

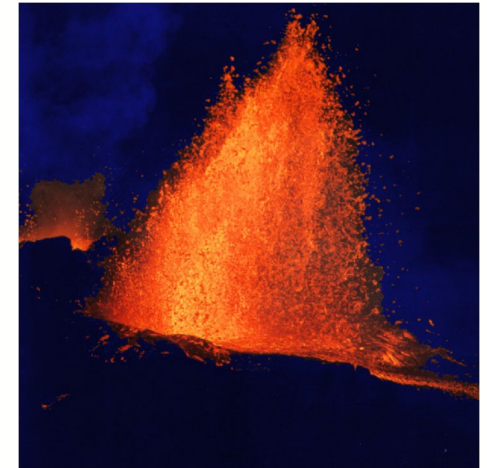
Heat Exchangers Keeps Things Cool

SPACE SHUTTLE TILES

MECH-420 Heat Transfer HEAT EXCHANGERS

Dr. K. J. Berry, P.E.
ASME FELLOW

1



Heat Exchangers (HX)

2

TYPES

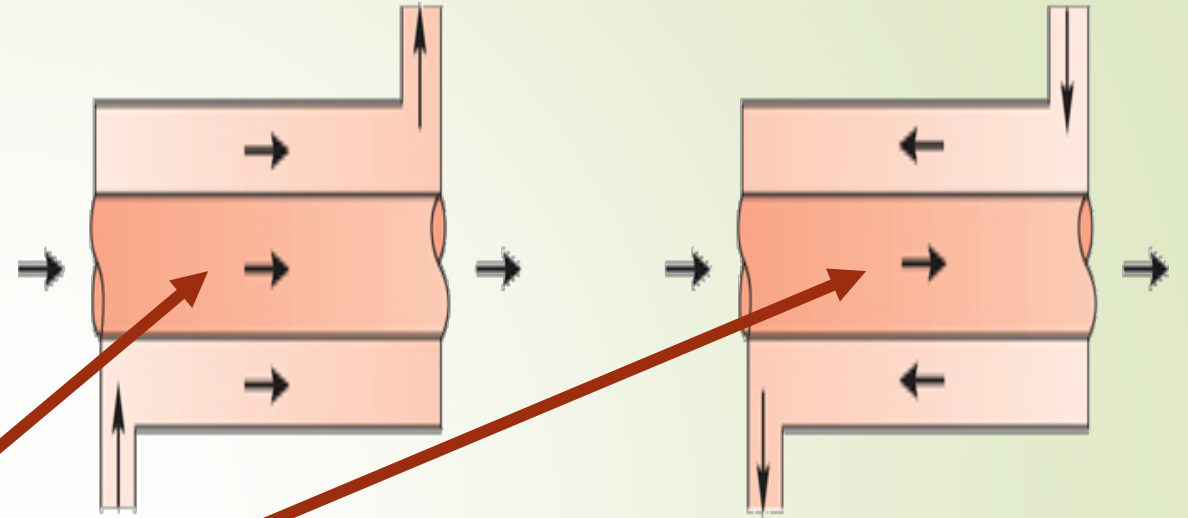
➤ A device used to implement the exchange of heat transfer between TWO fluids that are separated by a solid wall, and flowing at different mass flow rates and temperatures

COMMON HX TYPES

- Double Pipe: Two Concentric pipes or tubes
 - Parallel Flow (hot/cold fluids in same direction)
 - Counter Flow (hot/cold fluids in opposite directions)

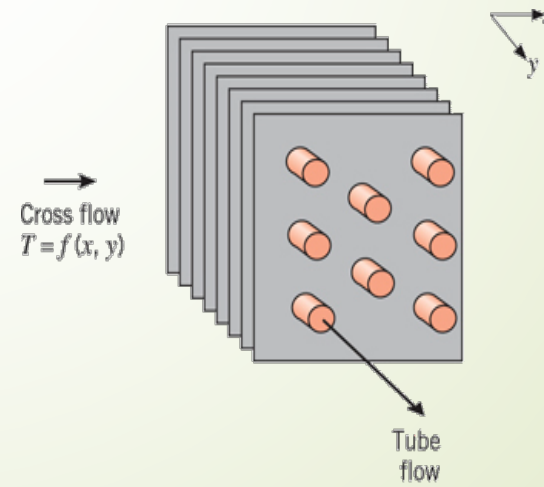
CROSS FLOW HX

- Two FLUIDS path cross each other, usually at right angles

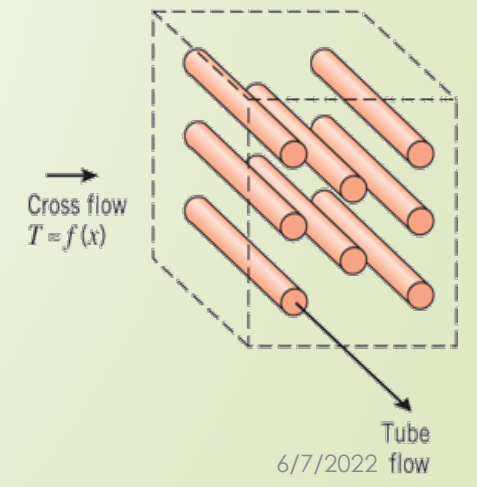


(a)

(b)



(a)



(b)

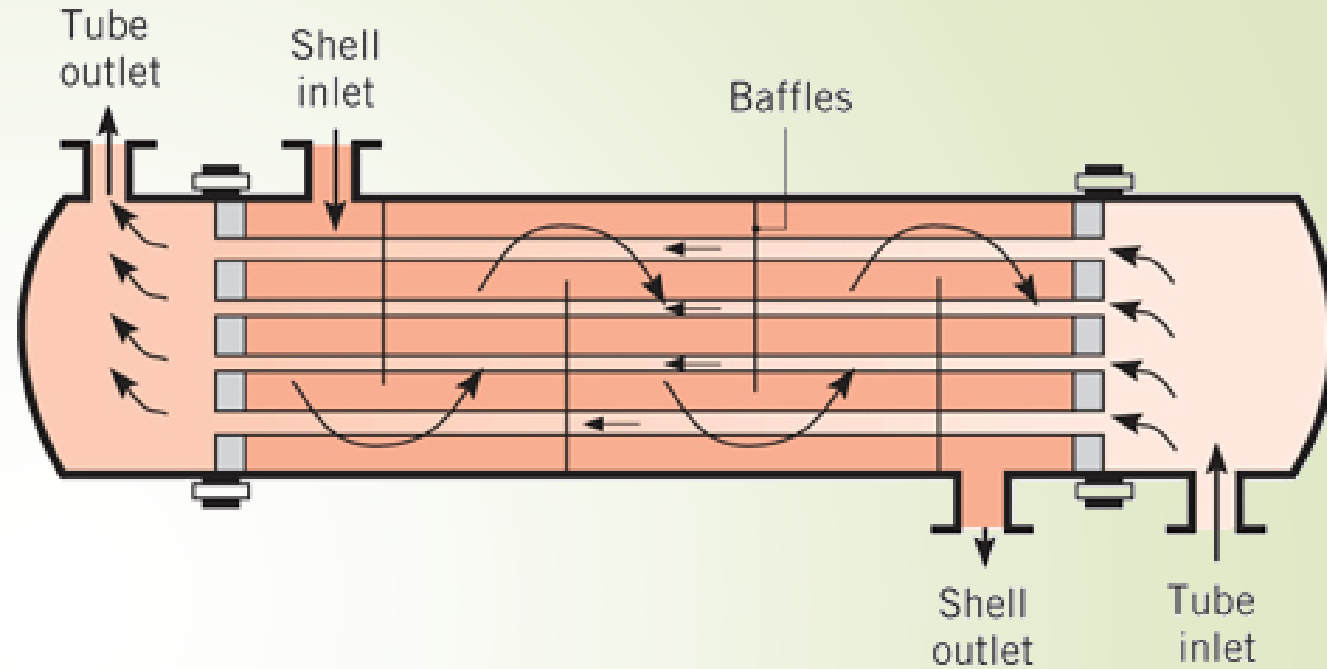
6/7/2022

Heat Exchangers

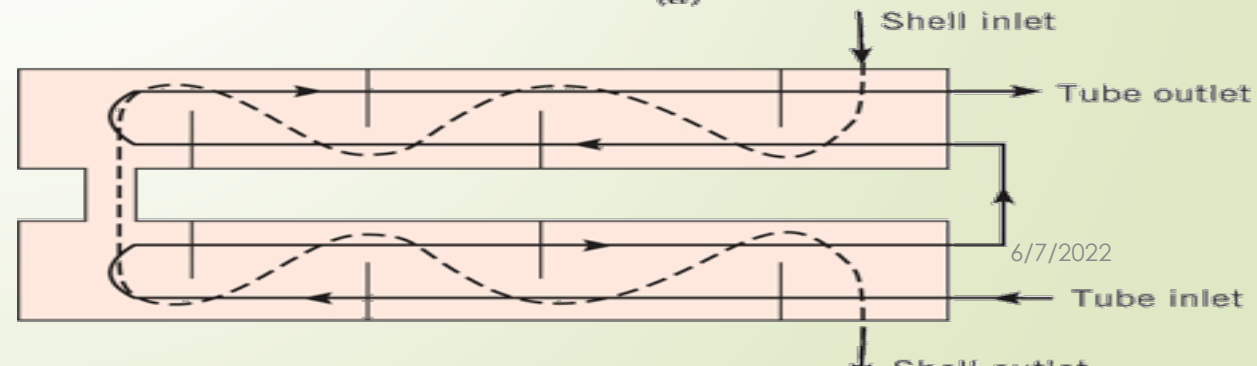
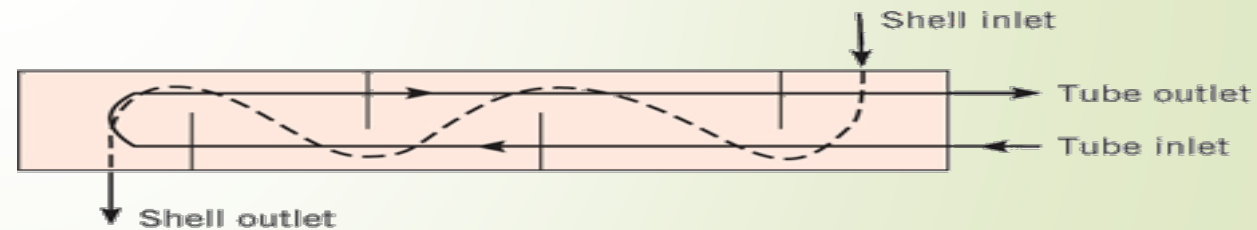
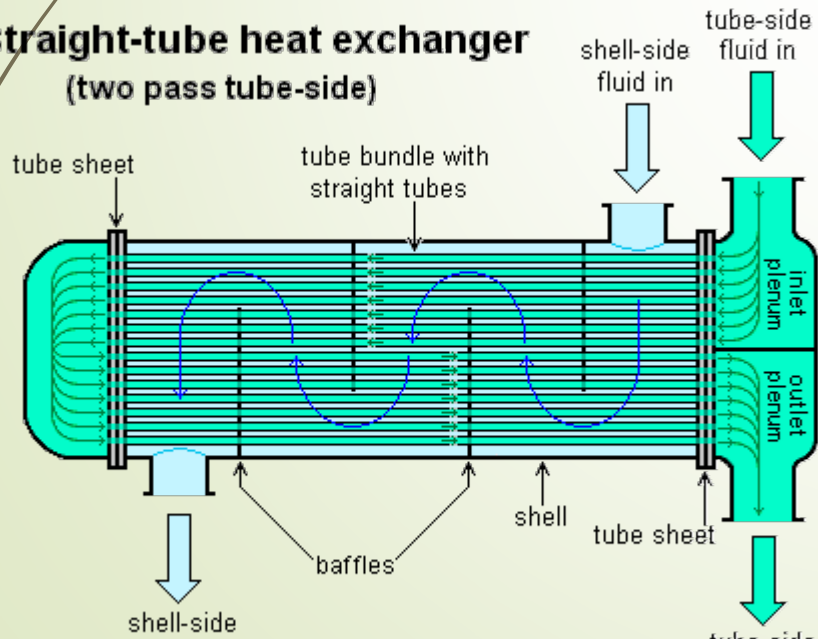
3

Type: Shell-n-Tube

- Shell-n-Tube: A huge outer cylinder, or SHELL, which contains many tubes.
- Can also have multiple shells and MULTIPLE tube passes per shell.

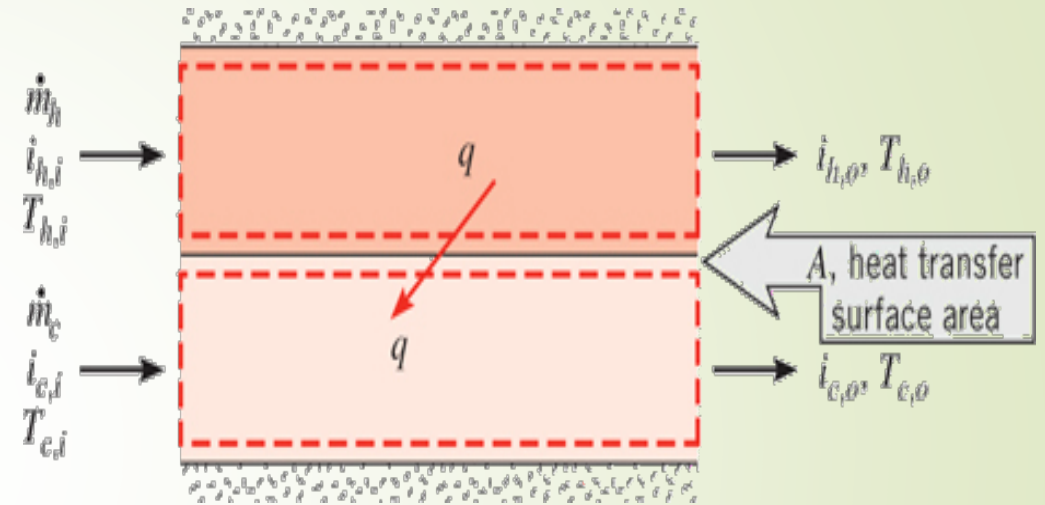


**Straight-tube heat exchanger
(two pass tube-side)**



Overall Heat Transfer Coefficient and Fouling Factors

- An essential part of any heat exchanger analysis is determination of the overall heat transfer coefficient.
- During normal operations, HOT and COLD surfaces are often subject to fouling by fouling impurities, rust formation, or their reactions between the fluid and the wall material.



$$UA = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{h_c A_c} + \frac{R_c'' m^2 - K}{A_c} + R_{t_{CONDUCTION}} + \frac{1}{h_h A_h} + \frac{R_h'' m^2 - K}{A_h}}$$

TABLE 11.1 Representative Fouling Factors [1]

Fluid	R_f'' ($m^2 \cdot K/W$)
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam (nonoil bearing)	0.0001

ANALYSIS: **METHOD #1: PARALLEL-FLOW** LOG MEAN TEMPERATURE DIFFERENCE

Assumptions

- Insulation from surroundings.
- Axial conduction is small
- Potential and kinetic energy changes are small
- Cp and Overall Heat Transfer Coefficient is constant

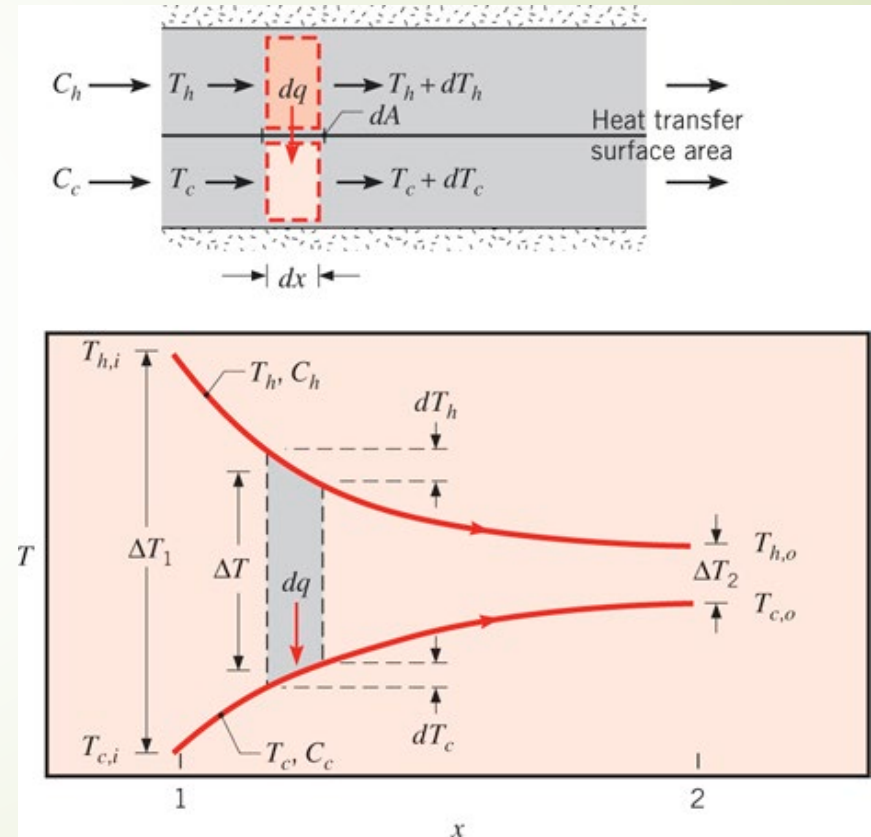
- 3 of 4 temperatures are **KNOWN**.

$$q = UA\Delta T_{LM} = (\dot{m}c_p)_{cold} \Delta T_{cold} = (\dot{m}c_p)_{hot} \Delta T_{hot}$$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

PARALLEL-FLOW HX

$$\Delta T_1 = T_{h,in} - T_{c,in} \quad , \quad \Delta T_2 = T_{h,out} - T_{c,out}$$



ANALYSIS: **METHOD #1: COUNTER-FLOW** LOG MEAN TEMPERATURE DIFFERENCE

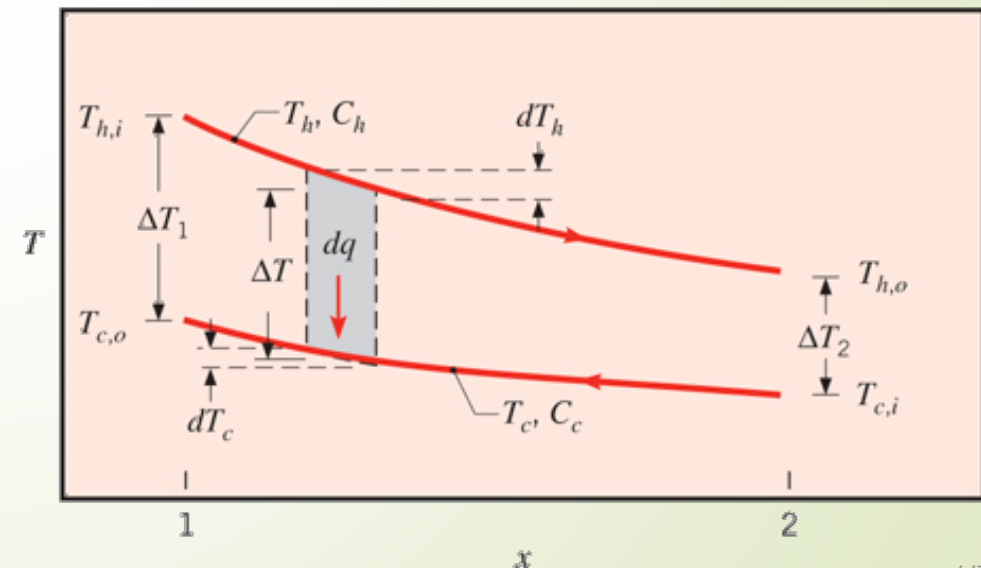
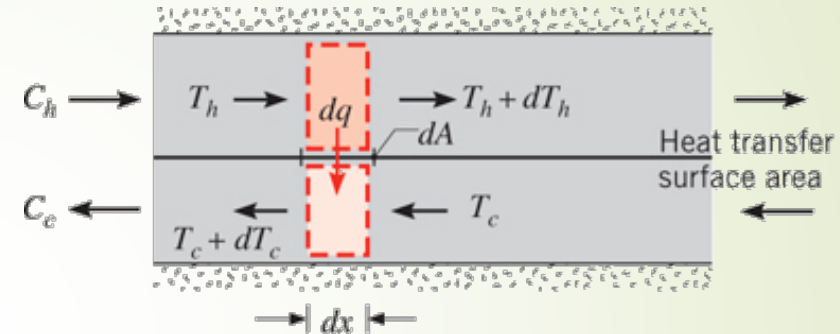
- Assumptions
 - Insulation from surroundings.
 - Axial conduction is small
 - Potential and kinetic energy changes are small
 - Cp and Overall Heat Transfer Coefficient is **constant (may really be a function of Temp)**
- 3 of 4 temperatures are **KNOWN**.

$$q = UA\Delta T_{LM} = (\dot{m}c_p)_{cold} \Delta T_{cold} = (\dot{m}c_p)_{hot} \Delta T_{hot}$$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

COUNTER-FLOW HX

$$\Delta T_1 = T_{h,in} - T_{c,out}, \quad \Delta T_2 = T_{h,out} - T_{c,in}$$

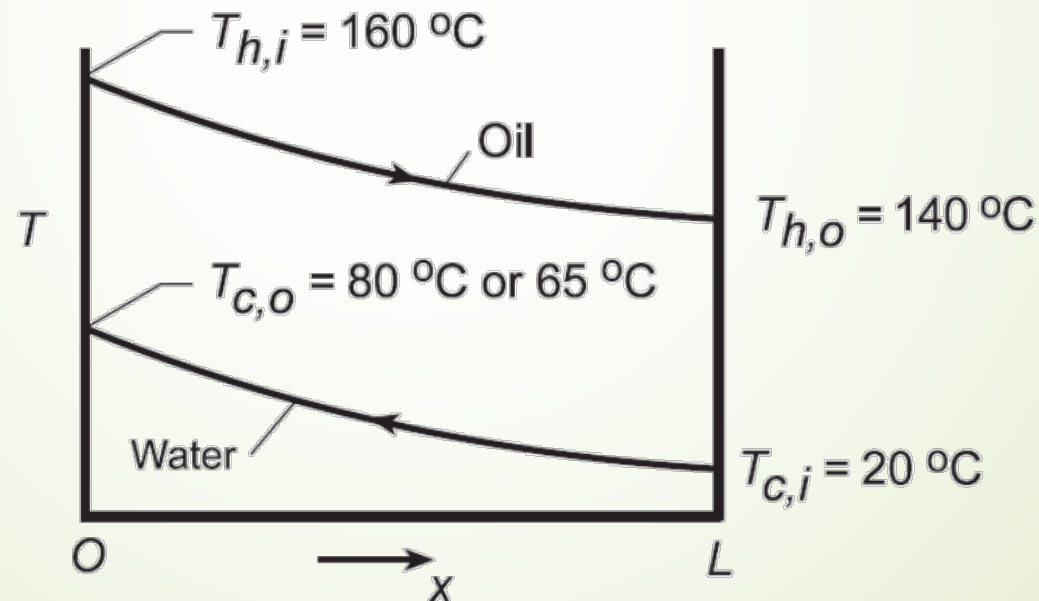


7

Known: Inner $D=0.02$, counterflow, concentric tube heat exchanger, thin wall. $U=500\text{W/m}^2\text{-K}$, $q=3000\text{W}$. Cold outlet temperature after 3 years, ($T_{c,o}=65\text{C}$).

Find; a) L , b) q , hot fluid out, U and FF after three years.

SCHEMATIC:

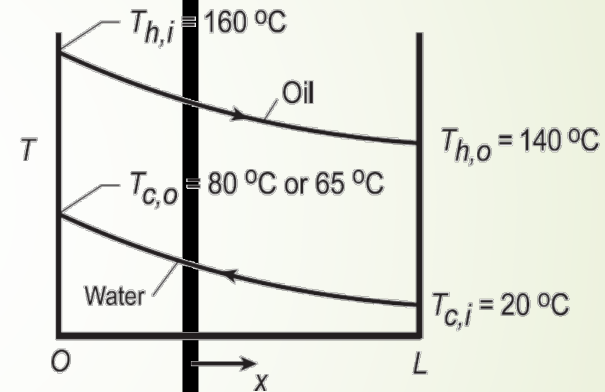


LMTD METHOD

$$q = UA\Delta T_{LM} = (\dot{m}cp)_{cold} \Delta T_{cold} = (\dot{m}cp)_{hot} \Delta T_{hot}$$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} = \frac{80 - 120}{\ln \left[\frac{80}{120} \right]} = 98.7C$$

SCHEMATIC:



COUNTER-FLOW HX

$$\Delta T_1 = T_{h,in} - T_{c,out} \quad , \quad \Delta T_2 = T_{h,out} - T_{c,in}$$

$$A = \pi DL$$

$$L = \frac{q}{(\pi D)U\Delta T_{LM}} = \frac{3000W}{(\pi \times 0.02m)500 \frac{W}{m^2 - K} \times 98.7C} = 0.968m$$

After 3 years, $T_{c,o}$ Drops from 80C to 65C.
Find q and U . (**FOULING COLD SIDE**)

$$q = UA\Delta T_{LM} = (\dot{m}c_p)_{cold} \Delta T_{cold} = (\dot{m}c_p)_{hot} \Delta T_{hot}$$

$$\frac{q_1}{q_3} = \frac{\left[\cancel{(\dot{m}c_p)_{cold}} \Delta T_{cold} \right]_1}{\left[\cancel{(\dot{m}c_p)_{cold}} \Delta T_{cold} \right]_3} = \frac{80 - 20}{65 - 20} = \frac{60C}{45C} = 1.333$$

$$q_3 = \frac{q_1}{1.333} = 2250W$$

$(T_{HOT OUT})_{Year 3}$

$$\frac{q_1}{q_3} = 1.333 = \frac{\left[\cancel{(\dot{m}c_p)_{hot}} \Delta T_{hot} \right]_1}{\left[\cancel{(\dot{m}c_p)_{hot}} (T_{h,i} - T_{h,o}) \right]_3} = \frac{160 - 140}{160 - T_{h,o}} \rightarrow T_{h,o} = 145C$$

$(U)_{YEAR 3}$

$$q_3 = (UA\Delta T_{LM})_3 \rightarrow U_3 = \frac{q_3}{(A\Delta T_{LM})_3} = \frac{q_3}{(\pi DL\Delta T_{LM})_3}$$

$$\Delta T_{LM} = \frac{125 - 95}{\ln \left[\frac{125}{95} \right]} = 109.3C$$

$$U_3 = \frac{q_3}{(\pi DL\Delta T_{LM})_3} = \frac{2250W}{\pi 0.02m \times 0.968m \times (109.3C)} = 338W / m^2 - K$$

FOULING FACTOR: COLD SIDE

$$UA = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{h_c A_c} + \frac{R_c'' \frac{m^2 - K}{W}}{A_c} + R_{t_{CONDUCTION}} + \frac{1}{h_h A_h} + \frac{R_h'' \frac{m^2 - K}{W}}{A_h}}$$

$$R_{t_{CONDUCTION}} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{wall} L} \rightarrow 0 \rightarrow \text{THIN WALL} \rightarrow \ln(1) \rightarrow 0$$

$$(UA)_{cold} = U_c A_c = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{h_c A_c} + \frac{R_c'' \frac{m^2 - K}{W}}{A_c} + \frac{1}{h_h A_h}}$$

$$U_c = \frac{1}{A_c \left(\frac{1}{h_c A_c} + \frac{R_c'' \frac{m^2 - K}{W}}{A_c} + \frac{1}{h_h A_h} \right)} = \frac{1}{\frac{1}{h_c} + \frac{R_c'' \frac{m^2 - K}{W}}{A_c} + \frac{A_c}{h_h A_h}}$$

$$\text{THIN WALL} \rightarrow \frac{A_c}{A_h} = 1$$

$$U_{3c} = \frac{1}{\frac{1}{h_c} + \frac{R_c'' \frac{m^2 - K}{W}}{A_c} + \frac{A_c}{h_h A_h}}$$

$$\frac{1}{U_{3c}} = \frac{1}{h_c} + \frac{R_c'' \frac{m^2 - K}{W}}{A_c} + \frac{1}{h_h}$$

$$R_c'' \frac{m^2 - K}{W} = \frac{1}{U_{3c}} - \left(\frac{1}{h_c} + \frac{1}{h_h} \right)$$

$$= \frac{1}{U_{c-\text{Year3}}} - \frac{1}{U_{c-\text{Year1}}} = \frac{1}{338} - \frac{1}{500} = 9.59 \times 10^{-4} \frac{m^2 - K}{W}$$

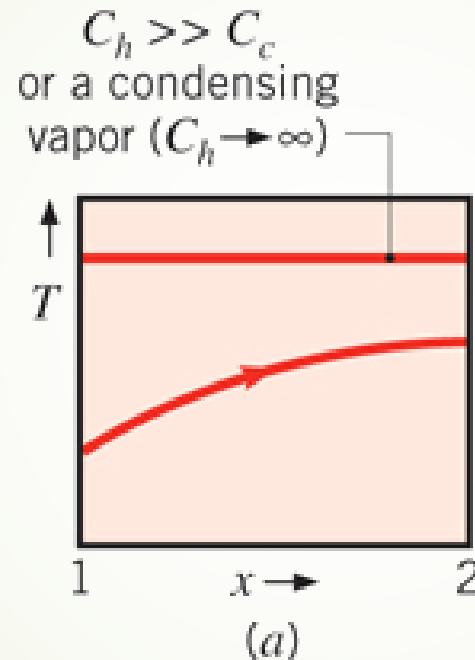
Special Operating Conditions Condensation and Evaporation

➤ **CONDENSATION** occurs at a constant temperature (i.e. **THERMODYNAMICS**)

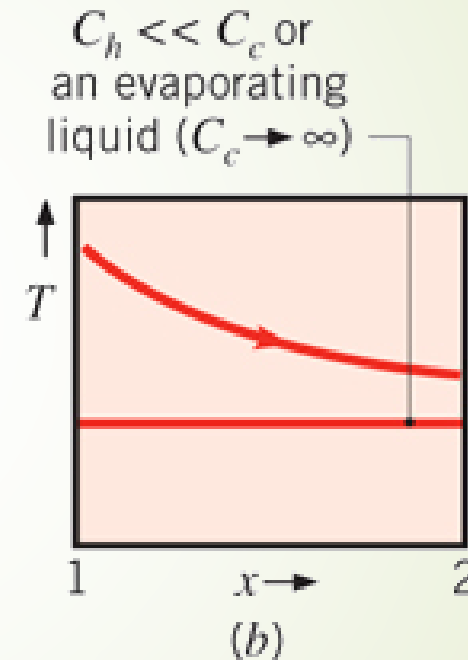
➤ Implies a **VERY LARGE THERMAL CAPACITY OF HOT FLUID** (C_h).

➤ **EVAPORATION** occurs at a constant temperature (i.e. **THERMODYNAMICS**)

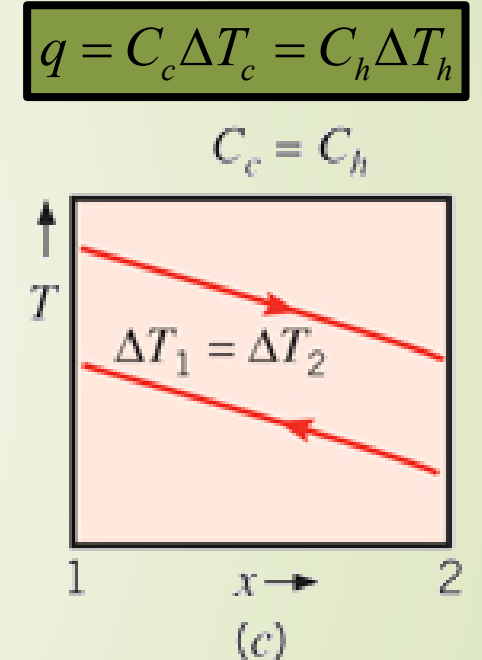
➤ Implies a **VERY LARGE THERMAL CAPACITY OF COLD FLUID** (C_c).



$$q = \dot{m}_h (\text{kg} / \text{s}) h_{fg} (\text{J} / \text{kg})$$



$$q = \dot{m}_c (\text{kg} / \text{s}) h_{fg} (\text{J} / \text{kg})$$



$$C \equiv \text{THERMAL CAPACITY} = \dot{m} c p$$

ANALYSIS: **METHOD #2: EFFECTIVENESS--NTU**

12

- ▶ ONLY **TWO** TEMPERATURES ARE REQUIRED (**Thi and Tci**)
 - ▶ MAJOR ADVANTAGE
- ▶ Definition: Heat Exchanger Effectiveness

$C \equiv \text{THERMAL CAPACITY} = \dot{m}cp$

$$C_h = (\dot{m}cp)_h, C_c = (\dot{m}cp)_c \rightarrow q = (\dot{m}cp)_{cold} \Delta T_{cold} = (\dot{m}cp)_{hot} \Delta T_{hot}$$

$$C_{\min} = \text{MINIMUM}(C_h, C_c)$$

$\varepsilon \equiv \text{HX EFFECTIVENESS}$

$$\begin{aligned} \varepsilon &\equiv \frac{q}{q_{\max}} = \frac{q}{C_{\min} (T_{h,i} - T_{c,i})} \\ &= \frac{C_h (T_{h,i} - T_{h,o})}{C_{\min} (T_{h,i} - T_{c,i})} = \frac{q_h}{C_{\min} (T_{h,i} - T_{c,i})} \\ &= \frac{C_c (T_{c,o} - T_{c,i})}{C_{\min} (T_{h,i} - T_{c,i})} = \frac{q_c}{C_{\min} (T_{h,i} - T_{c,i})} \end{aligned}$$

$C_{\min} (T_{h,i} - T_{c,i}) \rightarrow \text{MAXIMUM POSSIBLE FLUID HEAT TRANSFER}$

$$\varepsilon \rightarrow F\left(NTU, \frac{C_{\min}}{C_{\max}}\right), NTU \equiv \text{Number of Transfer Units (Dimensionless)}$$

PHASE CHANGE

13

(constant pressure and temperature heat transfer)

CONDENSATION, $C_h \rightarrow \infty$

$$\varepsilon \equiv \frac{q}{q_{\max}} = \frac{q}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})}, C_{\min} = C_c$$

$$= \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})}$$

$$q[W] = \dot{m}_h (kg/s) \cdot h_{fg} (J/kg)$$

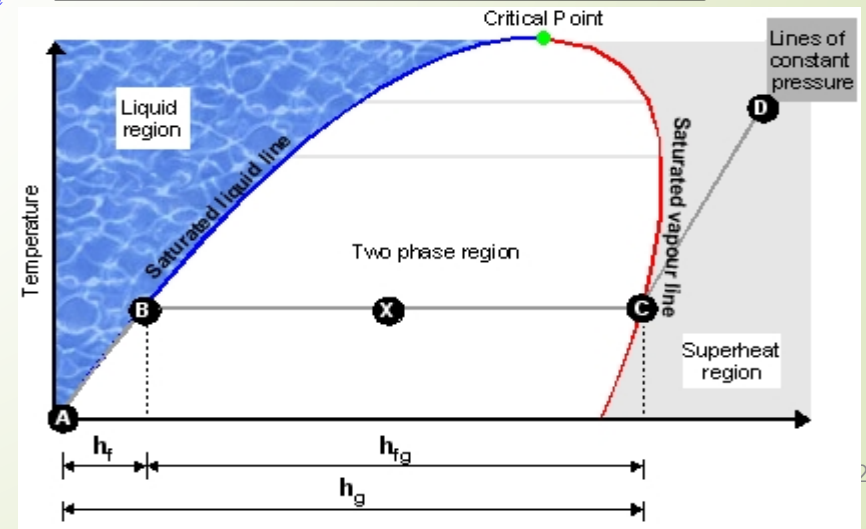
VAPORIZATION, $C_c \rightarrow \infty$

$$\varepsilon \equiv \frac{q}{q_{\max}} = \frac{q}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})}, C_{\min} = C_h$$

$$= \frac{(T_{h,i} - T_{h,o})}{(T_{h,i} - T_{c,i})}$$

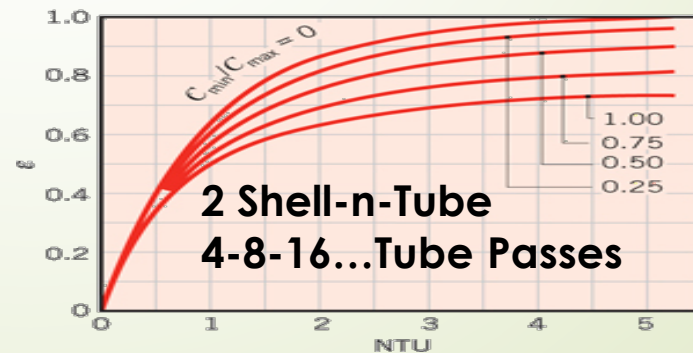
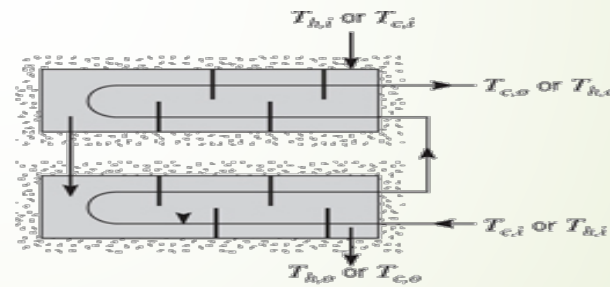
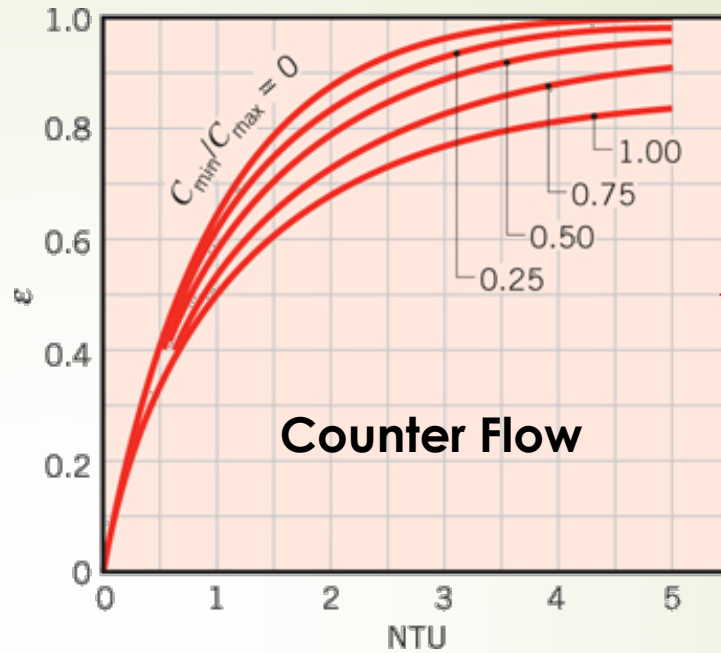
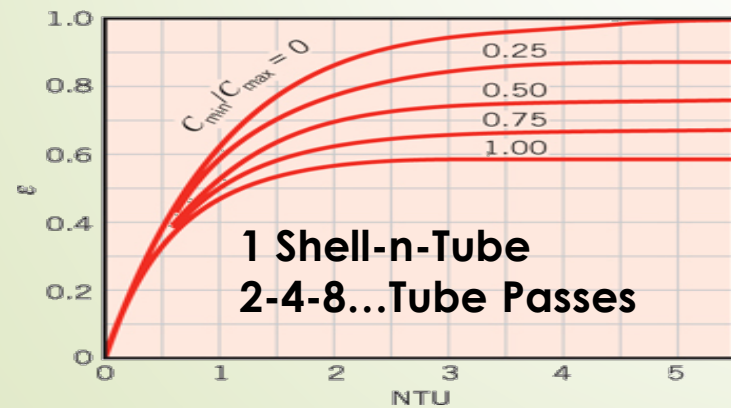
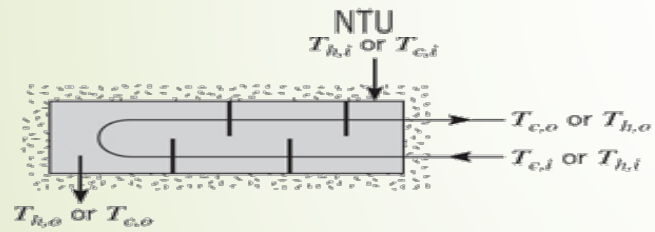
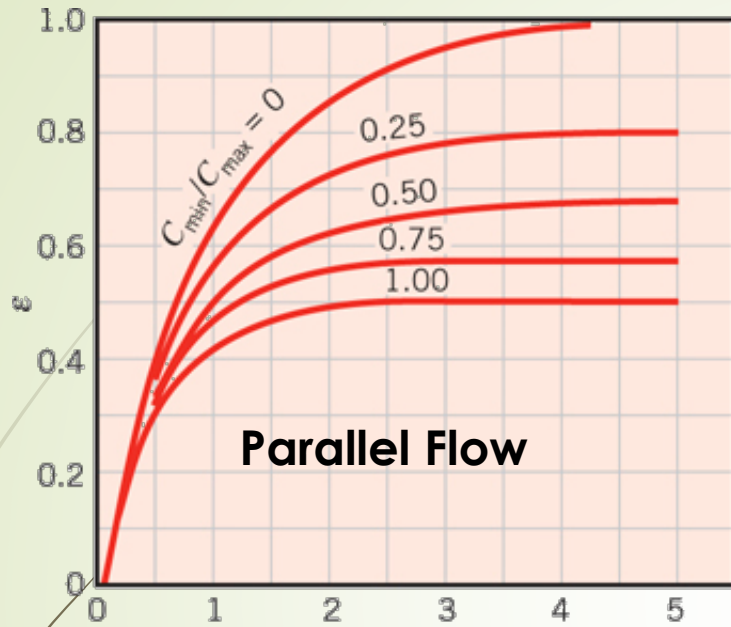
$$q[W] = \dot{m}_c (kg/s) h_{fg} (J/kg)$$

h_{fg} = latent heat of vaporization = $h_g - h_f$



EFFECTIVENESS

14



$$NTU \equiv \frac{UA}{C_{\min}}$$

NTU RELATIONSHIPS (sometimes easier)

$$C_r = \frac{C_{\min}}{C_{\max}} = 0 \rightarrow \text{ALL EXCHANGERS w/PHASE CHANGE}$$

$$\varepsilon = 1 - \exp(-NTU), NTU = -\ln(1 - \varepsilon)$$

COUNTER FLOW ($C_r = 1.0$)

$$\varepsilon = \frac{NTU}{1 + NTU}, NTU = \frac{\varepsilon}{1 - \varepsilon}$$

Parallel Flow

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$$

COUNTER FLOW ($C_r < 1.0$)

$$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}, NTU = \frac{1}{C_r - 1} \ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$$

ONE SHELL PASS ($n=2,4,8,16..$ tube passes)

$$NTU_1 = -(1 + C_r^2)^{-1/2} \ln\left(\frac{E - 1}{E + 1}\right), E = \frac{\frac{2}{\varepsilon_1} - (1 + C_r)}{(1 + C_r^2)^{+1/2}}, \varepsilon_1 = \frac{F - 1}{F - C_r}, F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} NTU = n(NTU)_1$$



NTU Relationships (CROSS FLOW HEAT EXCHNAGERS)

Cross-Flow (single pass)

C_{\max} (*mixed*), C_{\min} (*unmixed*)

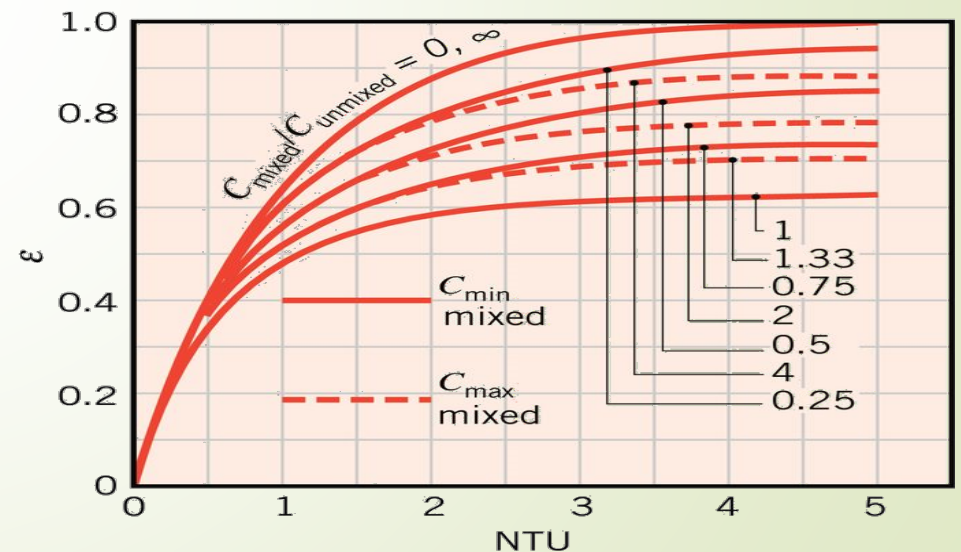
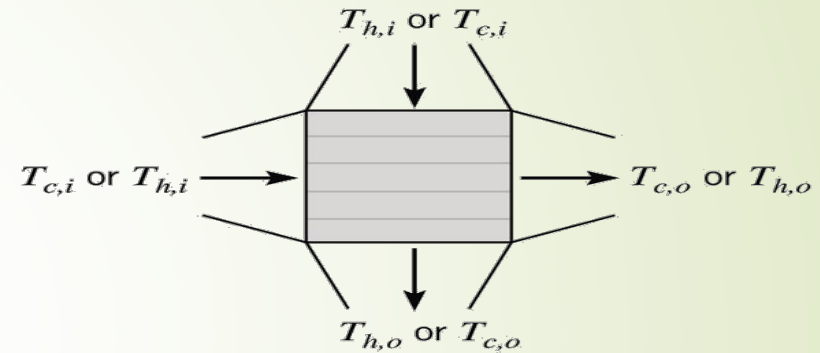
$$NTU = -\ln \left[1 + \left(\frac{1}{C_r} \right) \ln(1 - \varepsilon C_r) \right]$$

$$a = C_r (e^{-NTU} - 1), \varepsilon = \frac{1 - e^a}{C_r}$$

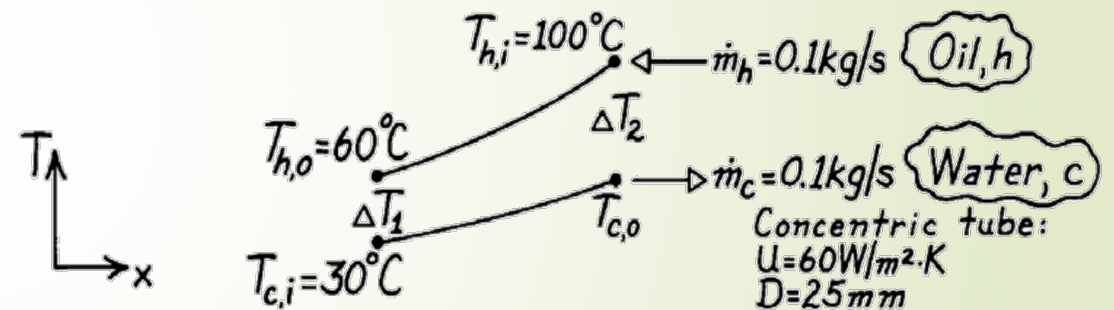
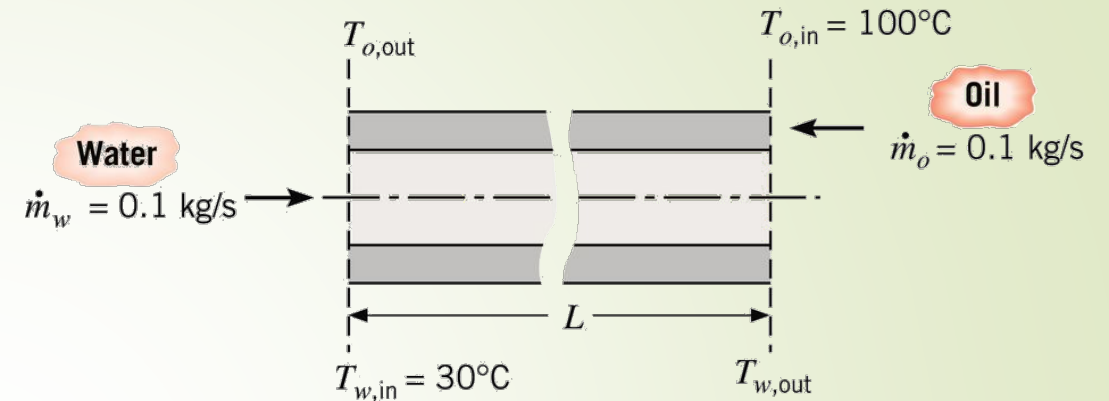
C_{\min} (*mixed*), C_{\max} (*unmixed*)

$$NTU = -\left(\frac{1}{C_r} \right) \ln [C_r \ln(1 - \varepsilon) + 1]$$

$$a = \frac{e^{-NTU \cdot C_r} - 1}{C_r}, \varepsilon = 1 - e^a$$



Concentric Tube Heat Exchanger. Thin walled inner tube of 25mm with water, and 45mm outer tube with oil. Counter flow with $U=60\text{W/m}^2\text{-K}$



Find:

- Total Heat Transfer and water outlet temperature if $T_{oil \text{ out}} = 60^\circ\text{C}$
- Tube Length.

PROPERTIES: (given):

	ρ (kg/m ³)	c_p (J/kg·K)	ν (m ² /s)	k (W/m·K)
Water	1000	4200	7×10^{-7}	0.64
Oil	800	1900	1×10^{-5}	140

OVERALL ENERGY BALANCE

$$q = (\dot{m}c_p)_{hot} (T_{h,i} - T_{h,o}) = (\dot{m}c_p)_{cold} (T_{c,o} - T_{c,i})$$

HOT : OIL

$$T_{h,i} = 100C, T_{h,o} = 60C, \dot{m}_h = 0.1kg / s$$

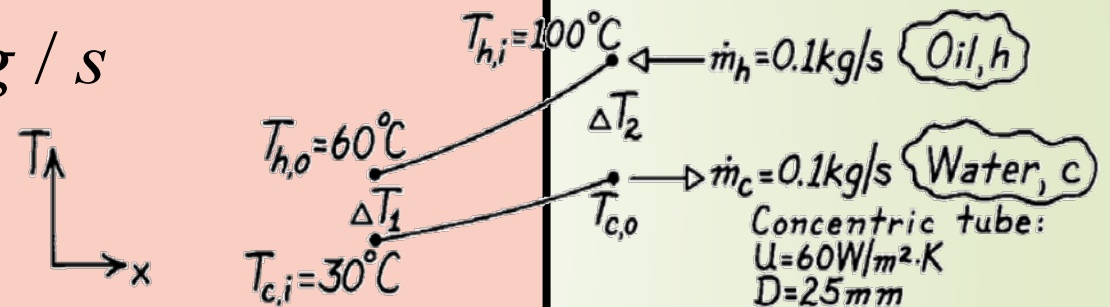
COLD : WATER

$$T_{c,i} = 30C, T_{c,o} = ?, \dot{m}_c = 0.1kg / s$$

$$T_{c,o} = \frac{(\dot{m}c_p)_{hot} (T_{h,i} - T_{h,o})}{(\dot{m}c_p)_{cold}} + T_{c,i}$$

$$= \frac{0.1kg / s \cdot 1900J / kg - K (100 - 60)C}{0.1kg / s \cdot 4200J / kg - K} + 30C$$

$$= 48.1C$$



LMTD METHOD #1

L=?

$$q = UA\Delta T_{\log\text{mean},CF} = U(\pi DL)\Delta T_{\log\text{mean},CF}$$

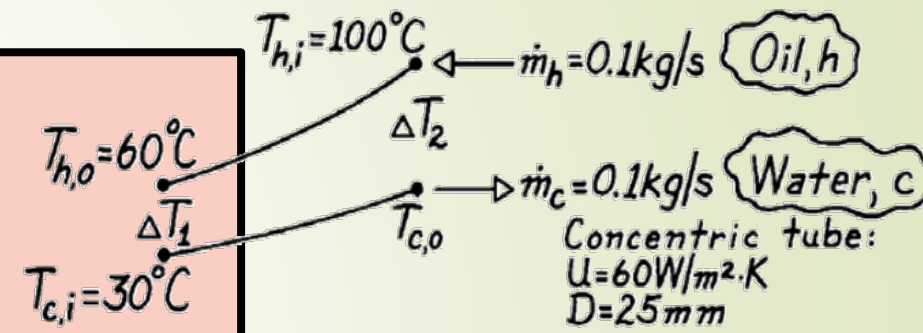
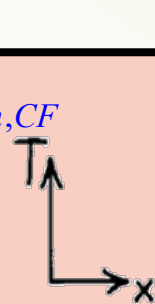
$$L = \frac{q}{U(\pi D)\Delta T_{\log\text{mean},CF}}$$

$$\Delta T_{\log\text{mean},CF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left[\frac{\Delta T_1}{\Delta T_2}\right]} = \frac{(60 - 30)C - (100 - 48.1)C}{\ln(30 / 51.9)} = 40.0C$$

$$q = (\dot{m}cp)_{\text{hot}} (T_{h,i} - T_{h,o}) = (\dot{m}cp)_{\text{cold}} (T_{c,o} - T_{c,i}) = 7600W$$

$$L = \frac{q}{U(\pi D)\Delta T_{\log\text{mean},CF}} = \frac{7600W}{60 \frac{W}{m^2 \cdot K} \cdot \pi \cdot 0.025m \cdot 40.0C}$$

$$L = 40.3m$$



COUNTER FLOW

$$\Delta T_1 = T_{h,o} - T_{c,i}$$

$$\Delta T_2 = T_{h,i} - T_{c,o}$$

PARALLEL FLOW

$$\Delta T_1 = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,o} - T_{c,o}$$

e-NTU METHOD #2

21

L=?

$$C_h = (\dot{m}c_p)_{hot} = 190W / K, C_c = (\dot{m}c_p)_{cold} = 420W / K$$

$$C_{min} = C_h$$

$$C_{max} = C_c$$

$$q_{max} = C_{min} (T_{h,i} - T_{c,i}) = 190W / K (100 - 30) = 13,300W$$

$$q = (\dot{m}c_p)_{hot} (T_{h,i} - T_{h,o}) = (\dot{m}c_p)_{cold} (T_{c,o} - T_{c,i}) = 7600W$$

$$\varepsilon = \frac{q}{q_{max}} = \frac{7600W}{13,300W} = 0.571$$

FIND NTU--EQN: 11.29b

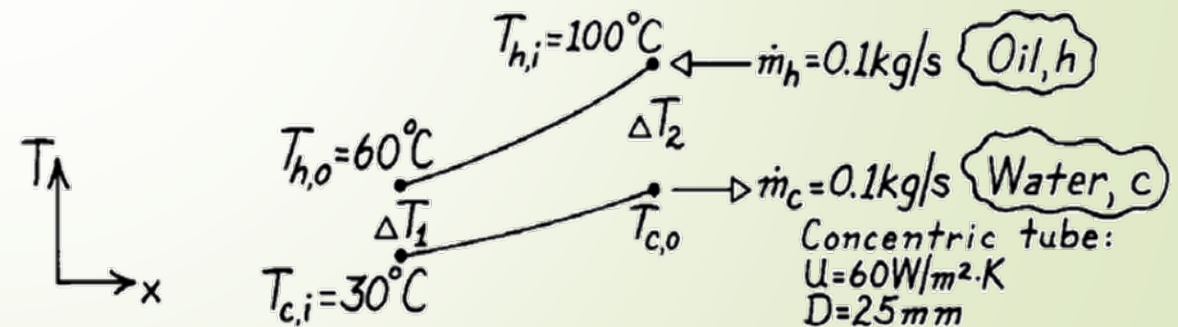
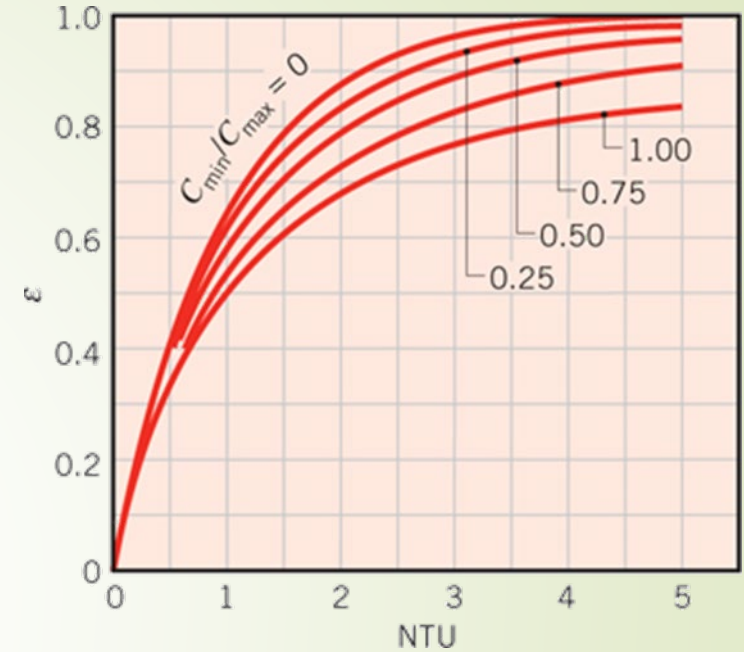
$$NTU = \frac{1}{C_r - 1} \ln \left(\frac{\varepsilon - 1}{\varepsilon C_r - 1} \right) = \frac{UA}{C_{min}}$$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{190}{420} = 0.452$$

$$NTU = \frac{1}{0.452 - 1} \ln \left(\frac{0.571 - 1}{0.571 \cdot 0.452 - 1} \right) = 1.00$$

$$1.00 = \frac{UA}{C_{min}} = \frac{U \pi D L}{C_{min}}$$

$$L = \frac{NTU \cdot C_{min}}{U \pi D} = \frac{1.0 \cdot 190W / K}{60W / m^2 \cdot K \cdot \pi \cdot 0.025m} = 40.3m$$



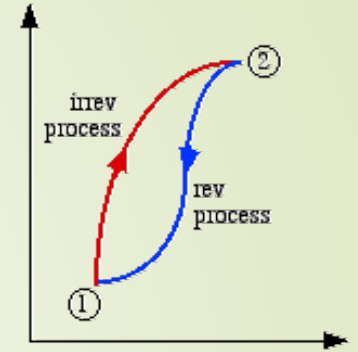
PARALLEL VS COUNTER FLOW

COUNTER FLOW is MORE EFFECTIVE: MORE HEAT TRANSFER w/SAME AREA

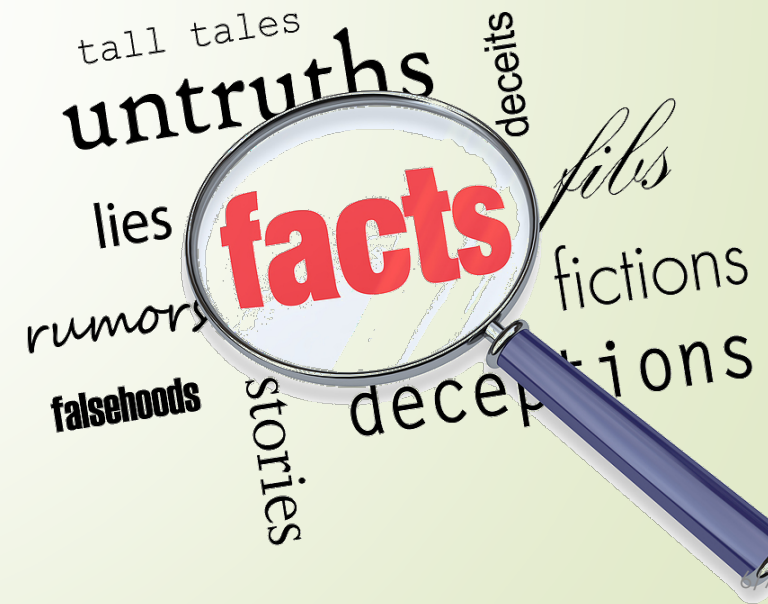
$$\oint \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q}{T} + \int_2^1 \frac{\delta Q}{T} < 0$$

$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 dS = \int_1^2 \frac{\delta Q}{T} + (S_1 - S_2) < 0$$

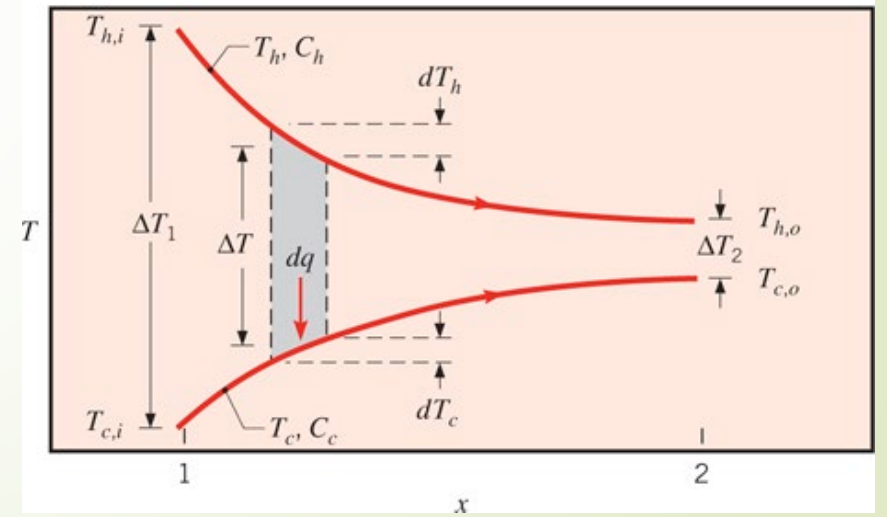
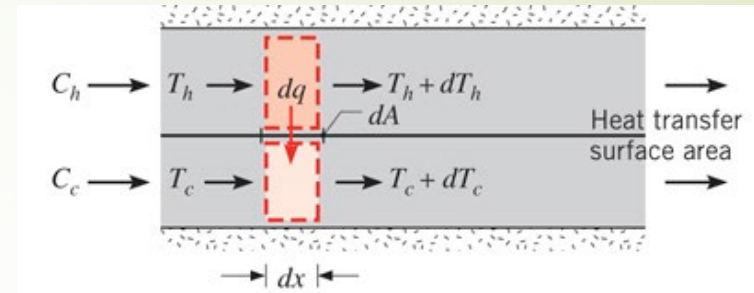
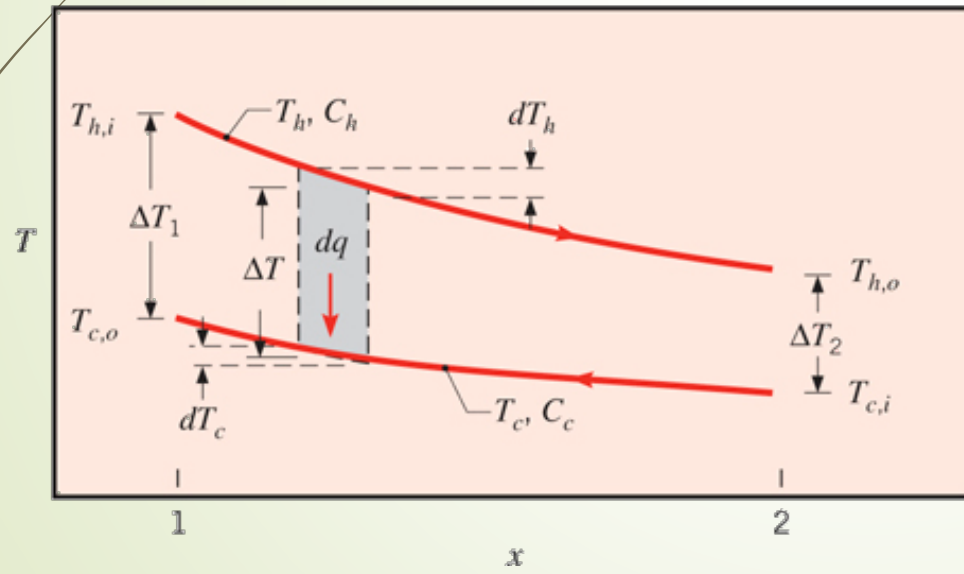
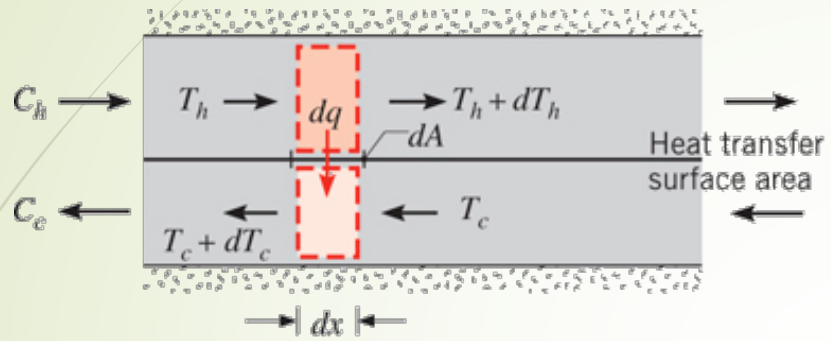
$$\Delta S = (S_2 - S_1) > \int_1^2 \frac{\delta Q}{T}$$



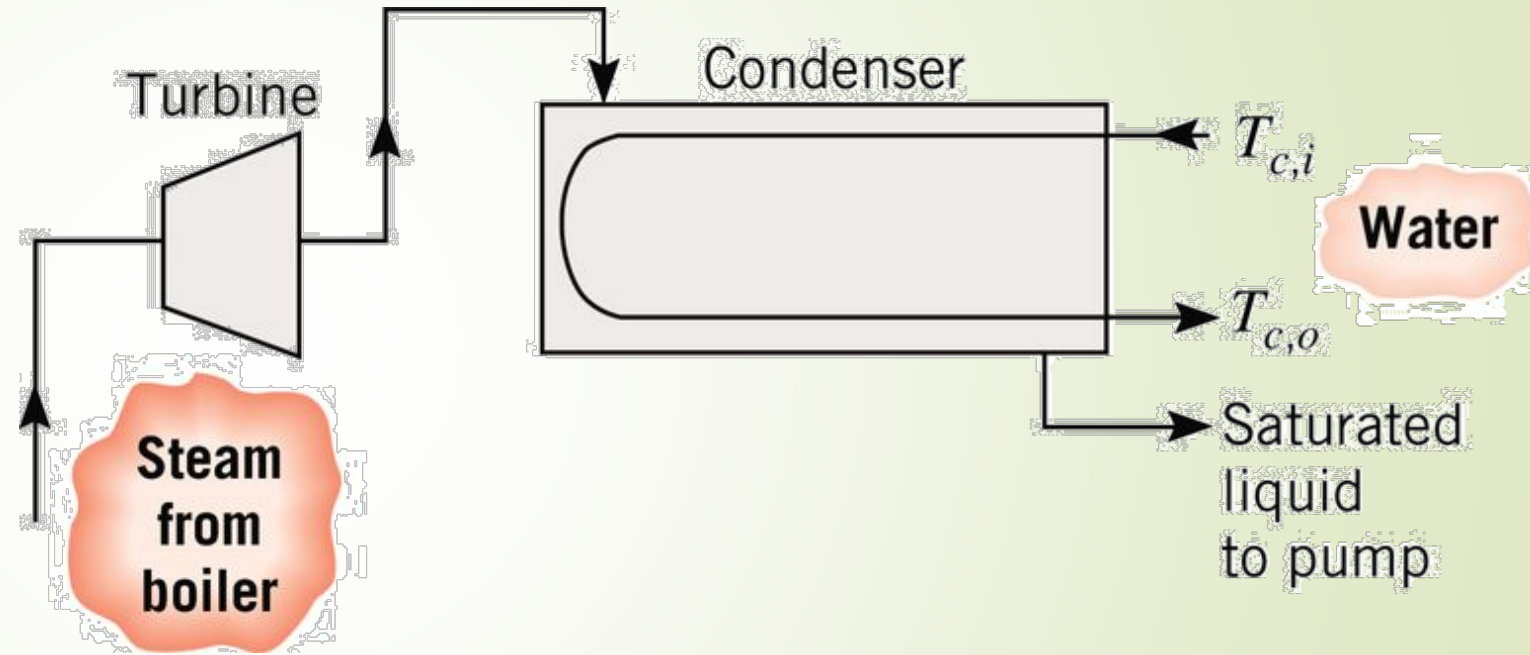
For adiabatic processes ($\delta Q = 0$) $\Rightarrow \Delta S \geq 0$



SMALLER dT OVER LENGTH and THUS LOWER THERMAL LOSSES DUE TO LOWER ENTROPY PRODUCTION



Steam Power Plant Condenser Design

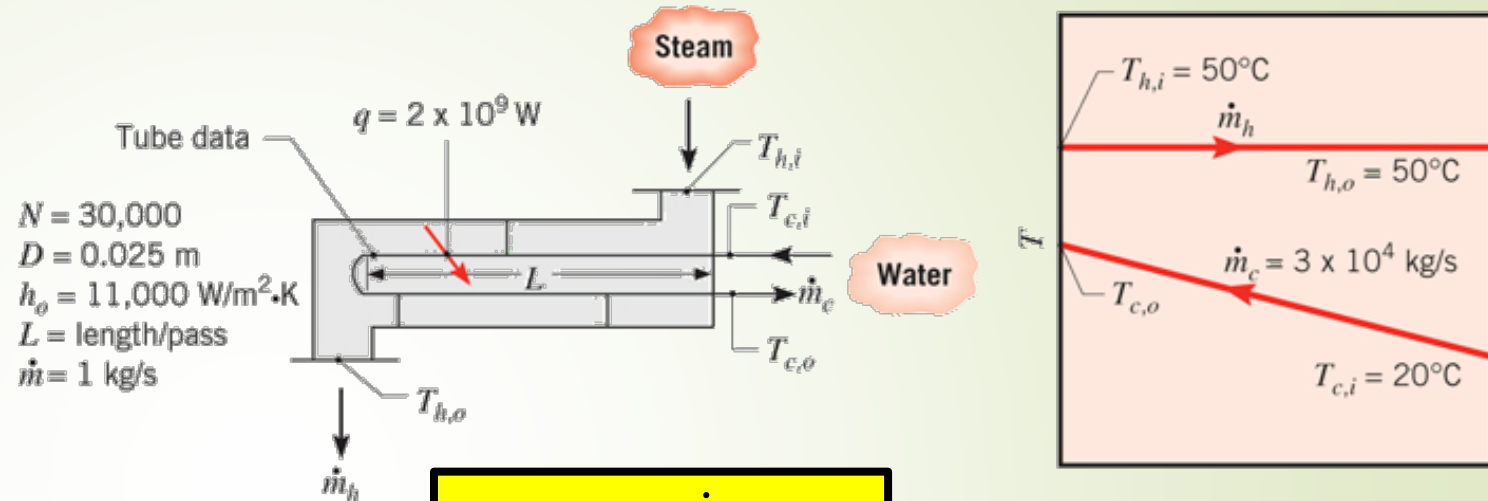


Problem

25

Large Power Plant with Steam Condenser

Find T_c cold out, and Tube Length ("L") Per Pass for required Heat Transfer



$$\dot{m}_{TUBE} = \frac{\dot{m}_{TOTAL}}{\#TUBES}$$

PROPERTIES

$$T_{c_{FILM}} = \frac{T_{c,i} + T_{c,o}}{2} = \frac{20 + ?}{2} \rightarrow \text{GUESS } T_{c,o} = 34C \rightarrow T_{c_{FILM}} = 300K$$

$$\text{water: } \rho = 997 \text{ kg/m}^3, cp = 4179 \text{ J/kg-K}, \mu = 855 \times 10^{-6} \text{ N-s/m}^2, k = 0.613 \text{ W/m-K}, Pr = 5.83$$

COLD ENERGY BALANCE

$$q = C_c (T_{c,o} - T_{c,i}) \rightarrow T_{c,o} = \frac{q}{C_c} + T_{c,i} = \frac{2 \times 10^9 \text{ W}}{3 \times 10^4 \text{ kg/s} \cdot 4179 \text{ J/kg-K}} + 20C = 36C$$

UA and Effectiveness

26

NTU DESIGN PROBLEM, Need UA (NO FOULING FACTORS)

$$UA = \left(\frac{1}{h_i A_i} + \frac{1}{h_o A_o} \right)^{-1} = \left(\frac{1}{A \left[\frac{1}{h_i} + \frac{1}{h_o} \right]} \right)^{-1} \rightarrow \text{THIN WALL TUBE} \rightarrow A = A_i = A_o$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}, h_o = 11,000 \text{ W/m}^2\text{-K} \rightarrow \text{GIVEN}$$

INTERNAL FLOW

$$Re_D = \left(\frac{4\dot{m}}{\pi D \mu} \right)_{\text{ONE TUBE}} = \frac{4 \cdot \frac{1 \text{ kg/s}}{\text{tube}}}{\pi \cdot 0.025 \cdot 855 \times 10^{-6} \text{ N-s/m}^2} = 59,567 \rightarrow \text{TURBULENT}$$

$$\overline{NU}_D = 0.023 Re_D^{4/5} Pr^{0.4} = \frac{\overline{h}_i D}{k_{\text{fluid}}} = 308 \rightarrow \overline{h}_i = 7543 \text{ W/m}^2\text{-K}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7543 \text{ W/m}^2\text{-K}} + \frac{1}{11,000 \text{ W/m}^2\text{-K}}} = 4474 \text{ W/m}^2\text{-K}$$

$$\varepsilon = 1 - \exp(-NTU), NTU = -\ln(1 - \varepsilon)$$

Because of STEAM CONDENSING

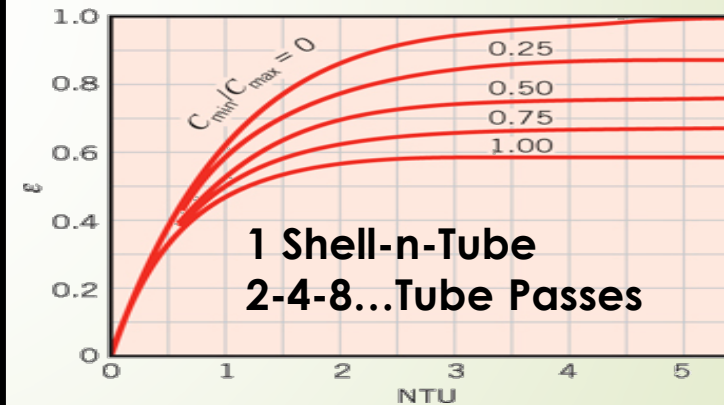
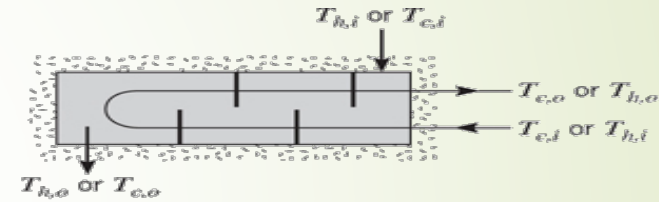
$$C_h \rightarrow C_{\max} \rightarrow \infty \rightarrow CMAX \rightarrow \frac{C_{\min}}{C_{\max}} = 0$$

Max Possible Heat Transfer

$$q_{\max} = C_{\min_{TOTAL}} (T_{h,i} - T_{c,i}) = C_c (T_{h,i} - T_{c,i}) = 3 \times 10^4 \text{ kg/s} \cdot 4179 \text{ J/kg-K} (50 - 20) \text{ K} = 3.76 \times 10^9 \text{ W}$$

Effectiveness

$$\varepsilon = \frac{q}{q_{\max}} = \frac{2 \times 10^9 \text{ W}}{3.76 \times 10^9 \text{ W}} = 0.532$$



$$NTU \left(\frac{C_{\min}}{C_{\max}} = 0, \varepsilon = 0.532 \right) \rightarrow 0.759 = \frac{UA}{C_{\min}}$$

TUBE LENGTH

$$NTU \left(\frac{C_{\min}}{C_{\max}} = 0, \varepsilon = 0.532 \right) \rightarrow 0.759 = \frac{U \cdot \# \text{ tubes} \cdot A_{TOTAL}}{C_{\min} \cdot \text{tube}}$$

$$A_{TOTAL} = \pi D \frac{L_{TOTAL}}{\text{pass}} \cdot \frac{\# \text{ passes}}{\text{shell}} \cdot \# \text{ shells}$$

$$\frac{L_{TOTAL}}{\text{pass}} = \frac{0.759 \cdot C_{\min}}{\frac{U}{\text{tube}} \left(\pi D \cdot \frac{\# \text{ passes}}{\text{shell}} \cdot \# \text{ tubes} \cdot \# \text{ shells} \right)}$$

$$\frac{\# \text{ passes}}{\text{shell}} = 2$$

$$\# \text{ tubes} = 30,000$$

$$\# \text{ shells} = 1$$

$$\frac{L_{TOTAL}}{\text{pass}} = 4.51 \text{ m per pass}$$

PHASE CHANGE

$$q = \dot{m}_h h_{fg} = (\dot{m}c_p)_{\text{cold}} \Delta T_{\text{cold}}$$

$$\dot{m}_h = \frac{(\dot{m}c_p)_{\text{cold}} \Delta T_{\text{cold}}}{h_{fg}}$$

Because of STEAM CONDENSING

$$C_h \rightarrow C_{\max} \rightarrow \infty \rightarrow C_{MAX} \rightarrow \frac{C_{\min}}{C_{\max}} = 0$$

Max Possible Heat Transfer

$$\begin{aligned} q_{\max} &= C_{\min} (T_{h,i} - T_{c,i}) = C_c (T_{h,i} - T_{c,i}) \\ &= 3 \times 10^4 \text{ kg/s} \cdot 4179 \text{ J/kg-K} (50 - 20) \text{ K} = 3.76 \times 10^9 \text{ W} \end{aligned}$$

Effectiveness

$$\varepsilon = \frac{q}{q_{\max}} = \frac{2 \times 10^9 \text{ W}}{3.76 \times 10^9 \text{ W}} = 0.532$$

$$NTU_{TOTAL} \equiv \frac{UA(TOTAL)}{C_{\min}(TOTAL)}$$

$UA \equiv$ TOTAL RESISTANCE--ALL TUBES

$C_{\min} \equiv$ TOTAL CAPACITANCE--ALL TUBES

$$NTU_{TOTAL} = \left(\frac{(U_{\text{one-tube}} \cdot \# \text{ tubes} \cdot A_{TOTAL})}{C_{\min-TOTAL}} \right)$$

Pressure Drop, Power, and COST

TURBULENT

$$Re_D > 2300$$

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left(\left(\frac{\varepsilon / D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right)$$

Smooth $\varepsilon=0$

$$f=0.0376,$$

$$L_{TOTAL} = \frac{L_{TOTAL}}{pass} \bullet \#passes = 9m$$

$$\frac{\Delta P}{TUBE} = \frac{fL}{D} \rho \frac{u_m^2}{2} = \frac{28,764 Pa}{TUBE}, \rightarrow \frac{\dot{m}}{TUBE} = \rho_{fluid} A_c \frac{u_m}{TUBE} \rightarrow A_c = \frac{\pi D^2}{4}$$

POWER

$$\dot{P}_{ower} = \frac{\Delta P}{TUBE} \frac{\dot{m}_{TOTAL}}{\rho_{water}} = \frac{28,764 Pa}{TUBE} \frac{30,000 kg / s}{997 kg / m^3} = 0.87 MW \rightarrow \text{TOTAL POWER REQUIRED-PUMP}$$

COST -- Electric Pump

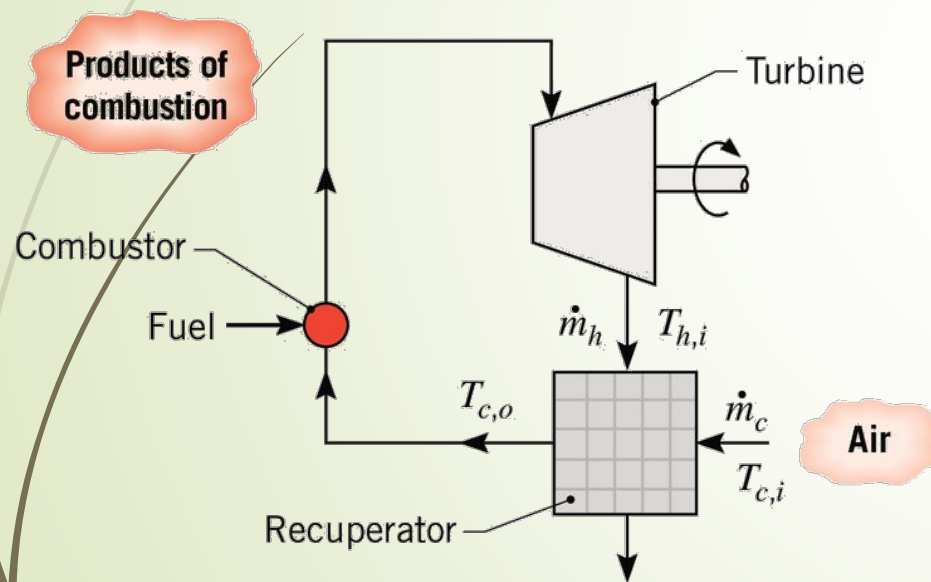
$$\eta_{pump} = 0.87$$

$$Electric = \frac{\$0.05}{kWh}$$

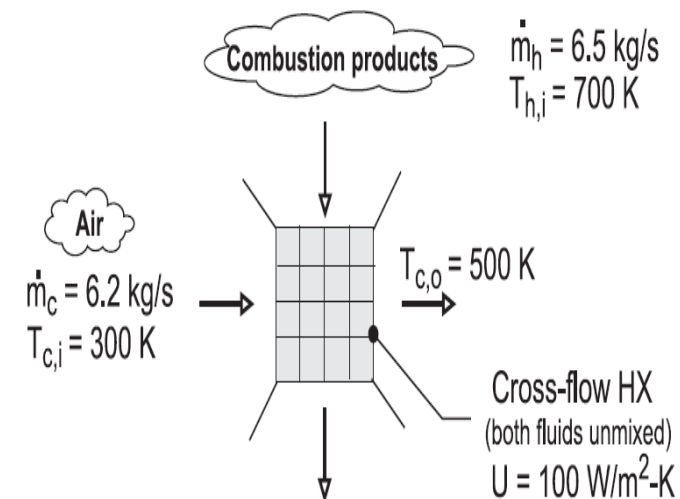
$$\text{Operating Cost} = \frac{0.87 \times 10^6 W \bullet \frac{\$0.05 \times 10^{-3}}{Wh} \bullet 8,760 \frac{hr}{yr}}{0.87} = \$438,000 / year$$

A recuperator is a heat exchanger that heats air used in a combustion process by extracting energy from the products of combustion as shown below. (air and combustion products have $c_p = 1040 \text{ J/kg-K}$)

Find: Surface Area as function of $T_{c,o}$.



SCHEMATIC:



How do you MAXIMIZE heat transfer rates?

Properties

$$cp_c = cp_h = 1040 \text{ J / kg} \cdot \text{K}$$

Analysis

$$C_{\min} = C_c = 6.2 \text{ kg / s} \cdot 1040 \text{ J / kg} \cdot \text{K} = 6,448 \text{ W / K}$$

$$C_{\max} = C_h = 6.5 \text{ kg / s} \cdot 1040 \text{ J / kg} \cdot \text{K} = 6,760 \text{ W / K}$$

$$C_r = \frac{C_{\min}}{C_{\max}} = 0.954$$

$$q_{\max} = C_{\min} (T_{h,i} - T_{c,i}) = 6,448 \text{ W / K} (400 \text{ K}) = 2.58 \times 10^6 \text{ W}$$

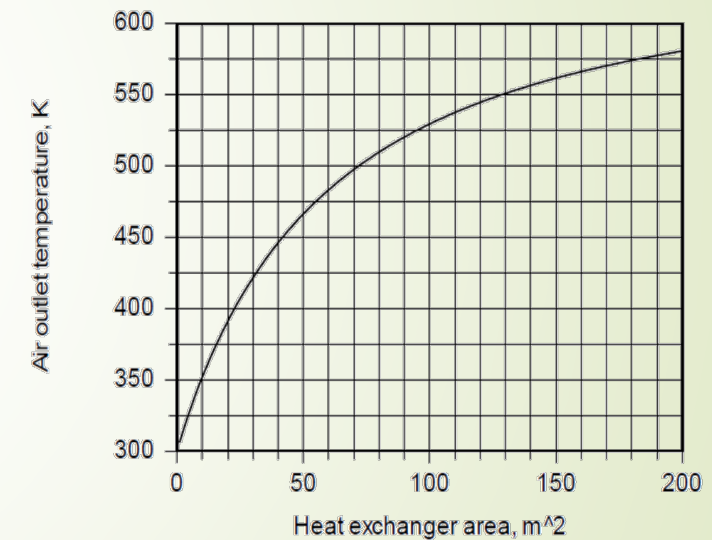
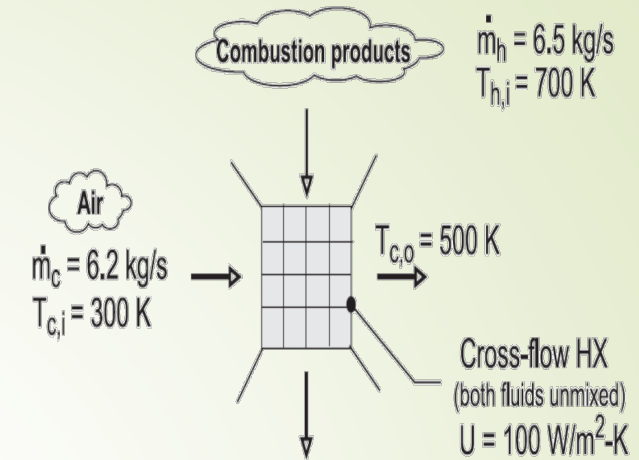
$$q = C_c (T_{c,o} - T_{c,i})$$

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{\min} (T_{h,i} - T_{c,i})} = \frac{6,448 \text{ W / K} (500 - 300)}{6,448 \text{ W / K} (700 - 300)} = 0.50$$

$$NTU \approx 1.10 = UA / C_{\min} \rightarrow \text{FIG : 11.14 (both unmixed)}$$

$$A = \frac{NTU \cdot C_{\min}}{U} = 70.9 \text{ m}^2$$

SCHEMATIC:



NOTE: As area **increases**, $T_{c,o}$ increases but the % increase is less and less as you approach $T_{h,i}$. To maximize $T_{c,o}$ (and q) would require large heat exchanger and costly.

PROBLEMS

11.10, 11.14, 11.18,
11.23, 11.32, 11.35

11.63a, 11.64,
11.76abc