MECH-322 Study Aid
MASS CONTINUITY

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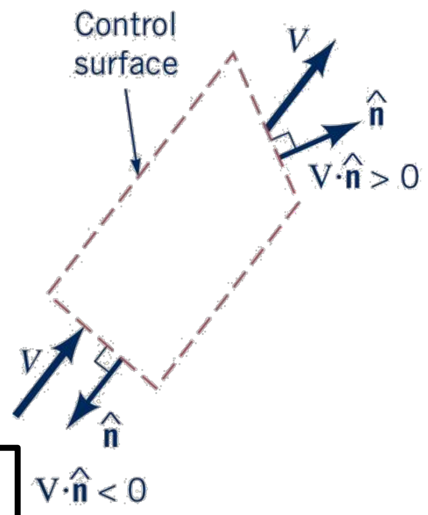
CONTINUITY



- Mass Conservation: "TIME RATE OF CHANGE OF SYSTEM MASS = 0"

$$\left.\frac{DM}{Dt}\right|_{SYSTEM} = 0;$$
$$M = system\ \text{MASS} = DENSITY(kg/m3)\ x\ VOLUME(m3)$$
$$M_{sys} = \int_{sys} \rho d\forall$$

- REYNOLDS TRANSPORT THM.

$$\left.\frac{DM}{Dt}\right|_{SYSTEM} \equiv 0 = \frac{\partial}{\partial t}\left[\int_{CV}\left(\rho\left[\frac{kg}{vol}\right]\right)d\forall\right] + \int_{CS}\rho\left(\vec{V}\bullet\hat{n}\right)dA$$

- Constant Properties & Flow

$\dot{m} = \rho A V_n = \rho Q$: MASS FLOW RATE (mass/time)

V_n = NORMAL VELOCITY @ SURFACE

$$0 = \frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} - \sum \dot{m}_{in}$$

- An aquarium is being emptying at a steady rate with a small pump. The water is pumped to a 12” diameter cylinder bucket, and its depth is increasing at a rate of 4”/min.

- Find the rate at which the aquarium water level is dropping if the aquarium measures 24” (wide) x 36” (long) x 18” (high).

MASS CONSERVATION

$$\frac{\partial}{\partial t}\left[\overbrace{\int_{cv}\rho d\forall}^{\rho\forall=\rho A h(t)} + \right]\sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

Aquarium

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall + \sum \dot{m}_{out} = 0$$

$$\rho A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} + \left(\sum \dot{m}_{out}\right)_{AQ} = 0;(1)$$

BUCKET

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall - \left(\sum \dot{m}_{in}\right)_{BU} = 0$$

$$\rho\frac{\pi D^2}{4}\left(\frac{dh}{dt}\right)_{BU} - \left(\sum \dot{m}_{in}\right)_{BU} = 0;(2)$$

EQUILIBIRUM

$$\left(\sum \dot{m}_{out}\right)_{AQ} = \left(\sum \dot{m}_{in}\right)_{BU} = \rho\frac{\pi D^2}{4}\left(\frac{dh}{dt}\right)_{BU}$$

$$\rho A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} + \left(\sum \dot{m}_{out}\right)_{AQ} = 0;(1)$$

$$\left(\sum \dot{m}_{out}\right)_{AQ} = \left(\sum \dot{m}_{in}\right)_{BU} = \rho\frac{\pi D^2}{4}\left(\frac{dh}{dt}\right)_{BU}$$

$$\not{\rho} A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} + \not{\rho}\frac{\pi D^2}{4}\left(\frac{dh}{dt}\right)_{BU} = 0 \rightarrow \text{PARAMETRIC FUNCTION}$$

$$A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} = -\frac{\pi D^2}{4}\left(\frac{dh}{dt}\right)_{BU} = -\frac{\pi\left(\frac{1}{12}\right)^2}{4} ft^2 \frac{4/12}{m}\frac{1m}{60s} ft = -3x10^{-5}\frac{ft^3}{s}$$

$$A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} = \text{Rate of AQ water level drop}$$

$$\text{Area}_{AQ} = 24\text{"}x36\text{"}in^2$$

$$\frac{dVol(t)_{AQ}}{dt} = \text{Area}_{AQ}\frac{dh}{dt}$$