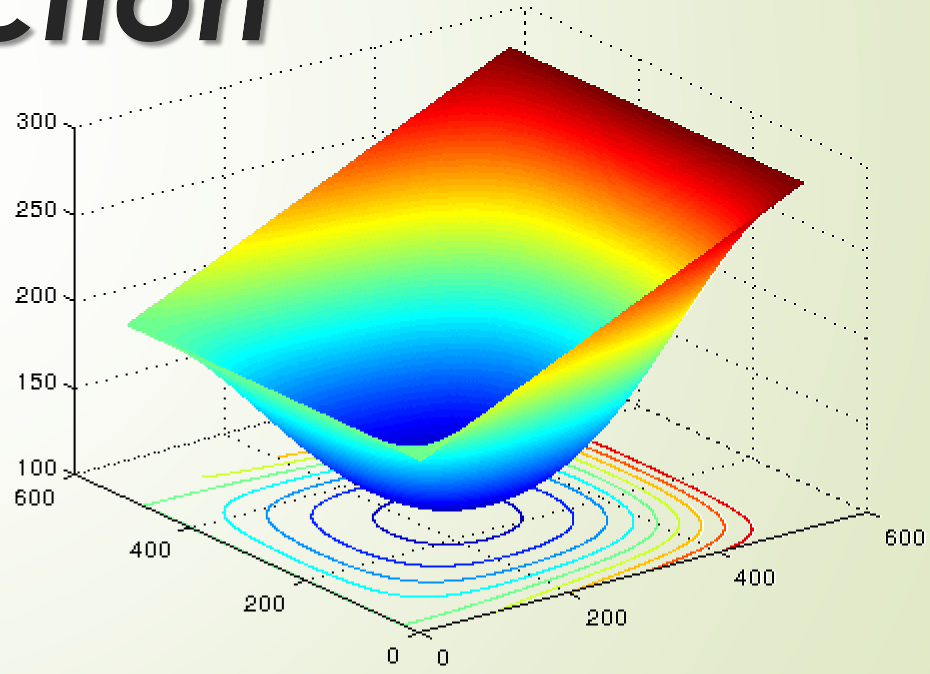
MECH-420 Heat Transfer Study Aid 1D Heat Conduction



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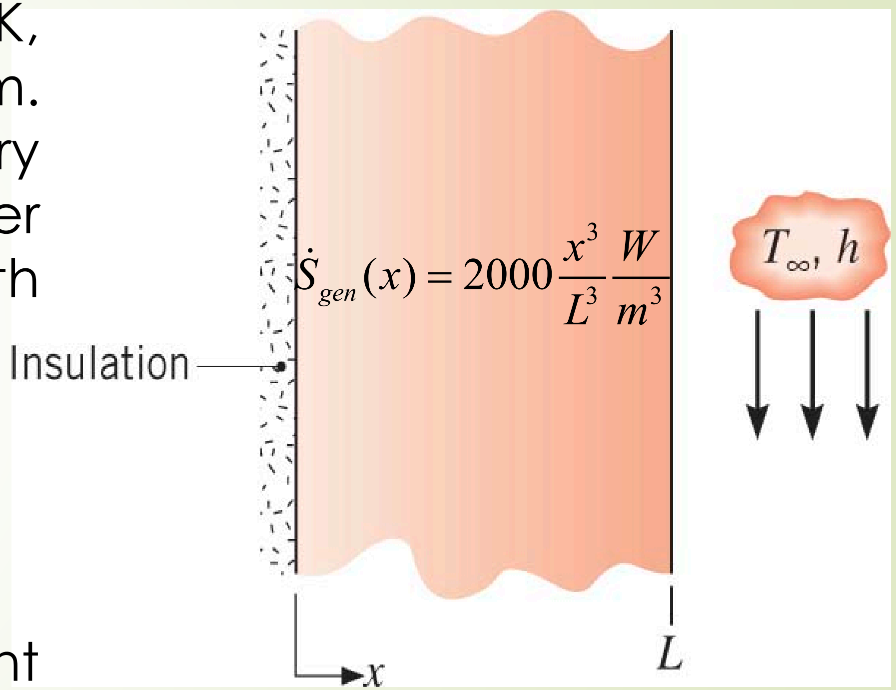
ASME FELLOW

1D Steady State Cartesian

Consider a 2D plane wall with 1D heat transfer in X with homogeneous thermal conductivity of K=25 W/m-K, and width L = 10m, height H = 5m, and depth D = 1m. The right boundary is insulated, and the left boundary (L=10) experiences a convective fluid with heat transfer coefficient h=10 W/m2-K and ambient fluid of 20C with internal generation rate of:

$$\dot{S}_{gen}(x) = 2000\frac{x^3}{L^3}\frac{W}{m^3}$$

a. What is the total internal generation rate in W?
b. Applying an overall control volume, what is the right boundary surface temperature?
c. What is the temperature of the insulated wall?



Total Internal Generation Rate (W) Plain Wall

$$\dot{E}_{gen}[W] = \int_0^L \dot{S}_{gen}(x)d\forall = \int_0^L \dot{S}_{gen}(x)(HD)dx = HD\int_0^L 2000\frac{x^3}{L^3}dx$$

$$= \left(2000HD\frac{x^4}{4L^3}\right)_{0-L}$$

$$= \left(2000\frac{W}{m^3}\right)\left(HDm^2\right)\frac{L}{4}m$$

$$= \left(2000\frac{W}{m^3}\right)\left(5\bullet 1m^2\right)\frac{10}{4}m$$

$$= 25,000W$$

"When" is the CV 1st Law Needed

- **To find SURFACE temperature only, Ts. (and heat flux is known at each surface)**
  - Apply 1st law to CV around entire object
- To find initial rate of change of temperature (dT/dt)
- When there is no spatial gradients of temperature "INSIDE" the body, i.e. a "thin walled tube".
- Apply at "surface" to find energy balance at surface to determine boundary conditions.

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Fluid Surface Temperature

OVERALL CONTROL VOLUME (METHOD #1)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \frac{dE_{st}}{dt}$$

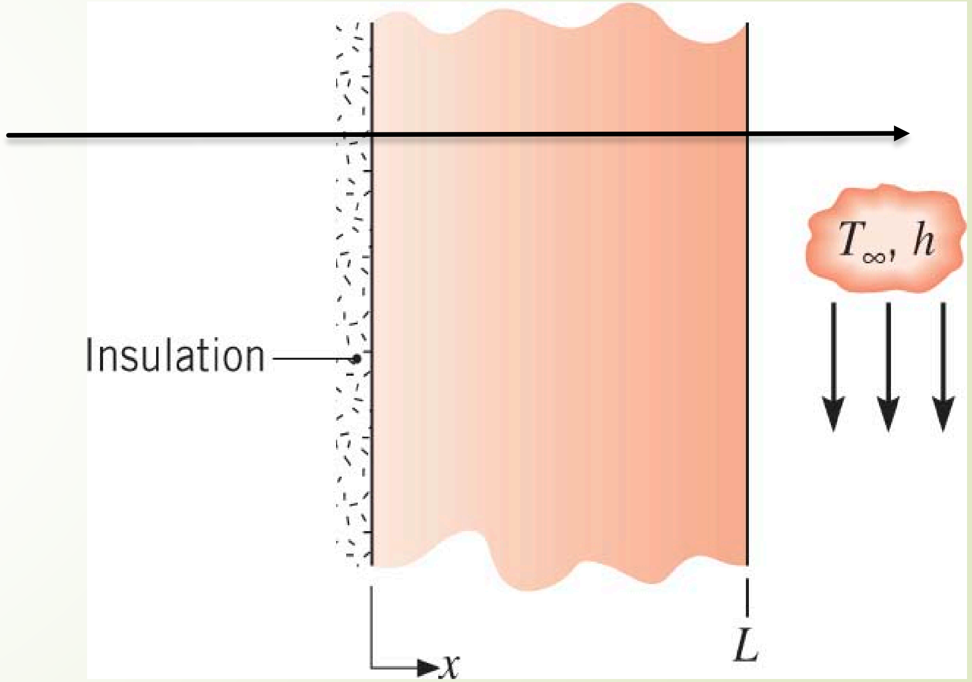
$$\dot{E}_{out} = \dot{E}_{gen}$$

$$hA_s(T_s - T_\infty) = \dot{E}_{gen}$$

$$T_s = \frac{\dot{E}_{gen}}{hA_s} + T_\infty, \; A_s = HD$$

$$= \frac{25{,}000W}{\left(10\frac{W}{m^2-K}\right)1m \bullet 5m} + 20C$$

$$= 520C$$



Solution: 1D, SS, Homogeneous, Cartesian Find Exact Solution

$$HDE \rightarrow CARTESIAN$$

$$\frac{d^2T}{dx^2} = \frac{-\dot{S}_{gen}(x)}{k} = -\dot{S}_o \frac{x^3}{kL^3}$$

$$INTEGRATION$$

$$\frac{dT}{dx} = -\dot{S}_o \frac{x^4}{4kL^3} + C_1$$

$$INTEGRATION$$

$$T(x) = -\dot{S}_o \frac{x^5}{20kL^3} + C_1 x + C_2 \rightarrow \text{MOST GENERAL SOLUTION}$$

Apply Boundary Conditions

$$T(x) = -\dot{S}_o \frac{x^5}{20kL^3} + C_1 x + C_2$$

$$\frac{dT}{dx} = -\dot{S}_o \frac{x^4}{4kL^3} + C_1$$

$$BC\#1, \frac{dT}{dx}_{(x=0)} = 0 = -\dot{S}_o \frac{0^4}{4kL^3} + C_1, \rightarrow C_1 = 0$$

$$BC\#2, -k\frac{dT}{dx}_{(x=L)} = h(T(x=L) - T_\infty) \rightarrow Conduction = Convection$$

$$-k(-\dot{S}_o \frac{L^4}{4kL^3}) = h(-\dot{S}_o \frac{L^5}{20kL^3} + C_2 - T_\infty)$$

$$\dot{S}_o \frac{L}{4} = h(-\dot{S}_o \frac{L^2}{20k} + C_2 - T_\infty)$$

$$C_2 = \dot{S}_o \frac{L}{4h} + \dot{S}_o \frac{L^2}{20k} + T_\infty$$

$$C_2 = \dot{S}_o(\frac{L}{4h} + \frac{L^2}{20k}) + T_\infty = 2000(\frac{10}{4 \bullet 10} + \frac{100}{20 \bullet 25}) + 20 = 920C$$

$$= \frac{W}{m^3}(\frac{m}{\frac{W}{m^2 - K}} + \frac{m^2}{\frac{W}{m - K}}) + K$$

Exact Solution

$$T(x) = -\dot{S}_o \frac{x^5}{20kL^3} + 920.0C$$

Surface Temperature

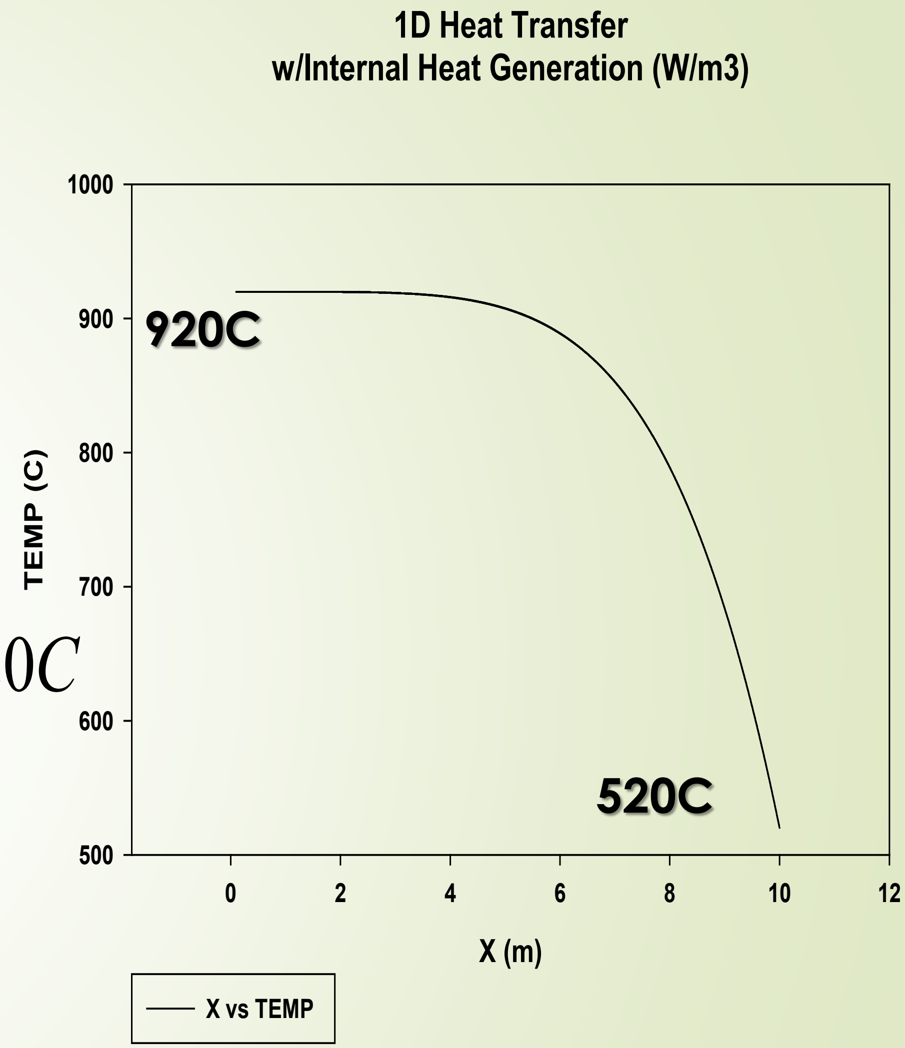
$$T_s = T(x=L) = -\dot{S}_o \frac{L^5}{20kL^3} + 920.0C = -2000\frac{100}{20 \bullet 25} + 920C$$

$= 520C \rightarrow$ SECOND METHOD

Wall Temperature

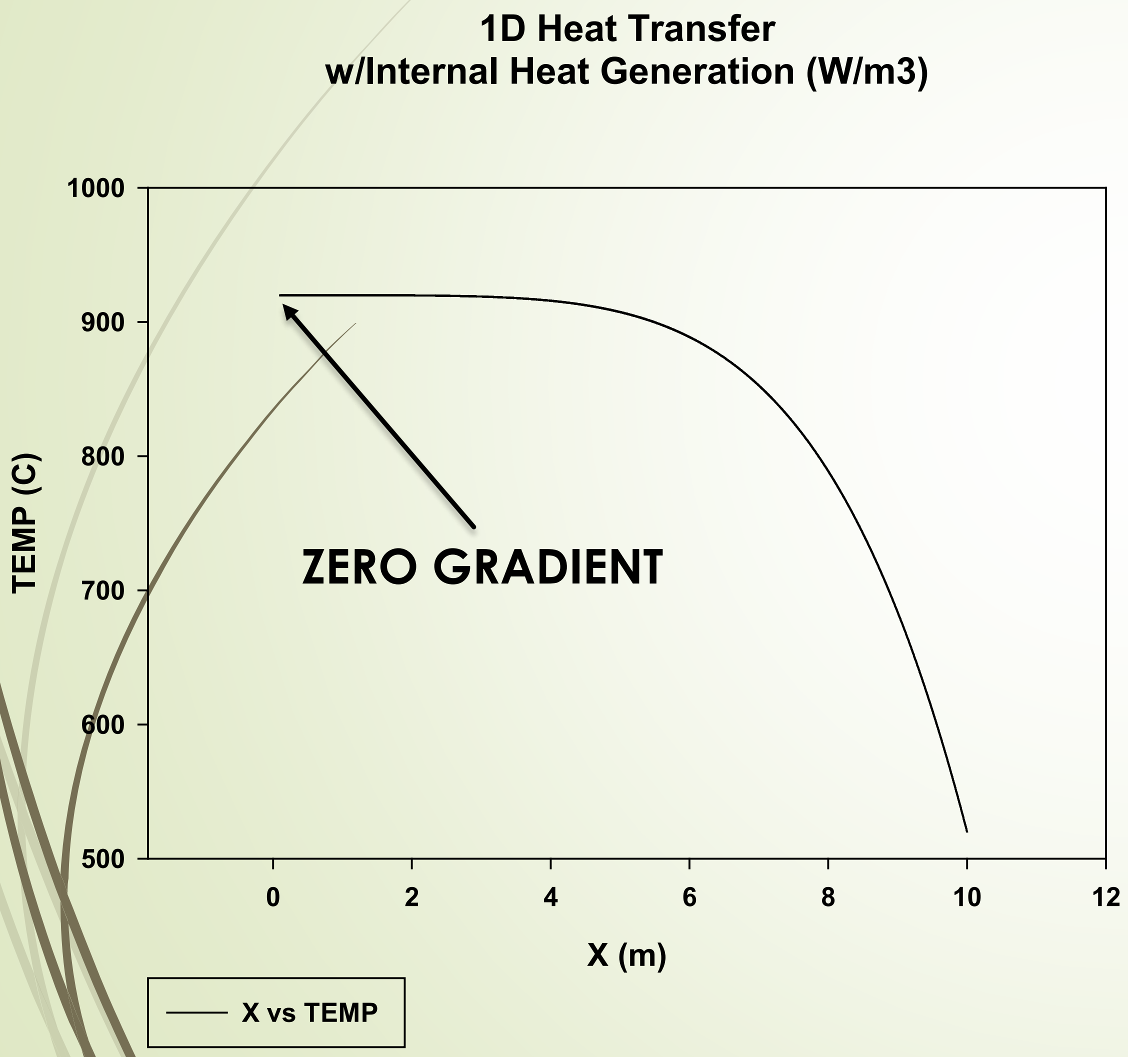
$$T_{wall} = T(x=0) = -\dot{S}_o \frac{0^5}{20kL^3} + 920.0C$$

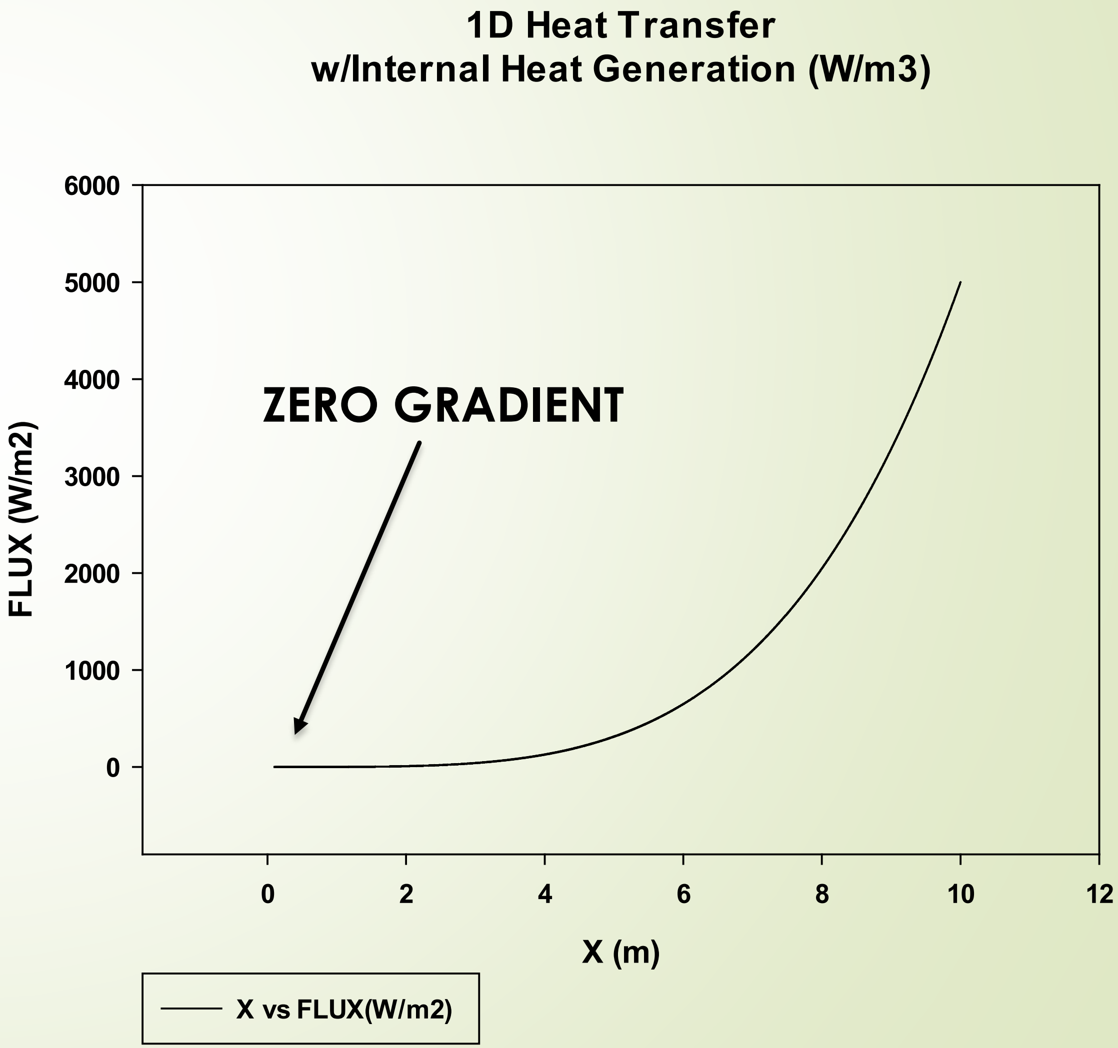
$= 920C$



$$T(x) = -\dot{S}_o \frac{x^5}{20kL^3} + 920.0C$$

$$q_x''(x) = -k\frac{dT}{dx} = \dot{S}_o \frac{x^4}{4L^3} \frac{W}{m^2}$$



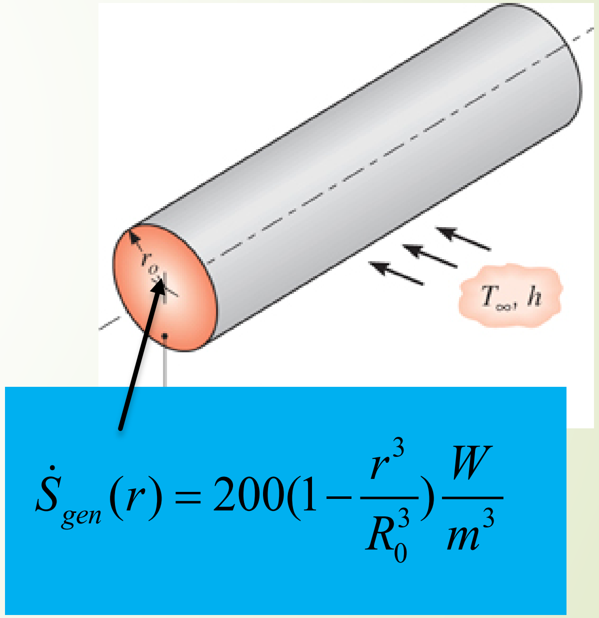


1D Steady State Cylindrical

Consider cylindrical radial heat transfer w/homogeneous thermal conductivity of K=25 W/m-K, and width radius R = 10m, and length L = 5m. The outer boundary experiences a convective fluid with heat transfer coefficient h=10 W/m2-K and an ambient fluid of 20C with internal generation rate of:

$$\dot{S}_{gen}(\mathrm{r}) = 200(1 - \frac{r^3}{R_0^3})\frac{W}{m^3}$$

a. What is the total internal generation rate in W?
b. Applying an overall control volume, what is the right boundary surface temperature?
c. What is the temperature at the center?



Total Internal Generation Rate (W) Cylinder

$$\dot{E}_{gen}[W] = \int_0^{R_0} \dot{S}_{gen}(r)d\forall = \int_0^{R_0} \dot{S}_{gen}(r)(2\pi r dr L) = 2\pi L \int_0^{R_0} 200(1 - \frac{r^3}{R_0^3}) r dr$$

$$= 2\pi L \bullet 200(\frac{r^2}{2} - \frac{r^5}{5R_0^3})_{0-R_0} = 2\pi L \bullet 200(\frac{R_0^{\ 2}}{2} - \frac{R_0^{\ 2}}{5}) = 2\pi L \bullet 200 R_0^2 (1/2 - 1/5)$$

$$= 2\pi L \bullet 200 R_0^2 (5/10 - 2/10)$$

$$= 2\pi L \bullet 200 R_0^2 \frac{3}{10}$$

$$= m \bullet \frac{W}{m^3} m^2$$

$$= 1.9x10^5 W$$

Fluid Surface Temperature

OVERALL CONTROL VOLUME (METHOD #1)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \frac{d\dot{E}_{st}}{dt}$$

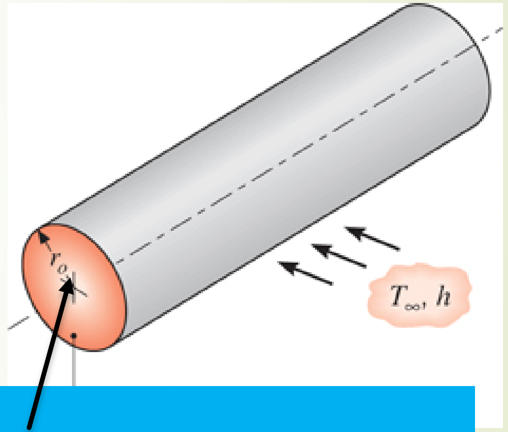
$$\dot{E}_{out} = \dot{E}_{gen}$$

$$hA_s(T_s - T_\infty) = \dot{E}_{gen}$$

$$T_s = \frac{\dot{E}_{gen}}{hA_s} + T_\infty, \; A_s = \pi DL$$

$$= \frac{1.9x10^5 W}{\left(10\frac{W}{m^2-K}\right)\pi \bullet 10 \bullet 2 \bullet 5} + 20C$$

$$= 80C$$



$$\dot{S}_{gen}(r) = 200\left(1 - \frac{r^3}{R_0^3}\right)\frac{W}{m^3}$$

Solution: 1D, SS, Homogeneous, Cartesian
Find Exact Solution

$HDE \rightarrow CARTESIAN$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) = \frac{-\dot{S}_{gen}(r)}{k} = \frac{-\dot{S}_0(1-\frac{r^3}{R_0^3})}{k}; \quad 0 \le r \le R_0$$

$MULTIPLY\ by\ "r"$

$$\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) = \frac{-\dot{S}_0}{k}(r - \frac{r^4}{R_0^3})$$

$INTEGRATE$

$$r\frac{\partial T}{\partial r} = \frac{-\dot{S}_0}{k}(\frac{r^2}{2} - \frac{r^5}{5R_0^3}) + C_1$$

$\div r$

$$\frac{\partial T}{\partial r} = \frac{-\dot{S}_0}{k}(\frac{r}{2} - \frac{r^4}{5R_0^3}) + \frac{C_1}{r}$$

$INTEGRATION$

$$T(r) = \frac{-\dot{S}_0}{k}(\frac{r^2}{4} - \frac{r^5}{25R_0^3}) + C_1\ln(r) + C_2 \rightarrow \text{MOST GENERAL SOLUTION}$$

Apply Boundary Conditions

$$T(r) = \frac{-\dot{S}_0}{k}\left(\frac{r^2}{4} - \frac{r^5}{25R_0^3}\right) + C_1 \ln(r) + C_2$$

$$\frac{dT}{dr} = \frac{-\dot{S}_0}{k}\left(\frac{r}{2} - \frac{r^4}{5R_0^3}\right) + \frac{C_1}{r}$$

$$BC\#1, \left.\frac{dT}{dr}\right|_{(r=0)} = 0 \rightarrow C_1 = 0 \text{ (or T(r=0) must be finite)}$$

$$BC\#2, -k\left.\frac{dT}{dr}\right|_{(r=R_0)} = h(T(r=R_0) - T_\infty) \rightarrow \text{Conduction} = \text{Convection}$$

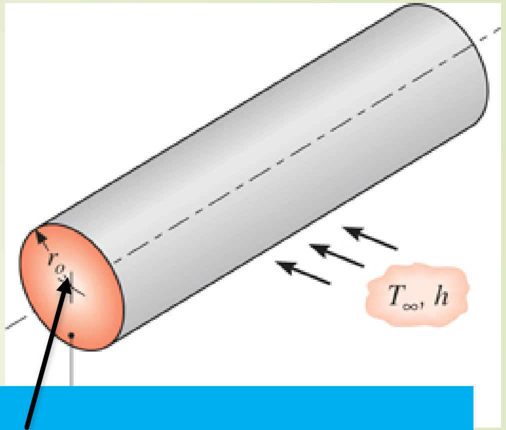
$$-k\left(\frac{-\dot{S}_0}{k}\left(\frac{R_0}{2} - \frac{R_0^4}{5R_0^3}\right)\right) = h\left(\frac{-\dot{S}_0}{k}\left(\frac{R_0^2}{4} - \frac{R_0^5}{25R_0^3}\right) + C_2 - T_\infty\right)$$

$$\left(\dot{S}_0 R_0\left(\frac{1}{2} - \frac{1}{5}\right)\right) = h\left(\frac{-\dot{S}_0}{k} R_0^2\left(\frac{1}{4} - \frac{1}{25}\right) + C_2 - T_\infty\right) = h\left(\frac{-\dot{S}_0}{k} R_0^2 \frac{21}{100} + C_2 - T_\infty\right)$$

$$\dot{S}_0 R_0 \frac{3}{10} = h\left(\frac{-21\dot{S}_0}{100k} R_0^2 + C_2 - T_\infty\right)$$

$$C_2 = \dot{S}_0\left(\frac{3R_0}{10h} + \frac{21R_0^2}{100k}\right) + T_\infty = 200(0.3 \bullet 10/10 + 21 \bullet 100/2500) + 20C = 248C$$

$$= \frac{W}{m^3}\left(\frac{m}{\frac{W}{m^2 - K}} + \frac{m^2}{\frac{W}{m - K}}\right) + K$$



$$\dot{S}_{gen}(r) = 200\left(1 - \frac{r^3}{R_0^3}\right)\frac{W}{m^3}$$

Exact Solution

$$T(r) = \frac{-\dot{S}_0}{k}\left(\frac{r^2}{4} - \frac{r^5}{25R_0^3}\right) + 248C$$

Surface Temperature

$$\mathrm{T}_s = T(r = R_0) = \frac{-\dot{S}_0 R_0^2}{k}\left(\frac{1}{4} - \frac{1}{25}\right) + 248C$$

$$= 80C \rightarrow \text{SECOND METHOD}$$

Center Temperature

$$\mathrm{T}_{Center} = T(r = 0) = 248C$$