

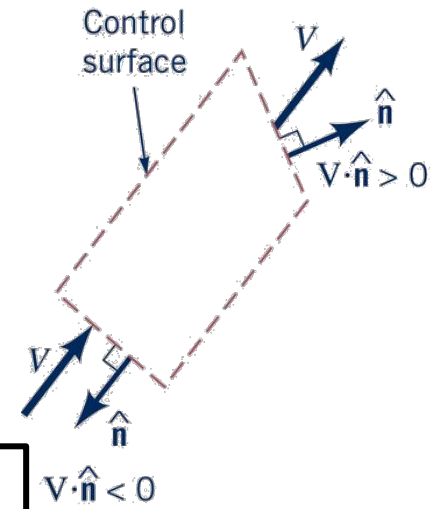
The background is a vibrant, abstract composition of various colors and patterns. It includes a large purple shape on the left with a dashed pattern, a yellow oval at the top center, a large tan shape on the right with a dotted pattern, a red shape at the bottom right with a wavy pattern, and a red shape at the bottom center with a plus sign pattern. There are also white wavy lines scattered throughout the design.

# MECH-322 Study Aid

# MASS CONTINUITY

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# CONTINUITY



- Mass Conservation: “TIME RATE OF CHANGE OF SYSTEM MASS = 0”

$$\frac{DM}{Dt}_{SYSTEM} = 0;$$

$$M = \text{system MASS} = \text{DENSITY (kg / m}^3\text{)} \times \text{VOLUME (m}^3\text{)}$$

$$M_{sys} = \int_{sys} \rho dV$$

- REYNOLDS TRANSPORT THM.


$$\frac{DM}{Dt}_{SYSTEM} \equiv 0 = \frac{\partial}{\partial t} \left[ \int_{CV} \left( \rho \left[ \frac{\text{kg}}{\text{vol}} \right] \right) dV \right] + \int_{CS} \rho (\vec{V} \cdot \hat{n}) dA$$

- Constant Properties & Flow

$\dot{m} = \rho A V_n = \rho Q$ : MASS FLOW RATE (mass/time)

$V_n$  = NORMAL VELOCITY @ SURFACE

$$0 = \frac{\partial}{\partial t} \int_{cv} \rho dV + \sum \dot{m}_{out} - \sum \dot{m}_{in}$$



- An aquarium is being emptied at a steady rate with a small pump. The water is pumped to a 12" diameter cylinder bucket, and its depth is increasing at a rate of 4"/min.

- Find the rate at which the aquarium water level is dropping if the aquarium measures 24" (wide) x 36" (long) x 18" (high).

## MASS CONSERVATION

$$\frac{\partial}{\partial t} \left[ \overbrace{\int_{cv} \rho d\forall}^{\rho \forall = \rho Ah(t)} + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0 \right]$$

Aquarium

$$\frac{\partial}{\partial t} \int_{cv} \rho d\forall + \sum \dot{m}_{out} = 0$$

$$\rho A_{AQ} \left( \frac{dh}{dt} \right)_{AQ} + \left( \sum \dot{m}_{out} \right)_{AQ} = 0; (1)$$

*BUCKET*

$$\frac{\partial}{\partial t} \int_{cv} \rho d\forall - \left( \sum \dot{m}_{in} \right)_{BU} = 0$$

$$\rho \frac{\pi D^2}{4} \left( \frac{dh}{dt} \right)_{BU} - \left( \sum \dot{m}_{in} \right)_{BU} = 0; (2)$$

*EQUILIBIRUM*

$$\left( \sum \dot{m}_{out} \right)_{AQ} = \left( \sum \dot{m}_{in} \right)_{BU} = \rho \frac{\pi D^2}{4} \left( \frac{dh}{dt} \right)_{BU}$$

$$\rho A_{AQ} \left( \frac{dh}{dt} \right)_{AQ} + \left( \sum \dot{m}_{out} \right)_{AQ} = 0; (1)$$

$$\left( \sum \dot{m}_{out} \right)_{AQ} = \left( \sum \dot{m}_{in} \right)_{BU} = \rho \frac{\pi D^2}{4} \left( \frac{dh}{dt} \right)_{BU}$$

$$\cancel{\rho} A_{AQ} \left( \frac{dh}{dt} \right)_{AQ} + \cancel{\rho} \frac{\pi D^2}{4} \left( \frac{dh}{dt} \right)_{BU} = 0 \rightarrow \text{PARAMETRIC FUNCTION}$$

$$A_{AQ} \left( \frac{dh}{dt} \right)_{AQ} = - \frac{\pi D^2}{4} \left( \frac{dh}{dt} \right)_{BU} = - \frac{\pi \left( \frac{1}{12} \right)^2}{4} \text{ft}^2 \frac{4/12 \text{ m}}{\text{m}} \frac{1 \text{ m}}{60 \text{ s}} \text{ft} = -3 \times 10^{-5} \frac{\text{ft}^3}{\text{s}}$$

$$A_{AQ} \left( \frac{dh}{dt} \right)_{AQ} = \text{Rate of AQ water level drop}$$

$$\text{Area}_{AQ} = 24" \times 36" \text{in}^2$$

$$\frac{dVol(t)_{AQ}}{dt} = \text{Area}_{AQ} \frac{dh}{dt}$$