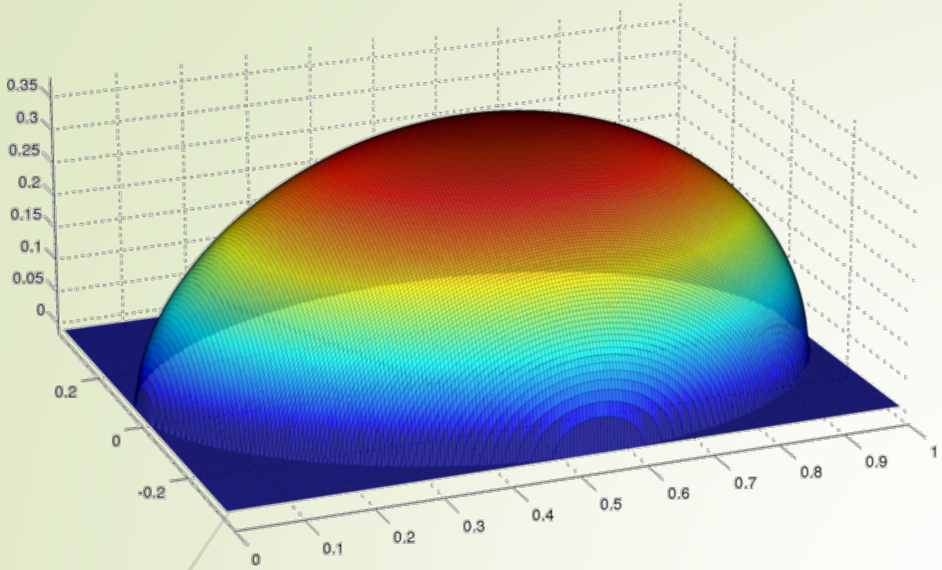

HDE STUDY AID
MECH-420 HEAT TRANSFER

Parametric Thinking & The Real World

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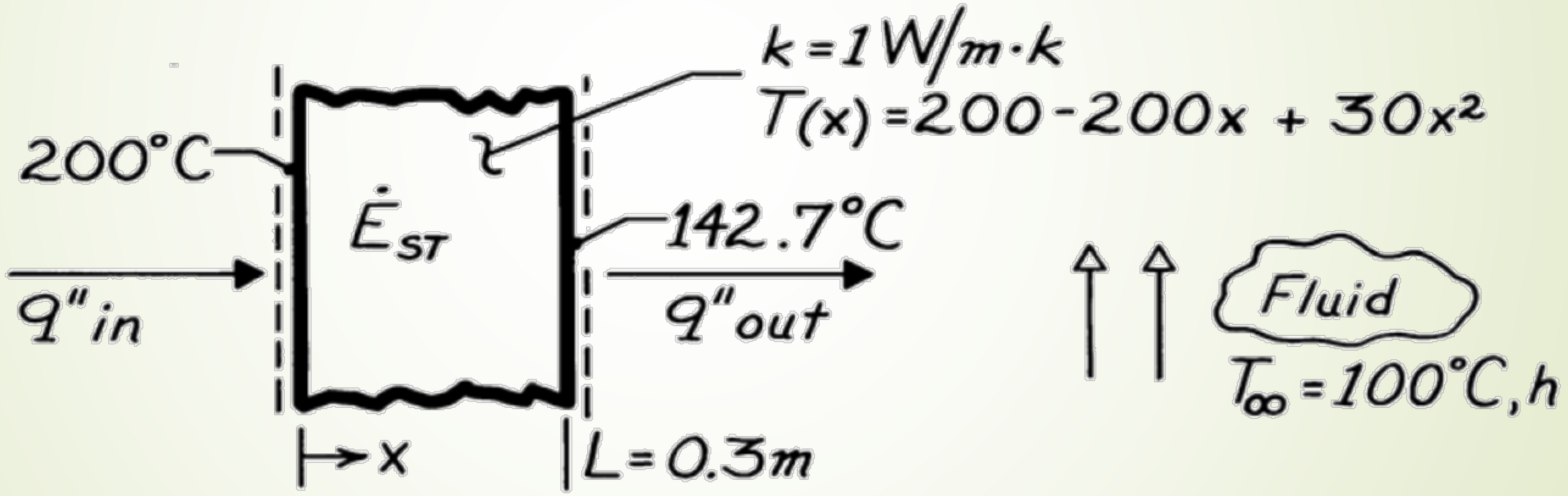
Purpose

- Provide an example of the solution to the Heat Diffusion Equation and to demonstrate concepts from Chapter One and Two.
- Show how heat transfer modeling is used to determine thermal performance of engineering systems.
- Show how when engineering, science and a little basic math comes together, the result highlights the behavior of the real world and engineering becomes FUN.

The temperature distribution accross a wall 0.3m thick (k=1 Wm-K) at steady state is: $T(x)=a+bx+cx^2$

where T is in C, x in meters: a=200C,b=-200C/m, and c=30/m^2



On a unit suface area basis, determine the rate of heat transfer into and out of the wall: (once T(x) is known, I can always find q(x))

$$q(x)'' = -k \frac{dT}{dx}_{x=X^*} = -k(b+2cx)\frac{W}{m^2}$$

$$q(x=0) = -kb = -1\frac{W}{m-K} \bullet (-200\frac{{}^0C}{m}) = 200\frac{W}{m^2}$$

$$q(x=L) = -k(b+2cL) = -k\frac{W}{m-K}(-200\frac{{}^0C}{m} + 2 \bullet 30\frac{{}^0C}{m^2} \bullet 0.3m)$$

$$= 182\frac{W}{m^2}$$

What is the internal heat generation rate: $\dot{S}_{gen}\ \dfrac{W}{m^3}$

From an overall wall control volume:

$$\dot{S}_{gen}\frac{W}{m^3}=\frac{\dot{E}_{gen}[W]}{\forall m^3}=\frac{\left[q''_{x-out}-q''_{x-in}\right]}{WIDTH}\left[\frac{W}{AREA}\right]=\frac{(182-200)W/m^2}{WIDTH}$$

$$=-18\frac{W/m^2}{UNIT\ \text{WIDTH}}\left[\frac{W}{m^3}\right]\rightarrow \text{HEAT SINK}$$

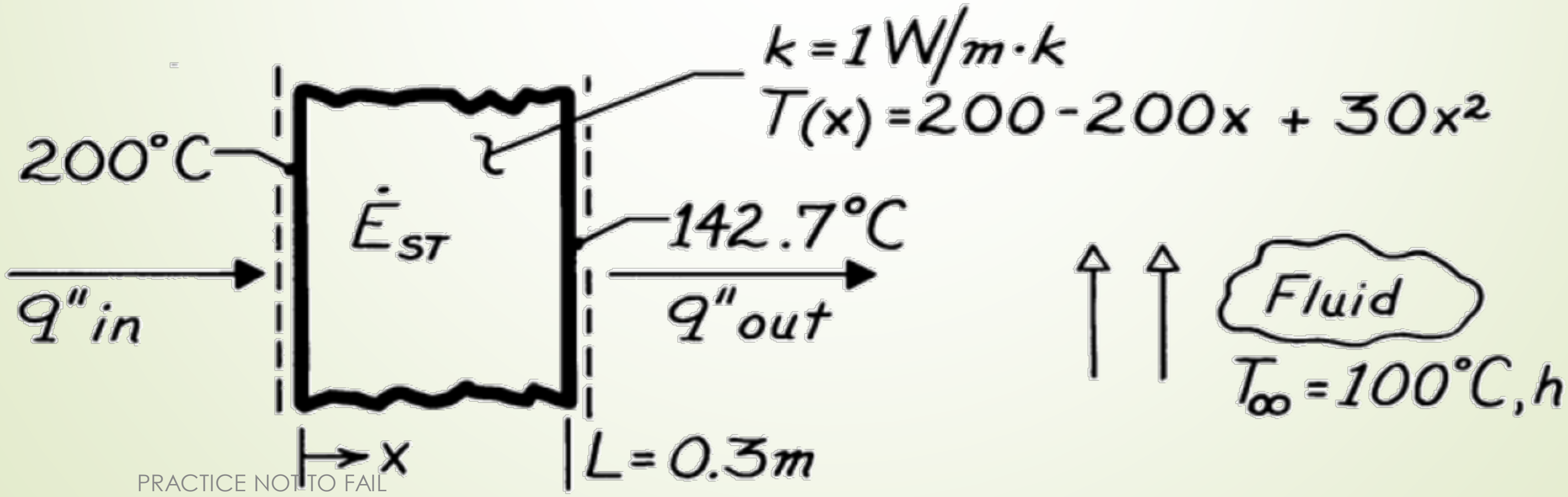
If the cold surface is exposed to 100C, determine the convective heat transfer coefficient.

Newton's Law of Cooling

$$q''(x=L)=h(T_s-T_\infty)$$

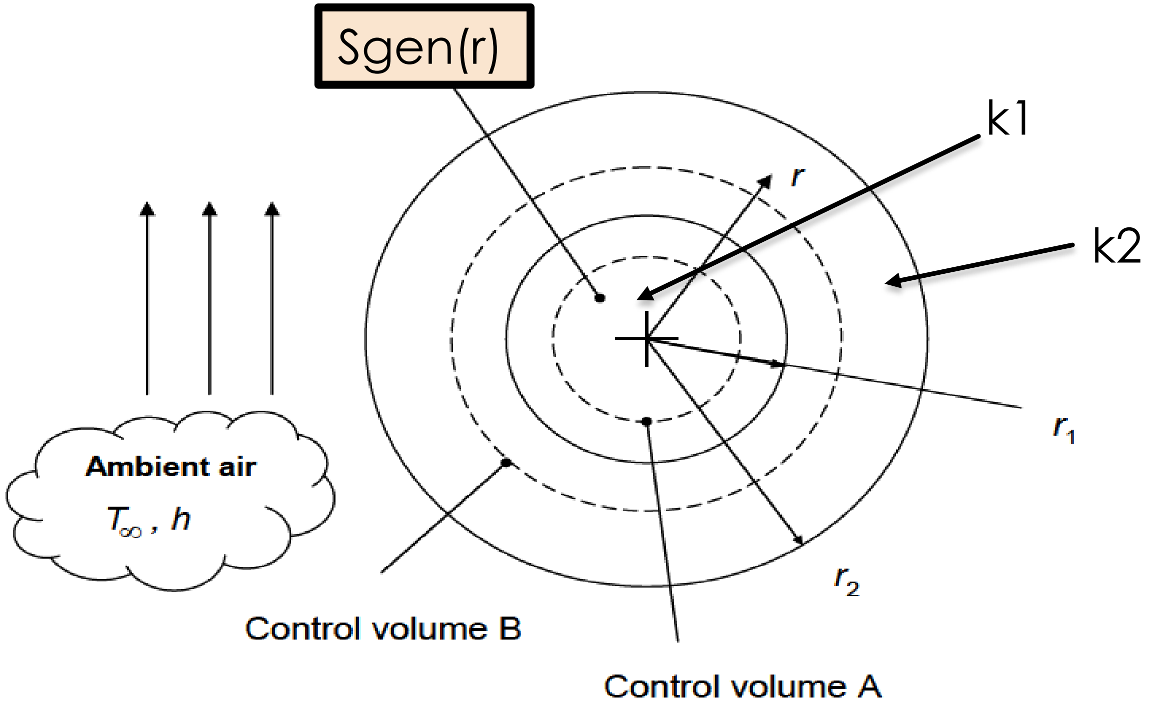
$$T_s=T(x=L)=a+bL+cL^2=142.7C$$

$$h=\frac{q''(x=L)}{(T_s-T_\infty)}=\frac{182\frac{W}{m^2}}{(142.7-100)K}=4.26\frac{W}{m^2-K}$$



A spherical nuclear containment vessel of radius "$r_1 = 1m$" experiences an internal heat generation rate of $S_{gen}(r) = S_0\left(\frac{r}{r_1}\right)^{3/2} \frac{W}{m^3} \rightarrow 0 \le r \le r_1$

If the thermal conductivity is "k_1 & k_2" as shown, determine expression for q(r), $0 \le r \le r_1$. Assume steady state and homegeneous medium.



1D HDE SPHERE $\rightarrow$

$$\frac{1}{r^2}\frac{d}{dr}(k_r r^2 \frac{dT}{dr}) + \dot{S}_{gen}(r,t) = \rho c_p \frac{dT}{dt} = 0$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \frac{dT}{dr}) = -\frac{\dot{S}_{gen}(r)}{k_r} = -\frac{S_0\left(\frac{r}{r_1}\right)^{3/2}}{k_r}$$

multiple by r^2

$$\frac{d}{dr}(r^2 \frac{dT}{dr}) = -\frac{S_0\left(\frac{r}{r_1}\right)^{3/2} r^2}{k_r} = -\frac{S_0}{k_r}\frac{r^{\frac{3}{2}+2}}{r_1^{3/2}} = -\frac{S_0}{k_r}\frac{r^{3.5}}{r_1^{3/2}}$$

INTEGRATE

$$r^2\frac{dT}{dr} = -\frac{S_0}{4.5k_r}\frac{r^{4.5}}{r_1^{3/2}} + C_1$$

DIVIDE by r^2

$$\frac{dT}{dr} = -\frac{S_0}{4.5k_r}\frac{r^{4.5}}{r_1^{3/2}}\frac{1}{r^2} + \frac{C_1}{r^2} = -\frac{S_0}{4.5k_r}\frac{r^{2.5}}{r_1^{3/2}} + \frac{C_1}{r^2}$$

INTEGRATE

$$T(r) = -\frac{S_0}{4.5 \bullet 3.5k_r}\frac{r^{3.5}}{r_1^{3/2}} - \frac{C_1}{r} + C_2 \rightarrow \text{MOST GENERAL SOLUTION}$$

$$q''(r)\frac{W}{m^2} = -k_r\frac{dT}{dr}$$

$$\frac{dT}{dr} = -\frac{S_0}{4.5k_r}\frac{r^{2.5}}{r_1^{3/2}} + \frac{C_1}{r^2}$$

$$q''(r)\frac{W}{m^2} = -k_r\left[-\frac{S_0}{4.5k_r}\frac{r^{2.5}}{r_1^{3/2}} + \frac{C_1}{r^2}\right]$$

For Solid SPHERE, T(r=0) must exist and be finite. $\rightarrow C_1 = 0$

$$q''(r)\frac{W}{m^2} = \left[\frac{S_0}{4.5}\frac{r^{2.5}}{r_1^{3/2}}\right] \rightarrow \frac{W}{m^3}\frac{m^{2.5}}{m^{1.5}} = \frac{W}{m^3}m = \frac{W}{m^2} \rightarrow \text{LIFE IS GOOD!!}$$

The nuclear containment vessel transfers heat by convection at "$r_2 = 2m$" to air at $T_\infty = 30C$, and h=200 $\frac{W}{m^2-K}$. If $S_0 = 50kW/m^3, k_2 = 2\frac{W}{m-K}$, determine the outer steady state suface temperature at $T(r=r_2)$.

Seeking surface temperature and know flux at boundries, consider 1st law applied to an overall control volume.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$\dot{E}_{in} = 0$$

$$\dot{E}_{out} = hA_s(T_s - T_\infty)$$

$$\dot{E}_{gen} = \int \dot{S}_{gen}(r)d\forall, \rightarrow \forall_{sphere} = \frac{4}{3}\pi r^3$$

$$d\forall_{sphere} = 4\pi r^2 dr$$

$$\dot{E}_{gen} = \int_0^{r_1} S_0\left(\frac{r}{r_1}\right)^{3/2} 4\pi r^2 dr = \left[\frac{4\pi S_0}{r_1^{3/2}}\frac{r^{4.5}}{4.5}\right]_{0-r_1} = \frac{4\pi S_0}{4.5}\frac{r_1^{4.5}}{r_1^{1.5}}$$

$$= \frac{4\pi S_0\left[\frac{W}{m^3}\right]}{4.5} r_1^3 \left[m^3\right]$$

$$\dot{E}_{gen} = 139.6kW$$

$$\dot{E}_{gen} = \frac{4\pi S_0}{4.5} r_1^3 = \dot{E}_{out} = hA_s(T_s - T_\infty)$$

$$T_s = \frac{\frac{4\pi S_0}{4.5} r_1^3}{hA_s} + T_\infty \rightarrow A_s = 4\pi r_2^2$$

$$T_s = \frac{\frac{4\pi S_0}{4.5} r_1^3 [W]}{h\left[\frac{W}{m^2-K}\right]4\pi r_2^2} + T_\infty \rightarrow \frac{W}{\frac{W-m^2}{m^2-K}} + K = 43.9C$$

Determine the interafce Temperature at $T(r=r_1)$

Must interface two HDE solutions. (General Sol's is "general" until BC's)

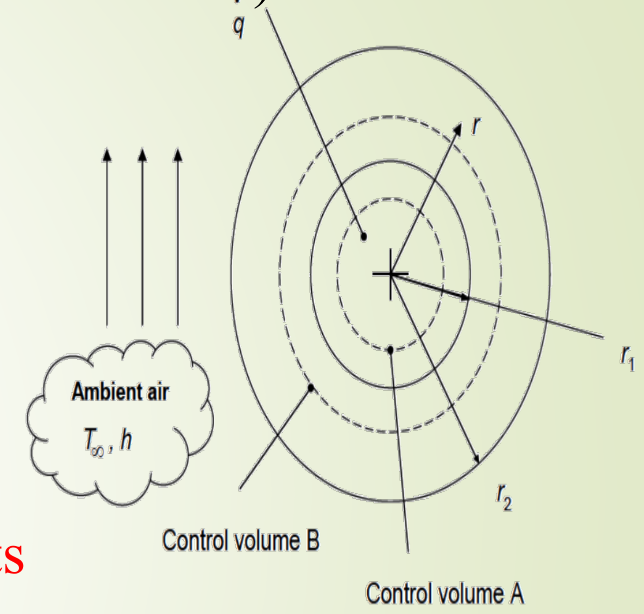
$0 \le r \le r_1$

$$T(r) = -\frac{S_0}{4.5 \bullet 3.5 k_1} \frac{r^{3.5}}{r_1^{3/2}} - \frac{C_1}{r} + C_2 \rightarrow S_{gen} \text{ only in this part.}$$

$$C_1 = 0 \rightarrow T(r=0) = \textit{finite}$$

$r_1 \le r \le r_2$

$$T(r) = -\frac{D_1}{r} + D_2 \rightarrow \text{Different medium needs different constants}$$



3 Constants $\rightarrow C_2, D_1, D_2$ to find from BOUNDARY CONDITIONS

BC#1: $T(r=r_1) = T(r=r_1) \rightarrow$ TEMPERATURE CONTINUITY

$BC\#2: T(r=r_2) = T_s \rightarrow$ Overall Energy Balance

$$BC\#3: -k_1 \frac{dT}{dr} = -k_2 \frac{dT}{dr} \rightarrow \text{FLUX CONTINUITY}$$

BC#1: T(r=r_1) $= T(r = r_1) \rightarrow$ TEMPERATURE CONTINUITY

$$-\frac{S_0}{4.5 \bullet 3.5k_1}\frac{r_1^{3.5}}{r_1^{3/2}} + C_2 = -\frac{D_1}{r_1} + D_2 \rightarrow \text{let } \beta = \frac{S_0}{4.5 \bullet 3.5k_1} r_1^2 \, [K]$$

$$\left\{ C_2 = -\frac{D_1}{r_1} + D_2 + \beta : (1) \right\}$$

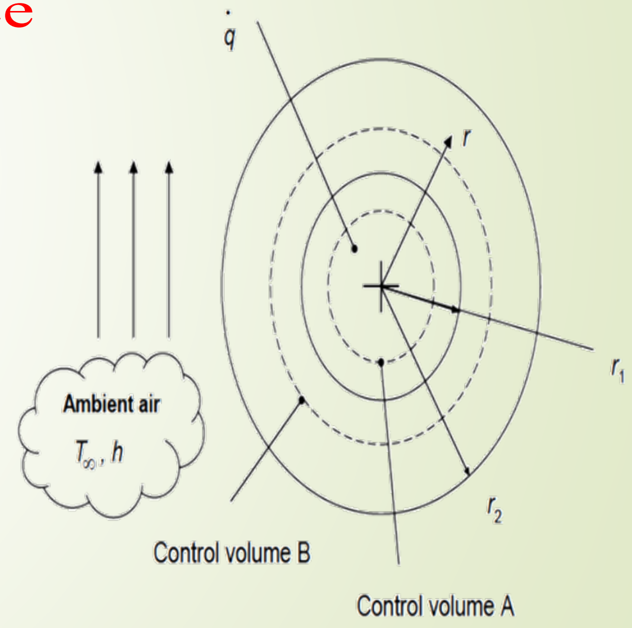
$BC\#2 : T(r = r_2) = T_s \rightarrow$ Overall Energy Balance

$$-\frac{D_1}{r_2} + D_2 = T_s$$

$$\left\{ D_2 = T_s + \frac{D_1}{r_2} : (2) \right\} \rightarrow \text{Insert into (1)}$$

$$C_2 = -\frac{D_1}{r_1} + T_s + \frac{D_1}{r_2} + \beta$$

$$\left\{ C_2 = D_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right) + T_s + \beta : (3) \right\}$$



$$BC\#3: -k_1 \frac{dT}{dr}_{r=r_1} = -k_2 \frac{dT}{dr}_{r=r_1}$$ → FLUX CONTINUITY

$$\frac{dT}{dr} = -\frac{S_0}{4.5k_1}\frac{r^{2.5}}{r_1^{3/2}} + \frac{C_1}{r^2}$$

$$k_1 \frac{S_0}{4.5k_1}\frac{r_1^{2.5}}{r_1^{3/2}} = -k_2 \frac{D_1}{r_1^2} \rightarrow \left[D_1 = \frac{-k_1}{k_2} 3.5\beta r_1^2 \left[K - m^2\right] \right]$$

INSERT INTO (3)

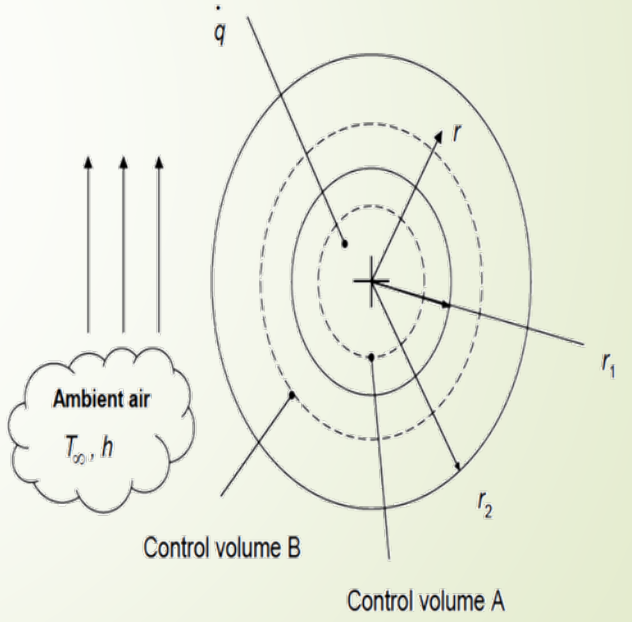
$$\left\{ C_2 = D_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right) + \beta : (3) \right\}$$

$$\left[C_2 = -\frac{k_1}{k_2} 3.5\beta r_1^2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right) + \beta \right] [K]$$

INSERT INTO (1)

$$\left\{ D_2 = T_s + \frac{D_1}{r_2} : (2) \right\} [K]$$

$$\left[D_2 = \left[T_s + \frac{-\frac{k_1}{k_2} 3.5\beta r_1^2}{r_2} \right] \right] [K]$$

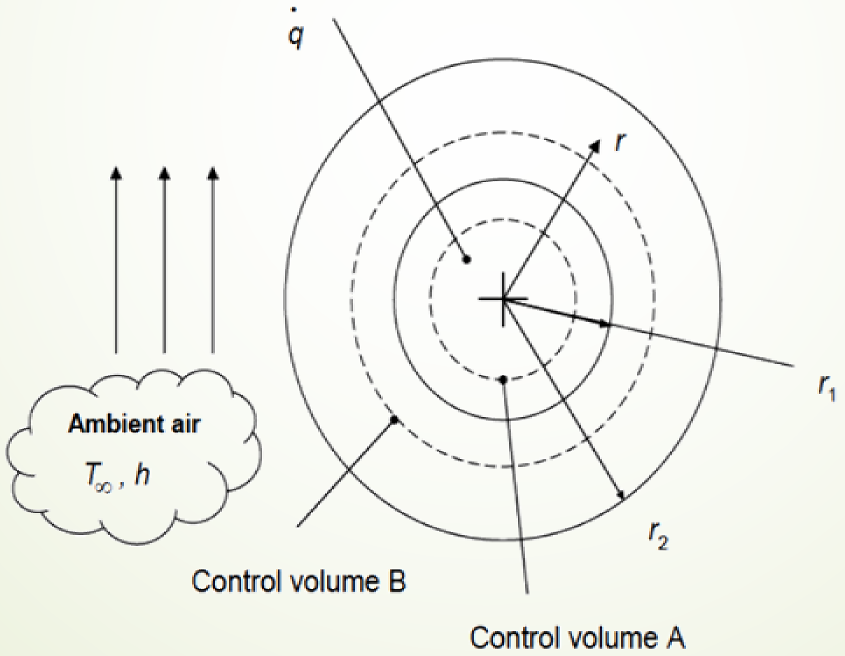


m	m	W/m-K	W/m-K	W/m3	C	W/m2-K	W	C	K	K-m2	K	K
r1	r2	k1	k2	S0	Tfluid	h	EGEN	Ts	BETA	D1	D2	C2
1	2	400	50	50000	30	20	**139,626.22**	168.89	7.936508	-222.222	57.77778	287.9365

m	C	W/m2	W
r1	**T(r)**	**q"(r)**	**q(r)**
0	287.9365	0	0
0.1	287.934	35.13642	4.4153688
0.2	287.9081	198.7616	99.908392
0.3	287.8191	547.7226	619.4591
0.4	287.6153	1124.365	2260.6688
0.5	287.235	1964.186	6170.6655
0.6	286.6086	3098.387	14016.759
0.7	285.6589	4555.149	28048.405
0.8	284.302	6360.371	51153.097
0.9	282.4477	8538.15	86907.705
1	280	11111.11	139626.22
r2	**T(r)**	**q"(r)**	**q(r)**
1	280	11111.11	**139,626.22**
1.1	259.798	9182.736	**139,626.22**
1.2	242.963	7716.049	**139,626.22**
1.3	228.7179	6574.622	**139,626.22**
1.4	216.5079	5668.934	**139,626.22**
1.5	205.9259	4938.272	**139,626.22**
1.6	196.6667	4340.278	**139,626.22**
1.7	188.4967	3844.675	**139,626.22**
1.8	181.2346	3429.355	**139,626.22**
1.9	174.7368	3077.87	**139,626.22**
2	168.8889	2777.778	**139,626.22**

NOTE: TOTAL HEAT TRANSFER (W) IS CONSTANT IN OUTER INSULATION LAYER: INDEPENDENT OF "r" DUE TO ENERGY CONSERVATION.

BUT HEAT FLUX (W/m2) IS A FUNCTION OF "r".



NOTE:

$$q''_{convection} = h(T_s - T_\infty) = 20\frac{W}{m^2-K}(168.89-30)$$

$$q''_{convection} = 2,778 W/m^2$$

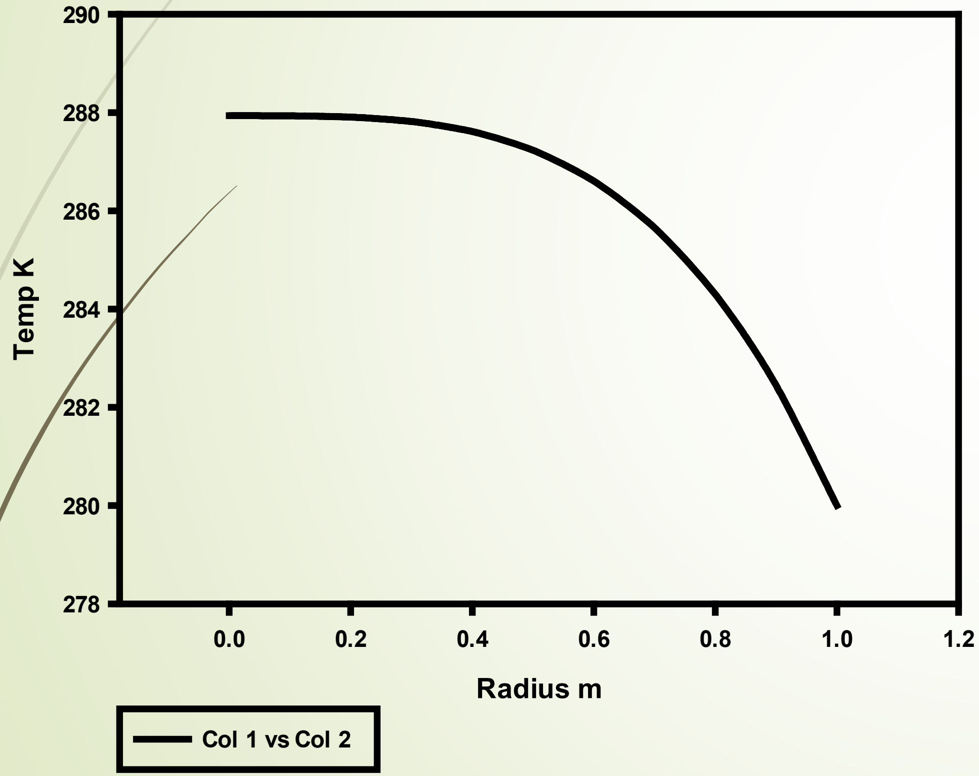
$$q_{convection}[W] = q''_{convection}\left[W/m^2\right] \bullet A_s\left[m^2\right]$$

$$= 2,778 W/m^2 \bullet 4\pi r_2^2$$

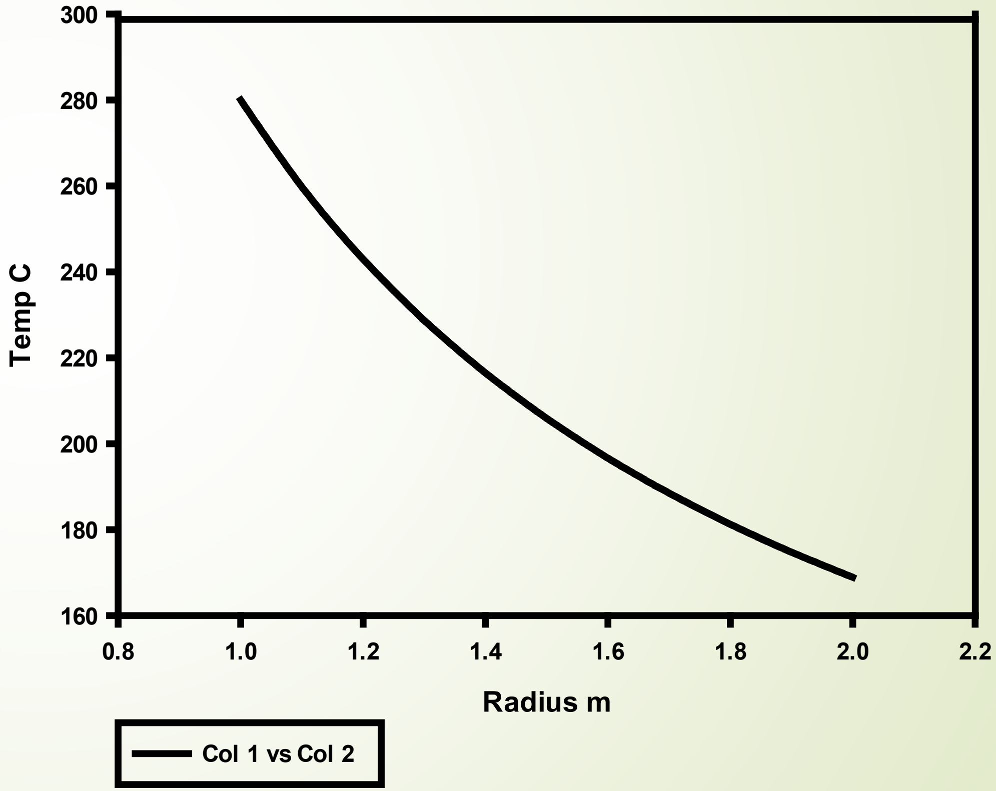
$$= 139,627 W$$

**RADIUS VS. TEMPERATURE
CORE SHELL**

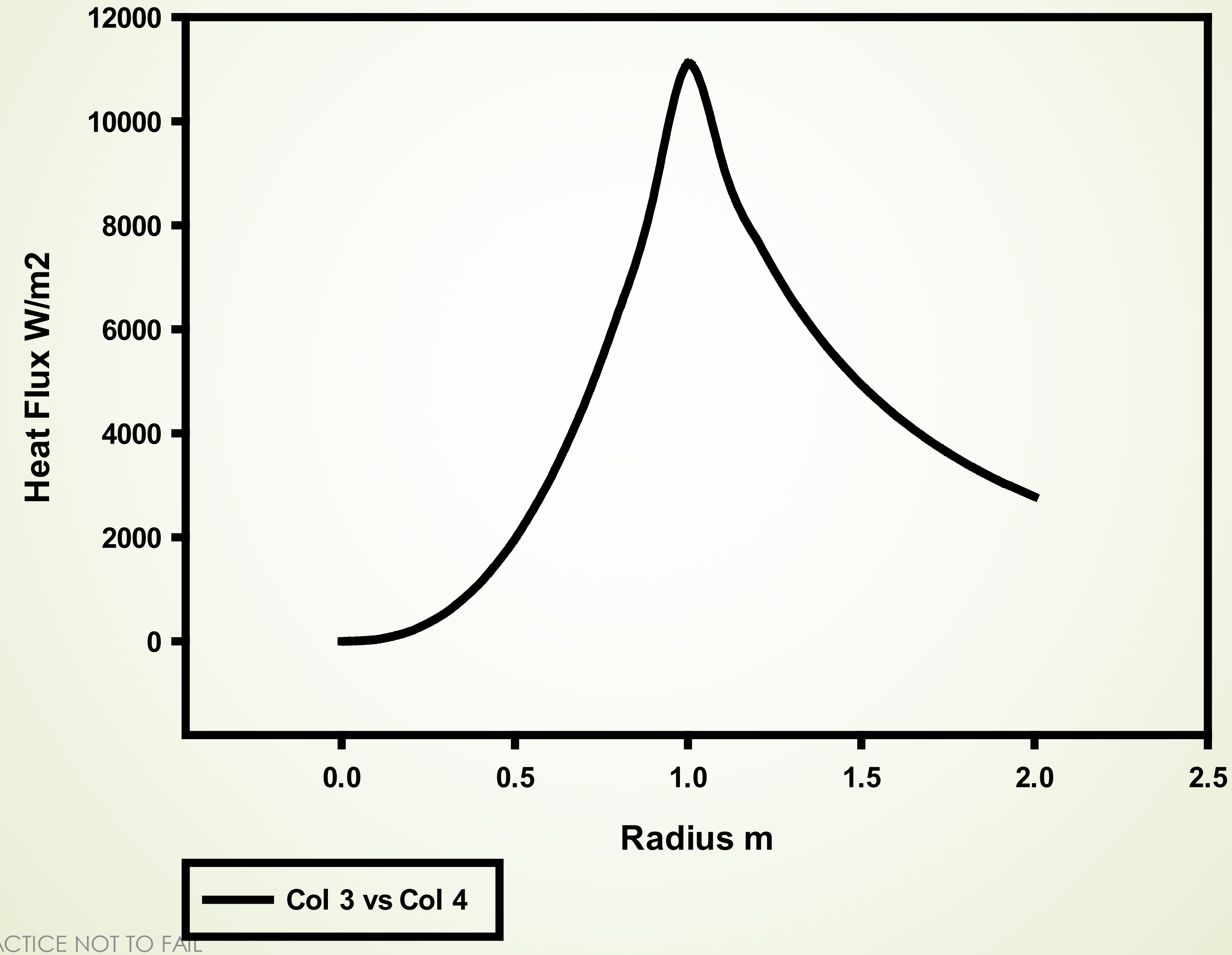
NOTE TEMP GRADIENTS



**RADIUS VS. TEMPERATURE
TER INSULATION**



RADIUS VS HEAT FLUX
NUCLEAR CONTAINMENT VESSEL



W/m2-K	C	C
h	Tcenter	Ts
1	2927	2807
5	705	586
10	427	308
15	334	215
20	288	169
25	260	141
30	242	123
50	205	86
75	186	67
100	177	58
200	163	44
300	158	39
500	154	36
1000	151	33