MECH-420 STUDY AID
HEAT DIFFUSION EQUATION
ENERGY CONSERVATION
THERMAL CIRCUITS

You can't BUILD a house if you don't have a blueprint, and you can't build a house if you don't know how to use a hammer or know what is the definition of a miter saw.

NAME____

MECH-420

QUIZ III -- SOLUTION

1. **Definitions (10 Points)**

- **What is the definition of steady state heat transfer?**
  - **After a long term, there is no change in temperature for any value of x,y,z,r, i.e.**

    $$\frac{\partial T}{\partial t} \to 0$$

- **When is it OK to apply an overall control volume and the Conservation of Energy to an object?**
  - **One can apply the conservation of energy for a control volume around an object with the HEAT FLUX is known as EVERY boundary. If only TEMPERATURE at a boundary is known, that is not enough. MUST KNOW FLUX upfront.**

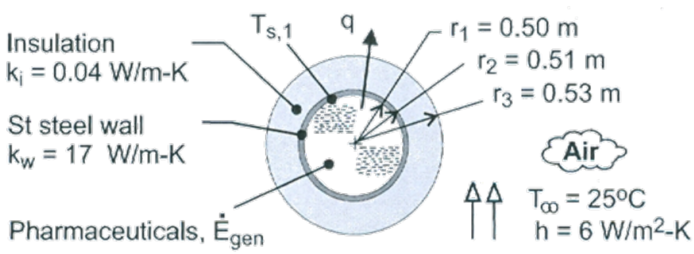
- **What is the definition of the Heat Diffusion Equation and what does it provide?**
  - **Heat Diffusion Equation (HDE) is a 2nd order partial differential equation (PDE) for temperature that when solved and combined with 2 boundary conditions provides the EXACT solution EVERYWHERE within the medium.**

- **Under what conditions can we use a thermal circuit, AND, what is the heat rate in each resistor element of the thermal circuit?**
  - **A thermal circuit can be used for 1D, steady state, homogeneous heat transfer. Used to define the resistance to heat transfer due to CONVECTION or RADIATION heat transfer at a boundary, OR, due to CONDUCTION heat transfer resistance for a solid medium either for a plain wall, cylindrical SHELL, or SPHERICAL shell. A thermal circuit CAN NOT BE used for a solid cylinder OR a solid sphere. Equivalent to an ELECTRICAL series circuit with the CURRENT (HEAT FLOW) being the same in each resistor of the series circuit.**

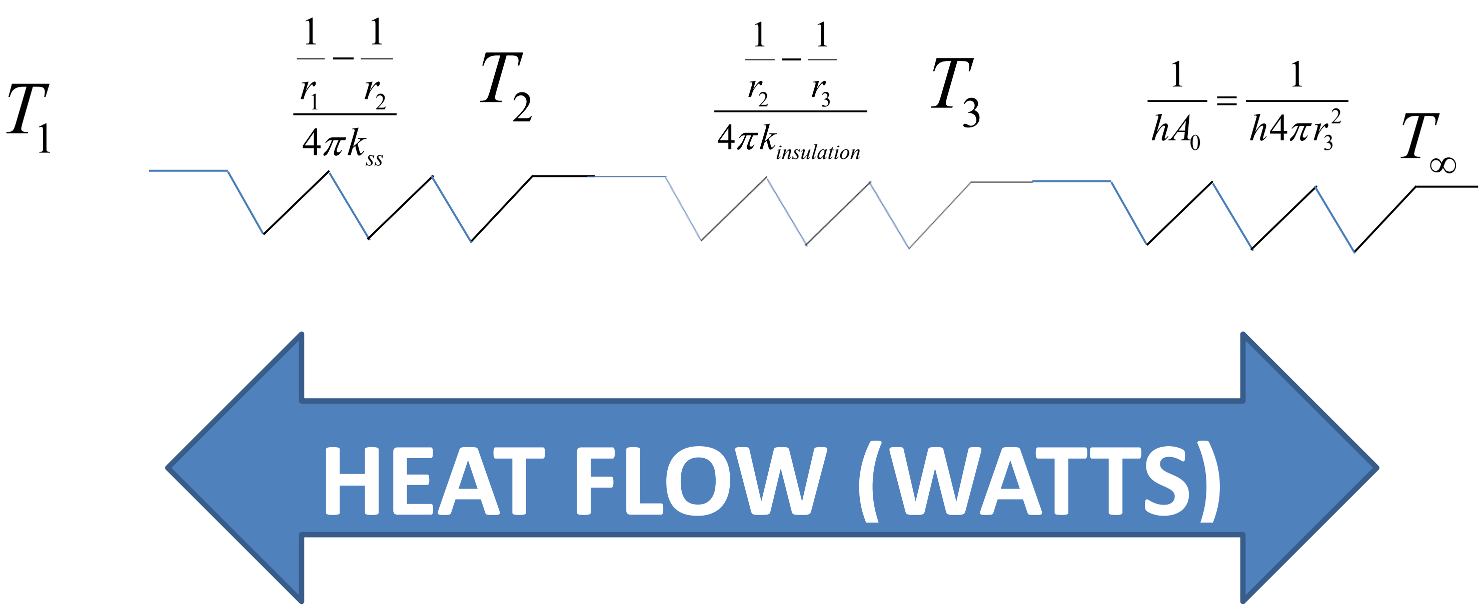
- **What is the steady state center temperature of a homogeneous sphere on the planet TAURUS IV in the DELTA quadrant of the universe with a gravitational pull of only 80% of Earth if the sphere surface temperature is 100C, and no internal heat generation?**
  - **For solid cylinder or sphere, with NO internal heat generation rate, the steady state temperature is a constant throughout, i.e., T_{center} = 100C. ON any planet and ANY universe.**

2. A "spherical" vessel used as a reactor for producing pharmaceuticals has a 10 mm thick stainless steel wall and an inner diameter of 1m, as shown below with insulation. The exterior surface of the vessel is exposed to ambient air with a convective heat transfer coefficient and the thermal conductivity of the pharmaceuticals is 8.55 W/m-K., with an internal heat generation rate of

$$S_{gen}(r) = S_o(2.5r - \left(\frac{r}{r_1}\right)^{2.5}); V = \frac{4}{3}\pi r^3, S_o = 50{,}000\frac{W}{m^3}$$



a) Draw the correct thermal circuit and label all components (5 Points)



b) What is the heat loss from the vessel [W]? (5 Points)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$S_{gen}(r) = S_o(2.5[1/m]r[m] - \left(\frac{r}{r_1}\right)^{2.5}); V = \frac{4}{3}\pi r^3, S_o = 50,000\frac{W}{m^3}$$

$$\dot{E}_{gen} = \dot{E}_{out} = \int_0^{r_1} S_{gen}(r)dV = \int_0^{r_1} S_{gen}(r)\left(4\pi r^2 dr\right)$$

$$\dot{E}_{gen} = \dot{E}_{out} = S_o 4\pi \int_0^{r_1} (2.5r - \left(\frac{r}{r_1}\right)^{2.5})r^2 dr$$

$$\dot{E}_{gen} = \dot{E}_{out} = S_o 4\pi \left[2.5\frac{r^4}{4} - \frac{r^{5.5}}{5.5 r_1^{2.5}}\right]_{0-r_1}$$

$$\dot{E}_{gen} = \dot{E}_{out} = S_o 4\pi \left[2.5\frac{r_1^4}{4} - \frac{r_1^{5.5}}{5.5 r_1^{2.5}}\right] = S_o 4\pi \left[2.5\frac{r_1^4}{4} - \frac{r_1^3}{5.5}\right]$$

$$\dot{E}_{gen} = \dot{E}_{out} = S_o\left[\frac{W}{m^3}\right]4\pi r_1^3[m^3](\frac{2.5[1/m]r_1[m]}{4} - \frac{1}{5.5}) = 10,263W$$

c) What is the exterior vessel surface temperature [C] (5 Points)

$$\dot{E}_{gen} = \dot{E}_{out} = S_o 4\pi r_1^3(\frac{2.5r_1}{4} - \frac{1}{5.5}) = 10,263W$$

$$= h(A_3)(T_3 - T_\infty)$$

$$T_3 = \frac{10,263W}{h(A_3 = 4\pi r_3^2)} + T_\infty = 509.6C$$

$$r_3 = 0.53m$$

$$h = 6\frac{W}{m^2 - K}, T_\infty = 25C$$

OR THERMAL CIRCUITS

$$q[\text{W}] = \frac{T_3 - T_\infty}{\sum R_t = \frac{1}{h4\pi r_3^2}} = 10,263W$$

$$T_3 = 10,263W \bullet \frac{1}{h4\pi r_3^2} + T_\infty$$

JUST UNDERSTAND THE LITTLE THINGS THAT ARE EXECUTED OVER AND OVER AND OVER. LEARNING IS NOT ROCKET SCIENCE

d) What is the temperature of the insulation [C] (5 Points)

THERMAL CIRCUITS

$$q[\mathrm{W}] = \frac{T_2 - T_3}{\dfrac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi k_{insulation}}} = 10,263W$$

$$\sum R_t = \frac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi k_{insulation}}$$

$$T_2 = 10,263W \bullet \frac{\frac{1}{r_2} - \frac{1}{r_3}}{4\pi k_{insulation}} + T_3 = 2,020C$$

$$r_2 = 0.51m, r_3 = 0.53m, k_{ins} = 0.04\frac{W}{m-K}, T_3 = 509.6C$$

e) What is the center temperature of the sphere? (5 Points)

HDE---SPHERE--1D--HOMEGENEOUS

$$S_{gen}(r) = S_o(2.5[1/m]r[m] - \left(\frac{r}{r_1}\right)^{2.5})$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dT}{dr}) = -\frac{S_{gen}(r)}{k_p} = \frac{S_o(2.5[1/m]r[m] - \left(\frac{r}{r_1}\right)^{2.5})}{k_p}$$

multiple by r^2

$$\frac{d}{dr}(r^2\frac{dT}{dr}) = \frac{S_o(2.5[1/m]r[m] - \left(\frac{r}{r_1}\right)^{2.5})}{k_p} \bullet r^2$$

$$\frac{d}{dr}(r^2\frac{dT}{dr}) = \frac{S_o(2.5[1/m]r^3[m] - \left(\frac{r^{4.5}}{r_1^{2.5}}\right))}{k_p}$$

INTEGRATE #1

$$r^2\frac{dT}{dr} = \frac{S_o(2.5\frac{r^4}{4} - \left(\frac{r^{5.5}}{5.5r_1^{2.5}}\right))}{k_p} + C_1$$

$\div r^2$

$$\frac{dT}{dr} = \frac{S_o(2.5\frac{r^2}{4} - \left(\frac{r^{3.5}}{5.5r_1^{2.5}}\right))}{k_p} + \frac{C_1}{r^2}$$

INTEGRATE #2

$$T(r) = \frac{S_o(2.5\frac{r^3}{12} - \left(\frac{r^{4.5}}{24.75r_1^{2.5}}\right))}{k_p} - \frac{C_1}{r} + C_2$$

MOST GENERAL SOLUTION

$$T(r)=\frac{S_o\left[\frac{W}{m^3}\right](2.5\left[\frac{1}{m}\right]\frac{r^3}{12}[m^3]-\left(\frac{r^{4.5}}{24.75r_1^{2.5}}\right)[m^2])}{k_p\left[\frac{W}{m-K}\right]}-\frac{C_1}{r}+C_2[K]$$

$0 \le r \le r_1$

MOST GENERAL SOLUTION

TO FIND EXAC SOLUTION, NEED BOUNDARY CONDITION

#1, T(r=0) $\rightarrow$ FINITE $\rightarrow$ SOLID SPHERE

#2, $T(r=r_1) = T_1$

$$\#1, T(r=0) \rightarrow FINITE : C_1 = 0$$

$$T(r=r_1) = T_1 = \frac{S_o(2.5\frac{r_1^3}{12}-\left(\frac{r_1^{4.5}}{24.75r_1^{2.5}}\right))}{k_p}+C_2$$

$$C_2 = T_1 - \frac{S_o(2.5\frac{r_1^3}{12}-\left(\frac{r_1^{4.5}}{24.75r_1^{2.5}}\right))}{k_p}$$

EXACT SOLUTION

$$T(r)=T_1+\frac{S_o(\frac{2.5}{12}\left[\frac{1}{m}\right](r^3-r_1^3)-\frac{1}{24.75r_1^{2.5}}\left(r^{4.5}-r_1^{4.5}\right))}{k_p}$$

CENTER TEMPERATURE

r=0

$$T(r=0)=T_1+\frac{S_o(\frac{2.5}{12}(0^2-r_1^3)-\frac{1}{24.75r_1^{2.5}}\left(0^{4.5}-r_1^{4.5}\right))}{k_p}$$

$$T_{center}(r=0)=T_1+\frac{S_o\left[\frac{\cancel{W}}{m^3}\right](\frac{2.5}{12}\left[\frac{1}{m}\right](-r_1^3[m^3])+\frac{1}{24.75r_1^{2.5}[m^{2.5}]}\left(r_1^{4.5}\right))}{k_p\left[\frac{\cancel{W}}{m-K}\right]}$$

HOW DO YOU FIND T1?

$$q = \frac{T_1 - T_2}{\frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k_{ss}}} = 10,263W$$

$$T_1 = 10,263W \bullet \frac{\left[\frac{1}{r_1} - \frac{1}{r_2}\right]\frac{1}{m}}{4\pi k_{ss}\frac{W}{m-K}} + T_2 = 2,210C \text{ (A Bit HOT)}$$

$$k_{ss} = 17\frac{W}{m-K}, r_1 = 0.50m, r_2 = 0.51m, T_2 = 2,020C$$