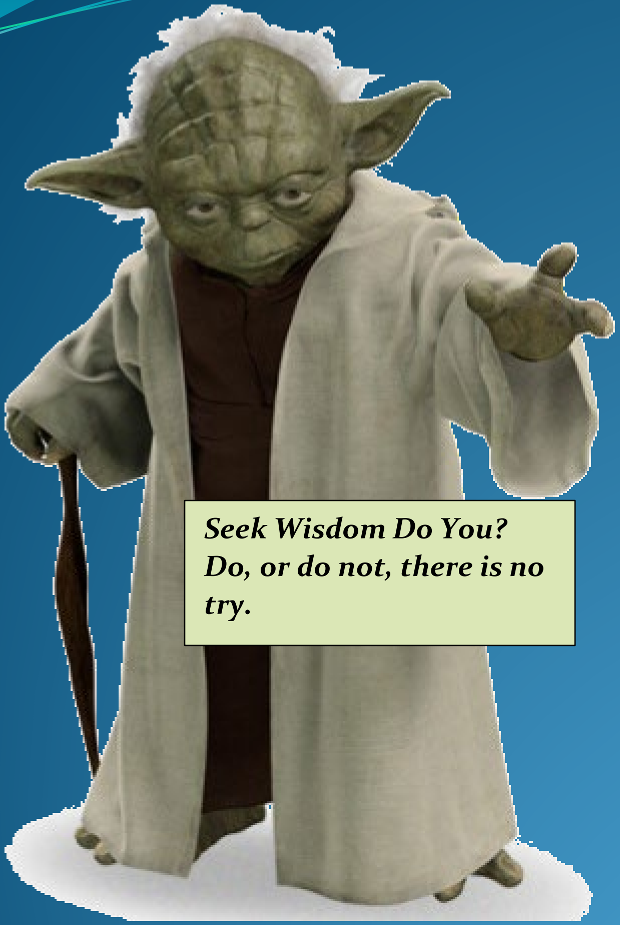

Heat Transfer: Physical Origins and Rate Equations

Chapter One
Sections 1.1-1.3

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ASME TECHNICAL FELLOW

TRUST THE PATH

Spring 2019/2020

1. Yes. If you would have told me that @ the end of the term I would be able to do all this, I wouldn't have believed you. If I can figure this out, I can figure anything out.
2. I think more engineering courses should focus on the way to think & develop processes, rather than plugging #'s into eqns.
3. This course has encouraged deeper thought and effort to solve problems. It emphasizes being able to see a problem and even not knowing an answer, and being able to start working towards it.
4. I think heat transfer is important in engineering design because heat transfer is everywhere and relates to everyday life. We should expect to encounter some similar problems in the future.
5. It has enhanced my ability and my understanding. The course required enhanced thoughts and incorporated technical aspects. It's a hard course with a knowledgeable professor. I did everything and still struggled.
6. MECH-420 is a difficult class and not all the answers are always available. This class has improved my skills by forcing me to *put more effort to actually learn the material*.

- ***NO SUCCESS WITHOUT PRACTICE***

- What is **heat transfer**?

Heat transfer is thermal energy in transit due to a **TEMPERATURE DIFFERENCE.**

What is the cause of the change in **TEMPERATURE?**

What is **thermal energy**?

Thermal energy is associated with the translation, rotation, vibration and electronic states of the atoms and molecules that comprise matter. It represents the cumulative effect of microscopic activities and is directly linked to the temperature of matter.

DO NOT confuse or interchange the meanings of **Thermal Energy, Temperature** and **Heat Transfer**

Quantity	Meaning	Symbol	Units
Thermal Energy[+]	Energy associated with microscopic behavior of matter	U or u	J or J/kg
Temperature	A means of indirectly assessing the amount of thermal energy stored in matter	T	K or °C
Heat Transfer	Thermal energy transport due to temperature gradients		
Heat	Amount of thermal energy transferred over a time interval $\triangle t > 0$	Q	J
Heat Rate	Thermal energy transfer per unit time	q	W
Heat Flux	Thermal energy transfer per unit time and surface area	q''	W/m^2

+
$U \rightarrow$ Thermal energy of system
$u \rightarrow$ Thermal energy per unit mass of system

CONSERVATION OF ENERGY (FIRST LAW OF THERMODYNAMICS)

- An important tool in heat transfer analysis, often providing the basis for determining the temperature of a system.
- Alternative Formulations

Time Basis:

At an instant
or
Over a time interval

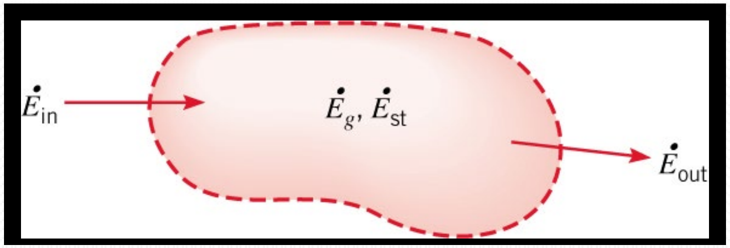
$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho e d\forall + \int_{CS} \rho e\left(\vec{V}\cdot d\vec{A}\right)$$

Type of System:

Control volume
Control surface

APPLICATION TO A CONTROL VOLUME

- At an Instant of Time:



Note representation of system by a control surface (dashed line) at the boundaries.

Surface Phenomena

$\dot{E}_{in}, \dot{E}_{out}$: rate of thermal and/or mechanical energy transfer across the control surface due to heat transfer, fluid flow and/or work interactions.

Volumetric Phenomena

$\dot{E}_g$: rate of thermal energy generation due to conversion from another energy form (e.g., electrical, nuclear, or chemical); energy conversion process occurs within the system

$\dot{E}_{st}$: rate of change of energy storage in the system.

Conservation of Energy

$$\overset{+}{\dot{E}_{in}} - \overset{+}{\dot{E}_{out}} + \overset{\pm}{\dot{E}_g} = \frac{dE_{st}}{dt} \equiv \overset{\pm}{\dot{E}_{st}}$$

Each term has units of J/s or W.

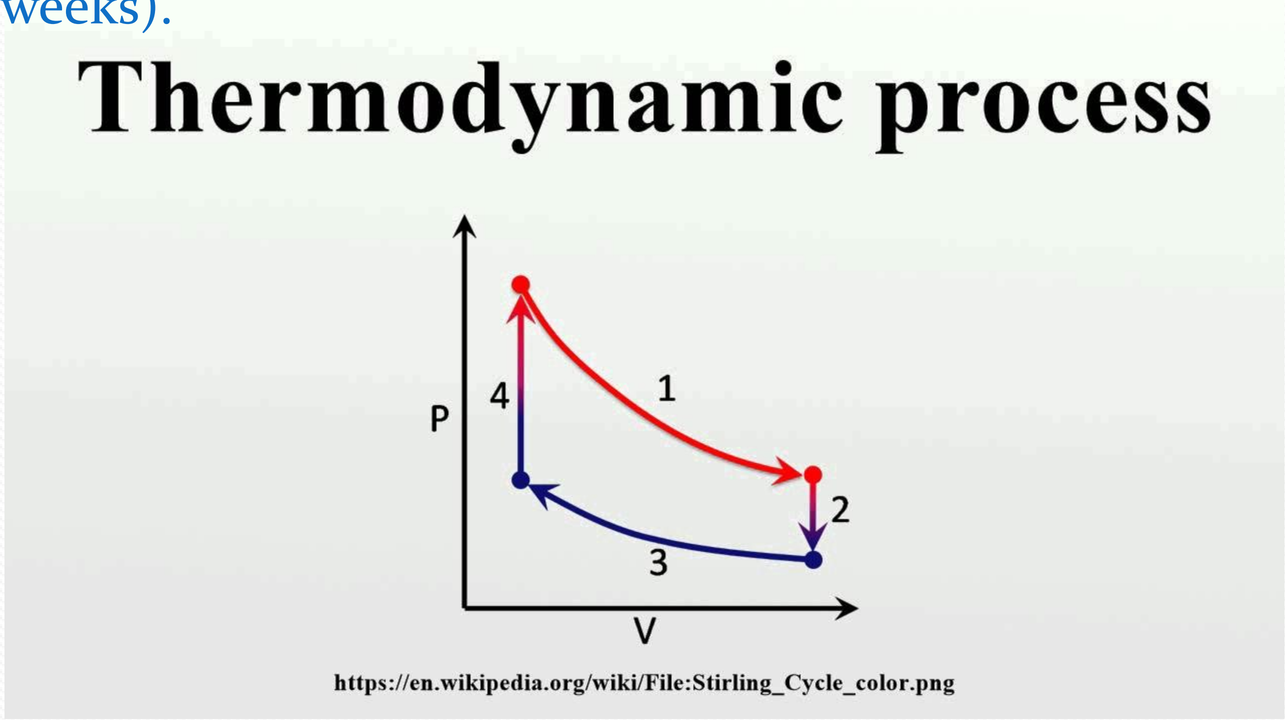
$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho e d\forall + \int_{CS} \rho e\left(\vec{V} \cdot d\vec{A}\right)$$

- **Over a Time Interval (Joules)**

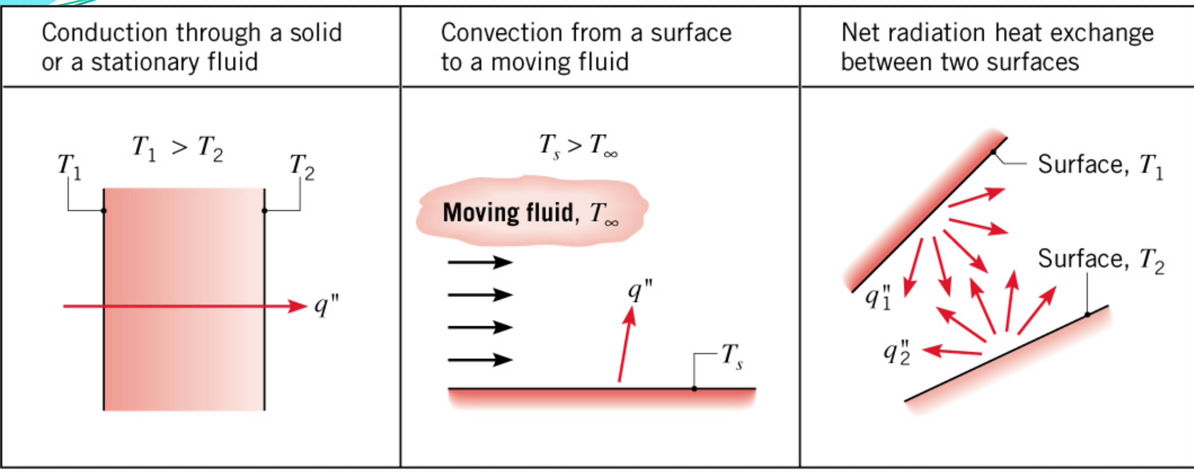
$$E_{in} - E_{out} + E_g = \Delta E_{st}$$

THERMO to HEAT TRANSFER

- In Thermodynamics, the CV temperature was assumed to be uniform throughout and we didn't care what thermal process was responsible for the change in the system's internal energy from state point 1 to 2.
- In Heat Transfer, we now compute the ***how much 'heat'*** is needed to cause the process change, and identify the different ***modes of heat transfer*** possible (11 weeks).



Modes of Heat Transfer



Conduction: Heat transfer in a solid or a stationary fluid (gas or liquid) due to the random motion of its constituent atoms, molecules and /or electrons.

Convection: Heat transfer due to the combined influence of bulk and random motion for fluid flow over a surface. (FREE AND FORCED)

Radiation: Energy that is emitted by matter due to changes in the electron configurations of its atoms or molecules and is transported as electromagnetic waves (or photons).

Conduction and Convection require the presence of temperature variations in a material.

Although radiation originates from matter, its transport does not require a material medium and occurs most efficiently in a vacuum.

https://www.youtube.com/watch?v=HpCvWuvCUoA

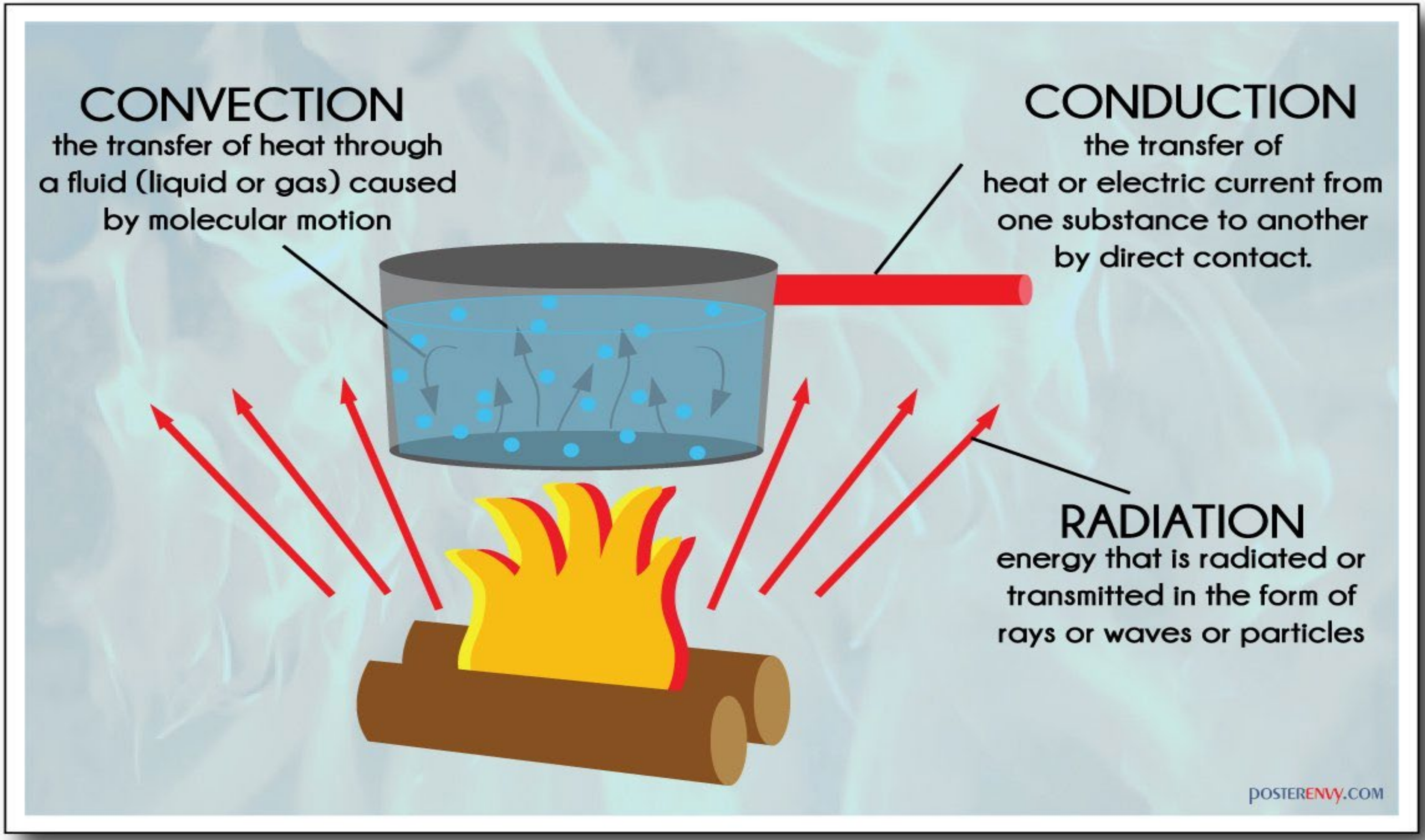
BOILING AND CONDENSATION:

LATENT HEAT EXCHANGE

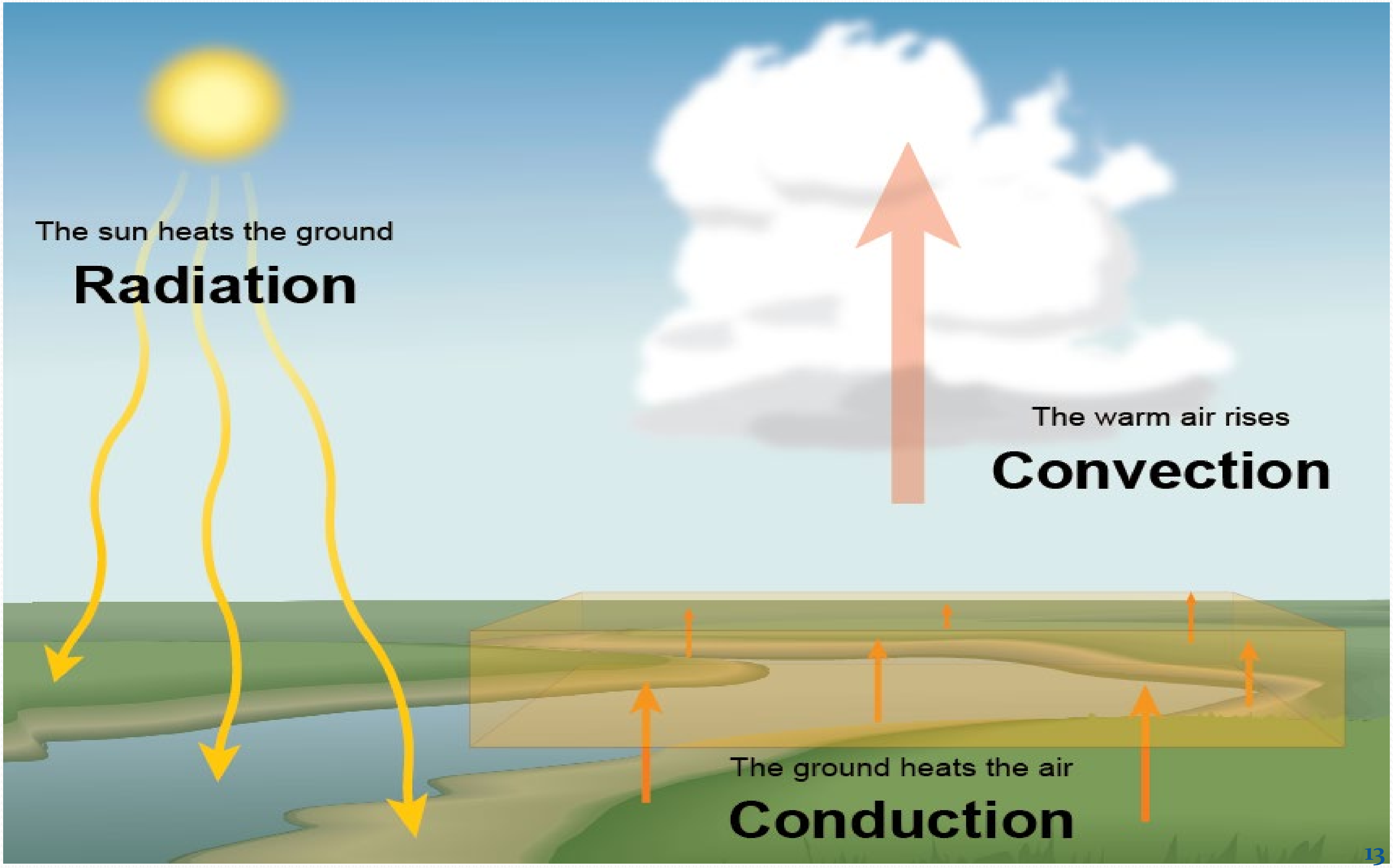
- Convection heat transfer with an addition of latent heat exchange, i.e., a phase change between the liquid and vapor of the fluid.
- **Boiling**: The change from the liquid to the vapor state due to **and** is sustained by heat transfer from the solid surface; **Condensation** of a vapor to the liquid state results in heat transfer to the solid surface.
  - **condensation** can be seen when drops of **water** form on the outside of a glass of ice **water**; also:
  - dew that forms on grass overnight

https://www.youtube.com/watch?v=xYU7RSoOZoU

Modes of Heat Transfer



Modes of Heat Transfer



Fusion Reactor and Clean Energy
How to Harness a “STAR” in a lab?

150 Million C to -269C

https://www.youtube.com/watch?v=ekub_xEiUww

Heat Transfer Rates

Conduction: **Fourier's Law**

General (vector) form of Fourier's Law:

$$\vec{q''} = -k\underbrace{\nabla T}$$

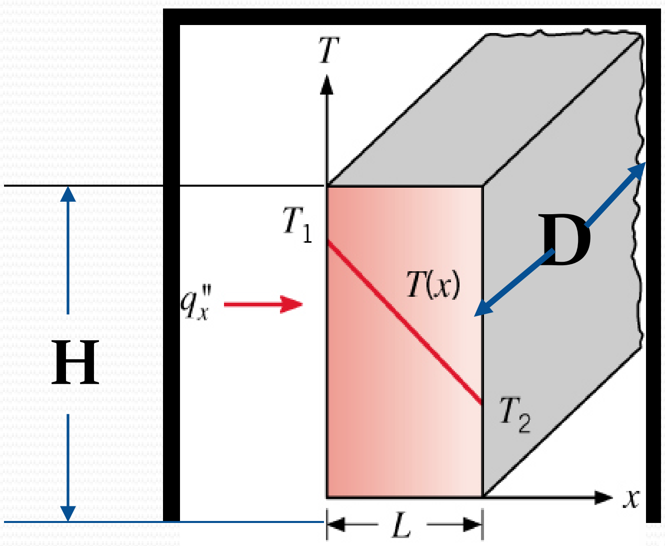
Heat flux W/m^2

Thermal conductivity $W/m \cdot K$

Temperature gradient $°C/m$ or K/m

q is a 3D vector
T is scalar

Application to one-dimensional, steady conduction across a plane wall of constant thermal conductivity:



3D Cartesian Heat Transfer

$$q'' = \left(-k_x \frac{\partial T}{\partial x}\right) i + \left(-k_y \frac{\partial T}{\partial y}\right) j + \left(-k_z \frac{\partial T}{\partial z}\right) k$$

1D Cartesian Heat Transfer

$$q''_x = -k_x \frac{dT}{dx} = -k_x \frac{T_2 - T_1}{L}$$

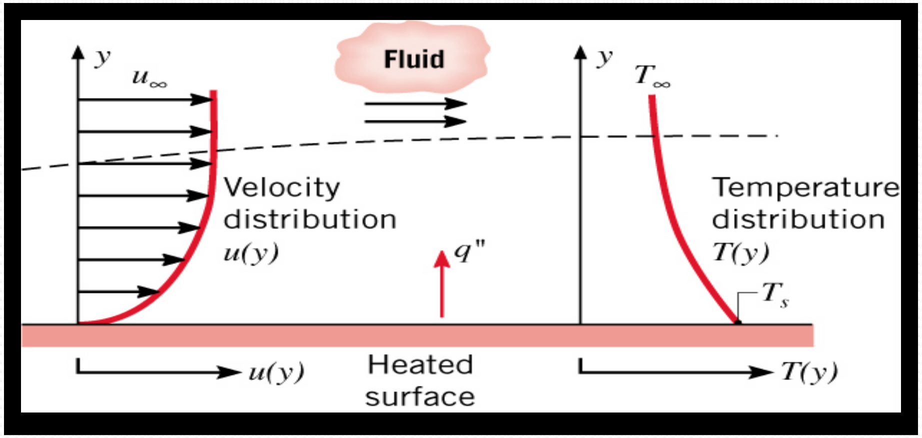
Heat rate (W):

"k" is a physical medium property and indicates the capacity of the medium for energy transport. W/m-K

Heat Transfer Rates

Convection: **Newton's Law Cooling**

Relation of convection to flow over a surface and development of velocity and thermal boundary layers:



Newton's law of cooling:

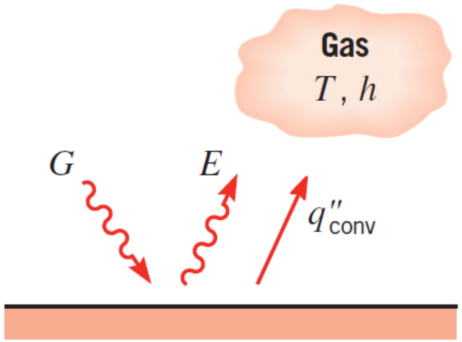
$$q''\left[\frac{W}{m^2}\right]=\bar{h}\left[\frac{W}{m^2-K}\right](T_s-T_\infty)$$

$$q=[W]=q''\left[\frac{W}{m^2}\right]A_s[m^2]=\bar{h}\left[\frac{W}{m^2-K}\right]A_{surface}\left[m^2\right](T_s-T_\infty)[K]$$

h : Convection heat transfer coefficient $(\mathrm{W/m^2\cdot K})$

Heat Transfer Rates

Radiation



Surface of emissivity ε, absorptivity α, and temperature T_s

Heat transfer at a gas/surface interface involves radiation emission from the surface and may also involve the absorption of radiation incident from the surroundings (irradiation, G), as well as convection $\left(\text{if } T_s \neq T_\infty\right)$.

Energy outflow due to emission:

$$E = \varepsilon E_b = \varepsilon \sigma T_s^4 \qquad (1.5)$$

E : Emissive power $\left(\mathrm{W/m^2}\right)$

ε : Surface emissivity $\left(0 \le \varepsilon \le 1\right)$

E_b : Emissive power of a blackbody (the perfect emitter)

σ : Stefan-Boltzmann constant $\left(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}\right)$

Energy absorption due to irradiation: G : Irradiation $\left(\mathrm{W/m^2}\right)$

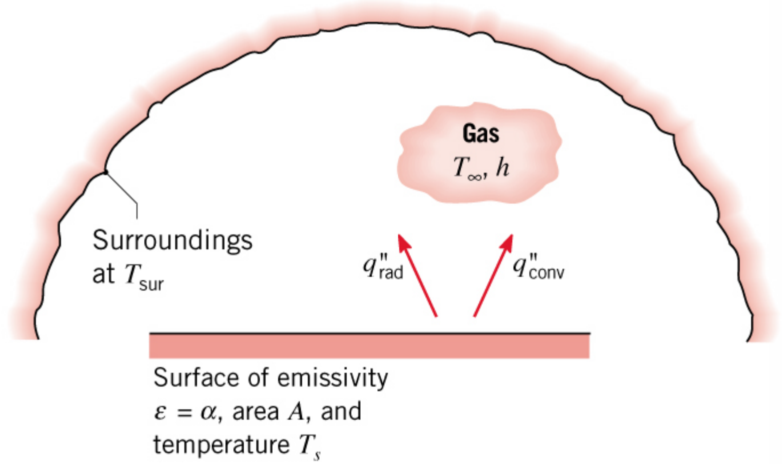
$$G_{abs} = \alpha G$$

G_{abs} :Absorbed incident radiation$\left(\mathrm{W/m^2}\right)$

α : Surface absorptivity $\left(0 \le \alpha \le 1\right)$

Heat Transfer Rates

Irradiation: Special case of surface exposed to large surroundings of uniform temperature, T_{sur}



(b)

$$G = G_{\text{sur}} = \sigma T_{\text{sur}}^4$$

If $\alpha = \varepsilon (GREY\ SURFACE)$, the net radiation heat flux from the surface due to exchange with the surroundings is:

$$q''_{\text{rad}} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{\text{sur}}^4\right) \quad (1.7)$$

Heat Transfer Rates

Alternatively,

$$q''_{\text{rad}} = h_r \left(T_s - T_{\text{sur}}\right) \tag{1.8}$$

h_r: Radiation heat transfer coefficient $\left(\mathrm{W/m^2 \cdot K}\right)$

$$h_r = \varepsilon\sigma\left(T_s + T_{\text{sur}}\right)\left(T_s^2 + T_{\text{sur}}^2\right) \tag{1.9}$$

For combined convection and radiation,

$$q'' = q''_{\text{conv}} + q''_{\text{rad}} = h\left(T_s - T_\infty\right) + h_r\left(T_s - T_{\text{sur}}\right) \tag{1.10}$$

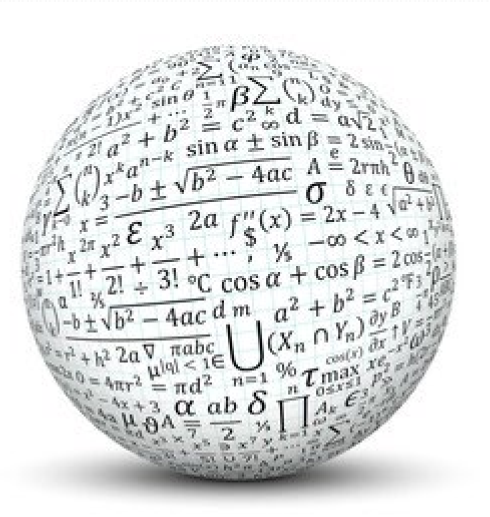
Any problem without correct units receive 0 points.

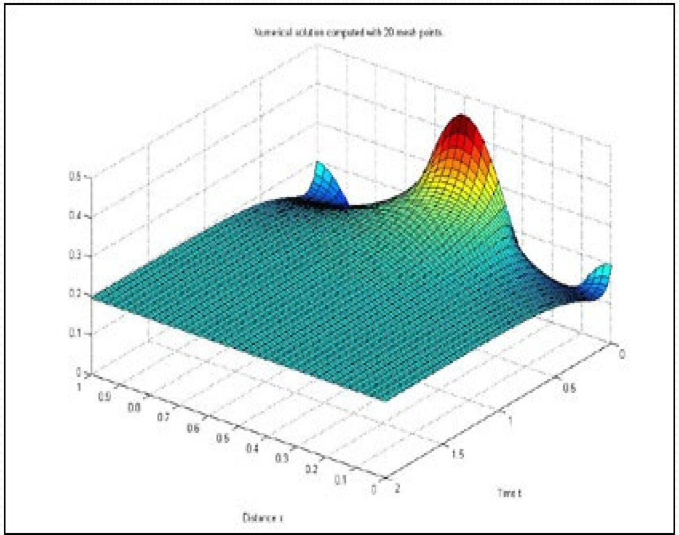
Quantity	Name Symbol	SI Units	English Units	Conversion
Force	Newton (N)	$\frac{m-kg}{s^2}$	lb_f	$1N = 0.224809 lb_f$
Pressure	Pascal (Pa)	$\frac{N}{m^2}$	$\frac{lb_f}{ft^2}$	$1PA = 0.020886 \frac{lb_f}{ft^2}$
Energy	Joules (J)	$N-m$	Btu	$1J = 0.000948 Btu$
Power	Watts (W)	$\frac{J}{\sec}$	hp	$1\frac{J}{\sec} = 1W = 0.00134 hp$
Thermal Conductivity	k	$\frac{W}{m-K}$	$\frac{Btu/hr}{ft-R}$	$1\frac{W}{m-K} = 0.57779 \frac{Btu}{hr-ft-R}$
Specific Heat	Cp	$\frac{J}{kg-K}$	$\frac{Btu}{slugs-R}$	$1\frac{J}{kg-K} = 7700 \frac{Btu}{slugs-R}$
Density	ρ	$\frac{kg}{m^3}$	$\frac{slugs}{ft^3}$	$1\frac{kg}{m^3} = 0.001939 \frac{slugs}{ft^3}$
Mass	m	kg	slugs	$1 slug = 32.2 lb_m = 14.6 kg$

http://www.digitaldutch.com/unitconverter/energy.htm

Differential Equations

A Differential Equation is a mathematical representation that governs/controls the physical behavior of real systems interacting with the real world and being ***CONSTRAINED*** by real world conditions. Real Engineers solve Real Problems by modeling.





GENERAL SOLUTION 1ST ORDER PDE

$$\frac{dy}{dx} + p(x)y = f(x)$$

HAS GENRAL SOLUTION OF:

$$y(x) = Ce^{-\int p(x)dx} + e^{-\int p(x)dx} \int e^{\int + p(x)dx} f(x)dx$$

$C \equiv$ Arbitrary Constant of Integration Obtained

From Initial Condition y(x=0)=Y_0

SPECIAL CASE: p(x) = 0

$$y(x)=C+\int f(x)dx$$

SPECIAL CASE: f(x) = 0

$$y(x)=Ce^{-\int p(x)dx}$$

First Order ODE Solution

$$y' + a = be^{-ct} \rightarrow \frac{dy}{dt} + a = be^{-ct}$$

$$dy = \left(be^{-ct} - a\right)dt \rightarrow \text{Separate Variables}$$

$$y(t) = \int \left(be^{-ct} - a\right)dt$$

$$y(t) = \left[\frac{-b}{c}e^{-ct} - at\right] + C \rightarrow \text{MOST GENERAL SOLUTION}$$

Initial Condition

$$\mathrm{Y(t{=}0){=}Y_0}$$

$$\mathrm{Y(t{=}0){=}Y_0} = \left[\frac{-b}{c}e^{-c0} - a0\right] + C$$

$$Y_0 = \frac{-b}{c} + C \rightarrow C = Y_0 + \frac{b}{c}$$

EXACT SOLUTION

$$y(t) = \left[\frac{-b}{c}e^{-ct} - at\right] + Y_0 + \frac{b}{c}$$

$$y(t) = \frac{b}{c}(1 - e^{-ct}) - at + Y_0$$

Q(t) 1st ORDER ODE

$$6\frac{dQ}{dt} + 0Q(t) + 12 = 24e^{-2t}$$

$$\frac{dQ}{dt} = -2 + 4e^{-2t}$$

$$\int dQ = \int (-2 + 4e^{-2t})dt \rightarrow \text{Separate Variables and Integrate}$$

$$Q(t) = -2t + \frac{4e^{-2t}}{-2} + C \rightarrow \text{Most General Solution (see Slide 22)}$$

Initial Condition;Q(t=0)=Q_0

$$Q_0 = \frac{4}{-2} + C \rightarrow C = Q_0 + 2$$

$$Q(t) = -2t + \frac{4e^{-2t}}{-2} + Q_0 + 2 \rightarrow \text{EXACT SOLUTION}$$

1st Order w/Forcing Function

$$\frac{dy(t)}{dt} + ky(t) = q(t); or \rightarrow k \text{ constant}$$

$$y' + ky = q(t)$$

has general solution of:

$$y(t) = e^{-kt}\int q(t)\, e^{+kt} dt + Ce^{-kt}; k > 0 \rightarrow$$ See SLIDE 22

where C is an arbitrary constant of integration

obtained from initial condition at t = 0.

Initial Condition: CASE A

$$y(t) = e^{-kt}\int q(t)\, e^{+kt} dt + Ce^{-kt}; k > 0$$

$$q(t) = Q_0 \text{(constant)}$$

$$e^{-kt}\int q(t)\, e^{+kt} dt = e^{-kt}\int Q_0\, e^{+kt} dt = \frac{Q_0 \bullet e^{-kt} e^{+kt}}{k} = \frac{Q_0}{k}$$

$$y(t) = \frac{Q_0}{k} + Ce^{-kt} \rightarrow \text{MOST GENERAL SOLUTION,q(t)=Q}_0$$

INITITAL CONDITION

$$\text{y(t=0)=Y}_0 = \frac{Q_0}{k} + Ce^{-k0} \rightarrow C = \text{Y}_0 - \frac{Q_0}{k}$$

EXACT SOLUTION

$$y(t) = \frac{Q_0}{k} + \left[\text{Y}_0 - \frac{Q_0}{k}\right] e^{-kt}$$

$$y(t) = \frac{Q_0}{k}(1 - e^{-kt}) + \text{Y}_0 e^{-kt}$$

USEFUL TRIG INTEGRATION RELATION

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} [a \sin bx - b \cos bx] + c$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx] + c$$

"a" and "b" are the ODE equation constants

"c" is the arbitrary constant of integration

HOMEWORK

FIND MOST GENERAL SOLUTION

$$\frac{dS}{dx} + 2S = \sin\left(\frac{3\pi x}{L}\right)$$

$$S(x) = ?$$

INITIAL CONDITION

S(x=0)=20

FIND EXACT SOLUTION

Plot S(x); $0 \le \frac{x}{L} \le 1$

ROAD MAP: FOLLOW THE PATH

$$\frac{dS}{dx} + 2S = \sin(\frac{3\pi x}{L})$$

$$y' + ky = q(t)$$

has general solution of:

$$y(t) = e^{-kt} \int q(t)\, e^{+kt} dt + Ce^{-kt}; k > 0$$

$$S(x) = e^{-kx} \int \sin(\frac{3\pi x}{L})\, e^{+kx} dx + Ce^{-kx}; k = 2$$

$$\boxed{\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}[a \sin bx - b \cos bx] + c}$$

$$\boxed{\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}[a \cos bx + b \sin bx] + c}$$

MOST GENERAL SOLUTION

$$S(x) = e^{-kx} \int \sin(\frac{3\pi x}{L})\, e^{+kx} dx + Ce^{-kx}; k = 2$$

In General

$$\int e^{ax} \sin(bx)\, dx = \frac{e^{ax}}{a^2 + b^2}\left[a \sin(bx) - b\cos(bx)\right] + C$$

$$a = k = 2, b = \frac{3\pi}{L}$$

Most General Solution

$$S(x) = e^{-2x}\left[\frac{e^{2x}}{2^2 + \left(\frac{3\pi}{L}\right)^2}\left[2\sin\left(\frac{3\pi}{L}x\right) - \frac{3\pi}{L}\cos\left(\frac{3\pi}{L}x\right)\right]\right] + Ce^{-2x}; k = 2$$

INITIAL CONDITION

$$S(x)=e^{-2x}\left[\frac{e^{2x}}{2^2+\left(\frac{3\pi}{L}\right)^2}\left[2\sin\left(\frac{3\pi}{L}x\right)-\frac{3\pi}{L}\cos\left(\frac{3\pi}{L}x\right)\right]\right]+Ce^{-2x}$$

$$S(x=0)=20$$

$$20=e^{0}\left[\frac{e^{0}}{2^2+\left(\frac{3\pi}{L}\right)^2}\left[2\sin\left(\frac{3\pi}{L}0\right)-\frac{3\pi}{L}\cos\left(\frac{3\pi}{L}0\right)\right]\right]+Ce^{0}\quad;\ \text{BUT } e^{0}=1$$

$$20=\left[\frac{1}{2^2+\left(\frac{3\pi}{L}\right)^2}\left[-\frac{3\pi}{L}\right]\right]+C\rightarrow C=20+\frac{\frac{3\pi}{L}}{2^2+\left(\frac{3\pi}{L}\right)^2}$$

EXACT SOLUTION

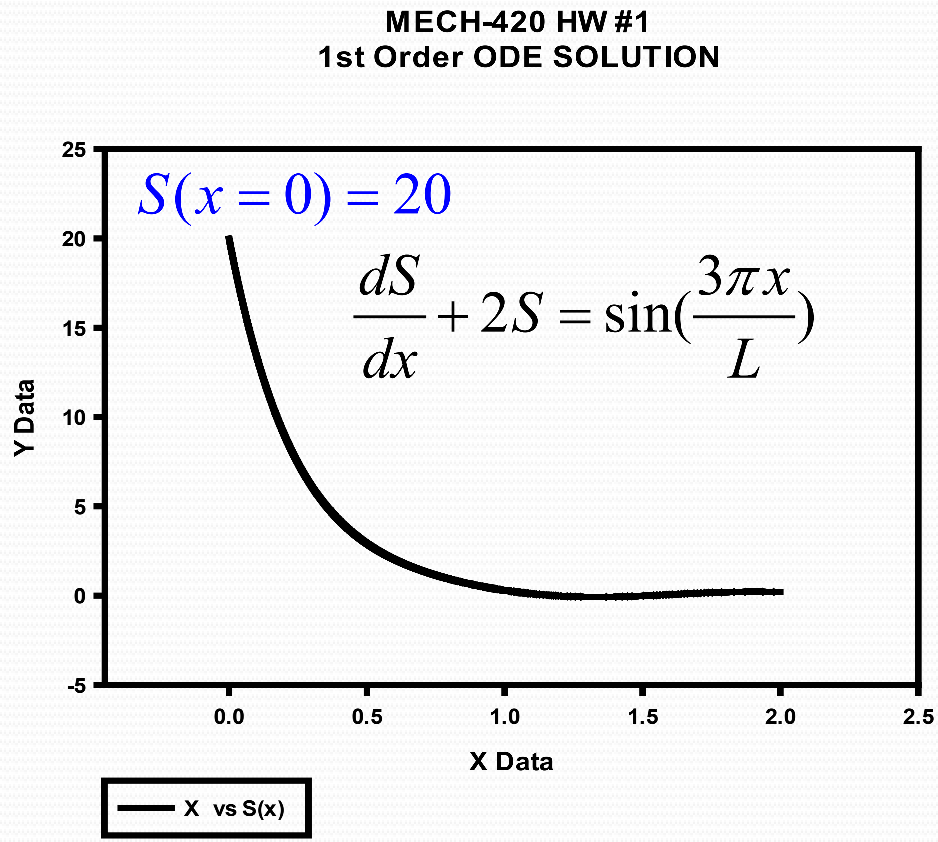
$$S(x)=e^{-2x}\left[\frac{e^{2x}}{2^2+\left(\frac{3\pi}{L}\right)^2}\left[2\sin\left(\frac{3\pi}{L}x\right)-\frac{3\pi}{L}\cos\left(\frac{3\pi}{L}x\right)\right]\right]+Ce^{-2x}$$

$$C=20+\frac{\frac{3\pi}{L}}{2^2+\left(\frac{3\pi}{L}\right)^2}$$

Let L = 2

Initial Solution ?,x=0

Steady State Solution ?,x $\rightarrow \infty$



$$S(x=0)=20$$

$$\frac{dS}{dx}+2S=\sin\left(\frac{3\pi x}{L}\right)$$

Fourier's Rate Equation: Conduction

Conduction: **Fourier's Law**

General (vector) form of Fourier's Law:

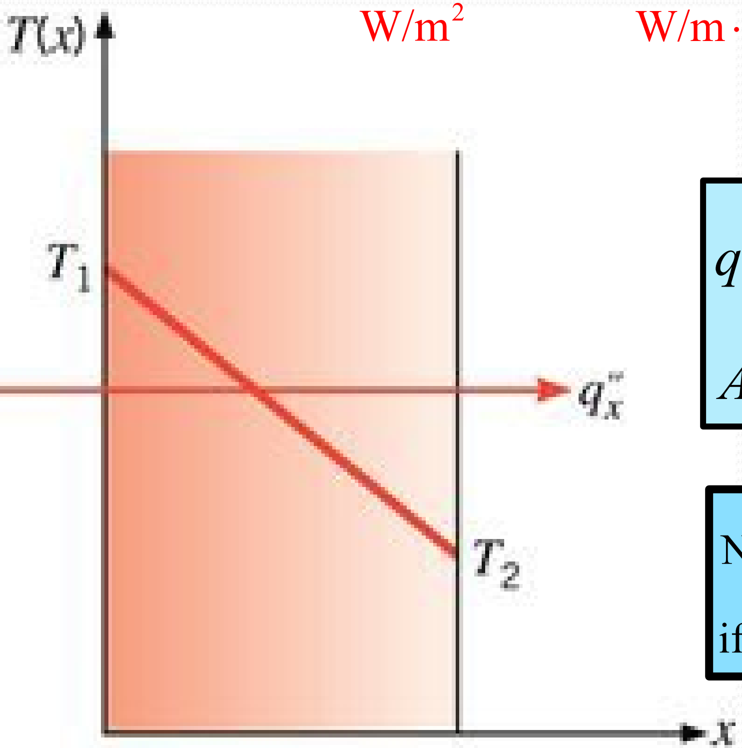
$$\overrightarrow{q''} = -k \underbrace{\nabla T}$$

Heat flux W/m^2

Thermal conductivity $W/m \cdot K$

Temperature gradient °C/m or K/m

$$\boxed{\rightarrow q[W] = q''\left[\frac{W}{m^2}\right] \bullet A_c[m^2]}$$



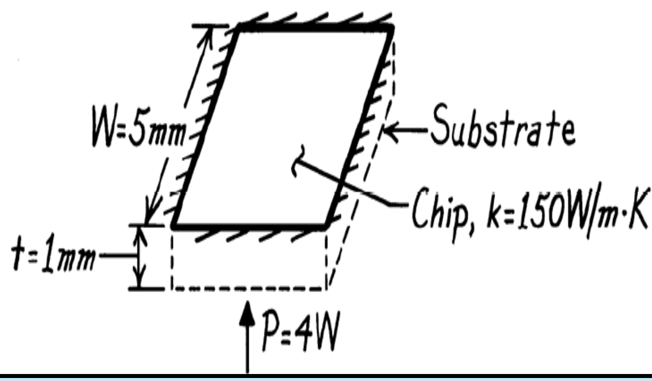
$$q_x[W] = -k\left[\frac{W}{m-K}\right] A_{normal}[m^2] \frac{dT}{dx}\left[\frac{K}{m}\right]$$

A_{normal} = Area Normal to HEAT TRANSFER (i.e. Q)

Note: Since HEAT flows from HIGH to LOW, $\frac{dT}{dx}$ is NEGATIVE if Δx is positive.

Square chip (k=150W/m-k) of w=5mm on a side and t=thickness = 1mm. Mounted on a substrate such that back side is $insulated\,(q=0)$, and front surface is exposed to coolant. If P=4W of power is to be dissipated in circuits mounted to back side of chip, what is ΔT temperature difference between back and front surfaces.

SCHEMATIC:



Analysis: Internal power produced must be dissipated through the solid medium by **<u>CONDUCTIVE</u>** heat transfer to the exposed surface. At the solid-fluid surface, heat transfer is by **<u>CONVECTION</u>** from the solid to the flowing fluid.

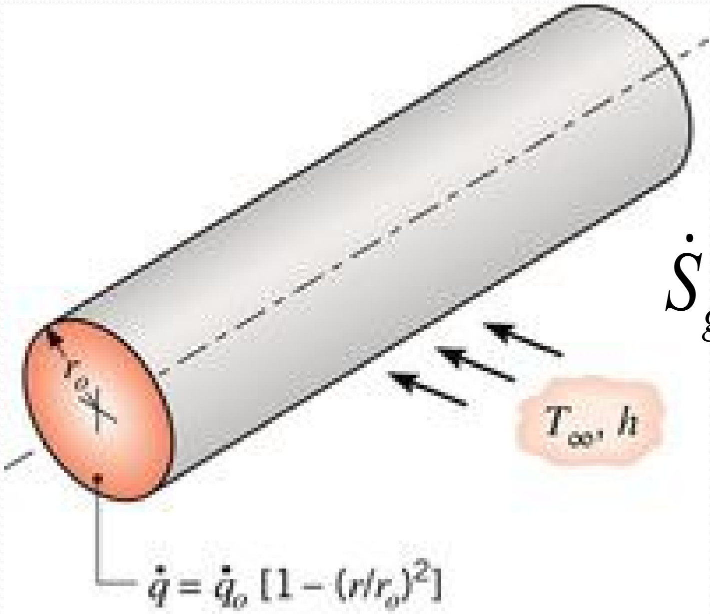
$$q[W] = -kA_{normal}\frac{dT}{dx} = -kA_{normal}\frac{\Delta T}{\Delta x}\left(\frac{\Delta T}{\Delta x} = \frac{T_2 - T_1}{x_2 - x_1}\right)$$

$$\Delta T = \frac{q[W]}{-kA_{normal}} \bullet \Delta x = \frac{W}{\frac{W}{m-K}m^2}m = K$$

$$\Delta T = \frac{4W}{-150\frac{W}{m-K}\left(\frac{5mm}{1000}\frac{5mm}{1000}\right)}\frac{1mm}{1000} = -1.07K$$

Note: Since HEAT AWAYS flows from HIGH to LOW, $\dfrac{dT}{dx}$ is negative.

Find TOTAL Internal Heat Generation (Watts)

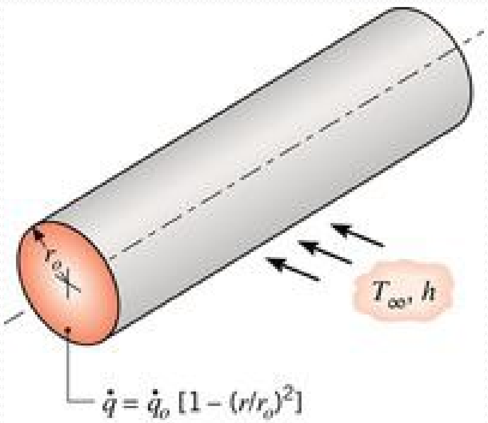


$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_o\left[\frac{W}{m^3}\right](1-\frac{r^2}{r_0^2});0 \geq r \leq r_0$$

$$\dot{E}_{gen}[W] = \int_V \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right]dV$$

$$= S_o\left[\frac{W}{m^3}\right]\int_0^{r_0}(1-\frac{r^2}{r_0^2})dV$$

Cylinder with variable INTERNAL HEAT GENERATION RATE W/m3. FIND TOTAL POWER [W].



$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_o\left[\frac{W}{m^3}\right](1-\frac{r^2}{r_0^2});0 \ge r \le r_0$$

$$\dot{E}_{gen}[W] = \int_V \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right]dV = S_o\left[\frac{W}{m^3}\right]\int_0^{r_0}(1-\frac{r^2}{r_0^2})dV$$

$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_o\left[\frac{W}{m^3}\right](1-\frac{r^2}{r_0^2});0 \ge r \le r_0$$

$$\dot{E}_{gen}[W] = \int_V \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right]dV = S_o\left[\frac{W}{m^3}\right]\int_0^{r_0}(1-\frac{r^2}{r_0^2})dV$$

$$\boxed{V_{cylinder} = \pi\, r^2 L \to \frac{dV}{dr} = 2\pi r \bullet L \to dV = 2\pi r \bullet dr \bullet L}$$

$$\dot{E}_{gen}[W] = \int_V \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right]dV = S_o\left[\frac{W}{m^3}\right]\int_0^{r_0}(1-\frac{r^2}{r_0^2})(2\pi r \bullet dr \bullet L)$$

MUST INTEGRATE FOR VARIABLE INTERNAL GENERATION RATE!!! NO OPTIONS

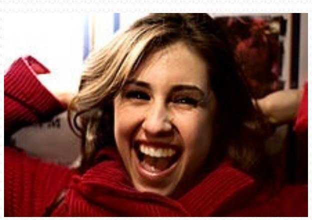
Cylinder with variable INTERNAL HEAT GENERATION RATE W/m3. FIND TOTAL POWER [W].

$$\dot{E}_{gen}[W] = 2\pi L S_o \left[\frac{W}{m^3}\right]\int_0^{r_0}(1-\frac{r^2}{r_0^2})(r \bullet dr) = 2\pi L S_o \left[\frac{W}{m^3}\right]\int_0^{r_0}(r-\frac{r^3}{r_0^2})(dr)$$

$$\dot{E}_{gen}[W] = 2\pi L S_o \left[\frac{W}{m^3}\right]\left(\frac{r^2}{2}-\frac{r^4}{4r_0^2}\right)_{0-r_0}$$

$$\dot{E}_{gen}[W] = 2\pi L S_o \left[\frac{W}{m^3}\right]\left(\frac{r_0^2}{2}-\frac{r_0^4}{4r_0^2}\right) = 2\pi L S_o \left[\frac{W}{m^3}\right]r_0^2(\frac{1}{2}-\frac{1}{4})$$

$$\dot{E}_{gen}[W] = \frac{2\pi L[m] S_o \left[\frac{W}{m^3}\right] r_0^2\left[m^2\right]}{4} = \frac{\pi L S_0 r_0^2}{2}$$



$\rightarrow$ UNIT CHECK

$\rightarrow L[m]S_0[W/m^3]r_0^2[m^2]$

$\rightarrow W$

THE SURFACE ENERGY BALANCE

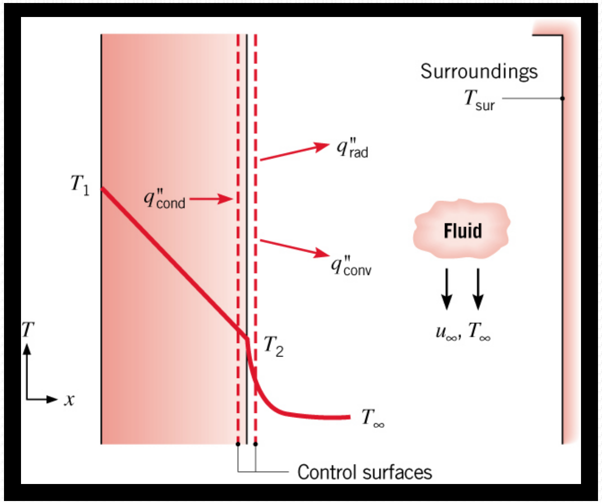
A special case for which no volume or mass is encompassed by the control surface.

Conservation of Energy (Instant in Time):

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0 \tag{1.13}$$

- Applies for steady-state and transient conditions.
- With no mass and volume, energy storage and generation are not pertinent to the energy balance, even if they occur in the medium bounded by the surface.

Consider surface of wall with heat transfer by conduction, convection and radiation.



$$k\frac{T_1 - T_2}{L} - h\left(T_2 - T_\infty\right) - \varepsilon_2\sigma\left(T_2^4 - T_{\text{sur}}^4\right) = 0$$

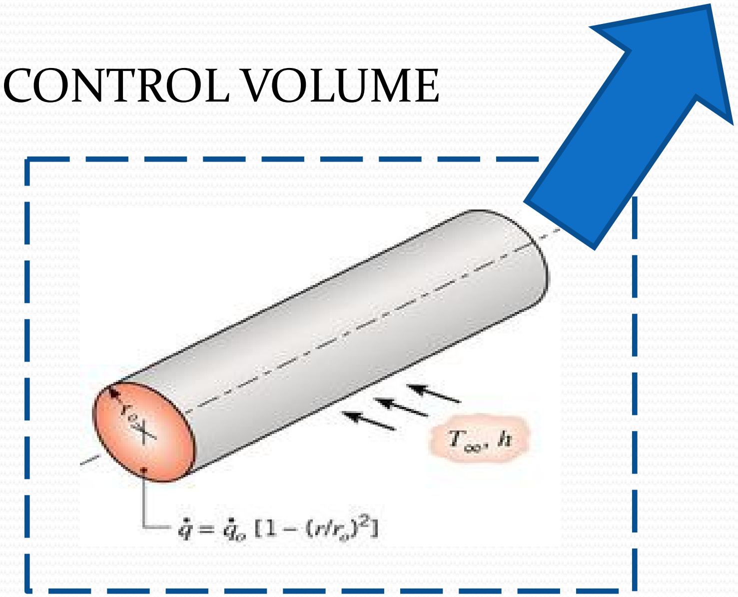
"When" is the CV 1st Law Needed

- To find SURFACE temperature only, Ts. (and heat flux is known at each surface)
  - Apply 1st law to CV around entire object
- To find initial rate of change of temperature (dT/dt)
- When there is no spatial gradients of temperature "INSIDE" the body, i.e. a "thin walled tube".
- Apply at "surface" to find energy balance at surface to determine boundary conditions.

For Cylinder with Variable internal heat generation rate and exposed to convective fluid, find **STEADY STATE** surface temperature.

CONVECTION

CONTROL VOLUME



$$\cancel{\overset{+}{\dot{E}}_{\mathrm{in}}} - \overset{+}{\dot{E}}_{\mathrm{out}} + \overset{\pm}{\dot{E}}_g = \frac{dE_{\mathrm{st}}}{dt} \equiv \cancel{\overset{\pm}{\dot{E}}_{\mathrm{st}}}$$

$$\overset{+}{\dot{E}}_{\mathrm{out}} = \overset{\pm}{\dot{E}}_g$$

$$\bar{h}\left[\frac{W}{m^2-K}\right]A_{surface}\left[m^2\right](T_s - T_\infty) = \overset{\pm}{\dot{E}}_g$$

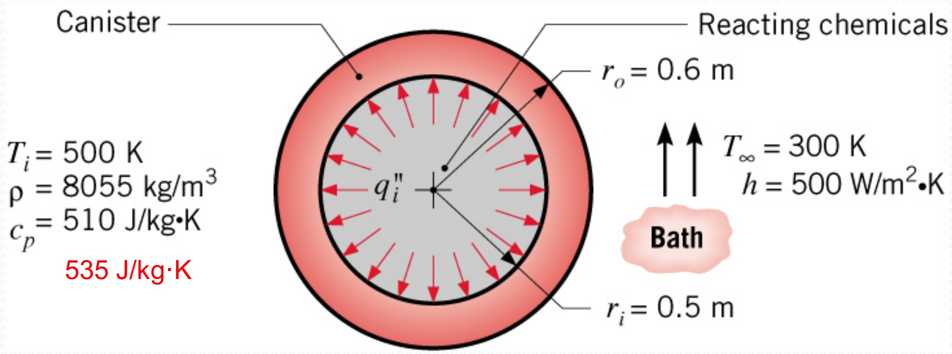
$$T_S = T_\infty + \frac{\overset{\pm}{\dot{E}}_g[W]}{\bar{h}A_{surface}[W/K]}$$

$$\dot{E}_{gen}[W] = \int_V \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right]dV = \frac{\pi L S_0 r_0^2}{2}$$

$A_{sutface} \equiv$ surface exposed to the fluid

$= \pi D \bullet L$

Problem 1.64: Cooling of spherical canister used to store reacting chemicals. Determine (a) *the initial rate of change of the canister temperature (dT/dt)*, (b) the steady-state temperature, and (c) the effect of convection on the steady-state temperature.



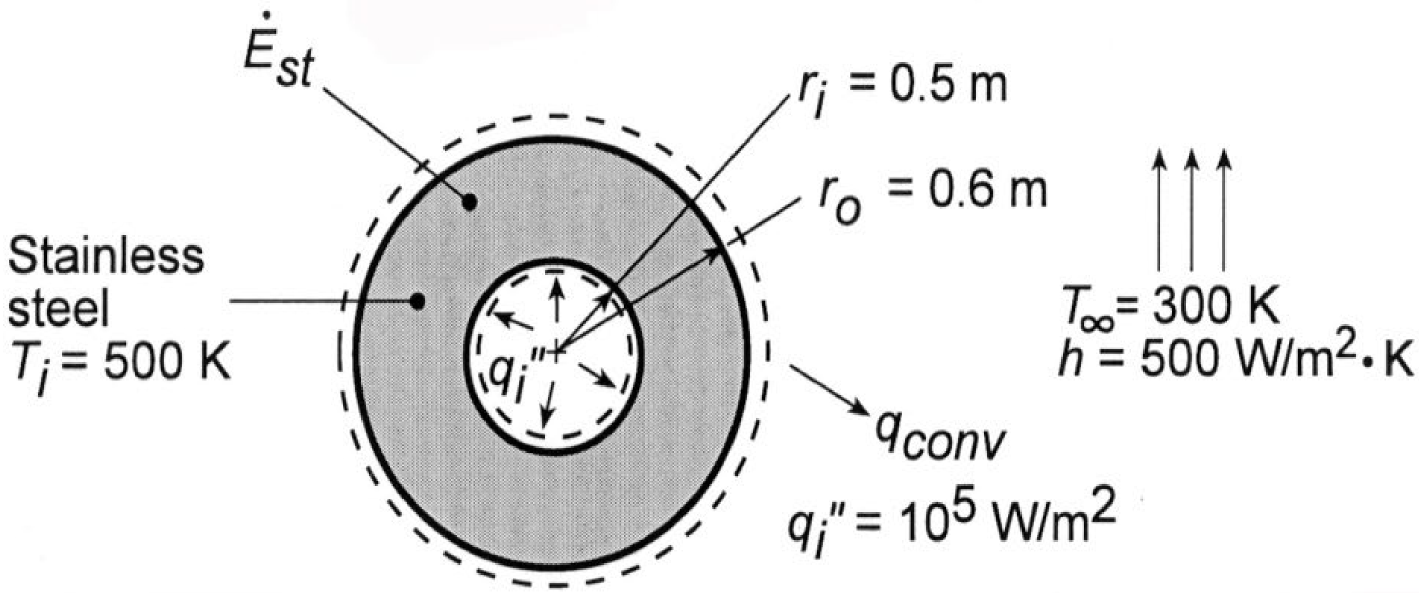
$$\overset{+}{\dot{E}}_{in} - \overset{+}{\dot{E}}_{out} + \overset{\pm}{\dot{E}}_{g} = \frac{dE_{st}}{dt} \equiv \overset{\pm}{\dot{E}}_{st}$$

CONTROL VOLUME IS OUTER SHELL

Known: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and materials

Find: a) Governing equation for variation of wall temperature with time and the initial rate of change, b) Steady State wall temperature and, c) Effect of convection coefficient on canister temperature

CV Applied to OUTER SHELLL Only



$$\overset{+}{\dot{E}}_{\text{in}} - \overset{+}{\dot{E}}_{\text{out}} + \overset{\pm}{\dot{E}}_{g} = \frac{dE_{\text{st}}}{dt} \equiv \rho \forall c_p \frac{dT}{dt}$$

Assume:

1. Constant Properties
2. Uniform Heat Flux at Inner Radius
3. Small Temperature Wall Gradients

Governing ODE for Time Varying Wall Temperature?

ROADMAP

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \frac{dE_{st}}{dt} = \rho \forall c_p \frac{dT}{dt}$$

$$\dot{E}_{in} = q_i''\left[\frac{W}{m^2}\right] \bullet A_i[m^2] = q_i''\left[\frac{W}{m^2}\right] \bullet 4\pi r_i^2[m^2]$$

$$\dot{E}_{out} = \bar{h}\left[\frac{W}{m^2-K}\right] \bullet A_0[m^2] \bullet (T(t) - T_\infty)[K]$$

$$\dot{E}_{out} = \bar{h}\left[\frac{W}{m^2-K}\right] \bullet 4\pi r_0^2[m^2] \bullet (T(t) - T_\infty)[K]$$

$$\dot{E}_{gen} = 0 \rightarrow \text{Internal Generation in Core Only (not shell)}$$

$$q_i''\left[\frac{W}{m^2}\right] \bullet 4\pi r_i^2[m^2] - \bar{h}\left[\frac{W}{m^2-K}\right] \bullet 4\pi r_0^2[m^2] \bullet (T(t) - T_\infty)[K] = \rho \forall c_p \frac{dT}{dt}$$

$$\rho \forall c_p \frac{dT}{dt}$$

$$\frac{kg}{m^3} m^3 \frac{J}{kg-K} \frac{K}{s} = \frac{J}{s} \equiv W$$

Initial Time Rate of Change of Temp

PARAMETRIC ROAD MAP

$$q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]-\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]\bullet(T(t)-T_\infty)[K]=\rho\forall c_p\frac{dT}{dt}$$

$$\left[\frac{dT}{dt}\right]_{t=0}=\frac{q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]-\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]\bullet(T(t=0)-T_\infty)[K]}{\rho\forall c_p}$$

$$\left[\frac{dT}{dt}\right]_{t=0}=\frac{q_i''\left[\frac{W=\cancel{J}/s}{m^2}\right]\bullet 4\pi r_i^2[m^2]-\bar{h}\left[\frac{W=\cancel{J}/s}{m^2-\cancel{K}}\right]\bullet 4\pi r_0^2[m^2]\bullet(T(t=0)-T_\infty)[\cancel{K}]}{\rho\left[\frac{\cancel{kg}}{\cancel{m^3}}\right]\bullet\frac{4}{3}\pi(r_0^3-r_i^3)\left[\cancel{m^3}\right]\bullet c_p\left[\frac{\cancel{J}}{\cancel{kg}-K}\right]}$$

$T(t=0)=T_i=500K$

Table A.1 Stainless Steel AISI 302

$$\rho=8055\frac{kg}{m^3},cp=535\frac{J}{kg-K}$$

$$\boxed{\left[\frac{dT}{dt}\right]_{t=0}=-0.084\frac{K}{s}=-0.084\frac{C}{s}}$$

Steady State Temperature

PARAMETRIC ROAD MAP

$$q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]-\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]\bullet(T(t)-T_\infty)[K]=\rho\forall c_p\frac{dT}{dt}$$

$$\text{Steady State}\rightarrow\text{No Storage}\rightarrow\frac{dT}{dt}=0\;\therefore$$

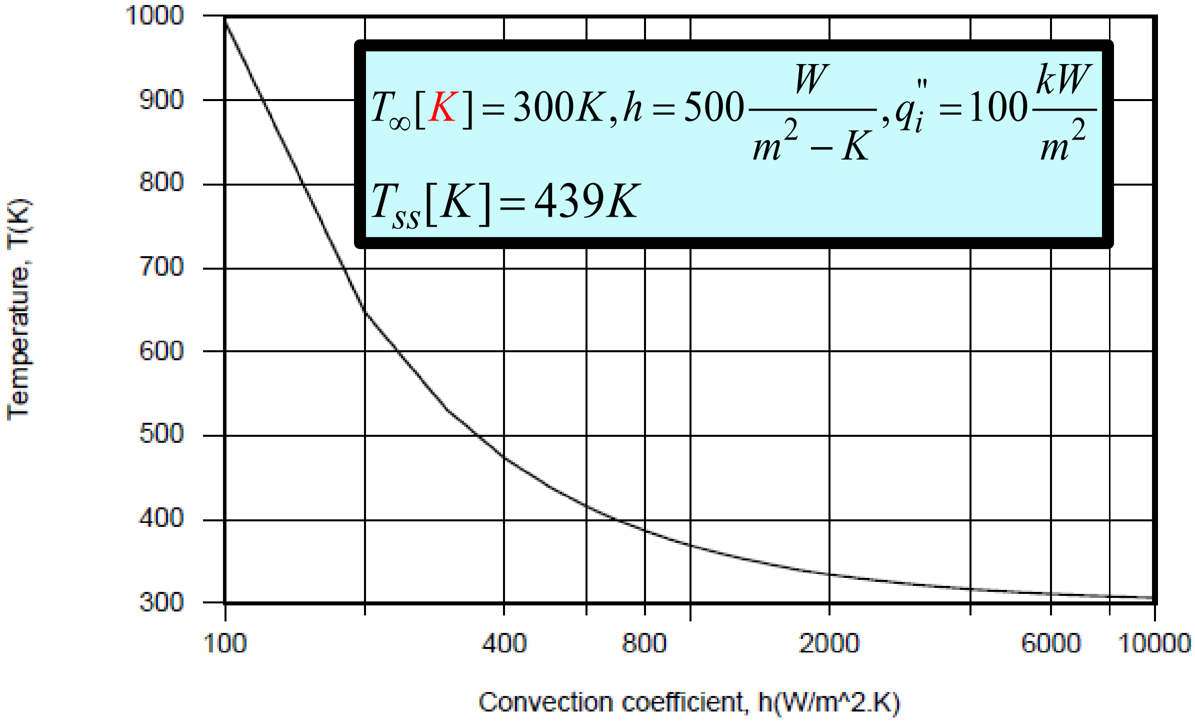
$$q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]-\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]\bullet(T(t\rightarrow\infty)-T_\infty)[K]=0$$

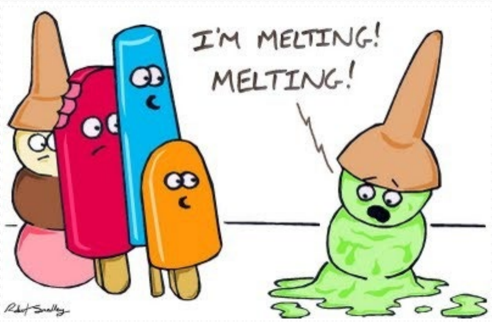
$$T(t\rightarrow\infty)=T_{ss}=\frac{q_i''\left[\frac{\cancel{W}}{\cancel{m^2}}\right]\bullet \cancel{4\pi}\, r_i^2[\cancel{m^2}]}{\bar{h}\left[\frac{\cancel{W}}{\cancel{m^2}-K}\right]\bullet \cancel{4\pi}\, r_0^2[\cancel{m^2}]}+T_\infty[K]$$ **SEE SLIDE #49**

$$T_\infty[K]=300K, h=500\frac{W}{m^2-K}, q_i''=100\frac{kW}{m^2}$$

$$T_{ss}[K]=439K$$

How does "Tss" vary with "h"?





1. In general, convective heat transfer coefficient value is a function of the design constraint of material property (ie. Melting Temp), or perhaps boiling, evaporation or freezing temperature of the liquid.
2. Higher "h" is driven by higher "velocity", which drives larger pumps/fans and increased system power requirements.

Determine Temperature Variation vs Time for Thin Shell, T(t).

PARAMETRIC ROAD MAP

$$q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]-\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]\bullet(T(t)-T_\infty)[K]=\rho\forall c_p\frac{dT}{dt}$$

let $\theta(\mathrm{t})=T(t)-T_\infty, \frac{d\theta}{dt}=\frac{dT(t)}{dt}-\cancel{\frac{dT_\infty}{dt}}$

$$\frac{d\theta}{dt}+\frac{\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]}{\rho\forall c_p}\theta=\frac{q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]}{\rho\forall c_p}$$

$$\frac{d\theta}{dt}+a\theta=b$$

$$y'+ky=q(t)$$

has general solution of:

$$y(t)=e^{-kt}\int q(t)\,e^{+kt}dt+Ce^{-kt};k>0$$

$$\theta(t)=e^{-at}\int be^{+at}dt+Ce^{-at};k>0$$

$\theta(t)=\frac{b}{a}+Ce^{-at}$, Initial Condition, t=0, $\theta(t=0)=\theta_i$

$$\theta_i=\frac{b}{a}+C, C=\theta_i-\frac{b}{a}$$

EXACT SOLUTION

$$\theta(t)=\frac{b}{a}(1-e^{-at})+\theta_i e^{-at}$$

$$a=\frac{\bar{h}\left[\frac{W}{m^2-K}\right]\bullet 4\pi r_0^2[m^2]}{\rho\forall c_p}$$

$$b=\frac{q_i''\left[\frac{W}{m^2}\right]\bullet 4\pi r_i^2[m^2]}{\rho\forall c_p}$$

Long Term Steady State Temperature

$$\theta(t) = \frac{b}{a}(1 - e^{-at}) + \theta_i e^{-at}, @t \to \infty, \theta(t) = \theta_{SS}$$

$$\theta_{SS} = (T_{SS} - T_\infty) = \frac{b}{a}$$

$$T_{SS} = \frac{b}{a} + T_\infty = \frac{\dfrac{q_i''\left[\dfrac{W}{m^2}\right] \bullet 4\pi r_i^2 [m^2]}{\rho \forall c_p}}{\dfrac{\bar{h}\left[\dfrac{W}{m^2 - K}\right] \bullet 4\pi r_0^2 [m^2]}{\rho \forall c_p}} + T_\infty$$

SEE SLIDE #46

$$a = \frac{\bar{h}\left[\dfrac{W}{m^2 - K}\right] \bullet 4\pi r_0^2 [m^2]}{\rho \forall c_p}$$

$$b = \frac{q_i''\left[\dfrac{W}{m^2}\right] \bullet 4\pi r_i^2 [m^2]}{\rho \forall c_p}$$

UNIT CHECK & TIME CONSTANT

$$a = \frac{\bar{h}\left[\dfrac{W}{m^2 - K}\right] \bullet 4\pi r_0^2 [m^2]}{\rho \forall c_p}$$

$$e^{-at} \rightarrow \text{Exponent Must Always be Unitless}$$

$$a = \frac{\left[\dfrac{W = J/s}{m^2 - K}\right][m^2]}{\dfrac{kg}{m^3} m^3 \dfrac{J}{kg - K}} = \frac{1}{s} \rightarrow \text{TIME CONSTANT}$$

$$b = \frac{q_i''\left[\dfrac{W}{m^2}\right] \bullet 4\pi r_i^2 [m^2]}{\rho \forall c_p}$$

$$\theta(t) = \frac{b}{a}(1 - e^{-at}) + \theta_i e^{-at}$$

$$0 \le t \le \infty$$

$$b = \frac{\left[\dfrac{W = J/s}{m^2}\right] \bullet [m^2]}{\dfrac{kg}{m^3} m^3 \dfrac{J}{kg - K}} = \frac{K}{s}$$

NO FLUX CONDITION

Parametric Road Map

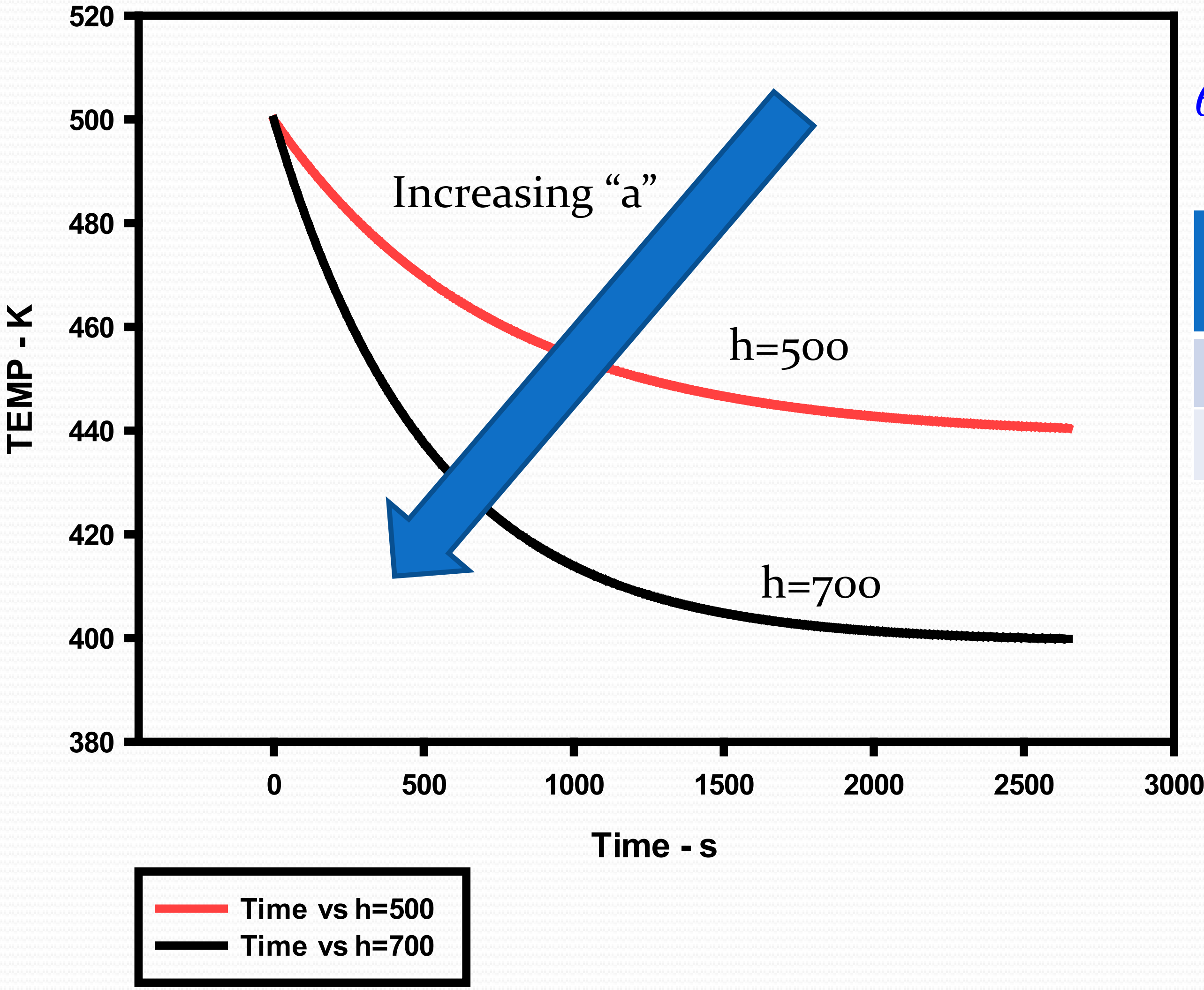
$$\theta(t) = \frac{b}{a}(1 - e^{-at}) + \theta_i e^{-at}$$

Note if b=0 (no flux),

$$\theta(t) = \theta_i e^{-at}, \text{ and, as } t \to \infty$$

$$\theta(t \to \infty) = T(t) - T_\infty = 0, \text{ or, } T(t \to \infty) = T_\infty$$

Time vs Temperature
Spherical Containment Vessel



$$\theta(t) = \frac{b}{a}(1 - e^{-at}) + \theta_i e^{-at}$$

h W/m2-K	Tss K	a 1/s
500	438	0.001377
700	399	0.001928

Find "TIME" AT *KNOWN* TEMPERATURE

$$\theta(t) = \frac{b}{a}(1 - e^{-at}) + \theta_i e^{-at} = T(t) - T_\infty$$

$$\theta(t) = e^{-at}(\theta_i - \frac{b}{a}) + \frac{b}{a}$$

$$\frac{\theta(t) - \frac{b}{a}}{(\theta_i - \frac{b}{a})} = e^{-at}$$

$$\frac{-1}{a}\ln\left[\frac{\theta(t) - \frac{b}{a}}{(\theta_i - \frac{b}{a})}\right] = t(s)$$

Chapter 1 Homework 8th Edition

1.1, 1.4, 1.6, 1.7, 1.10, 1.13, 1.16, 1.19, 1.21,1.23,1.24
1.26, 1.35, 1.36, 1.39,1.40, 1.48, 1.50, 1.51, 1.56

SPECIAL TOPICS PROBLEM

A spherical shell of inner radius "r_1" and outer radius "r_2" serves as radiation containment vessel is exposed to a convective fluid, $T_\infty(r_2) = 300K$, and convective heat transfer coeff. "h=25W/m^2-K". The internal material has a volumetric heat generation rate defined as:

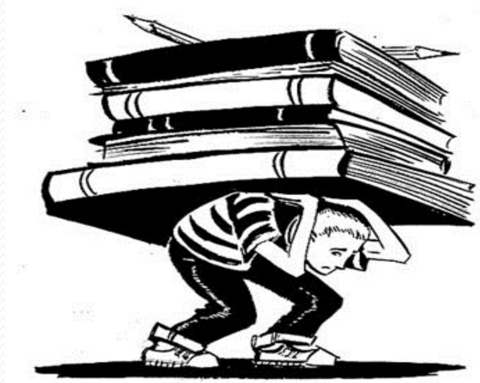
$$\dot{q}(r) = \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0\left[\frac{W}{m^3}\right](2.0 + \frac{r^4}{r_1^4}); 0 \le r \le r_1, r_1 = 2.5m, r_2 = 5m, S_0 = 20kW/m^3.$$

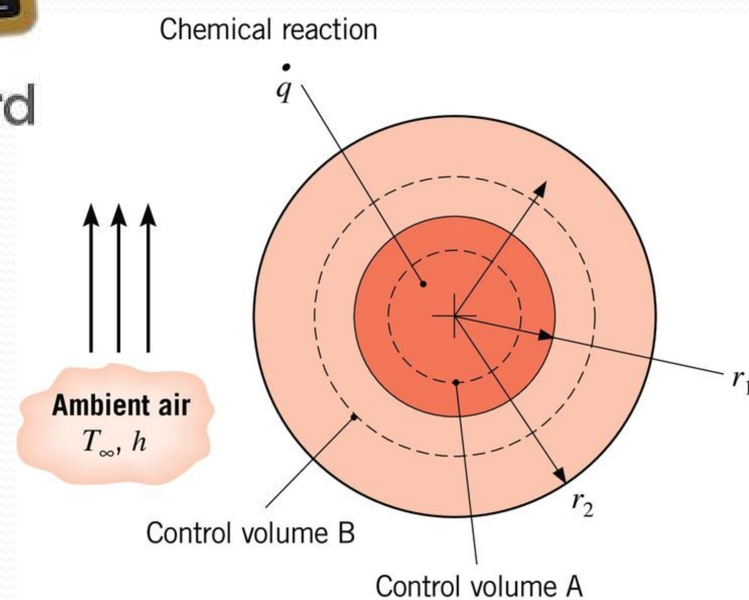
If $V_{sphere} = \frac{4}{3}\pi r^3$, find:

a) Total heat rate generated [W],

b) steady state surface temperature $T_s(r_1)$

c) Plot $T_s(r_1)$ and $q''(r_1)$ vs r_1 as $0 \le r_1 \le 4.5m$





Submit typed solution and spreadsheet/MATLAB

HOMEWORK

Blackboard

FIND MOST GENERAL SOLUTION

$$\frac{d\psi}{dr} + 2\psi = e^{(3\alpha\frac{r}{R})};0 \le \alpha \le 0.01$$

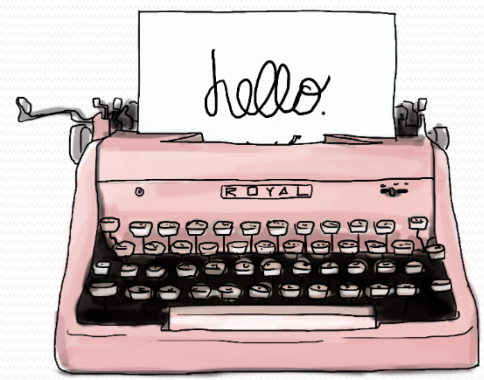
Submit typed solution and spreadsheet/MATLAB

$\psi(r) = ?, \rightarrow$ Cosmic Gravtation Neutron Flux ($\mathrm{TW/m^2}$)

INITIAL CONDITION

$\psi(\mathrm{r{=}0}) \rightarrow 20$

FIND EXACT SOLUTION



Plot $\Psi(\mathrm{r},\alpha);0 \le \frac{r}{R} \le 1, \alpha = 0, \rightarrow 0.01, step = 0.001$

- ***NO SUCCESS WITHOUT PRACTICE***