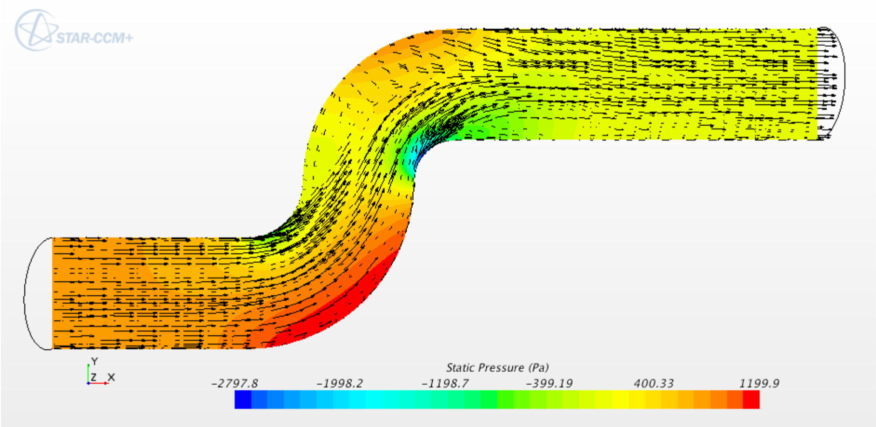
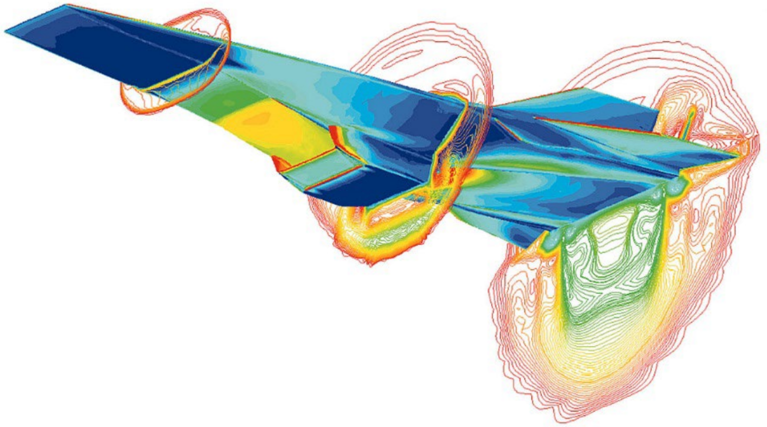
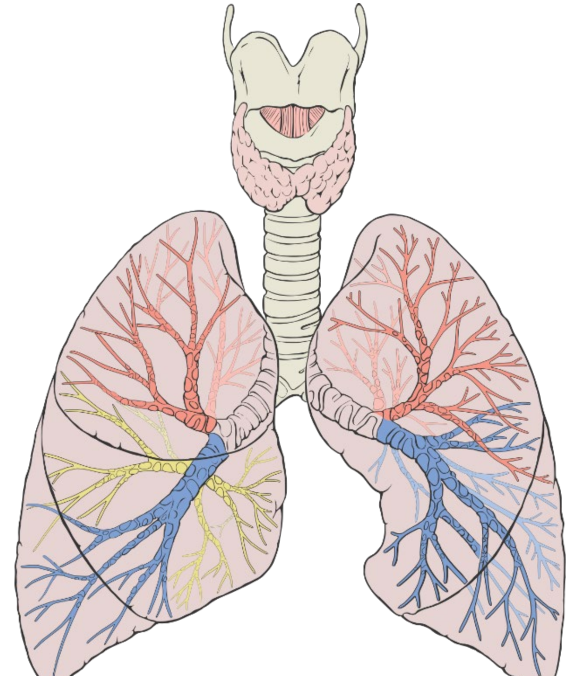




Chapter 8

Viscous Flow in Pipes

Laminar and Turbulent Flows




LAMINAR VS. TURBULENT FLOW

Objectives

- Identify and explain various characteristics of pipe flow.
- Understand and discuss main properties of laminar and turbulent flow.
- Calculate losses in straight pipes and components and determine pressure drops and require pumping power to overcome pressure drop and loses.
- Be able to use Moody diagram and Haaland equation to Iteratively solve for FLOW RATE and Diameters.

Energy Equation
Real Systems—SINGLE IN/OUT

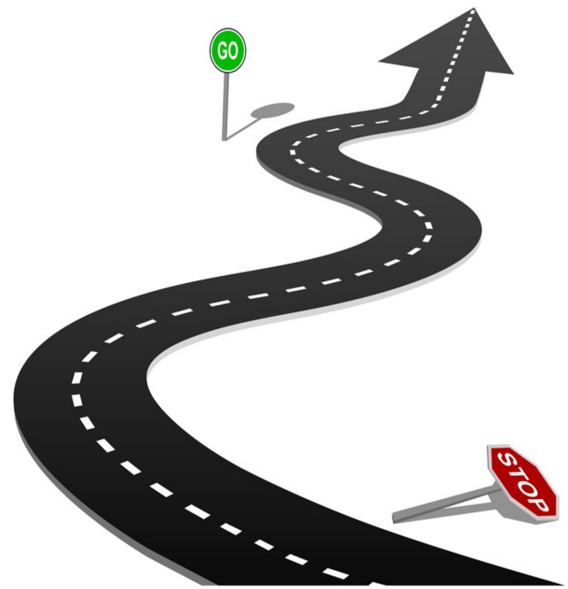
$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_{p_{IDEAL}}=h_{T_{IDEAL}}+\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_q+h_{L_{major}}+h_{L_{\text{minor}}}$$

$\rightarrow$ All terms "ft" or "m"

$h_p \equiv IDEAL\ PUMP\ POWER$

$h_T \equiv IDEAL\ TURBINE\ POWER$

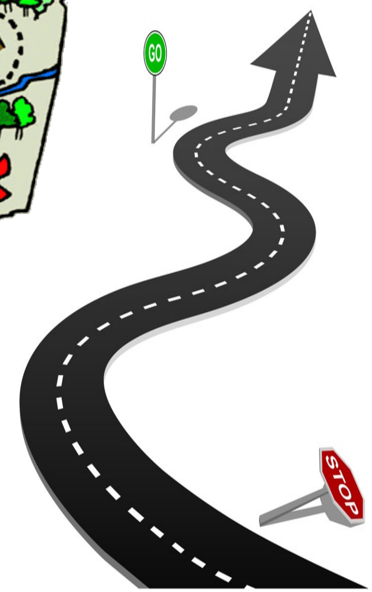
$$h_q \equiv \frac{u_2-u_1-\frac{\dot{Q}}{\dot{m}}}{g} \rightarrow \text{THERMAL LOSS}$$



Energy Equation
Real Systems—MULTIPLE IN/OUT

$$\sum_{in}\dot{m}g\left(\frac{P_{in}}{\gamma}+\frac{V_{in}^{2}}{2g}+z_{in}\right)+\dot{m}g(h_{p_{IDEAL}})=\dot{m}g(h_{T_{IDEAL}})+\sum_{out}\dot{m}g\left(\frac{P_{out}}{\gamma}+\frac{V_{out}^{2}}{2g}+z_{out}\right)+\dot{m}g\left(h_{TOTAL}\right)$$

$$\dot{m}g(h)=\gamma Q(h)\rightarrow WATTS, or, \frac{ft-lbf}{s}$$



$h_p \equiv$ *IDEAL PUMP POWER* HEAD (m or ft)

$h_T \equiv$ *IDEAL TURBINE POWER* HEAD (m or ft)

$h_{TOTAL} = h_q + h_{L_{major}} + h_{L_{minor}}$ → TOTAL POWER FLOW LOSS (m or ft)

$$h_q \equiv \frac{(u_2-u_1)\left[\frac{J}{kg}\right]-\frac{\dot{Q}}{\dot{m}}\left[\frac{J}{kg}\right]}{g}$$ → THERMAL LOSS

Pump and Turbine Power & Efficiency

$$W_{P_{ACTUAL}} = \frac{\gamma Q h_{P_{IDEAL}}}{\eta_p} = \frac{Q\Delta P}{\eta_p} \rightarrow PUMP$$

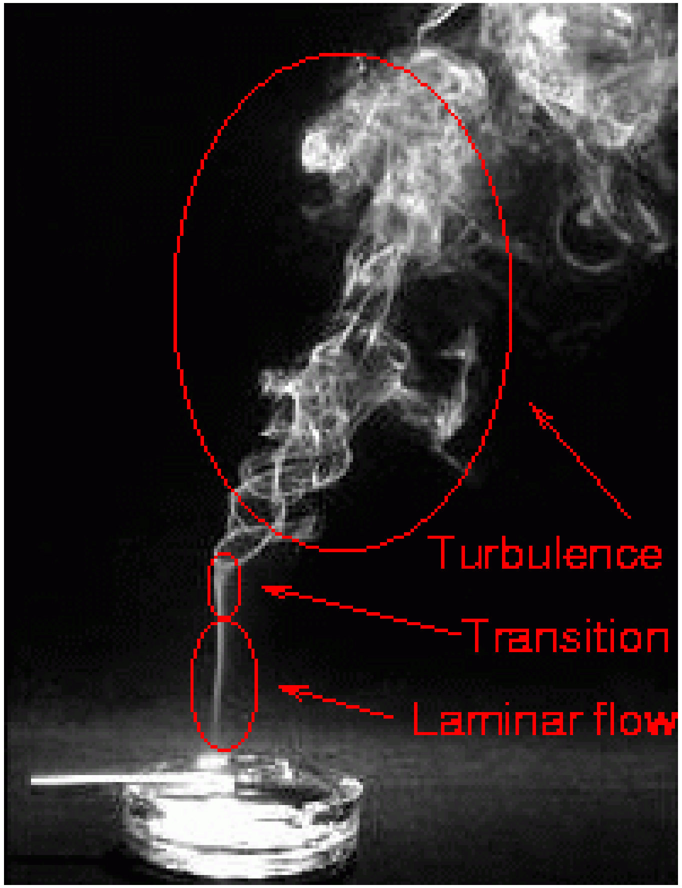
$$W_{T_{ACTUAL}} = \gamma Q h_{T_{IDEAL}} \eta_t \rightarrow TURBINE$$

$$\eta_p \equiv \text{pump efficiency} = \frac{W_{ideal}}{W_{actual}} = \frac{W_{actual} - Loss}{W_{actual}}$$

$$\eta_t \equiv \text{turbine efficiency} = \frac{W_{actual}}{W_{ideal}} = \frac{W_{actual}}{W_{actual} + Loss}$$

Flow Visualization

- **Reynolds Experiment (Pipe Flow)**

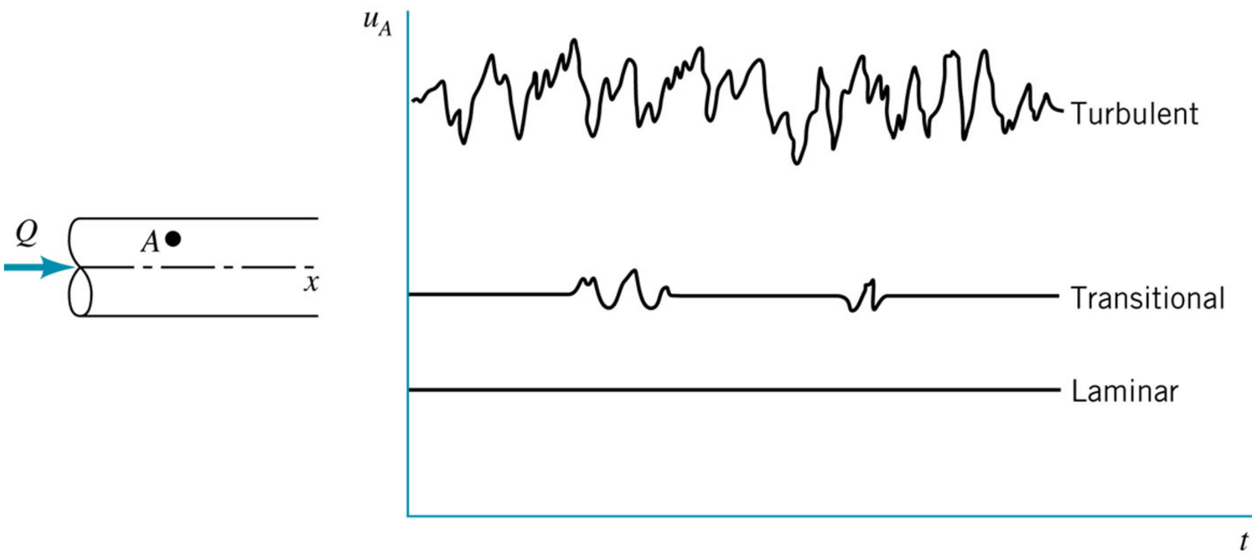


https://www.youtube.com/watch?v=9A-uUG0WR0w

Fluid Dynamic Instabilities!

PLEASE DO NOT SMOKE !!!

Laminar or Turbulent Flow



- **Reynolds Number** $$Re_D = \frac{\rho V D_H}{\mu} = \frac{V D_H}{\nu}$$

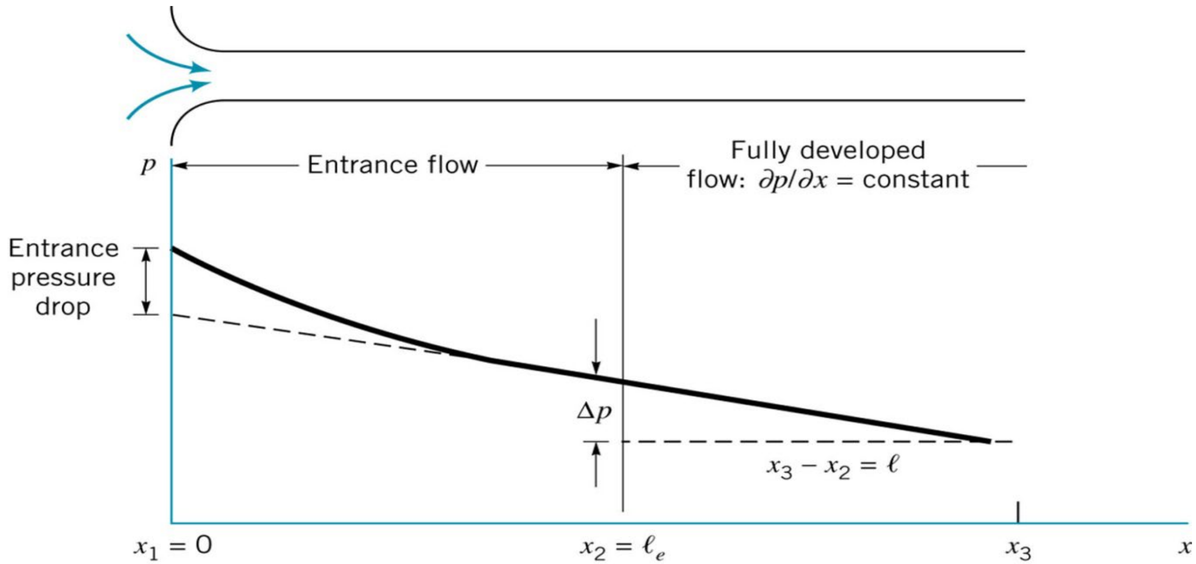
Laminar Flow: $Re_{D_H} \le 2100$

Transitional Flow: $2100 < Re_{D_H} \le 4000$

Turbulent Flow: $Re_{D_H} > 4000$

Entrance Length, Fully Developed Flow

What Causes fluid to flow through the pipe?



- **For horizontal pipe flow, gravity has no effect except for a hydrostatic pressure variation across the pipe.**

- **The pressure difference, $\Delta p = p_1 - p_2$ forces the fluid through the pipe.**

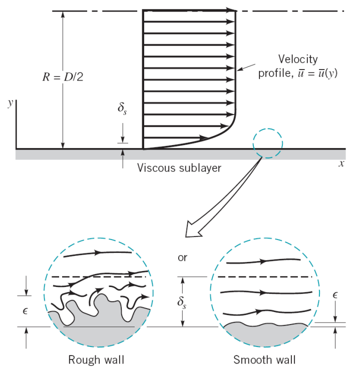
- **Pressure force is needed to overcome the viscous forces generated.**

- **Nature of the pipe flow is dependent on whether the flow is laminar or turbulent.**

Friction factor & head loss

- **Friction factor for laminar/turbulent pipe flow:**

$$f_{LAM} = \frac{8\tau_w}{\rho V^2} = \frac{64}{\mathrm{Re}_{D_H}}$$



$$f_{TURB} = \phi\left(\mathrm{Re}_{D_H}, \varepsilon/D_H\right)$$

$$\varepsilon/D_H \equiv \text{Relative Roughness (pipe wall interior)}$$

- **Pressure Drop & Head Loss:**

$$\Delta p = f\left(\frac{l}{D_H}\right)\frac{\rho V^2}{2}\left(pressure\ drop - Pa; \frac{lbf}{ft^2}\right)$$

$$\Rightarrow h_L = \frac{\Delta p}{\gamma} = f\left(\frac{l}{D_H}\right)\frac{V^2}{2g}(\text{energy loss} - m; ft)$$

Hydraulic Diameter: $D_H = \frac{4A_c}{P}$, A_c = CROSS-SECTIOPN *area*; P = *Wetted* Perimeter

(non-circular pipe)

12

Friction factor & head loss

Example Problem: **A fluid flows through a 0.1in diameter pipe. When the Reynolds number is 1500, the head loss over a 20ft long pipe is 64ft. Determine the fluid velocity.**

Solution: **Since Reynolds number = 1500 < 2100. The flow is Laminar. Hence**

$$f = \frac{64}{\mathrm{Re}_D} = \frac{64}{1500}$$

$$\Rightarrow f = 0.043$$

Now

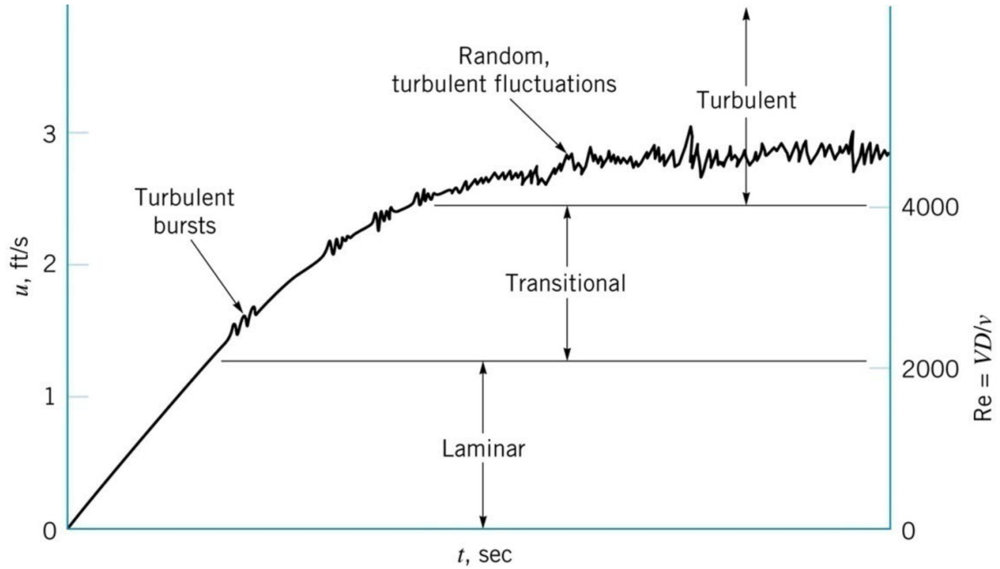
$$h_L = f\left(\frac{l}{D}\right)\frac{V^2}{2g}$$

$$\Rightarrow 64\,ft = 0.043\left(\frac{20\,ft}{0.1in/12in/ft}\right)\frac{V^2}{2\left(32.2\,ft/s^2\right)}$$

$$\Rightarrow V = 6.32\,ft/s$$

13

Fully Developed Turbulent Flow



- **Turbulent is a very complex flow pattern**
- **Mixing processes (heat, mass transfer) are considerably enhanced due to the macroscopic scale of randomness.**
- **Without turbulence it would be virtually impossible to carry out life as we now know it.**
- **In some situations turbulent flow is desirable and in others they are a disadvantage.**

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1+\frac{h_{p_{IDEAL}}}{\eta_p}=\eta_t h_{T_{IDEAL}}+\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_q+h_L$$

$$h_L = h_{L_{\text{major}}} + h_{L_{\text{minor}}}$$

$$h_q \equiv \frac{u_2 - u_1 - \frac{\dot{Q}}{\dot{m}}}{g}$$

- **Most turbulent pipe flow information is based on experimental data.**

- **It is often necessary to determine the head loss that occurs in a pipe flow**

- **The overall head loss for the pipe system consists of the head loss due to viscous effects in the straight pipes, termed the major loss and the head loss in the various pipe components termed the minor loss.**

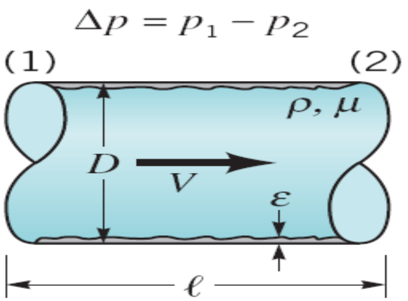
15

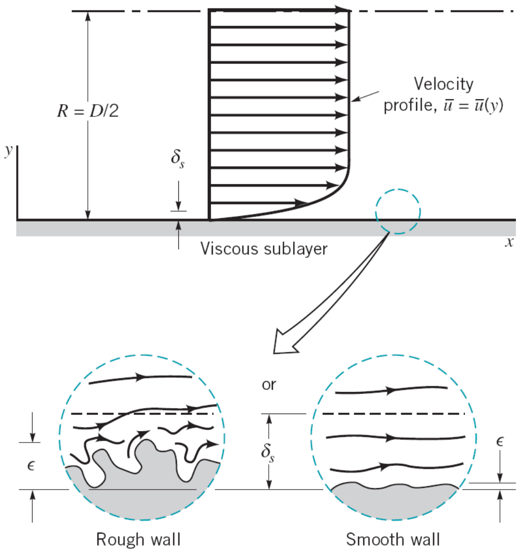
Major Losses

- **Turbulent pipe flow properties depend on the fluid density and the pipe roughness.**

- **For turbulent flow the pressure drop is expected to be a function of the wall roughness, ε.**

- **The relative roughness (ε/D) for typical constant diameter pipes is between 0 and 0.05.**



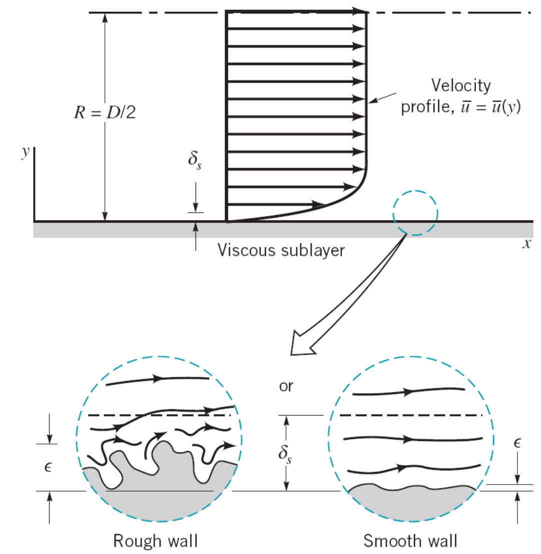


$$\frac{\delta_s}{D} << 1$$

$$0 \leq \varepsilon / D \leq 0.05$$

Friction factor for Turbulent flow

- Turbulent flow, the functional dependence of the friction factor on the Reynolds number and the relative roughness is a rather complex one that cannot, as yet be obtained from a theoretical analysis.



$$h_{L,major} = f \frac{\ell}{D} \frac{V^2}{2g}$$

$$f_{TURB} = \phi\left(\mathrm{Re}_D, \varepsilon/D\right)$$

TABLE 8.1

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

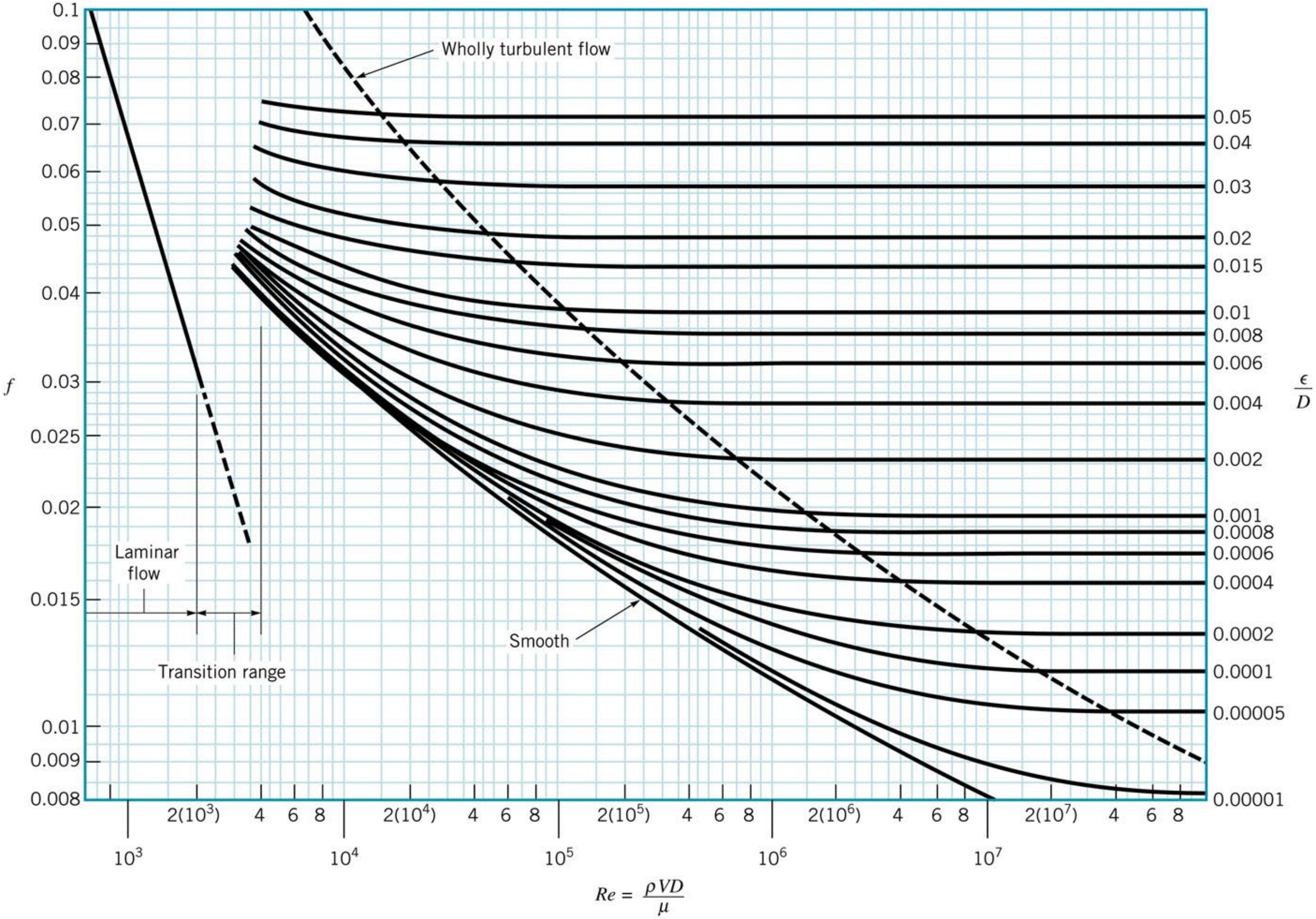
Moody diagram

$f = \frac{64}{Re_D}$; Laminar Flow

Wall roughness & friction factor

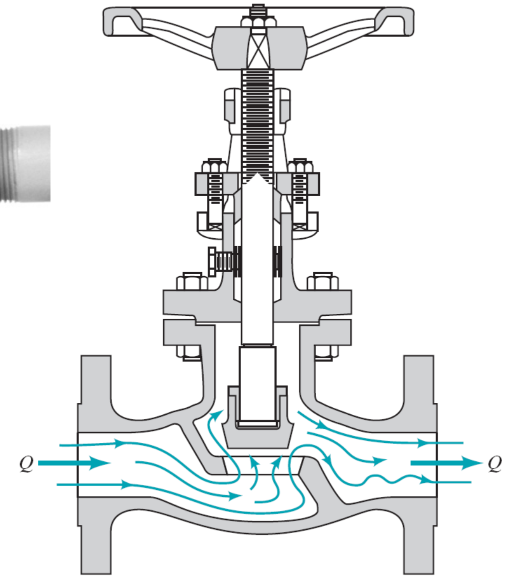
Haaland EQN. formula for turbulent flow

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$$



Minor Losses

(a)



- Most pipe systems, consist of **components (valves, bends, tees and the like)** that adds to the overall head loss of the system.

- The head losses from these components are generally termed **minor losses**.

- **Loss coefficient K_L** is used for these head losses or pressure drops

$$\sum K_L = \frac{h_{L\,minor}}{\left(V^2/2g\right)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

$$\Delta p = \left(\sum k_L\right)\frac{1}{2}\rho V^2$$

$$or\ h_{Lminor} = \left(\sum k_L\right)\frac{V^2}{2g}$$

19

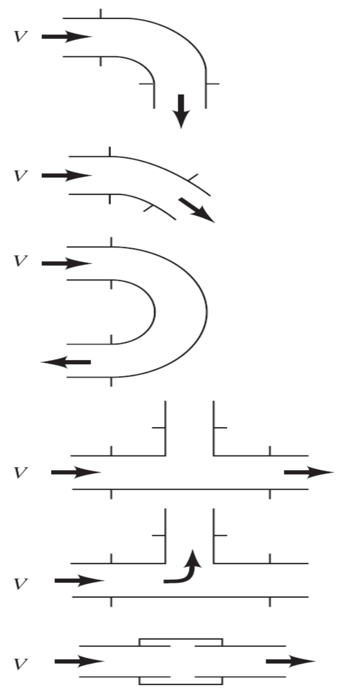
Loss coefficient for piping components

■ **TABLE 8.2**

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ **(Data from Refs. 5, 10, 27)**

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

*See Fig. 8.32 for typical valve geometry.



$$K_{L_{inlet}} = 0.5$$

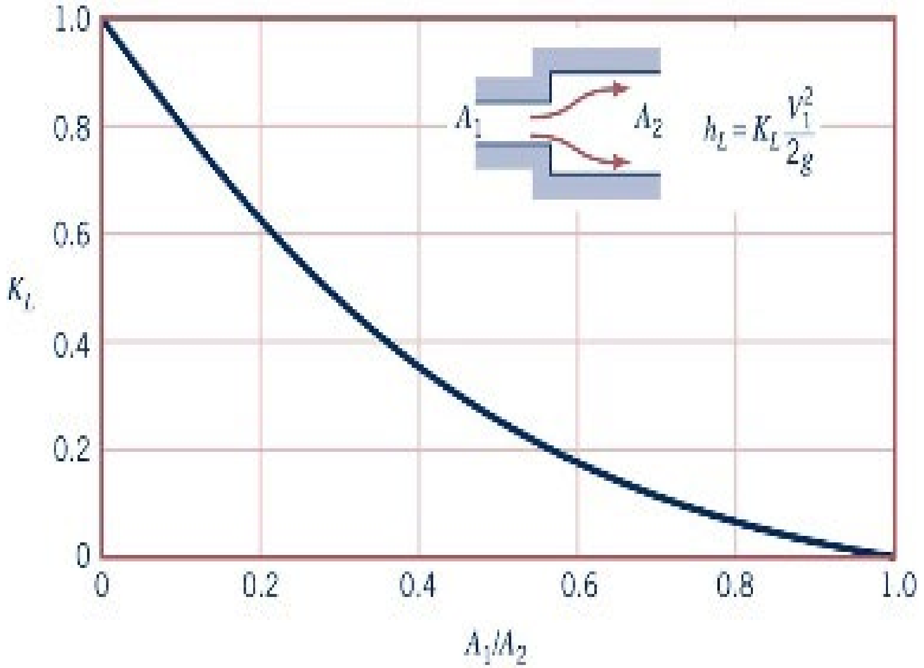
$$K_{L_{exit}} = 1.0$$

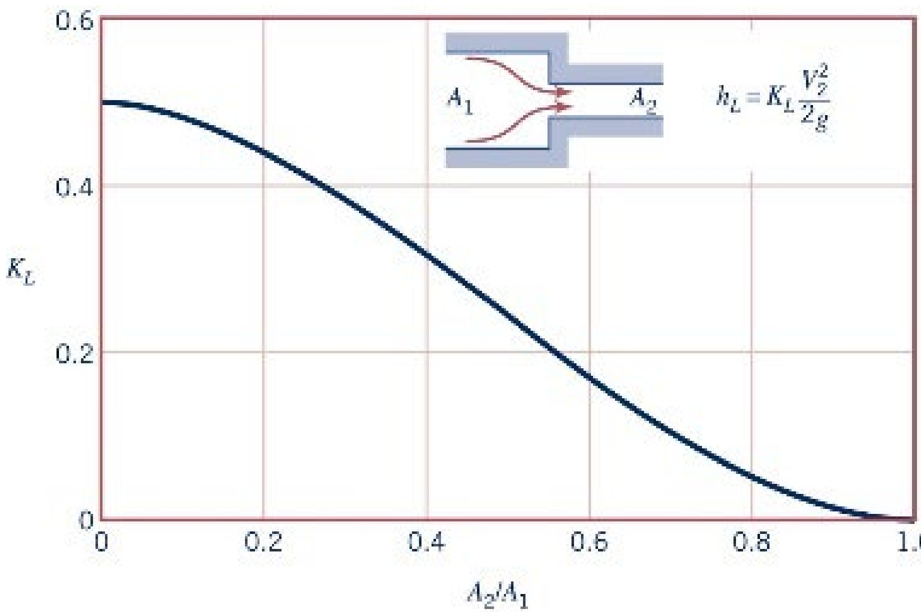
For multi-component systems

$$h_{L,multiple}\Big|_{\min or} = h_{L1} + h_{L2} + h_{L3} + \ldots..$$

20

Sudden Contraction/Expansion Loss





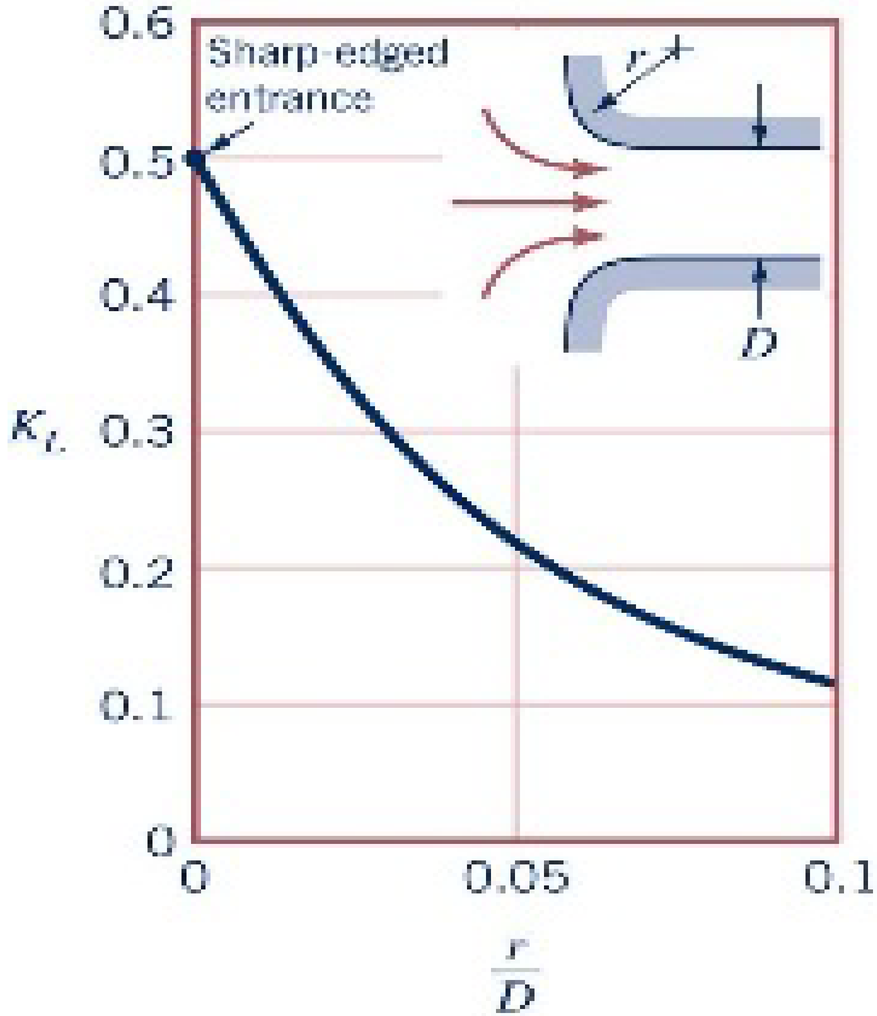
NOTE

$$Let : \frac{A_1}{A_2} \rightarrow 0, i.e. \text{ Exit Loss, } \mathrm{K}_L = 1.0$$

NOTE

$$Let : \frac{A_2}{A_1} \rightarrow 0, i.e. \text{ Inlet Loss, } \mathrm{K}_L = 0.5$$

Loss Coefficient vs Radius



Total Head Loss in Pipe Flows

$\Delta P = \gamma h_L \rightarrow$ Pressure Drop Due to Friction

$\dot{W}_p = \gamma h_L Q = \Delta P Q \rightarrow$ Minimum Pumping Power Required

$$h_L\big|_{total} = h_{L_{\text{major}}} + h_{L_{\text{minor}}} \rightarrow \text{UNITS = LENGTH}$$

$$\Rightarrow h_L\big|_{total} = \sum_{j=1}^{m} f_j \frac{L_j}{D_j}\frac{V_j^2}{2g} + \sum_{i=1}^{n}\left(k_{L_{\text{minor}}}\right)_i\left(\frac{V_i^2}{2g}\right)$$

Major losses due to friction in straight pipe sections

Minor losses due to bending, joints, etc. "connecting" straight pipe sections

23

Energy Equation

$$h_1 + h_{p\,IDEAL} = h_{T_{IDEAL}} + h_2 + h_q + \sum_{j=1}^{m}\left(\frac{fL}{D}\frac{V^2}{2g}\right)_j + \sum_{i=1}^{n}\left(\frac{k_{L\text{minor}}V^2}{2g}\right)_i$$

$$h_q = \frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g} \rightarrow \text{Flow w/Heat Transfer}$$

$$h_{1,2} = \frac{P}{\gamma} + \frac{V^2}{2g} + z;\, or$$

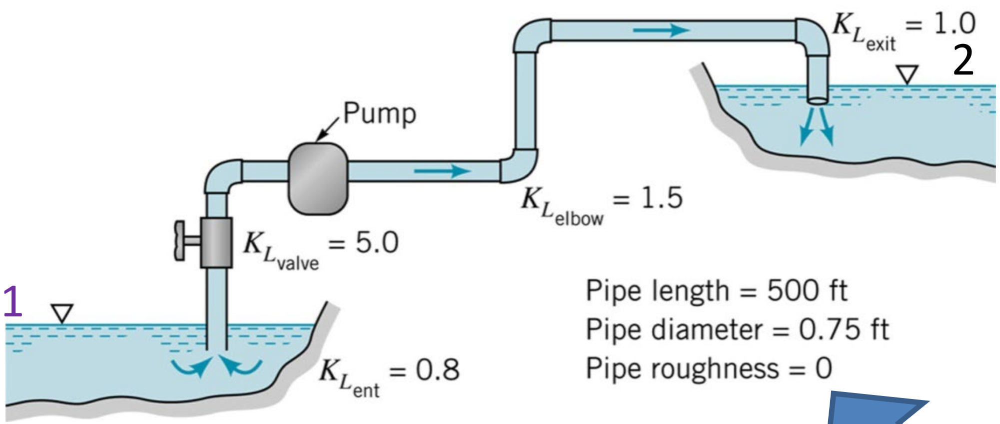
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + \frac{h_{p_{IDEAL}}}{\eta_p} = h_q + \eta_T h_{T_{IDEAL}} + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum_{j=1}^{m}\left(\frac{fL}{D}\frac{V^2}{2g}\right)_j + \sum_{i=1}^{n}\left(\frac{k_L V^2}{2g}\right)_i;\, where$$

h_p = isentropic (ideal) pump INPUT head

h_T = isentropic (ideal) turbine OUTPUT head

Calculation of head losses

Example – Problem # 8.97: **The pump shown in Figure below delivers a head of 250ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200ft.**



Solution: **The viscous pipe flow equation with losses can be written as**

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L \qquad (1)$$

$$where\ p_1 = p_2 = 0;\ V_1 = V_2 = 0;\ z_1 = 0;\ z_2 = 200\,ft;\ h_p = 250\,ft.$$

25

Calculation of head losses

CONSTANT PIPE DIAMETER

Thus (1) reduces to:

$$-h_L + h_p = z_2$$

$$\Rightarrow -\left(f_{guess} \frac{L}{D} \frac{V^2}{2g} + \sum_{i=1}^{n} \left(k_{Lminor}\right)_i \left(\frac{V^2}{2g}\right)\right) + h_p = z_2 \qquad (2)$$

Now $\sum_{i=1}^{n} \left(k_{Lminor}\right)_i \left(\frac{V^2}{2g}\right) = \left(0.8 + 5.0 + 4\times(1.5) + 1.0\right)\left(\frac{V^2}{2g}\right) = 12.8\left(\frac{V^2}{2g}\right)$

Therefo $-\left(\left[f_{GUESS} \frac{(500\,ft)}{(0.75\,ft)} + 12.8\right] \frac{V^2}{2\left(32.2\,ft/s^2\right)}\right) + 250\,ft = 200\,ft$

$$\Rightarrow \left(667 f_{GUESS} + 12.8\right) V^2 = 3220 \qquad (3):$$

Friction Characteristic Equation $\rightarrow$

$$V(f_{GUESS}) = \left(\frac{3220}{667 f_{GUESS} + 12.8}\right)^{1/2} \rightarrow \text{Guess f: } 0.01 \le f_{GUESS} \le 0.03$$

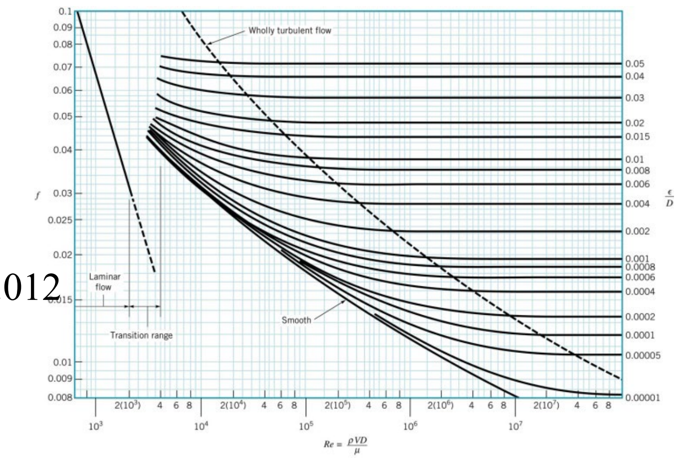
Calculation of head losses

Now $V = \left(\dfrac{3220}{667\, f_{GUESS} + 12.8} \right)^{1/2}$ (3)

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{\left(1.94\, slug/ft^3\right) V \left(0.75\, ft\right)}{\left(2.34\times10^{-5}\, lb\cdot s/ft^2\right)} = 6.22\times10^4 \bullet V \quad (4)$$

Use Moody's diagram -

Trial & error method:



$Assume\ f_1 = 0.02 \rightarrow V = 11.1\, ft/s \rightarrow \mathrm{Re}_D = 6.9\times10^5 \rightarrow f_{moody} = 0.012 \neq 0.02$

$Assume\ f_2 = 0.012 \rightarrow V = 12.4\, ft/s \rightarrow \mathrm{Re}_D = 7.7\times10^5 \rightarrow f_{moody} = 0.0121 \simeq 0.012$

$Thus\ \ V = 12.4\, ft/s$

Hence the power added to the water is

$$\dot{W}_S = Q(\gamma h_p) = (VA)(\gamma h_p) = (12.4\, ft/s)\left(\pi/4\left[0.75\, ft\right]^2\right)\left(62.4\, lb/ft^3\right)(250\, ft) = 8.55\times10^4\, \frac{ft.lb}{s}$$

$$\Rightarrow \dot{W}_S = 8.55\times10^4\, \frac{ft.lb}{s} \times \frac{1hp}{(550\, ft.lb/s)} = 155hp$$

You may also use HAALAND formula to find *f*.

27

HALLAND EQUATION
Good for Programming Calculator

$$V=\left(\frac{3220}{667\,f_{guess}+12.8}\right)^{1/2}\quad(3)$$

$$\mathrm{Re}_D=\frac{\rho VD}{\mu}=\frac{\left(1.94\,slug/ft^3\right)V\left(0.75\,ft\right)}{\left(2.34\times10^{-5}\,lb\cdot s/ft^2\right)}=6.22\times10^4 V\quad(4)$$

$$\frac{1}{\sqrt{f}}=-1.8\log_{10}\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{\mathrm{Re}}\right)$$

HALLAND EQUATION

HAALLAND EQUATION														
	DENS	Visc	D			DENS	Visc	D			DENS	Visc	D	
e/D	slugs/ft3	lb-s/ft2	ft	f (guess)	e/D	slugs/ft3	lb-s/ft2	ft	f (guess)	e/D	slugs/ft3	lb-s/ft2	ft	f (guess)
0	1.94	2.34E-05	0.75	0	0	1.94	2.34E-05	0.75	0.01	0	1.94	2.34E-05	0.75	1
f	V	Re	f (FALLAND)		f	V	Re	f (FALLAND)		f	V	Re	f (FALLAND)	
0	15.86	9.862E+05	0.0116		0.01	12.86	7.996E+05	0.0120		1	2.18	1.353E+05	0.0168	
0.0116	12.52	7.784E+05	0.0121		0.0120	12.43	7.731E+05	0.0121		0.0168	11.59	7.206E+05	0.0123	
0.0121	12.42	7.724E+05	0.0121		0.0121	12.42	7.723E+05	0.0121		0.0123	12.39	7.705E+05	0.0121	
0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121	
0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121	

Calculation of head losses/pressure drop

- EXAMPLE: Water at 0.02 m3/s flows through 350m of horizontal cast iron pipe (D=20.27 cm). Determine the head loss and pressure drop?

$$\nu = \frac{\mu}{\rho} = 9.569x10^{-7}\frac{m^2}{s}$$

$$V = \frac{Q}{A} = \frac{0.02\frac{m^3}{s}}{0.032270m^2} = 0.62\frac{m}{s} \rightarrow \mathrm{Re} = \frac{VD}{\nu} = 1.31x10^5 \rightarrow TURBULENT$$

Table 8.1

ε = Relative Roughness = 0.026 cm

$$\frac{\varepsilon}{D} = \frac{0.026cm}{20.27cm} = 0.0012$$

Moody Diagram $(\mathrm{Re}, \frac{\varepsilon}{D}), f = 0.022$; (or Haaland Equation)

HEAD LOSS

$$\mathrm{h}_L = \frac{fL}{D}\frac{V^2}{2g} = \frac{0.022x350}{0.2027}\frac{(0.62)^2}{2(9.81)} = 0.744 \text{ m of water}$$

PRESSURE DROP

$$\Delta P = \gamma \mathrm{h}_L = 9800\frac{N}{m^3} \bullet 0.744m = 7,280\frac{N}{m^2}$$

Problem Types

TYPES

1.Given: L, ν, ε, Q, D $\rightarrow$ FIND $h_L = f(V, \frac{\varepsilon}{D}) \frac{L}{D} \frac{V^2}{2g}$

2.Given: L, ν, ε, D, $h_L (Q, D) \rightarrow$ FIND Q $\rightarrow$ TRIAL-n-ERROR

3. GIVEN: L, ν, ε, Q, $h_L (Q, D) \rightarrow$ FIND D $\rightarrow$ TRIAL-n-ERROR

$$\nu = \frac{\mu(Dynamic\ Viscosity)}{\rho(Density)} \rightarrow Kinematic\ Viscosity$$

Calculation of head losses/pressure drop

- EXAMPLE: A flowrate of 3.5 ft3/s is to be maintained in a horizontal aluminum pipe with an inlet pressure of 65 psig, and an outlet pressure of 30 psig. The pipe length is 500ft. Determine the pipe diameter.

$$h_1 + \cancel{h_{P_{IDEAL}}} = \cancel{h_{T_{IDEAL}}} + h_2 + \cancel{h_q} + \sum_{j=1}^{m}\left(\frac{f_j L_j}{D_j}\frac{V_j^2}{2g}\right) + \cancel{\sum_{i=1}^{n}\left(k_{Lminor}\frac{V^2}{2g}\right)_i}$$

$$V = \frac{Q}{A} = \frac{3.5}{\pi\frac{D^2}{4}} = \frac{4.46}{D^2}$$

$$\frac{p_1 - p_2}{\gamma} = \frac{fL}{D}\frac{V^2}{2g} = \frac{fL}{D}\frac{\left(\frac{4.46}{D^2}\right)^2}{2g}$$

$$\frac{(65-30)psig \bullet 144\frac{in^2}{ft^2}}{62.4\frac{lbf}{ft^3}} = \frac{f \bullet 500}{D}\frac{\left(\frac{4.46}{D^2}\right)^2}{2 \bullet 32.2}$$

solving for D(f)=

$D=1.138f^{1/5}$

TRIAL-n-ERROR $\rightarrow$ f(V,D,$\frac{\varepsilon}{D}$)

SOLUTION METHOD

$\varepsilon = 5x10^{-6} ft$

1. Guess $f_1 = 0.02$, $D_1 = 0.520 ft$, $Re = \frac{VD}{\nu} = 7.11x10^5$, $\frac{\varepsilon}{D} = 9.6x10^{-4}$

2. Check GUESS $\rightarrow f_{1_{MOODY}} = 0.0128 \neq 0.02 \rightarrow$ MUST ITERATE AGAIN

3. Guess $f_2 = 0.0128$, $D_2 = 0.476 ft$, $Re_2 = \frac{VD}{\nu} = 7.77x10^5$, $\frac{\varepsilon}{D_2} = 1.1x10^{-5}$

4.Check GUESS $\rightarrow f_{2_{MOODY}} = 0.0128 = f_2 \rightarrow STOP$

ANSWER

$D = 0.476 ft$

Water is pumped from the tank and exits as a free jet, and the pump manufacture has provided the pump head vs volume flow rate curve as shown.

Determine the flow rate through the 100 meter straight smooth pipe section.

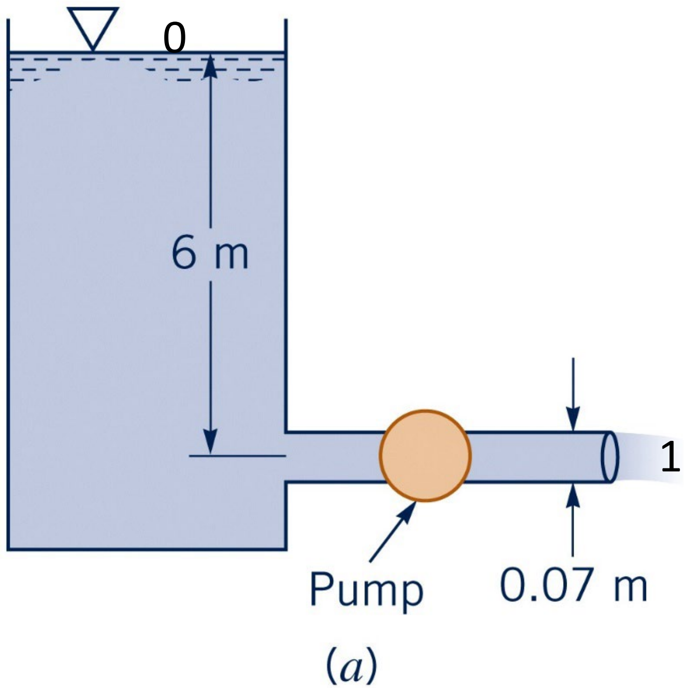


Figure P5.118a

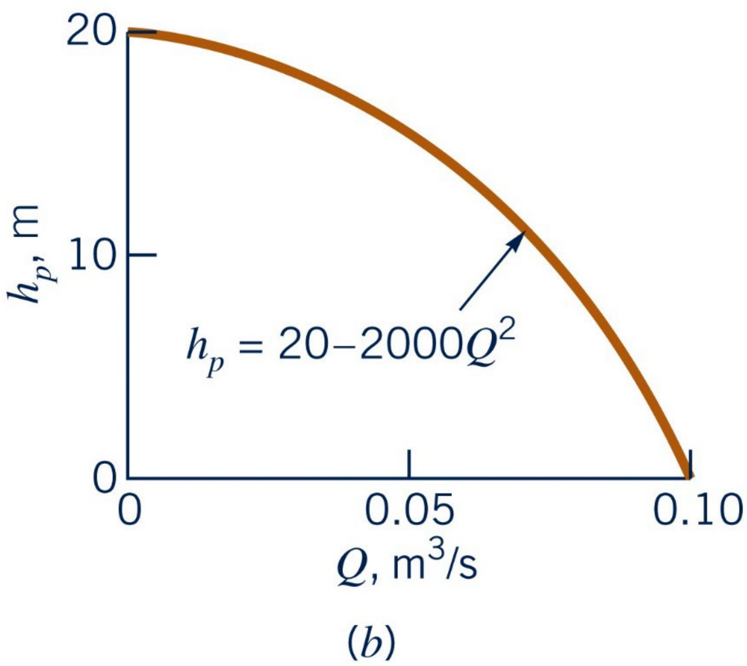


Figure P5.118b

ENERGY CONSERVATION (Single Inlet/Exit)

$$\frac{p_0}{\gamma} + z_0 + \frac{V_0^2}{2g} + h_p = h_T + \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}} \qquad (1)$$

$$p_0 = p_1 = V_0 = z_1 = h_T = 0$$

$$z_0 = 6m$$

$$6 + h_p = \frac{V_1^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}}$$

MASS CONSERVATION

$$V_1 = \frac{Q}{A_1} \qquad (2)$$

Combine MASS with Energy & Pump Head

$$6 + h_p = \frac{\left[\frac{Q}{A_1}\right]^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}}$$

$$6 + (20 - 2000Q^2) = \frac{\left[\frac{Q}{A_1}\right]^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}} \qquad (3)$$

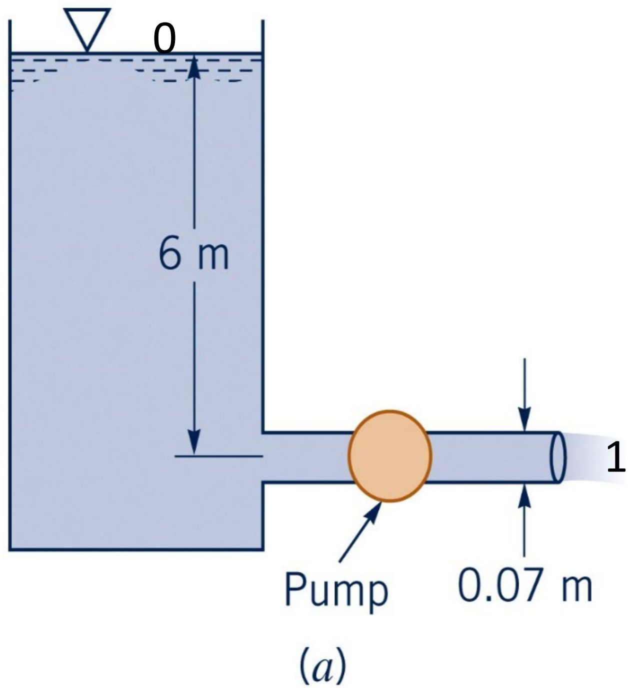


Figure P5.118a

PUMP HEAD UNITS

$$hp[m] = 20 - 2000Q^2$$

$$= 20[m] - 2000[\,]Q^2\left[\frac{m^3}{s}\right]^2$$

UNIT ANALYSIS

$$m = 20[m] - 2000[\,]Q^2\frac{m^6}{s^2}$$

$$m\frac{s^2}{m^6} = [\,]$$

$$\frac{s^2}{m^5} = [\,]$$

$$hp[m] = 20[m] - 2000\left[\frac{s^2}{m^5}\right]Q^2\left[\frac{m^3}{s}\right]^2$$

$$6+(20-2000Q^2)=\frac{\left[\frac{Q}{A_1}\right]^2}{2g}+h_{L_{MINOR}}+h_{L_{MAJOR}} \qquad (3)$$

MINOR and MAJOR LOSSES

$$h_{L_{MINOR}}=(k_{inlet}+k_{exit})\frac{V^2}{2g}=(0.5+1.0)\frac{\left[\frac{Q}{A_1}\right]^2}{2g}$$

$$h_{L_{MAJOR}}=f\frac{L}{D}\frac{V^2}{2g}=f\frac{L}{D}\frac{\left[\frac{Q}{A_1}\right]^2}{2g}$$

COMBINE WITH ENERGY EQUATION

$$6+(20-2000Q^2)=\frac{\left[\frac{Q}{A_1}\right]^2}{2g}+(0.5+1.0)\frac{\left[\frac{Q}{A_1}\right]^2}{2g}+f\frac{L}{D}\frac{\left[\frac{Q}{A_1}\right]^2}{2g}$$

$$6+(20-2000Q^2)=Q^2\frac{(2.5+f\frac{L}{D})}{A_1^2 2g}$$

$$26=Q^2\left(\frac{(2.5+f\frac{L}{D})}{A_1^2 2g}+2000\right)$$

$$\sqrt{\frac{26[m]}{\left(\frac{(2.5+f\frac{L}{D})}{A_1^2 2g[m^4\frac{m}{s^2}]}+2000\left[\frac{s^2}{m^5}\right]\right)}}=Q \rightarrow \text{FRICTION CHARACTERISTIC EQUATION}$$

$$\sqrt{\frac{m}{\frac{s^2}{m^5}}}=\sqrt{\frac{m^6}{s^2}}=\frac{m^3}{s}=Q$$

Trial-n-Error Solution

$$\sqrt{\frac{26[m]}{\left(\frac{(1.5+f\frac{L}{D})}{A_1^2 2g[m^4\frac{m}{s^2}]}+2000\left[\frac{s^2}{m^5}\right]\right)}}=Q \rightarrow \text{FRICTION CHARACTERISTIC EQUATION}$$

1. guess "f^1_{guess}"
2. Solve for Q
3. Solve for V=Q/A
4. Solve for $Re_D = \frac{\rho V D}{\mu}$
5. Check "f^1_{guess}" guess with HAALAND eqn.

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right)$$

$$f^1_{\text{halland}} = \left[\frac{1}{-1.8\log_{10}\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right)}\right]^2$$

6. Compare $f_{halland}$ to f_{guess}
7. If not close(x.xxx), re-iterate
8. NEW $f^2_{guess} = f^1_{\text{halland}} \rightarrow$ GO TO 2

HAALAND EQUATION				
	DENS	Visc	D	
e/D	kg/m3	Pa-s	m	**f (guess)**
0	999	1.12E-03	0.07	**0**
		L	A	
		m	m2	
	m3/s	100	0.00384845	m/s
f	**Q**	**Re**	**f (FALLAND)**	**V**
0	0.060252	9.775E+05	0.0116	15.65607
0.0116	0.020102	3.261E+05	0.0141	5.223338
0.0141	0.018423	2.989E+05	0.0144	4.787201
0.0144	0.018288	2.967E+05	0.0144	4.752007
0.0144	0.018276	2.965E+05	0.0144	4.749025

HAALAND EQUATION				
	DENS	Visc	D	
e/D	kg/m3	Pa-s	m	**f (guess)**
0	999	1.12E-03	0.07	**1**
		L	A	
		m	m2	
	m3/s	100	0.003848451	m/s
f	**Q**	**Re**	**f (FALLAND)**	**V**
1	0.002298	3.728E+04	0.0222	0.59713
0.0222	0.014967	2.428E+05	0.0149	3.889074
0.0149	0.017964	2.914E+05	0.0144	4.667822
0.0144	0.018249	2.961E+05	0.0144	4.741799
0.0144	0.018273	2.965E+05	0.0144	4.748156

HAALLAND EQUATION				
	DENS	Visc	D	
e/D	kg/m3	Pa-s	m	**f (guess)**
0	999	1.12E-03	0.07	**0**
		L	A	
		m	m2	
	m3/s	100	0.00384845	m/s
f	**Q**	**Re**	**f (FALLAND)**	**V**
0	0.060252	9.775E+05	0.0116	15.65607
0.0116	0.020102	3.261E+05	0.0141	5.223338
0.0141	0.018423	2.989E+05	0.0144	4.787201
0.0144	0.018288	2.967E+05	0.0144	4.752007
0.0144	0.018276	2.965E+05	0.0144	4.749025

HAALLAND EQUATION				
	DENS	Visc	D	
e/D	kg/m3	Pa-s	m	**f (guess)**
0	999	1.12E-03	0.07	**1**
		L	A	
		m	m2	
	m3/s	100	0.003848451	m/s
f	**Q**	**Re**	**f (FALLAND)**	**V**
1	0.002298	3.728E+04	0.0222	0.59713
0.0222	0.014967	2.428E+05	0.0149	3.889074
0.0149	0.017964	2.914E+05	0.0144	4.667822
0.0144	0.018249	2.961E+05	0.0144	4.741799
0.0144	0.018273	2.965E+05	0.0144	4.748156

Calculation of head losses/pressure drop

- EXAMPLE: Water to be moved from a large tank in which the air pressure is 20 psig into a large open tank through 2000 ft. of smooth pipe at the rate of 3 ft3/s. The open fluid level is 150 ft. below the level in the closed tank. Determine the pipe diameter.

ASSUME TURBULENT FLOW

Neglect "MINOR" losses due to length of pipe

$$h_1 + \cancel{h_{p\,IDEAL}} = \cancel{h_{T\,IDEAL}} + h_2 + \cancel{h_q} + \sum_{j=1}^{m}\left(\frac{f_j L_j}{D}\frac{V_j^2}{2g}\right)_j + \cancel{\sum_{i=1}^{n}\left(\frac{k_{Lminor} V^2}{2g}\right)_i}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D}\frac{V^2}{2g}$$

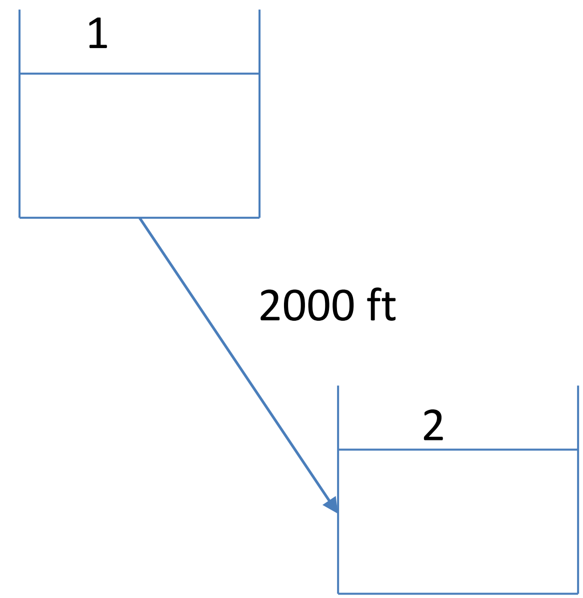
$$V_1 = V_2 = p_2 = 0; z_1 - z_2 = 150\,ft; p1 = 20\,psi \bullet \frac{144 in^2}{ft^2}; V = Q/A = \frac{3\,ft/s}{\frac{\pi D^2}{4}} = \frac{3.82}{D^2}$$

Combine

$$\frac{20\,psi \bullet \frac{144 in^2}{ft^2}}{\gamma = 62.4 lb/ft^3} + 150\,ft = \frac{fL}{D}\frac{1}{2g}\left(\frac{3.82}{D^2}\right)^2$$

or

$$D = 1.18 f^{1/5} \text{ AND } \mathrm{Re} = \frac{VD}{\nu} = \frac{\frac{3.82}{D^2} \bullet D}{\frac{2.34x10^{-5}}{1.94}} = \frac{3.17x10^5}{D}$$



TRIAL-n-ERROR

fguess					
0.02					
f	D	Re	HAALLAND		
0.02000000	0.53962	587,450	0.012698126		
0.01269813	0.492754	643,323	0.012497282		
0.01249728	0.491185	645,378	0.01249032		
0.01249032	0.49113	645,450	0.012490077		
0.01249008	0.491128	645,452	0.012490068		
fguess					
0.1					
f	D	Re	HAALLAND	Velocity	hl
0.10000000	0.74453	425,772	0.013450067	6.891271	198.0892
0.01345007	0.498456	635,964	0.012522452	15.3748	198.0892
0.01252245	0.491383	645,118	0.012491198	15.82062	198.0892
0.01249120	0.491137	645,441	0.012490108	15.83645	198.0892
0.01249011	0.491129	645,452	0.01249007	15.837	198.0892

HALLAND

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11} + \frac{6.9}{\text{Re}}\right); smooth, \varepsilon = 0$$

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\frac{6.9}{\text{Re}}\right)$$

$$D = 1.18 f^{1/5}$$

$$V = Q / A = \frac{3\,ft/s}{\frac{\pi D^2}{4}} = \frac{3.82}{D^2}$$

Power Loss Due to Friction = Minimum Pump Power

$$\dot{W}_{loss} = \gamma Q h_l$$
$$= 62.4\frac{lbf}{ft^3} \bullet 3\frac{ft^3}{s} \bullet 198.09\,ft$$
$$= 3.7x10^4\frac{lbf-ft}{s} \bullet \frac{1HP}{550\frac{lbf-ft}{s}}$$
$$= 67.4HP$$

Multiple Pipe Systems
Series vs Parallel

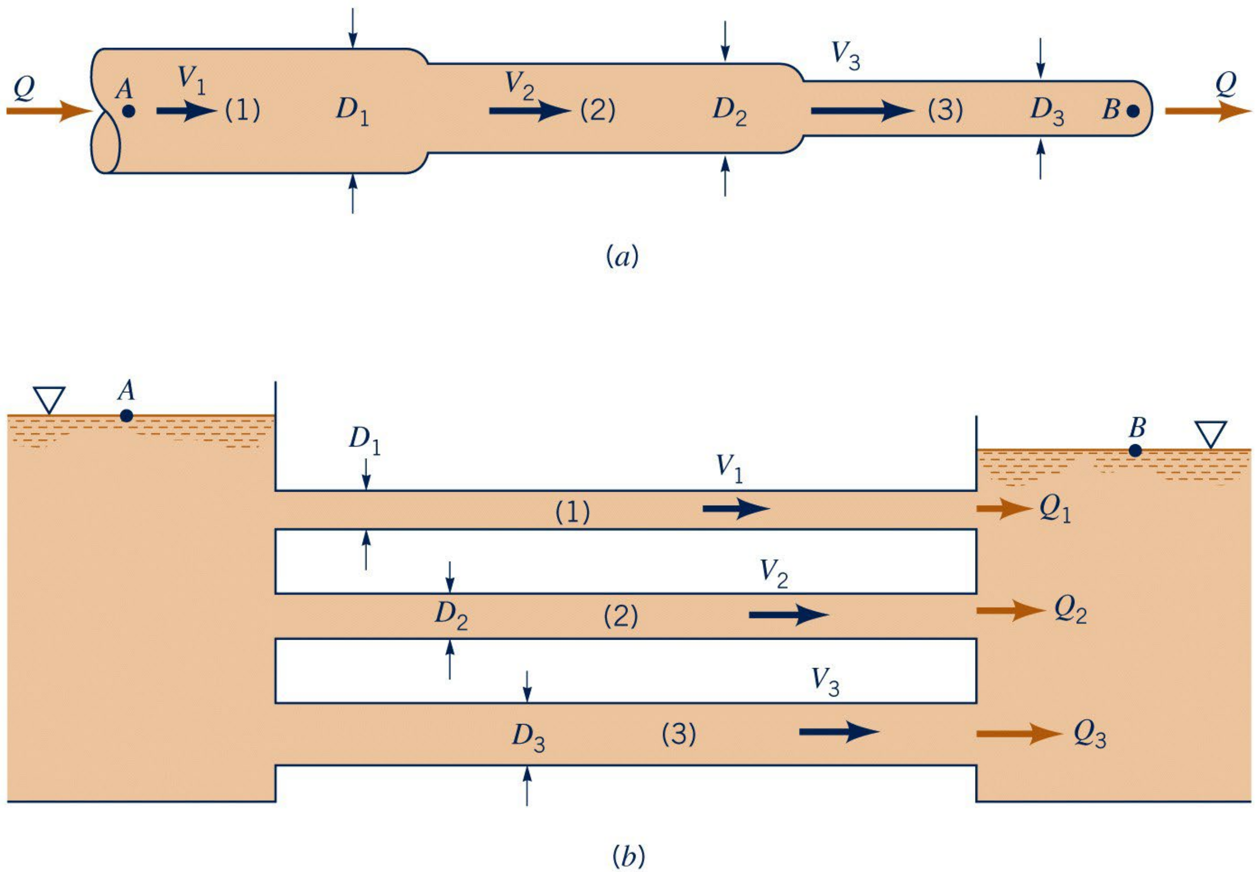


Figure 8.35

Multiple Pipe Systems
Series vs Parallel

SERIES

$$Q_1 = Q_2 = Q_3$$

$$h_{total} = h_1 + h_2 + h_3 = \sum f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum K_{L_i} \frac{V_i^2}{2g}$$

PARALLEL

$$\mathrm{Q}_{total} = Q_1 + Q_2 + Q_3$$

$$h_1 = h_2 = h_3$$

$$f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g}$$