$$E = mc^2$$

MASS CONTINUITY AID #2 STUDY AID

MECH-322 Fluid Mechanics

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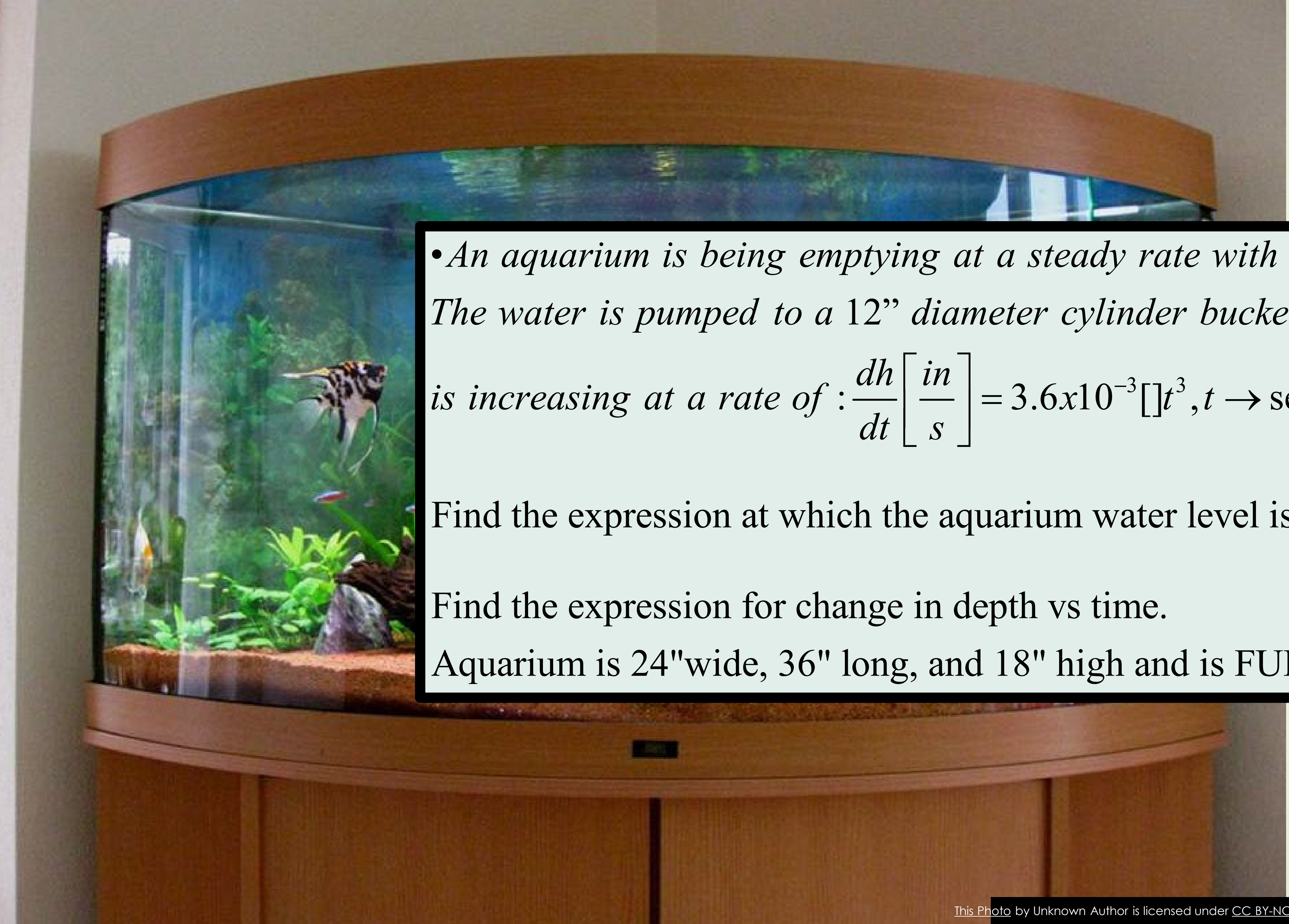

• *An aquarium is being emptying at a steady rate with a small pump. The water is pumped to a 12" diameter cylinder bucket, and its depth is increasing at a rate of* : $\frac{dh}{dt}\left[\frac{in}{s}\right] = 3.6x10^{-3}[\,]t^{3}, t \rightarrow \sec.$

Find the expression at which the aquarium water level is dropping $\left(\frac{ft^{3}}{s}\right)$

Find the expression for change in depth vs time.

Aquarium is 24"wide, 36" long, and 18" high and is FULL.

UNITS

$$\frac{dh}{dt}\left[\frac{in}{s}\right] = 3.6x10^{-3}[\,]t[s]^3$$

$$\frac{in}{s} = [\,]s^3 \rightarrow [\,] = \frac{\frac{in}{s}}{s^3} = \frac{in}{s^4}$$

$$\frac{dh}{dt}\left[\frac{in}{s}\right] = 3.6x10^{-3}\left[\frac{in}{s^4}\right]t[s]^3$$

MASS CONTINUITY

$$\frac{\partial}{\partial t}\left[\overbrace{\int_{cv}\rho d\forall}^{\rho\forall=\rho A h(t)}\right]+\sum \dot{m}_{out}-\sum \dot{m}_{in}=0$$

Aquarium

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall+\sum \dot{m}_{out}=0;\left(\sum \dot{m}_{in}\right)_{AQ}=0$$

$$\rho A_{AQ}\left(\frac{dh}{dt}\right)_{AQ}+\left(\sum \dot{m}_{out}\right)_{AQ}=0;(1)$$

BUCKET

$$\frac{\partial}{\partial t}\int_{cv}\rho d\forall-\left(\sum \dot{m}_{in}\right)_{BU}=0;\left(\sum \dot{m}_{out}\right)_{BU}=0$$

$$\rho\frac{\pi D^2}{4}\left(\frac{dh}{dt}\right)_{BU}-\left(\sum \dot{m}_{in}\right)_{BU}=0;(2)$$

EQUILIBIRUM

$$\left(\sum \dot{m}_{out}\right)_{AQ}=\left(\sum \dot{m}_{in}\right)_{BU}=\rho\frac{\pi D^2}{4}\left(\frac{dh}{dt}=3.6x10^{-3}[\frac{in^4}{s}]t^3[s^3]\right)_{BU}$$

$$\left(\frac{dh}{dt}\left[\frac{in}{s}\right]\right)_{BUCKET} = 3.6x10^{-3}[\frac{in}{s^4}]t[s]^3$$

$$\rho A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} + \rho\frac{\pi D^2}{4}\left(\sum \dot{m}_{out}\right)_{AQ} = 0$$

$$\not{\rho} A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} = -\not{\rho}\frac{\pi D_{BU}^2}{4}\left(3.6x10^{-3}[\frac{in}{s^4}]t[s]^3\right)$$

$$\underbrace{A_{AQ}[ft^2]\left(\frac{dh}{dt}\left[\frac{ft}{s}\right]\right)_{AQ}}_{\frac{ft^3}{s}} = \underbrace{-\frac{\pi D_{BU}^2}{4}\left[ft^2\right]\overbrace{\left(3.6x10^{-3}[\frac{in}{s^4}]t[s]^3\right)}^{\left[\frac{in}{s}\right]}\bullet\left[\frac{1ft}{12in}\right]}_{\left[\frac{ft^3}{s}\right]}$$

$$Q_{AQ}\left[\frac{ft^3}{s}\right] = -0.2356x10^{-3}\left[\frac{ft^3}{s^4}\right]t^3[s^3]$$

AQUARIUM HEIGHT VS TIME

$$A_{AQ}\left(\frac{dh}{dt}\right)_{AQ} = -\frac{\pi D_{BU}^2}{4}\left(3.6x10^{-3}[\frac{in}{s^4}]t[s]^3\right)$$

$$\int_{18"}^{h(t)} dh = C\int_0^{t^*} t^3 dt, C = \frac{\frac{\pi D_{BU}^2}{4}[in^2]\bullet 3.6x10^{-3}[\frac{in}{s^4}]}{A_{AQ}[in^2]}\left[\frac{in}{s^4}\right]$$

$$h(t)_{AQ}[in] = 18"[in] - \frac{C\left[\frac{in}{s^4}\right]t^4[s^4]}{4}[in]$$

Continuity Study Aid