MECH-420 Equations Sheet

Fourier's Law

$$\vec{q}_{conduction}[W] = -kA_n \nabla T$$

Newton's Law of Cooling

$$\text{q}_{convection}[W] = h_c A_s (T_s - T_\infty)$$

$h_c \equiv$ Convective Heat Transfer Coef.

Radiation

$$\text{q}_{radiation}[W] = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4); or\ alternatively$$

$$= h_r A_s (T_s - T_{surr}); where$$

$$h_r = \varepsilon\sigma (T_s + T_{surr})(T_s^2 + T_{surr}^2)$$

$h_r \equiv$ Radiation Heat Transfer Coeff.

1^{st} Law

$$\dot{\text{E}}_{in}[W] - \dot{\text{E}}_{out}[W] + \dot{\text{E}}_{gen}[W](\pm) = \dot{\text{E}}_{st}[W] = \frac{dE_{cv}}{dt} = \rho\forall c_p \frac{dT}{dt}[W]$$

Volume

$$Cylinder : \pi r^2 L; \quad Sphere : \frac{4}{3}\pi r^3$$

Surface Area

$$Cylinder : 2\pi r L; \quad Sphere : 4\pi r^2$$

1. You may use equation on each exam.
2. Do not write on any sheet of equation sheet.
3. After printing, sign below and submit with each exam.
4. It will be returned after each exam.

NAME________________________________

Any problem without correct units receive 0 points.

Quantity	Name Symbol	SI Units	English Units	Conversion
Force	Newton (N)	$\frac{m-kg}{s^2}$	lb_f	$1N = 0.224809 lb_f$
Pressure	Pascal (Pa)	$\frac{N}{m^2}$	$\frac{lb_f}{ft^2}$	$1PA = 0.020886 \frac{lb_f}{ft^2}$
Energy	Joules (J)	$N-m$	Btu	$1J = 0.000948 Btu$
Power	Watts (W)	$\frac{J}{\sec}$	Hp	$1\frac{J}{\sec} = 1W = 0.00134 Hp$
Thermal Conductivity	k	$\frac{W}{m-K}$	$\frac{Btu/hr}{ft-R}$	$1\frac{W}{m-K} = 0.57779 \frac{Btu}{hr-ft-R}$
Specific Heat	Cp	$\frac{J}{kg-K}$	$\frac{Btu}{slugs-R}$	$1\frac{J}{kg-K} = 7700 \frac{Btu}{slugs-R}$
Density	ρ	$\frac{kg}{m^3}$	$\frac{slugs}{ft^3}$	$1\frac{kg}{m^3} = 0.001939 \frac{slugs}{ft^3}$
Mass	m	kg	slugs	$1slug = 32.2 lb_m = 14.6 kg$

http://www.digitaldutch.com/unitconverter/energy.htm

Heat Diffusion Equation: 1D, Transient, Constant Properties (Homogeneous)

Cartesian

$$\frac{d^2T}{dx^2} + \frac{\dot{S}_{gen}}{k_x} = \frac{\rho c_p}{k_x}\frac{dT}{dt}$$

Cylindrical

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{S}_{gen}}{k_r} = \frac{\rho c_p}{k_r}\frac{dT}{dt}$$

Spherical

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{S}_{gen}}{k_r} = \frac{\rho c_p}{k_r}\frac{dT}{dt}$$

Thermal Resistance: 1D Heat Transfer, Steady State, No Internal Heat Generation, Homogenous

$$q = \frac{\Delta T}{\sum R_t}$$

Cartesian

$$R_t = \frac{L}{kA}, q = \frac{\Delta T}{\frac{L}{kA}} \rightarrow \text{HEAT RATE \& } \dot{\text{S}}_{gen} = 0$$

Cylindrical SHELL

$$R_t = \frac{\ln(r_2/r_1)}{2\pi Lk}, q = \frac{\Delta T}{\frac{\ln(r_2/r_1)}{2\pi Lk}} \rightarrow \text{HEAT RATE \& } \dot{\text{S}}_{gen} = 0$$

Spherical SHELL

$$R_t = \frac{(1/r_1)-(1/r_2)}{4\pi k}, q = \frac{\Delta T}{\frac{(1/r_1)-(1/r_2)}{4\pi k}} \rightarrow \text{HEAT RATE \& } \dot{\text{S}}_{gen} = 0$$

Convection / Radiation

$$R_t = \frac{1}{hA}$$

Series Circuit

$$\text{R}_{eq} = \sum R_t$$

Parallel Circuit

$$\text{R}_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots\frac{1}{R_n}}$$

THERMAL CIRCUITS SUMMARY

$\Delta T = T_2 - T_1$	Plane Wall	Cylindrical	Spherical
Heat Equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Profile T(x/r)	$T_1 - \Delta T\frac{x}{L}$	$T_2 + \Delta T\frac{\ln\left(\frac{r}{r_2}\right)}{\ln\left(\frac{r_1}{r_2}\right)}$	$T_1 - \Delta T\left[\frac{1-\frac{r_1}{r}}{1-\frac{r_1}{r_2}}\right]$
Flux ($q''\left[\frac{W}{m^2}\right]$)	$k\frac{\Delta T}{L}$	$\frac{k\Delta T}{r\ln\left(\frac{r_2}{r_1}\right)}$	$\frac{k\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Rate ($q[W]$)	$kA_c\frac{\Delta T}{L}$	$\frac{(2\pi rL)k\Delta T}{r\ln\left(\frac{r_2}{r_1}\right)}$	$\frac{(4\pi r^2)k\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Resistance $\left[\frac{K}{W}\right]$	$\frac{L}{kA_c}$	$\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$	$\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}$

Solution of 1st Order ODE:

$$\frac{d\Theta(t)}{dt} + a\Theta(t) = b(t); or$$

$$a = \frac{hA}{\rho V c}; b(t) = \frac{S_{gen}(t)}{\rho c}$$

has general solution of:

$$\Theta(t) = T(t) - T_{\infty} = e^{-at} \int b(t)\, e^{+at} dt + Ce^{-at}; a > 0$$

where C is an arbitrary constant of integration obtained from initial condition at t = 0.

For constant "b":

$$\Theta(t) = \frac{b}{a} + Ce^{-at}; \text{solving for time, t:}$$

$$\text{t} = \frac{\ln\left(\frac{\Theta(t) - \frac{b}{a}}{C}\right)}{-a}$$

Case	Tip Conditions	Temperature Distribution $\frac{\theta(x)}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection $h\theta(L) = -k\frac{d\theta}{dx}_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\frac{\sinh m(L-x) + (h/mk)\cosh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $\frac{d\theta}{dx}_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed Temp. $\theta(L) = \theta_L$	$\frac{\frac{\theta_L}{\theta_b}\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh(mL) - \frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite Fin mL > 4.6	e^{-mx}	M

$$\theta(x) = T(x) - T_\infty$$

$$\theta_b = \theta(0) = T_b - T_\infty$$

$$m = \left(\frac{hP}{kA_c}\right)^{1/2}$$

$$M = \left(hPkA_c\right)^{1/2}\theta_b$$

$$A_c = Cross\ Section\ Area$$

$$P = Perimeter$$

Temperature distribution and heat loss for fins on uniform cross section

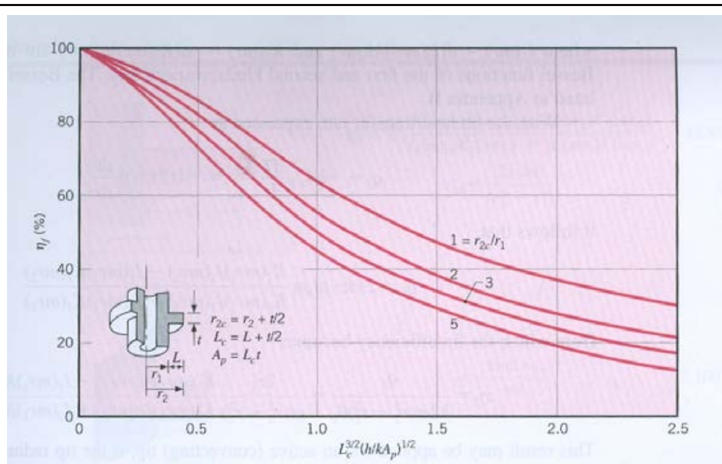


FIGURE 3.20 Efficiency of annular fins of rectangular profile.

Fin Performance

Overall Surface Eficiency

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = Fin\ effectiveness$$

$$\eta_f = \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f\theta_b} = Fin\ efficiency$$

$$q_t = q_{f_{Total}} + q_{wall_{W/O\ fins}} = Nq_f + (A_{wall} - A_{c,b}N)h\theta_b$$

$$= Nq_f + (H - Nt)Ph\theta_b$$

q_t = total heat transfer from fins AND exposed wll surface area

$A_{c,b}$ = cross section of fin at wall/tube base

A_f = total surafce area of fin exposed to fluid

H = height of exposed wall/tube

P = fin perimeter at base of fin

= πD; Pin Fin

= $2w + 2t$; Square Fin

N= number of fins

t = fin thickness

Annular Fin

$$A_f = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$

Geometry

Volume

$$Cylinder: \frac{\pi D^2}{4}L;\ \ Sphere: \frac{4}{3}\pi r^3$$

Surface Area

$$Cylinder: \pi DL;\ \ Sphere: 4\pi r^2$$

Transient Conduction (LUMPED): T(time) Only

$$Bi = Biot\,\# = \frac{UL_c}{k} < 0.1;$$
$U =$ Total resistance to heat transfer at solid boundary
$$L_c = \text{L;Plane wall}$$
$$= \frac{r_0}{2}; Cylinder$$
$$= \frac{r_0}{3}; Spere$$
$$UA \equiv \text{OVERALL THERMAL RESISTANCE=} \frac{1}{\sum R_{th}}\left[\frac{W}{K}\right]$$
$$U \equiv \text{OVERALL HEAT TRANSFER COEFFICIENT=} \frac{1}{A\sum R_{th}}\left[\frac{W}{m^2 - K}\right]$$

$$-E_{out} = E_{st}$$
$$-hA_s(T(t) - T_\infty) = \rho\forall c\frac{dT}{dt}$$
Solution w/o INTERNAL HEAT GENERATION,

Time:
$$t = \frac{\rho\forall c}{hA_s}\ln(\frac{\Theta_i}{\Theta(t)})$$
$$\Theta(t) = T(t) - T_\infty$$
or, Temperature
$$\frac{\Theta(t)}{\Theta_i} = \frac{T(t) - T_\infty}{T_i - T_\infty} = \exp[-(\frac{hA_s}{\rho\forall c})t] = \exp[-\frac{t}{\tau}], \tau = \frac{\rho\forall c}{hA_s}[\text{sec}] \rightarrow \text{TIME CONSTANT}$$
Total Energy
$$Q(t) = (\rho\forall c)\Theta_i[1 - \exp(-\frac{hA_s}{\rho\forall c}t)]$$
Solution WITH INTERNAL HEAT GENERATION

Time:
$$\text{t} = -\frac{1}{a}\ln\left[\frac{\Theta(t) - \frac{b}{a}}{(\Theta_i - \frac{b}{a})}\right]$$
or, Temperature
$$\Theta(t) = T(t) - T_\infty = \frac{b}{a} + \left(\Theta_i - \frac{b}{a}\right)e^{-at}$$
$$b = \frac{\dot{S}_{gen}\left[\frac{W}{m^3}\right]}{\rho c}; a = \frac{hA_s}{\rho\forall c}$$

Spatial Effects (Bi, > 0.1, Fo > 0.2)

$$F_o = \frac{\alpha t}{L^2} \equiv Fourier\ Number$$

$$Bi = \frac{hL_c}{k_{solid}}$$

$$L_c = L; Plane\ Wall$$

$$= \frac{r_0}{2}; Cylinder$$

$$= \frac{r_0}{3}; Sphere$$

INTERPOLATION

$$\text{C=C}_1 + \frac{C2 - C1}{B2 - B1}(B^* - B_1)$$

$$\Theta^*(x^*, t^*) = \frac{T(t) - T_\infty}{T_i - T_\infty} = f(x^*, Bi, Fo)$$

Table 5.1

C_1, Z_1

Table B.4

J_0, J_1

$$Infinite\ Plane\ Wall(Bi = \frac{hL}{k_{solid}}; F_0 = \frac{\alpha t}{L^2}); 0 \le x^* = \frac{x}{L} \le 1.0$$

$$\Theta^*(x^*, t^*) = \Theta_0^*(t^*)\cos(\xi_1 x^*)$$

$$\Theta_0^*(t^*) = C_1 \exp(-\xi_1^2 F_0) \rightarrow \text{CENTERLINE}$$

$$Infinite\ Cylinder(Bi = \frac{hr_0}{k_{solid}}; F_0 = \frac{\alpha t}{r_o^2}); 0 \le r^* = \frac{r}{r_0} \le 1.0$$

$$\Theta^*(r^*, t^*) = \Theta_0^*(t^*) J_0(\xi_1 r^*)$$

$$\Theta_0^*(t^*) = C_1 \exp(-\xi_1^2 F_0)$$

$$Infinite\ Sphere(Bi = \frac{hr_0}{k_{solid}}; F_0 = \frac{\alpha t}{r_o^2}); 0 \le r^* = \frac{r}{r_0} \le 1.0$$

$$\Theta^*(r^*, t^*) = \Theta_0^*(t^*) \frac{1}{\xi_1 r^*} \sin(\xi_1 r^*)$$

$$\Theta_0^*(t^*) = C_1 \exp(-\xi_1^2 F_0)$$

Total Energy

Plane Wall

$$\frac{Q(t)}{Q_0} = 1 - \frac{\sin(\xi_1)}{\xi_1}\Theta_0^*$$

Infinite Cylinder

$$\frac{Q(t)}{Q_0} = 1 - \frac{2\Theta_0^*}{\xi_1} J_1(\xi_1)$$

Sphere

$$\frac{Q(t)}{Q_0} = 1 - \frac{3\Theta_0^*}{\xi_1^3}[\sin(\xi_1) - \xi_1 \cos(\xi_1)]$$

$$Q_0 = \rho \forall c \Theta_i$$

SEMI-INFINITE SOLID

$Case\ 1:$ Constant Surface Temperature:T(0,t)=T_s

$Table\ B.2: erf()$

$$\frac{T(x,t)-T_s}{T_i-T_s}=erf(\frac{x}{2\sqrt{\alpha t}})$$

$$q_s^{''}(t)=\frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}[W/m^2]$$

$$Q[J]\equiv \text{TOTAL ENERGY TRANSFER=}A_s\int_0^{t^*} q_s^{''}(t)dt$$

$$=\frac{A_s k(T_s-T_i)}{\sqrt{\pi\alpha}}\int_0^{t^*} t^{-1/2}dt=\frac{2A_s k(T_s-T_i)}{\sqrt{\pi\alpha}}\sqrt{t}$$

$Case\ 2:$ Constant Surface Heat Flux:$q_s^{''}(x=0)=q_0^{''}$

$$\text{T(x,t)-T}_i=\frac{2q_0^{''}(\alpha t/\pi)^{1/2}}{k}\exp(\frac{-x^2}{4\alpha t})-\frac{q_0^{''}x}{k}erfc(\frac{x}{2\sqrt{\alpha t}})$$

Case 3: Surface Convetion

$$\frac{T(x,t)-T_s}{T_i-T_s}=erfc(\frac{x}{2\sqrt{\alpha t}})-[\exp(\frac{hx}{k}+\frac{h^2\alpha t}{k^2})][erfc(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k})]$$

EXTERNAL FORCED CONVECTION

$$\mathrm{Re}_x \equiv \mathrm{Renolyds\#} = \frac{\rho U_\infty x}{\mu}, \delta(x) = \frac{5x}{\sqrt{\mathrm{Re}}}$$

LAMINAR FLOW--ISO THERMAL PLATE

$$\mathrm{Re}_x < 5x10^5$$

$$NU_x = \frac{h_x x}{k_{fluid}} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}, 0.6 \le \mathrm{Pr} \le 50,$$

$$NU_x = \frac{h_x x}{k_{fluid}} = \frac{0.3387\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.0468}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}; \mathrm{Pr} \ge 100$$

LAMINAR FLOW-CONSTANT HEAT FLUX PLATE

$$NU_x = \frac{h_x x}{k_{fluid}} = 0.453\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}, \mathrm{Pr} \ge 0.6$$

$$h_x = \frac{NU_x \bullet k_{fluid}}{x} \rightarrow \text{LOCAL HEAT TRANSFER COEFF.}$$

$$\mathrm{Pr} \equiv \mathrm{Prandtl\#} = \frac{\mu c_p}{k_{fluid}} = \nu/\alpha \equiv \frac{\text{Diffusivity of Momentun}}{\text{Diffusivity of Heat}}$$

Thermal Boundary Layer

$$\delta_t(x) \approx \frac{\delta(x)}{\mathrm{Pr}^{1/3}} = \frac{1}{\mathrm{Pr}^{1/3}}\frac{5x}{\sqrt{\mathrm{Re}}}$$

PROPERTIES

$$\mathrm{T}_{film} = \frac{T_\infty + T_s}{2}$$

AVERAGE

$$\bar{h}_{x^*} = \frac{1}{x^*}\int_0^{x^*} h_x dx = 2h_x \rightarrow \overline{NU}_{x^*} = \frac{\bar{h}_x x^*}{k_{fluid}} = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}, 0.6 \le \mathrm{Pr} \le 50$$

TURBULENT FLOW--ISO THERMAL HEAT FLUX PLATE

$\mathrm{Re}_x > 5x10^5$

$$NU_x = \frac{h_x x}{k_{fluid}} = 0.029\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}; 0.6 \le \mathrm{Pr} \le 60$$

TURBULENT FLOW--CONSTANT HEAT FLUX

$$NU_x = \frac{h_x x}{k_{fluid}} = 0.0308\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}; 0.6 \le \mathrm{Pr} \le 60$$

$c_{f,x} = 0.0592\,\mathrm{Re}_x^{-1/5}, 5x10^5 \le \mathrm{Re}_{x^*} \le 10^8$

$\delta(x) = 0.37\,\mathrm{Re}_x^{-1/5}$

*Due to enhanced mixing, the turbulent boundary layer grows more rapidly and has larger friction friction and convection coefficicents (i.e. more heat transfer and more friction)

MIXED CONDITIONS - LAMINAR and TURBULENT

$$\overline{\mathrm{NU}}_{x^*} = (0.037\,\mathrm{Re}^{4/5}{}_{x,c} - A)\,\mathrm{Pr}^{1/3} = \frac{h_{x^*} x^*}{k_{fluid}}; 0.6 \le \mathrm{Pr} \le 60, 5x10^5 \le \mathrm{Re}_{x^*} \le 10^8$$

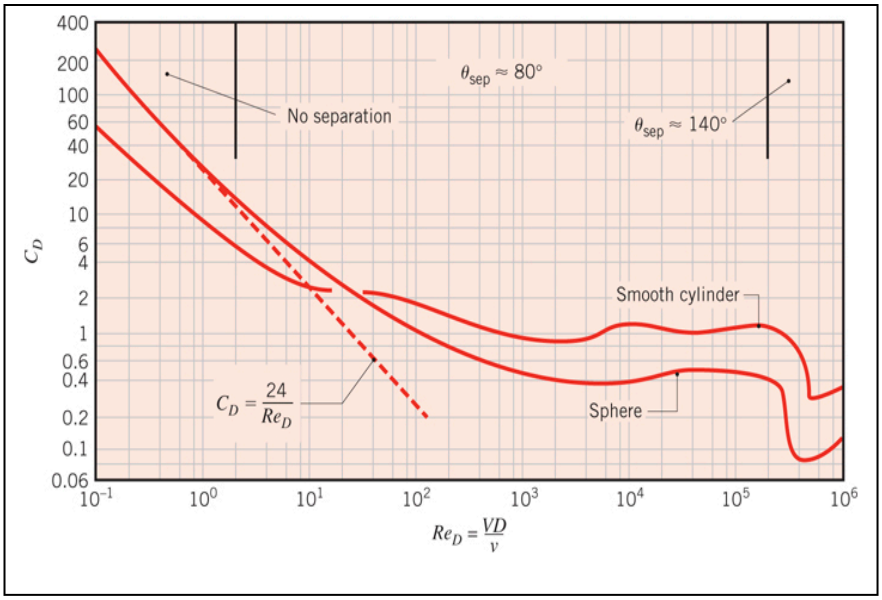
$A = 0.037\,\mathrm{Re}^{4/5}{}_{x,c} - 0.664\,\mathrm{Re}^{1/2}{}_{x,c}$ → FOR TRIPPED TURB BOUNDARY, A=0.0

CYLINDERS

RELATIONS DRAG

$$\mathrm{Re}_D = \frac{\rho V D}{\mu_{fluid}}$$

$$C_D = \frac{F_D}{A_f \dfrac{\rho V^2}{2}}$$



$$C_D = \frac{24}{\mathrm{Re}_D} \rightarrow \text{CREEPING FLOWS} \rightarrow \mathrm{Re}_D \le 0.5$$

CYLINDER/SPHERE IN CROSS FLOW HEAT TRANSFER

CYLINDER

$$\overline{NU}_D = \frac{\bar{h}_D D}{k_{fluid}} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Re}_D = \frac{\rho \bar{V} D}{\mu}$$

PROPERTIES @ T_{FILM}

PRANDTL > 0.7

TABLE 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

MORE ACCURATE

$\mathrm{Re}_D\mathrm{Pr} \geq 0.2$

$$\overline{NU}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282,000}\right)^{5/8}\right]^{4/5} = \frac{\bar{h}_D D}{k_{fluid}}$$

SPHERE IN CROSS FLOW

$$\overline{NU}_D = \frac{\bar{h}_D D}{k_{fluid}} = 2 + (0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3})\,\mathrm{Pr}^{0.4}\left[\frac{\mu(T_\infty)}{\mu(T_s)}\right]^{1/4}$$

$$\mathrm{Re}_D = \frac{\rho \bar{V} D}{\mu}$$

All Other Properties Evaluated at T_∞

INTERNAL FLOW---HYDRODYNAMICS

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu_{fluid}}, u_m \equiv \text{mean velocity}$$

$$\dot{m} \equiv \text{mass flow rate} = \rho u_m A_c$$

$$A_c \equiv \text{duct cross section area:} \frac{\pi D^2}{4}$$

Pressure Drop & Friction Coefficient

$$\Delta P = \mathrm{f}\,\frac{\rho u^2{}_m}{2}\frac{\Delta x}{D}, \mathrm{c}_f \equiv \frac{\tau_s}{\frac{\rho u^2{}_m}{2}} = \frac{f}{4}$$

Power

$$P = \frac{\dot{m}\Delta P}{\rho} = Q\Delta P$$

LAMINAR

$$0 \le \mathrm{Re}_D \le 2300$$

Friction Factor

$$\mathrm{f} = \frac{64}{\mathrm{Re}_D}$$

TURBULENT

$$\mathrm{Re}_D > 2300$$

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left(\left(\frac{\varepsilon / D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right)$$

INTERNAL FLOW—HEAT TRANSFER

Newton's Law of Cooling

$$q_s = hA_s(T_s - T_m)[W] = \dot{m}c_p(T_{m,out} - T_{m,in})[W]$$

Energy Balance

$$dq_{conv} = q_s^{"}Pdx = \dot{m}c_p dT_m$$

Combining

$$\frac{dT_m}{dx} = \frac{q_s^{"}P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m)$$

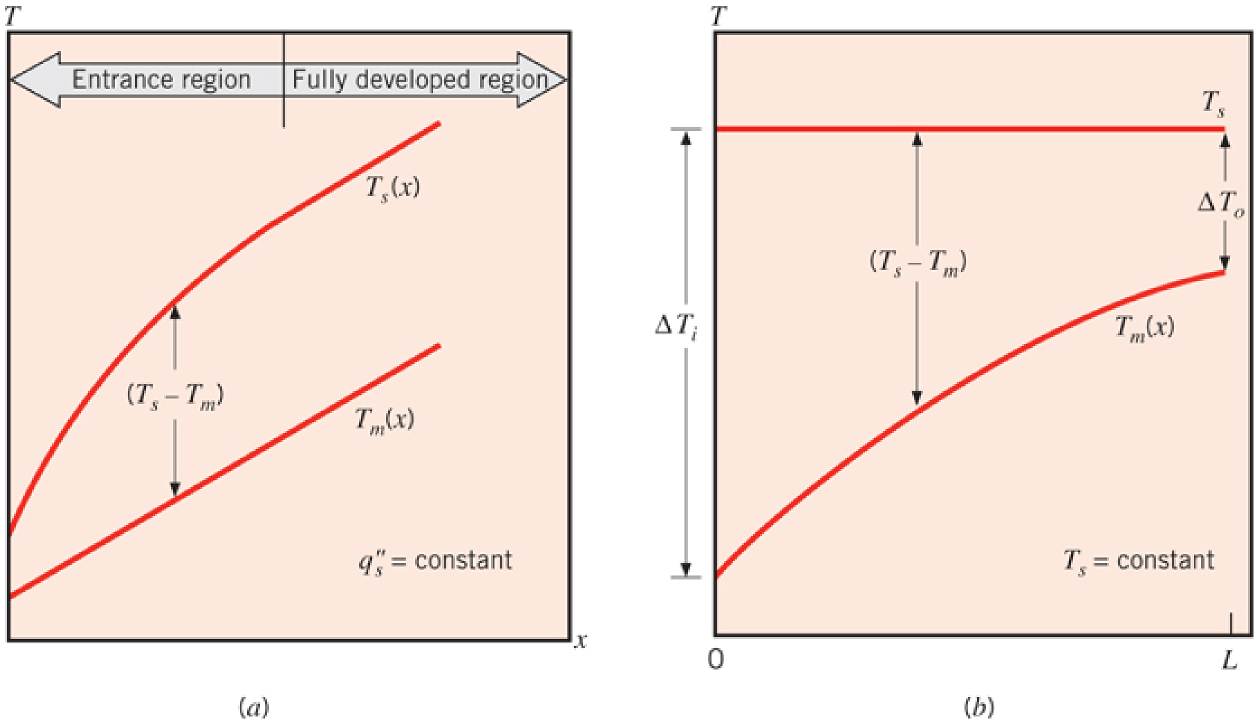
Constant Surface Heat Flux

$$\frac{dT_m}{dx} = \frac{q_s^{"}P}{\dot{m}c_p} \neq f(x) \rightarrow \text{Full Developed Flow}$$

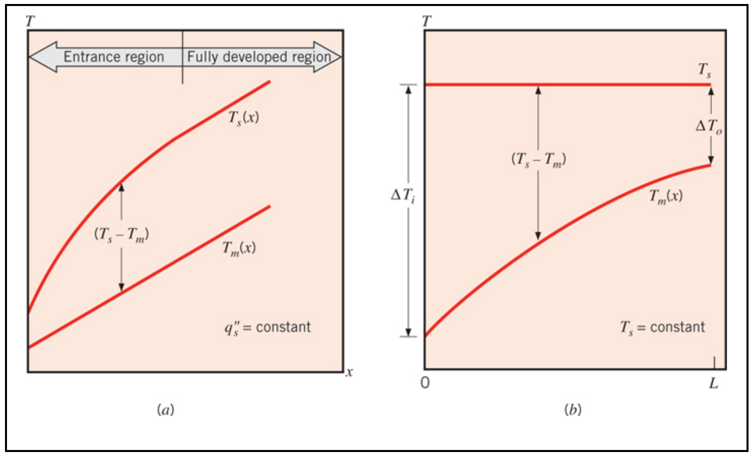
$P \equiv \text{PERIMETER} = \pi D$

INTEGRATING

$$T_m(x) = T_{m,i} + \frac{q_s^{"}P}{\dot{m}c_p} \bullet x \quad \rightarrow q_s^{"} = \text{constant}$$



INTERNAL FLOW—HEAT TRANSFER



CONSTANT SURFACE TEMPERATURE

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\Delta T$$

Seperating Variables

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}c_p}\int_0^L hdx$$

$$(1): \ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left[\frac{1}{L}\int_0^L hdx\right] = -\frac{PL}{\dot{m}c_p}\overline{h_L} = -\frac{A_s}{\dot{m}c_p}\overline{h_L} \rightarrow T_s = \text{CONSTANT}$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left[-\frac{A_s\overline{h_L}}{\dot{m}c_p}\right] \rightarrow \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{R_t}\right] \rightarrow T_s = \text{CONSTANT}$$

Heat Transfer

$$(2): q_{conv} = \dot{m}c_p[(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

BUT:

$$\dot{m}c_p = -\frac{A_s\overline{h_L}}{\ln\frac{\Delta T_o}{\Delta T_i}} \text{(From 1:)} \rightarrow \textit{SUB} \text{ INTO (2)}$$

$$q_{conv} = \dot{m}c_p(\Delta T_i - \Delta T_o) = A_s\overline{h_L}\frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}} = A_s\overline{h_L}\Delta T_{LM} = \frac{\Delta T_{LM}}{R_t}$$

SUMMARY

CONSTANT HEAT FLUX

Newton's Law of Cooling

$$q_s = hA_s(T_s - T_m)[W] = \dot{m}c_p(T_{m,out} - T_{m,in})[W]$$

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m}c_p}\bullet x$$

CONSTANT TEMPERATURE

$$q_{conv} = \dot{m}c_p(T_{m,out} - T_{m,in}) = \dot{m}c_p(\Delta T_i - \Delta T_o) = A_s\overline{h_L}\bullet\left[\frac{\Delta T_o - \Delta T_i}{\ln\frac{\Delta T_o}{\Delta T_i}}\right] = A_s\overline{h_L}[\Delta T_{LM}] = \frac{\Delta T_{LM}}{R_t}$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left[-\frac{P\bullet x}{\dot{m}c_p}\overline{h_x}\right] = \exp\left[-\frac{A_s\overline{h_x}}{\dot{m}c_p}\right] = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{R_t}\right]$$

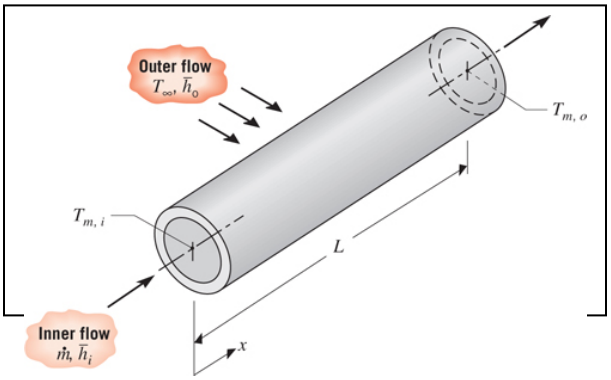
$$P = \pi D, PL = \text{AREA} \rightarrow A_s$$

SUMMARY – SPECIAL CASE: INTERNAL FLOW/EXTERNAL CONVECTION

CONSTANT TEMPERATURE

$$q_{conv} = A_s \overline{h_L} \frac{\Delta T_o - \Delta T_i}{\ln \frac{\Delta T_o}{\Delta T_i}} = \frac{\Delta T_{LM}}{\sum R_{th}}$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left[-\frac{1}{\dot{m}c_p}\frac{1}{\sum R_{th}}\right] = \exp\left[-\frac{UA}{\dot{m}c_p}\right]$$



INTERNAL FLOW: HEAT TRANSFER

LAMINAR

$0 \le \mathrm{Re}_D \le 2300$

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 4.36 \rightarrow \text{Constant Heat Flux}$$

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 3.66 \rightarrow \text{Constant Surface Temperature}$$

Evaluate Properties at T_{mean}

TURBULENT

$$\overline{NU_D} = \frac{\bar{h}D}{k_{fluid}} = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n \rightarrow \text{DITTUS-BOELTER}$$

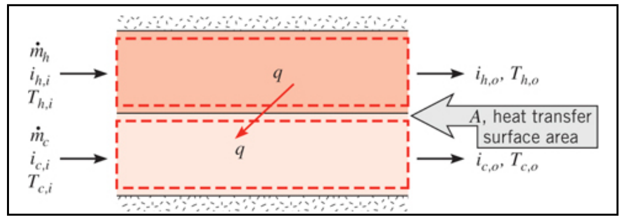
n=0.4 $\rightarrow$ Heating ($T_s > T_m$)

n=0.3 $\rightarrow$ Cooling ($T_s < T_m$)

Evaluate Properties at T_{mean}

HEAT EXCHANGERS

Overall Heat Transfer Coefficient & Fouling Factors



- **An essential part of any heat exchanger analysis is determination of the overall heat transfer coefficient.**
- **During normal operations, HOT and COLD surfaces are often subject to fouling by fouling impurities, rust formation, or their reactions between the fluid and the wall material.**

TABLE 11.1 Representative Fouling Factors [1]

Fluid	R''_f ($m^2 \cdot K/W$)
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam (nonoil bearing)	0.0001

$$UA = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{h_c A_c} + \frac{R''_c \frac{m^2-K}{W}}{A_c} + R_{t_{CONDUCTION}} + \frac{1}{h_h A_h} + \frac{R''_h \frac{m^2-K}{W}}{A_h}}$$

Heat Exchangers

LMTD METHOD

$$q = \overline{U} A \Delta T_{LM} = \overline{U_o} A_o \Delta T_{LM} = \overline{U_i} A_i \Delta T_{LM}$$

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln(\frac{\Delta T_1}{\Delta T_2})}$$

Parallel – Flow

$$\Delta T_1 = T_{h,in} - T_{c,in}$$

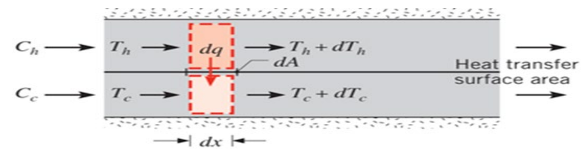
$$\Delta T_2 = T_{h,out} - T_{c,out}$$

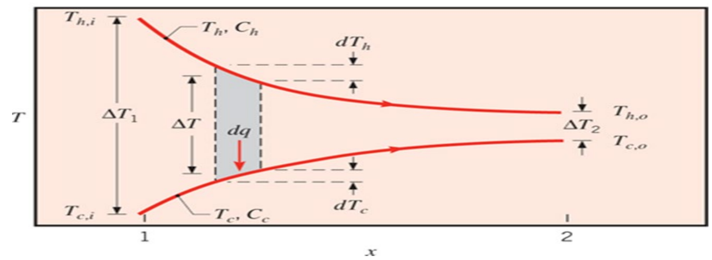
$$T_{h,out} > T_{c,out}$$

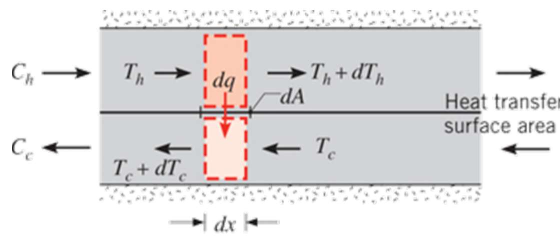
Counter – Flow

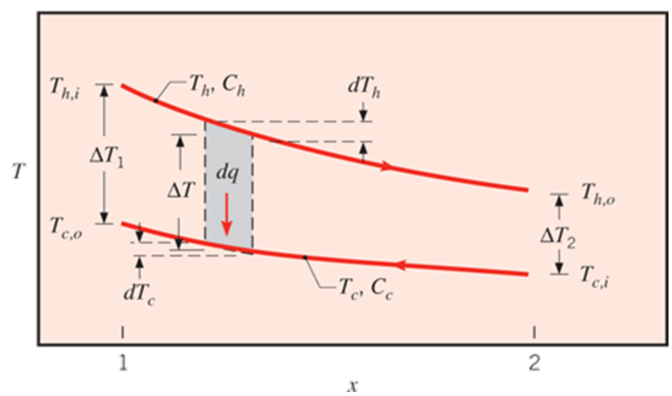
$$\Delta T_1 = T_{h,in} - T_{c,out}$$

$$\Delta T_2 = T_{h,out} - T_{c,in}$$





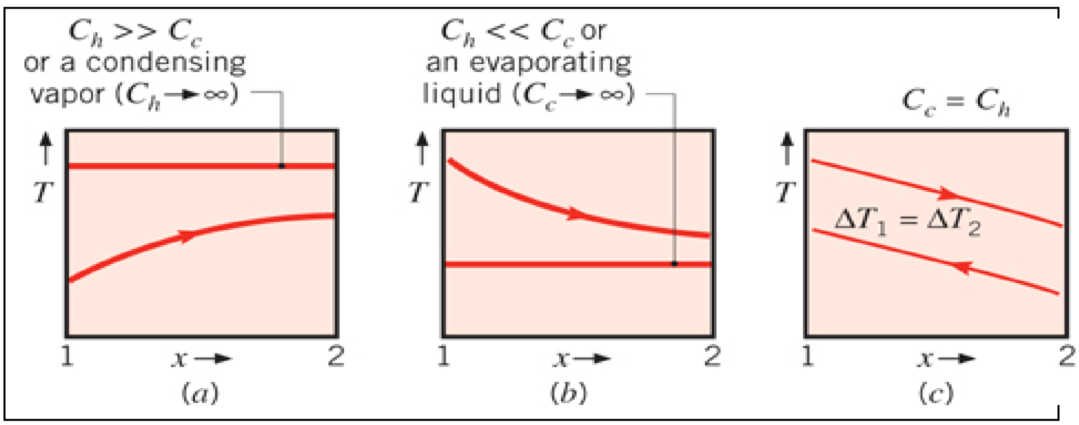




SPECIAL CASE

$C \equiv \text{THERMAL CAPACITY} = \dot{m}cp$

- ***CONDENSATION*** occurs at a constant temperature (i.e. ***THERMODYNAMICS***)
  - Implies a ***VERY LARGE*** THERMAL CAPACITY OF ***HOT*** FLUID (Ch).
- ***EVAPORATION*** occurs at a constant temperature (i.e. THERMODYNAMICS)
  - Implies a ***VERY LARGE*** THERMAL CAPACITY OF ***COLD*** FLUID (Cc).



$$q = \dot{m}_h \bullet h_{fg}$$

NTU METHOD

$$C \equiv \text{THERMAL CAPACITY} = \dot{m}cp$$

$$C_h = (\dot{m}cp)_h, C_c = (\dot{m}cp)_c \rightarrow q = (\dot{m}cp)_{cold}\,\Delta T_{cold} = (\dot{m}cp)_{hot}\,\Delta T_{hot}$$

$$C_{\min} = \text{MINIMUM}(C_h, C_c)$$

$$\varepsilon \equiv \text{ HX EFFECTIVENESS}$$

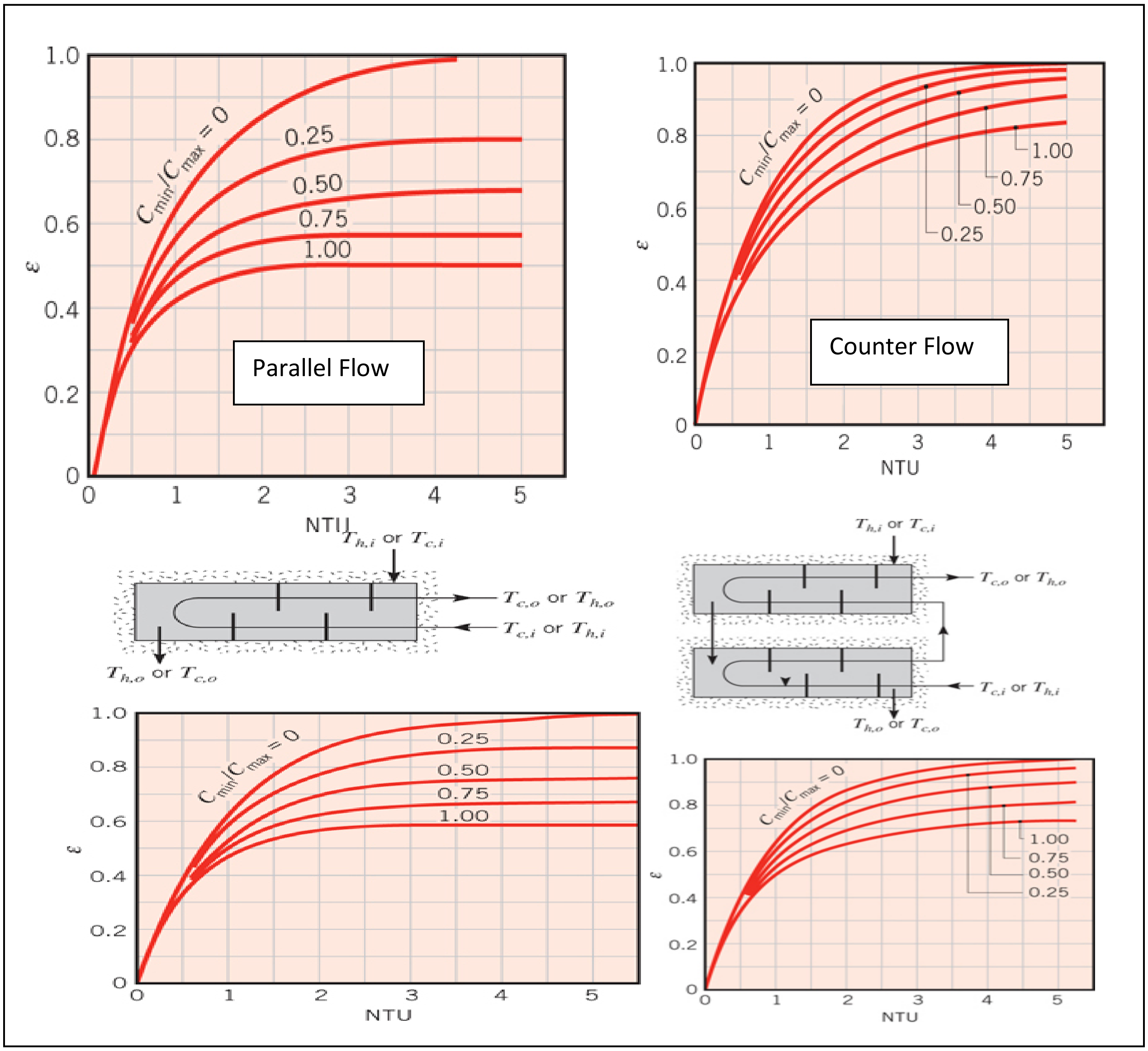
$$\varepsilon \equiv \frac{q}{q_{\max}} = \frac{q}{C_{\min}(T_{h,i} - T_{c,i})}$$

$$= \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})}$$

$$= \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})}$$

$C_{\min}(T_{h,i} - T_{c,i}) \rightarrow$ MAXIMUM POSSIBLE FLUID HEAT TRANSFER

$\varepsilon \rightarrow F(NTU, \frac{C_{\min}}{C_{\max}}), NTU \equiv$ Number of Transfer Units (Dimensionless)



$$NTU_{TOTAL} \equiv \frac{UA(TOTAL)}{C_{min}(TOTAL)}$$

$UA \equiv$ TOTAL RESISTANCE--ALL TUBES

$C_{min} \equiv$ TOTAL CAPACITANCE--ALL TUBES

$$NTU_{TOTAL} = \left(\frac{UA}{C_{min}}\right)_{TOTAL}$$

$$A = \text{TOTAL AREA} = \pi DL \bullet \#\text{tubes}$$

$$L = \frac{Length}{tube-pass} = \frac{A}{\pi D \bullet \#\text{tubes} \bullet \#shells \bullet pass/shell}$$

NTU RELATIONSHIPS (sometimes easier)

$$C_r = \frac{C_{\min}}{C_{\max}} = 0 \rightarrow \text{ALL EXCHANGERS}$$

$$\varepsilon = 1 - \exp(-NTU), NTU = -\ln(1-\varepsilon)$$

COUNTER FLOW$(C_r = 1.0)$

$$\varepsilon = \frac{NTU}{1+NTU}, NTU = \frac{\varepsilon}{1-\varepsilon}$$

Parallel Flow

$$\varepsilon = \frac{1-\exp\left[-NTU(1+C_r)\right]}{1+C_r}$$

COUNTER FLOW$(C_r < 1.0)$

$$\varepsilon = \frac{1-\exp\left[-NTU(1-C_r)\right]}{1-C_r\exp\left[-NTU(1-C_r)\right]}, NTU = \frac{1}{C_r-1}\ln\left(\frac{\varepsilon-1}{\varepsilon C_r-1}\right)$$

ONE SHELL PASS (n=2,4,8,16..tube passes)

$$NTU_1 = -(1+C_r^2)^{-1/2}\ln\left(\frac{E-1}{E+1}\right), E = \frac{\frac{2}{\varepsilon_1}-(1+C_r)}{(1+C_r^2)^{+1/2}}, \varepsilon_1 = \frac{F-1}{F-C_r}, F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad NTU = n(NTU)_1$$

B.2 *Gaussian Error Function*[1]

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679
0.06	0.06762	0.44	0.46622	1.16	0.89910
0.08	0.09008	0.48	0.50275	1.20	0.91031
0.10	0.11246	0.52	0.53790	1.30	0.93401
0.12	0.13476	0.56	0.57162	1.40	0.95228
0.14	0.15695	0.60	0.60386	1.50	0.96611
0.16	0.17901	0.64	0.63459	1.60	0.97635
0.18	0.20094	0.68	0.66378	1.70	0.98379
0.20	0.22270	0.72	0.69143	1.80	0.98909
0.22	0.24430	0.76	0.71754	1.90	0.99279
0.24	0.26570	0.80	0.74210	2.00	0.99532
0.26	0.28690	0.84	0.76514	2.20	0.99814
0.28	0.30788	0.88	0.78669	2.40	0.99931
0.30	0.32863	0.92	0.80677	2.60	0.99976
0.32	0.34913	0.96	0.82542	2.80	0.99992
0.34	0.36936	1.00	0.84270	3.00	0.99998

[1]The Gaussian error function is defined as

$$\text{erf}\, w = \frac{2}{\sqrt{\pi}} \int_0^w e^{-v^2}\, dv$$

The complementary error function is defined as

$$\text{erfc}\, w \equiv 1 - \text{erf}\, w$$

B.4 Bessel Functions of the First Kind

x	$J_0(x)$	$J_1(x)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

Table 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation		
Parallel flow	$\text{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$		(11.28b)
Counterflow	$\text{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$	$(C_r < 1)$	
	$\text{NTU} = \dfrac{\varepsilon}{1-\varepsilon}$	$(C_r = 1)$	(11.29b)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$(\text{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$		(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1+C_r)}{(1+C_r^2)^{1/2}}$		(11.30c)
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with		
	$\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \text{NTU} = n(\text{NTU})_1$		(11.31b, c, d)
Cross-flow (single pass)			
C_{max} (mixed), C_{min} (unmixed)	$\text{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$		(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\text{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$		(11.34b)
All exchangers ($C_r = 0$)	$\text{NTU} = -\ln(1-\varepsilon)$		(11.35b)

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728
1200	0.2902	1.175	473.0	162.9	76.3	224	0.728
1300	0.2679	1.189	496.0	185.1	82	257	0.719
1400	0.2488	1.207	530	213	91	303	0.703
1500	0.2322	1.230	557	240	100	350	0.685
1600	0.2177	1.248	584	268	106	390	0.688
1700	0.2049	1.267	611	298	113	435	0.685
1800	0.1935	1.286	637	329	120	482	0.683
1900	0.1833	1.307	663	362	128	534	0.677
2000	0.1741	1.337	689	396	137	589	0.672
2100	0.1658	1.372	715	431	147	646	0.667
2200	0.1582	1.417	740	468	160	714	0.655
2300	0.1513	1.478	766	506	175	783	0.647
2400	0.1448	1.558	792	547	196	869	0.630
2500	0.1389	1.665	818	589	222	960	0.613
3000	0.1135	2.726	955	841	486	1570	0.536
Ammonia (NH_3), $\mathcal{M}$ = 17.03 kg/kmol							
300	0.6894	2.158	101.5	14.7	24.7	16.6	0.887
320	0.6448	2.170	109	16.9	27.2	19.4	0.870
340	0.6059	2.192	116.5	19.2	29.3	22.1	0.872
360	0.5716	2.221	124	21.7	31.6	24.9	0.872
380	0.5410	2.254	131	24.2	34.0	27.9	0.869

TABLE A.6 Thermophysical Properties of Saturated Water[a]

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	Specific Volume (m³/kg) v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	Specific Heat (kJ/kg · K) $c_{p,g}$	Viscosity (N · s/m²) $\mu_f \cdot 10^6$	Viscosity (N · s/m²) $\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k_f \cdot 10^3$	Thermal Conductivity (W/m · K) $k_g \cdot 10^3$	Prandtl Number Pr_f	Prandtl Number Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K⁻¹)	Temperature, T (K)
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05	273.15
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74	275
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04	280
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1	285
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0	290
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5	295
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1	300
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6	305
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9	310
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4	315
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7	320
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2	325
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504.0	330
335	0.2167	1.018	7.09	2354	4.186	1.920	453	10.49	656	22.0	2.88	0.916	65.8	535.5	335
340	0.2713	1.021	5.74	2342	4.188	1.930	420	10.69	660	22.3	2.66	0.925	64.9	566.0	340
345	0.3372	1.024	4.683	2329	4.191	1.941	389	10.89	664	22.6	2.45	0.933	64.1	595.4	345
350	0.4163	1.027	3.846	2317	4.195	1.954	365	11.09	668	23.0	2.29	0.942	63.2	624.2	350
355	0.5100	1.030	3.180	2304	4.199	1.968	343	11.29	671	23.3	2.14	0.951	62.3	652.3	355
360	0.6209	1.034	2.645	2291	4.203	1.983	324	11.49	674	23.7	2.02	0.960	61.4	697.9	360
365	0.7514	1.038	2.212	2278	4.209	1.999	306	11.69	677	24.1	1.91	0.969	60.5	707.1	365
370	0.9040	1.041	1.861	2265	4.214	2.017	289	11.89	679	24.5	1.80	0.978	59.5	728.7	370
373.15	1.0133	1.044	1.679	2257	4.217	2.029	279	12.02	680	24.8	1.76	0.984	58.9	750.1	373.15
375	1.0815	1.045	1.574	2252	4.220	2.036	274	12.09	681	24.9	1.70	0.987	58.6	761	375
380	1.2869	1.049	1.337	2239	4.226	2.057	260	12.29	683	25.4	1.61	0.999	57.6	788	380
385	1.5233	1.053	1.142	2225	4.232	2.080	248	12.49	685	25.8	1.53	1.004	56.6	814	385
390	1.794	1.058	0.980	2212	4.239	2.104	237	12.69	686	26.3	1.47	1.013	55.6	841	390
400	2.455	1.067	0.731	2183	4.256	2.158	217	13.05	688	27.2	1.34	1.033	53.6	896	400
410	3.302	1.077	0.553	2153	4.278	2.221	200	13.42	688	28.2	1.24	1.054	51.5	952	410
420	4.370	1.088	0.425	2123	4.302	2.291	185	13.79	688	29.8	1.16	1.075	49.4	1010	420
430	5.699	1.099	0.331	2091	4.331	2.369	173	14.14	685	30.4	1.09	1.10	47.2		430

THERMAL *RADIATION*

MECH-420 Heat Transfer

Dr. K. J. Berry

NET RADIATION EXCHANGE

- Consider a small blackbody object at Temperature Ts and completely enclosed and exchanging radiation with the surroundings at Temperature Tsur < Ts as shown below.

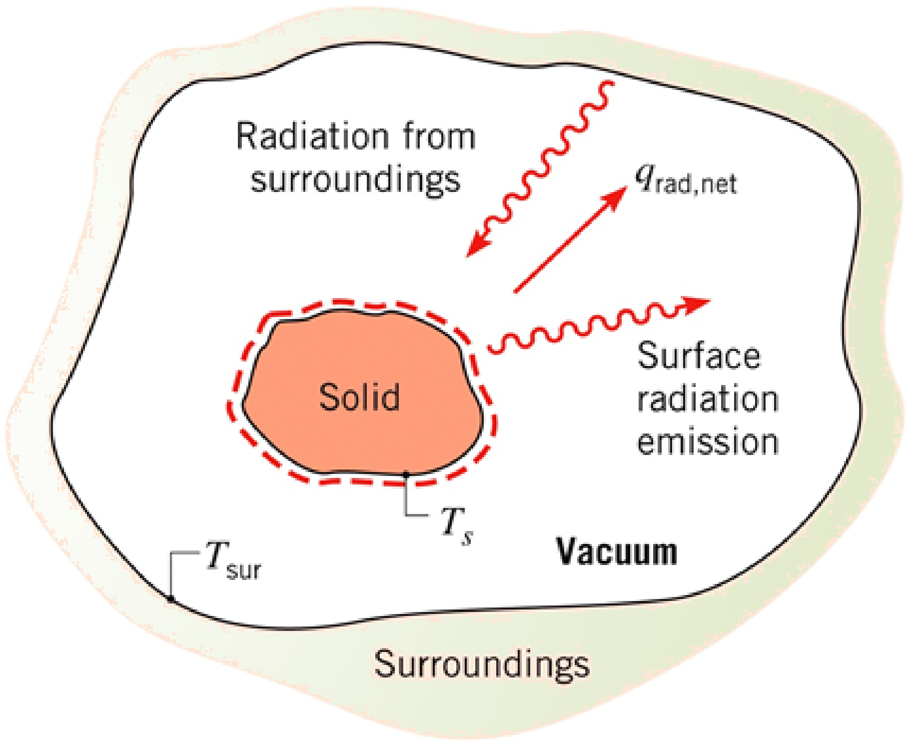


Figure 12.1

The 'net' radiation exchange between the blackbody and the surrounding enclosure is:

$$q''_{rad}\left[\frac{W}{m^2}\right] = \sigma \frac{W}{m^2 - K^4}(T_s^4 - T_{surr}^4)$$

REAL SURFACE RESISTANCE

Every "***real***" surface has a resistance to thermal radiation emission. This resistance and net radiation heat transfer exchange can be expressed by:

$$q_{net} = \frac{E_b - J}{\dfrac{1-\varepsilon}{\varepsilon A}}$$

Where $E_b - J$ is the driving surface potential and where J is known as the surface Radiosity (W/m2) and $\frac{1-\varepsilon}{\varepsilon A}$ is the surface resistance to radiation emission. Note for a blackbody $\varepsilon = 1$, and the resistance goes to zero.

$\frac{1}{A_i F_{ij}}$

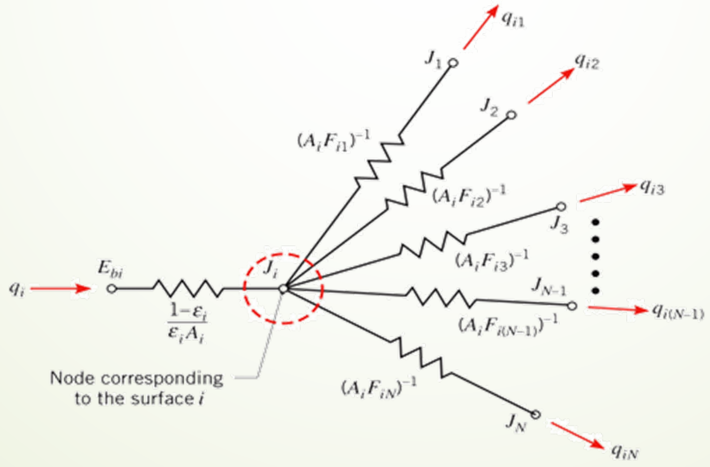
"n" Surfaces Exchange

To complete the exchange analysis we need to consider a radiation energy balance for each surface shown above to the right. Due to the distance between surface and the ***RELATIVE SHAPE*** of each surface, not all the energy that is emitted by surface "1", say will reach surface "2".

This distance and geometry differences result in a "surface" resistance for surface "i" of the form: $\frac{1}{A_i F_{ij}}$

. Where F_{ij} (shape/view factor) is the fraction of energy that leaves surface "I", and strikes surface "j" directly.

So a radiation balance of an arbitrary surface "I" and exchanging radiation with "n" other surfaces (including itself) becomes:



$$\frac{E_{bi}-J_i}{\frac{1-\varepsilon_i}{\varepsilon_i A_i}} = \sum_{j=1}^{n} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}; \text{applied to every surface}$$

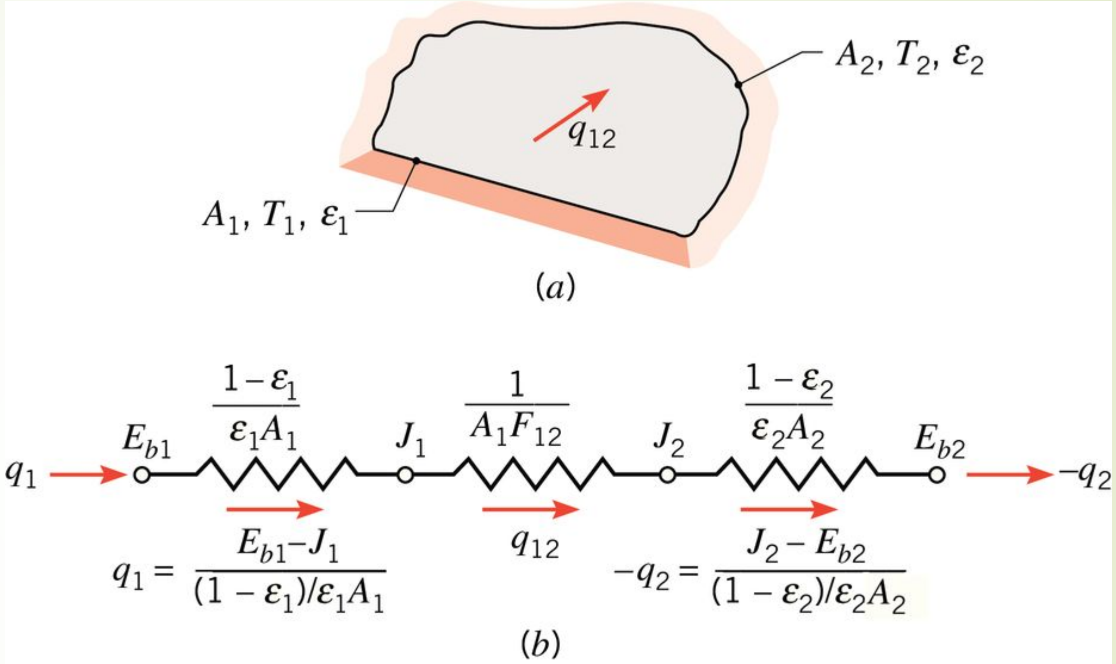
and once J's are known:

$$q_i = \sum_{j=1}^{n} \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}}$$

Radiation Balance – 2 Surface Problem

$$1: q_1 = \frac{E_{b1} - J_1}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_1 - J_2}{\dfrac{1}{A_1 F_{12}}}$$

$$2: q_2 = \frac{E_{b2} - J_2}{\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \frac{J_2 - J_1}{\dfrac{1}{A_2 F_{21}}}$$



Two equations and two unknowns for J_1 and J_2.

Assuming T_1 and T_2 are known.

THREE REAL SURFACES

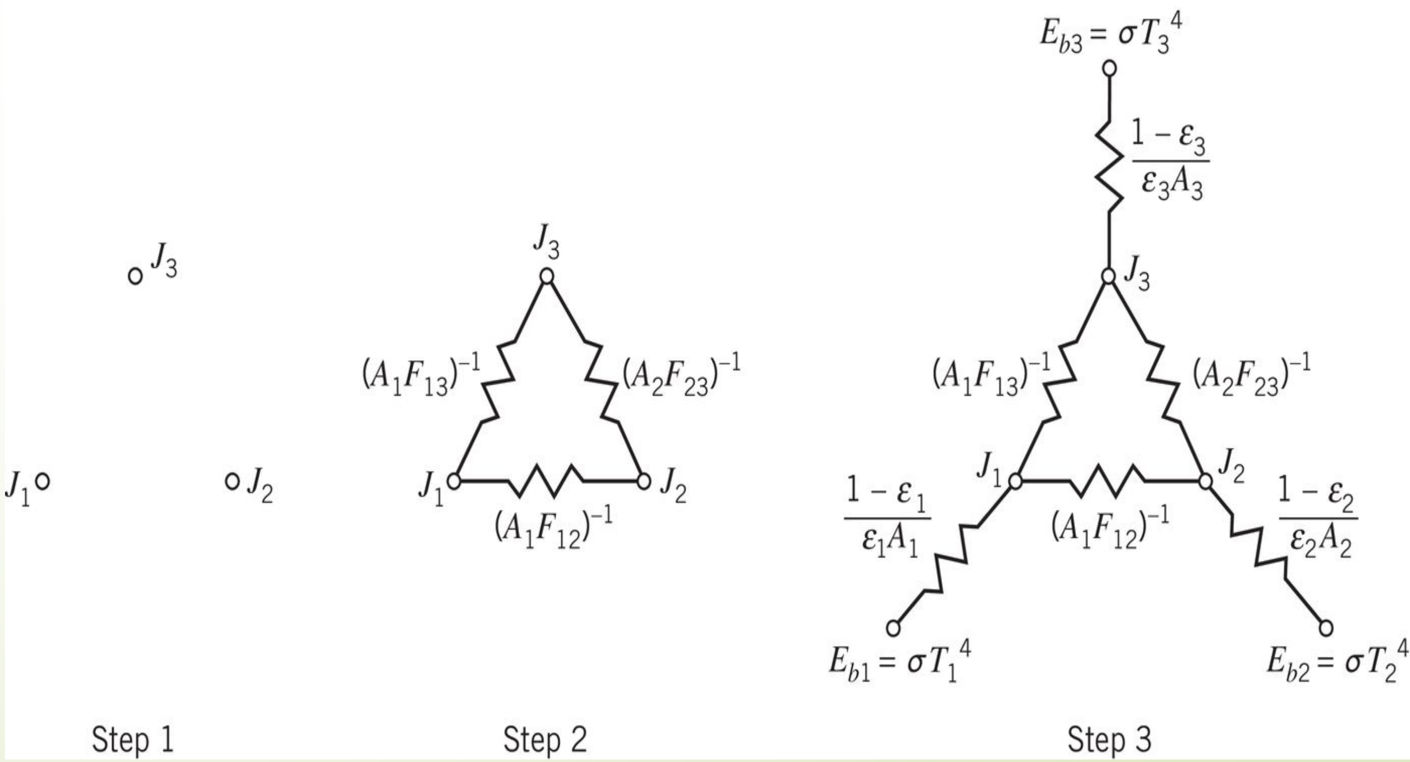
Consider two "real" surface that only exchange radiation with each other. The net radiation from each surface and the next exchange between each surface can be expressed as:

$$q_1 = \frac{E_{b1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}}; \text{net radiation to/from surface 1}$$

$$q_2 = \frac{E_{b2} - J_2}{\frac{1-\varepsilon_2}{\varepsilon_2 A_2}}; \text{net radiation to/from surface 2}$$

$$q_1 = -q_2$$

$$E = \sigma T^4$$



SHAPE FACTORS

SHAPE FACTORS--RECIPROCITY

The shape factor A_iF_{ij} is the fraction of energy that leaves surface “I” and strikes surface “j”. Of course this must be a reciprocal relationship, i.e.:

In general for any two arbitrary surfaces

$$A_iF_{ij} = A_jF_{ji}$$

Example, for surfaces 1 *and* 2

$$A_1F_{12} = A_2F_{21}$$

Also, since the “F” represents a fraction of the total energy leaving a surface and since energy is conserved the following summation rule applies as:

$$\sum_{j=1}^{n} F_{ij} = 1; \text{for every surface}$$

for example for surface 1:

$$F_{11}+F_{12}=1; \text{and}$$

for surface 2, etc.

$$F_{21}+F_{22}=1$$

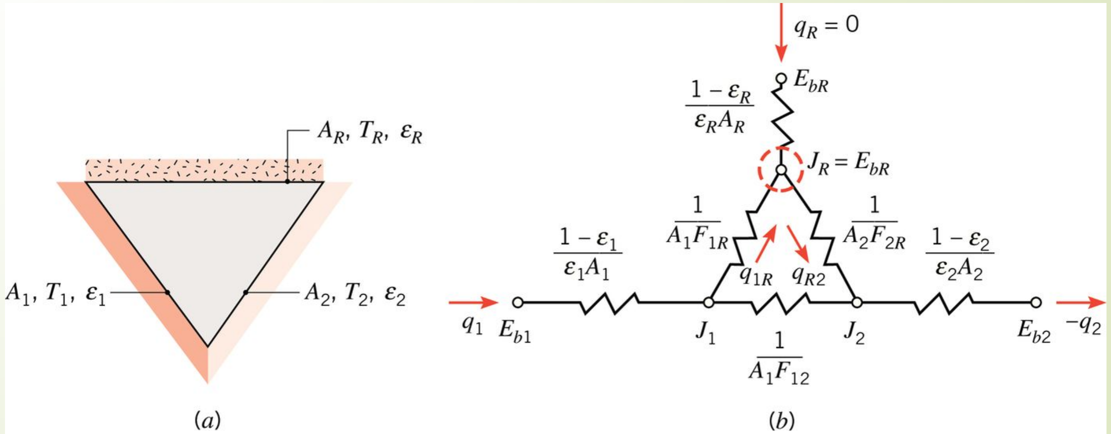
INSULATED SURFACES AND SURFACES WITH LARGE AREAS

- For ***insulated*** surfaces (re-re-radiating), this behaves "like" a ***blackbody*** and as such the surface resistance go to zero and Eb=J (q=0).

- Likewise for "large" areas (i.e. ***LARGE ROOM***), the surface resistance approach zero and once again the reduction in the thermal circuit becomes Eb=J (q=0). For example, consider the thermal circuit for the following 3-surface problem with one insulated surface.

Surface "r" Balance

$$\frac{J_r - J_1}{\frac{1}{A_r F_{r1}}} + \frac{J_r - J_2}{\frac{1}{A_r F_{r2}}} = 0$$



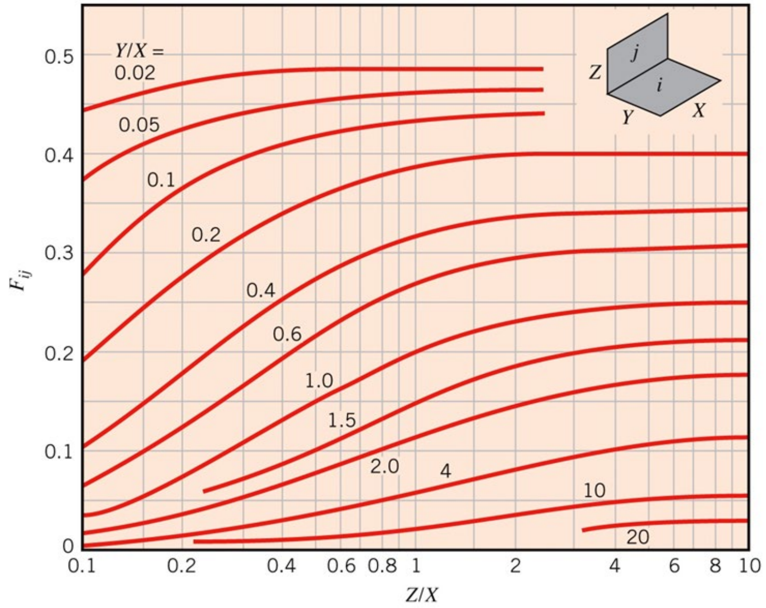


Figure 13.6

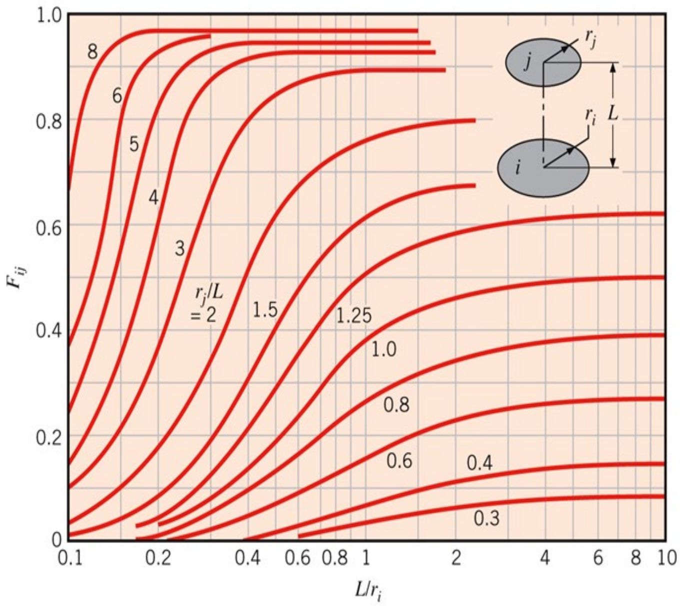


Figure 13.5

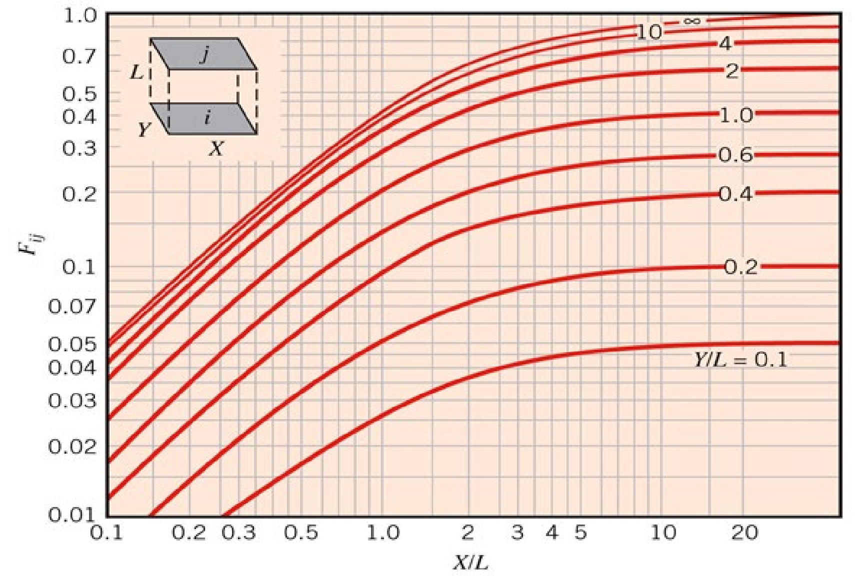


Figure 13.4

15 View Factors Standard Surfaces