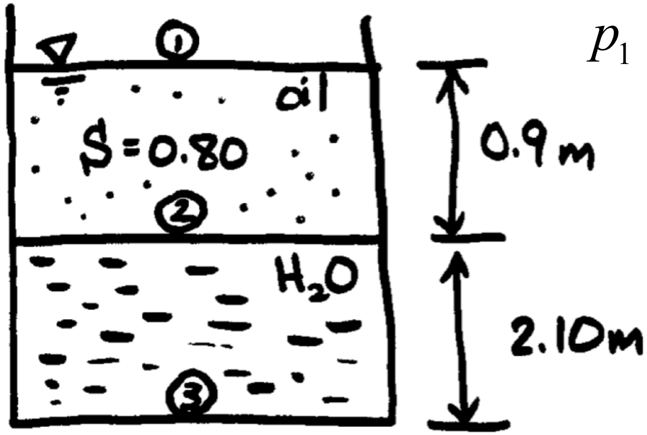
Identify START and END POINT.

IDENTIFY and MISSING dimensions from START to END with dummy variable.

Transverse fluid circuit from START POINT to END POINT and apply POINT-TO-POINT method for LAW OF HYDROSTATICS to solve for unknowns,

Pressure Variation with Elevation - Example

Problem:



Find pressure @ point (3).

Now work from pt 1 to pt 3:

$$p_1 + \gamma_{oil} 0.9m + \gamma_{H2O} 2.1m = p_3$$

$$0 + 7{,}060 Pa + 9810\, N/m^3 \times 2.10m$$

$$27{,}700 Pa\ (gage) = p_3$$

$$p_3 > p_2$$

Solution: @ point 1,

$$p_1 = p_{atm} = 0\, gage = 14.696\ PSIA = 101{,}325 PA$$

$$S_{oil} = \frac{\gamma_{oil}}{\gamma_{water}} \Rightarrow \gamma_{oil} = S_{oil} \times \gamma_{water}$$

$$p_1 + \gamma_{oil} h = p_2$$

$$0 + (S_{oil} \times \gamma_{water}) \times h = p_2$$

$$0.80 \times 9810\, N/m^3 \times 0.9m = 7{,}060 Pa\ (gage)$$

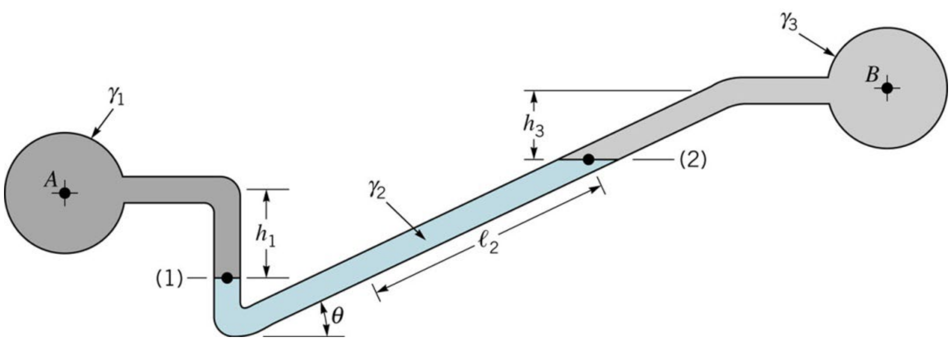
$$p_2 > p_1$$

***Note:* Does the answer make sense?**

Inclined-Tube Manometer

To measure small pressure changes

One leg of the manometer is inclined at an angle θ, and the differential reading l_2 is measured along the inclined tube.



The difference in pressure can be expressed as

$$p_A + \gamma_1 h_1 - \gamma_2 l_2 \sin\theta - \gamma_3 h_3 = p_B$$

$$\Rightarrow p_A - p_B \text{ (pressure difference)} = \gamma_2 l_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

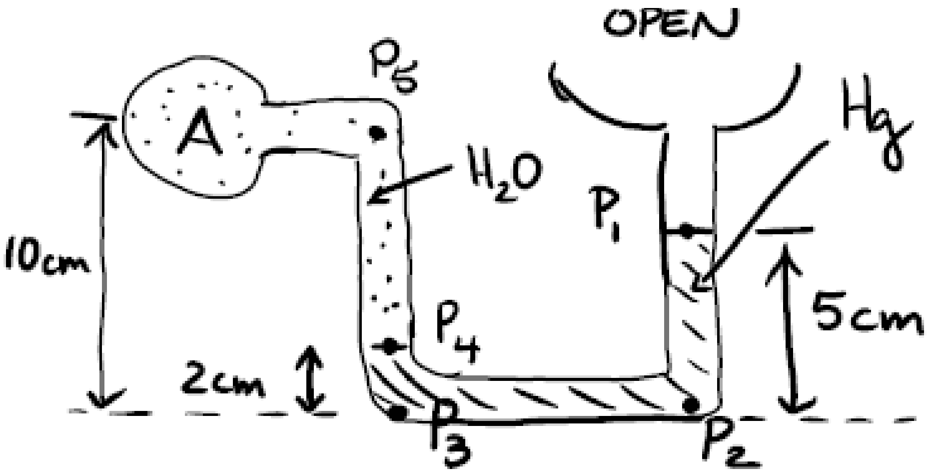
Often used to measure small changes in gas pressure, if pipes A and B contain a gas then

$$p_A - p_B \text{ (pressure difference)} = \gamma_2 l_2 \sin\theta$$

Gas column height h_1 and h_3 neglected

Pressure Measurement using Manometer

Problem: Find the pressure P_A.



$$\gamma_{Hg} = 133\times 10^3\ N/m^3$$
$$\gamma_{H_2O} = 9810\ N/m^3$$

$$p_A + \Delta P_{A-1} = p_1$$
$$p_A + \gamma_{H20}\frac{10-2}{100}m - \gamma_{hg}\frac{5-2}{100}m = p_1 = 0$$
$$p_A = 0 + \gamma_{hg}\frac{5-2}{100}m - \gamma_{H20}\frac{10-2}{100}m$$
$$p_A = 3,205 Pa$$

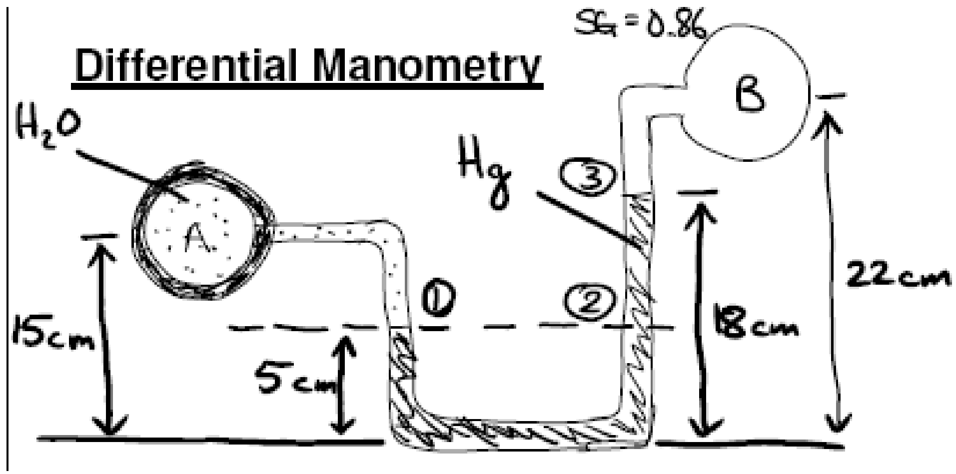
Solution: We know $p_1 = p_{atm} = 0\ gage$

$$p_2 = p_1 + \gamma_{Hg}\times\Delta h = 0 + \left(133\times 10^3\ N/m^3\right)\times\left(0.05m\right) = 6650\ N/m^2$$
$$p_2 = p_3 \text{ (Pascal's Law)}$$

$$p_A = p_5 = 3205\ N/m^2 = 3,205 Pa \text{ (Pascal's Law)}$$

Pressure Measurement using Manometer

Problem: Find the pressure P_B - P_A.



$$\gamma_{Hg} = 133\times10^3\ N/m^3$$

$$\gamma_{H_2O} = 9810\,N/m^3$$

Solution: We will use POINT-TO-POINT method

$$p_A + \gamma_{H_2O}(0.10m) - \gamma_{Hg}(0.13m) - \gamma(0.4m) = p_B$$

$$p_B - p_A = 980Pa - 17300Pa - 340Pa = -16,600Pa$$

$$\gamma = S.G.\times\gamma_{H_2O} = 0.86\times9810\,N/m^3 = 8436.6\,N/m^3$$

FIND "Δh".

ROAD MAP

Identify and label various elevation levels associated with different fluids and understanding Pascal's Law.

Identify Starting Point and Ending Point.

Identify any "MISSING" dimensions from Start Point to End Point, ΔS.

Transverse circuit from Start Point and Apply POINT-TO-POINT method for Law of Hydrostatics

Solve for unknown.

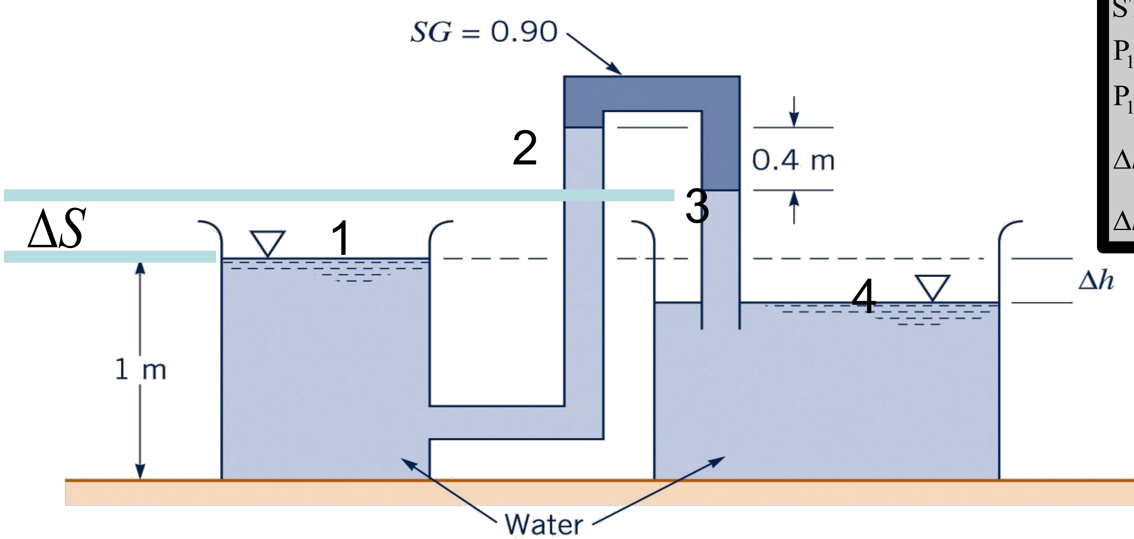


Figure P2.47

START 1, END 4

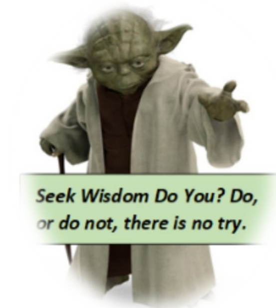
$$P_1 - \Delta S \gamma_{H20} - 0.4m\gamma_{H20} + 0.4\gamma_f + (\Delta S + \Delta h)\gamma_{H20} = P_4$$

$$P_1 = P_4 = 0\text{gauge}$$

$$\Delta h = \frac{0.4m(\gamma_{H20} - \gamma_f)}{\gamma_{H20}} = 0.4m(1 - S_{g_f}) \rightarrow \text{Parametric Equation}$$

$$\Delta h = 0.4m(1 - 0.90)$$

PROBLEM 2.48



IDENTIFY LEVELS and ANY MISSING DINENSIONS

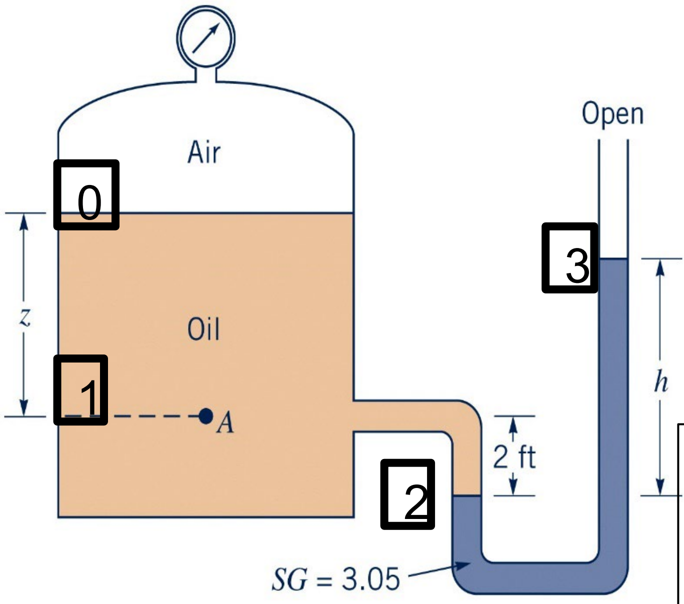


Figure P2.42

$$\gamma_{oil}\left[\frac{lbf}{ft^3}\right] = 54.0\frac{lbf}{ft^3}$$

$$P_A\left[\frac{lbf}{ft^2}\right] = 2.0\frac{lbf}{\cancel{in^2}} \bullet \frac{144\,\cancel{in^2}}{ft^2}$$

$$P_{AIR}\left[\frac{lbf}{ft^2}\right] = 0.50\frac{lbf}{\cancel{in^2}} \bullet \frac{144\,\cancel{in^2}}{ft^2}$$

Determine Height Z

Law of Hydrostatics Drives Fluid Principles

Start Point: 0, End Point 1

$$P_0 + \gamma_{oil} z = P_A$$

$$z[ft] = \frac{(P_A - P_0)\left[\frac{lbf}{ft^2}\right]}{\gamma_{oil}\left[\frac{lbf}{ft^3}\right]}[ft]$$

$$z[ft] = \frac{(2.0 - 0.5)\frac{lbf}{in^2} \bullet \frac{144 in^2}{ft^2}}{54.0\frac{lbf}{ft^3}} = 4\,ft$$

Determine Manometer Deflection "h"

Start: 0, End 3 → Apply Hydrostatics

$$P_0 + \gamma_{oil}(z+2) - \gamma_m h = P_3$$

$$h[ft] = \frac{(P_0 - P_3)\left[\frac{lbf}{ft^2}\right] + \gamma_{oil}\left[\frac{lbf}{ft^3}\right](z+2)[ft]}{\gamma_m\left[\frac{lbf}{ft^3}\right]}[ft]$$

$$h[ft] = \frac{(P_0 + \gamma_{oil} z - P_3)\left[\frac{lbf}{ft^2}\right] + 2[ft]\gamma_{oil}\left[\frac{lbf}{ft^3}\right]}{\gamma_m\left[\frac{lbf}{ft^3}\right]}$$

$$h[ft] = \frac{P_A - P_3 + 2[ft]\gamma_{oil}\left[\frac{lbf}{ft^3}\right]}{\gamma_m\left[\frac{lbf}{ft^3}\right]}[ft]$$

Problem 2.42

OR PICK OTHER POINTS

Start: A, End 3 → Apply Hydrostatics

$$P_A + 2\gamma_{oil} - \gamma_m h = P_3$$

$$h[ft] = \frac{(P_A - P_3)\left[\frac{lbf}{ft^2}\right] + 2[ft]\gamma_{oil}\left[\frac{lbf}{ft^3}\right]}{\gamma_m\left[\frac{lbf}{ft^3}\right]}[ft]$$

IDENTIFY LEVELS and
ANY MISSING DINENSIONS

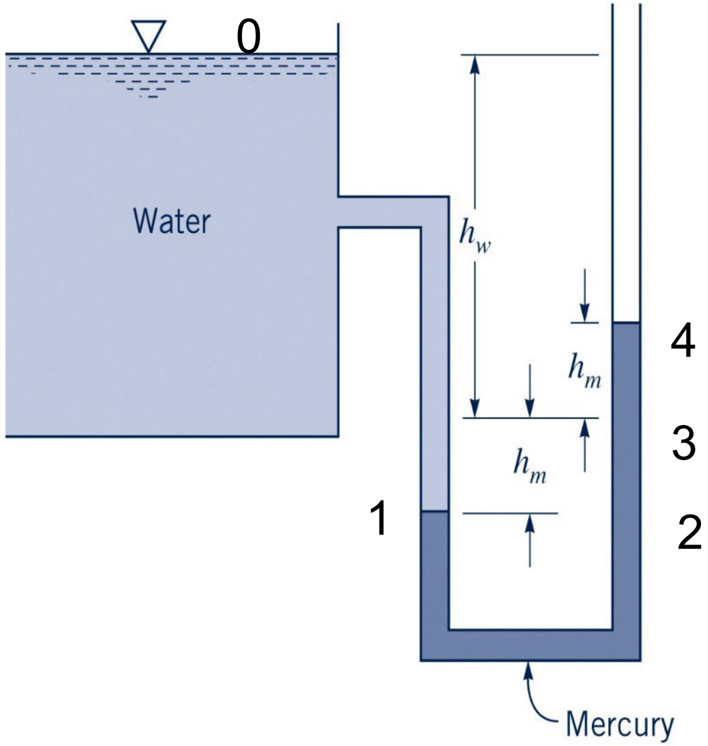


Figure P2.35

Problem 2.36

Determine $\frac{h_w}{h_m}$

Law of Hydrostatics Drives Fluid Principles

$P_1 \rightarrow$ MUST BE SAME AS P_2 (SAME FLUID/SAME ELEVATION)

Start Point 0: End Point 4

$$P_0 + \gamma_w(h_w + h_m) - \gamma_m(2h_m) = P_4$$

$$\gamma_w(h_w + h_m) - \gamma_m(2h_m) = P_4 - P_0$$

$\div h_m$

$$\frac{\gamma_w(h_w + h_m)}{h_m} - 2\gamma_m = \frac{P_4 - P_0}{h_m}$$

$\div \gamma_w$

$$\frac{h_w}{h_m} + 1 = \frac{2\gamma_m}{\gamma_w} + \frac{P_4 - P_0}{\gamma_w h_m}$$

$$\frac{h_w}{h_m} = \frac{2\gamma_m}{\gamma_w} - 1 + \frac{P_4 - P_0}{\gamma_w h_m}$$

$$\frac{h_w}{h_m} = 2(Sg)_m - 1 + \frac{(P_4 - P_0)}{\gamma_w h_m}$$

$$\frac{h_w}{h_m} = 2 \bullet 13.55 - 1 = 26.1$$

A flowrate measurement device in installed in a horizontal pipe through which water is flowing. A U tube manometer is connected through pressure taps with specific weight of the manometer fluid as 112 lbf/ft3.

Determine the differential reading of the manometer corresponding to a pressure drop between the taps of 0.5 lbf/in2.

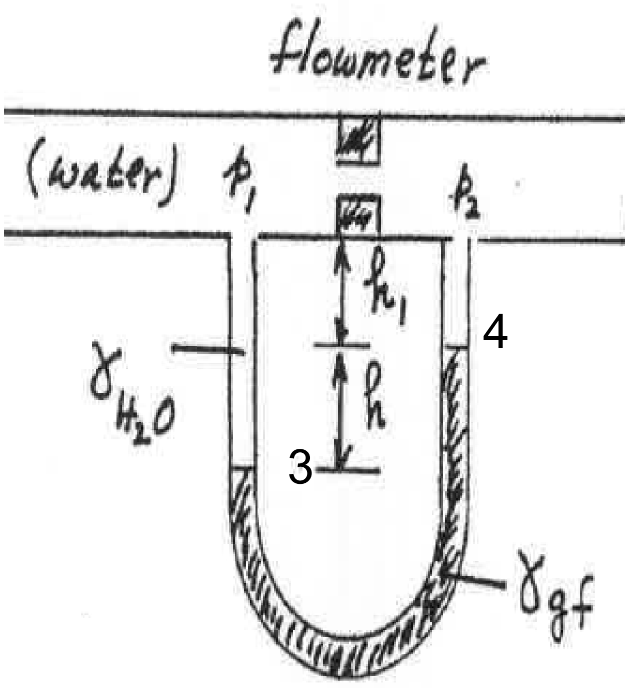
APPLY HYDROSTATICS

Start 1, End 2

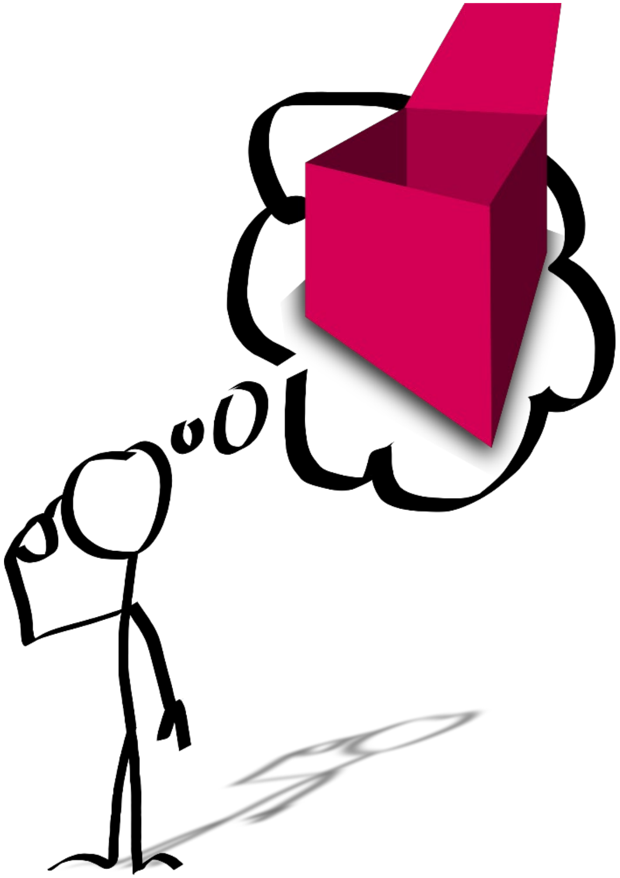
$$P_1 + (h_1 + h + D/2)\gamma_{H_2O} - (h)\gamma_m - (h_1 + D/2)\gamma_{H_2O} = P_2$$

$$h[ft] = \frac{[P_1 - P_2]\left[\frac{lbf}{ft^2}\right]}{[\gamma_m - \gamma_{H_2O}]\left[\frac{lbf}{ft^3}\right]}[ft]$$

$$h[ft] = \frac{0.5\frac{lbf}{in^2} \bullet \frac{144 in^2}{ft^2}}{[112 - 62.4]\left[\frac{lbf}{ft^3}\right]} = 1.45 ft$$



Problem 2.44



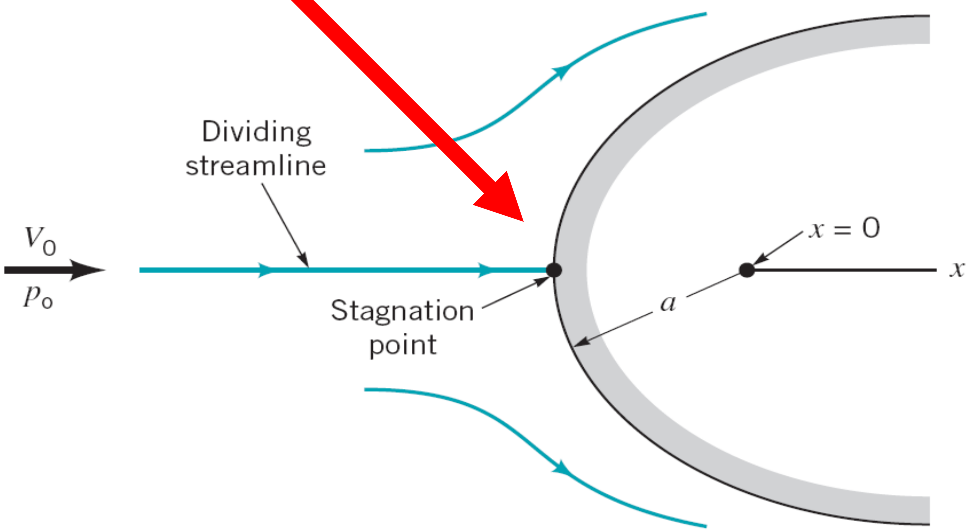
- *Manometry, Manometry, Manometry!!! Learn it at the beginning, because you will still be using it the exact same way all the way through the final. Geometry is irrelevant, if you know the process to solve the type of problem that is presented, and follow the path. If you don't know the definitions, it is improbable that you will be able to define the problem. Define the problem & develop the path of attack, BEFORE you start randomly writing down equations.*

- ***FLUIDS STUDENT FALL 2023***

STAGNATION OR TOTAL PRESSURE

$$\underbrace{P_{stag}}_{TOTAL} = \underbrace{P_0}_{STATIC} + \underbrace{\frac{1}{2}\rho V_0^2}_{DYNAMIC}$$

$$V_0 = \sqrt{\frac{2\left(P_{stag} - P_0\right)}{\rho}}$$



At **STAGNATION POINT**, **STATIC PRESSURE** and Pressure induced by Velocity (**DYNAMIC PRESSURE**) is converted to TOTAL or **STAGNATION PRESSURE**

MANOMETER can be used to measure SPEED of object.