MOMEMTUM/ ENERGY/ WORK STUDY AID

MECH-322 Fluid Mechanics

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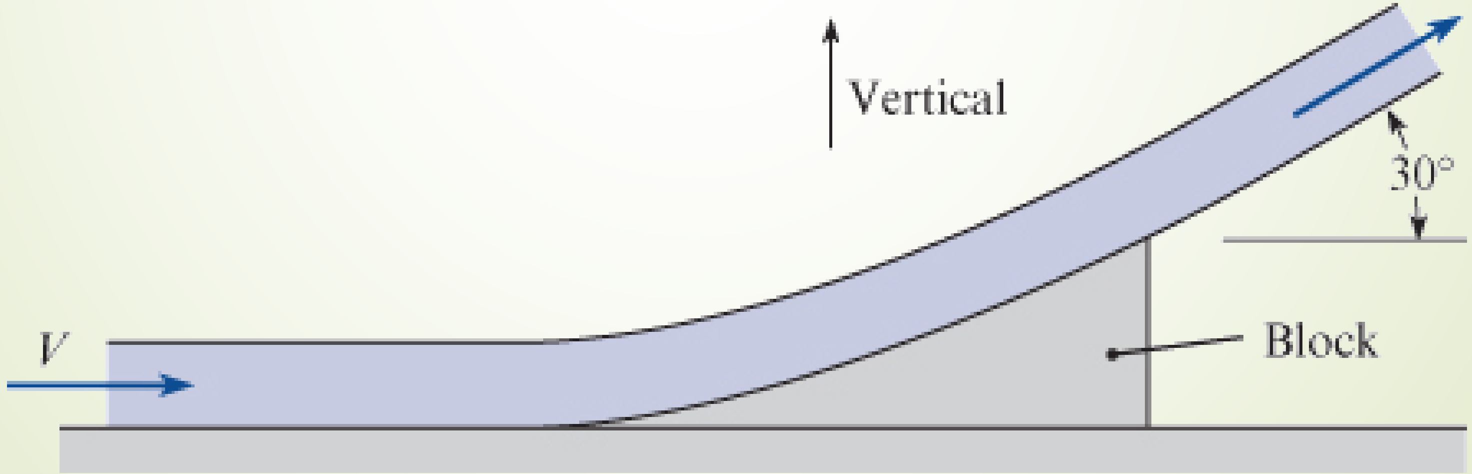
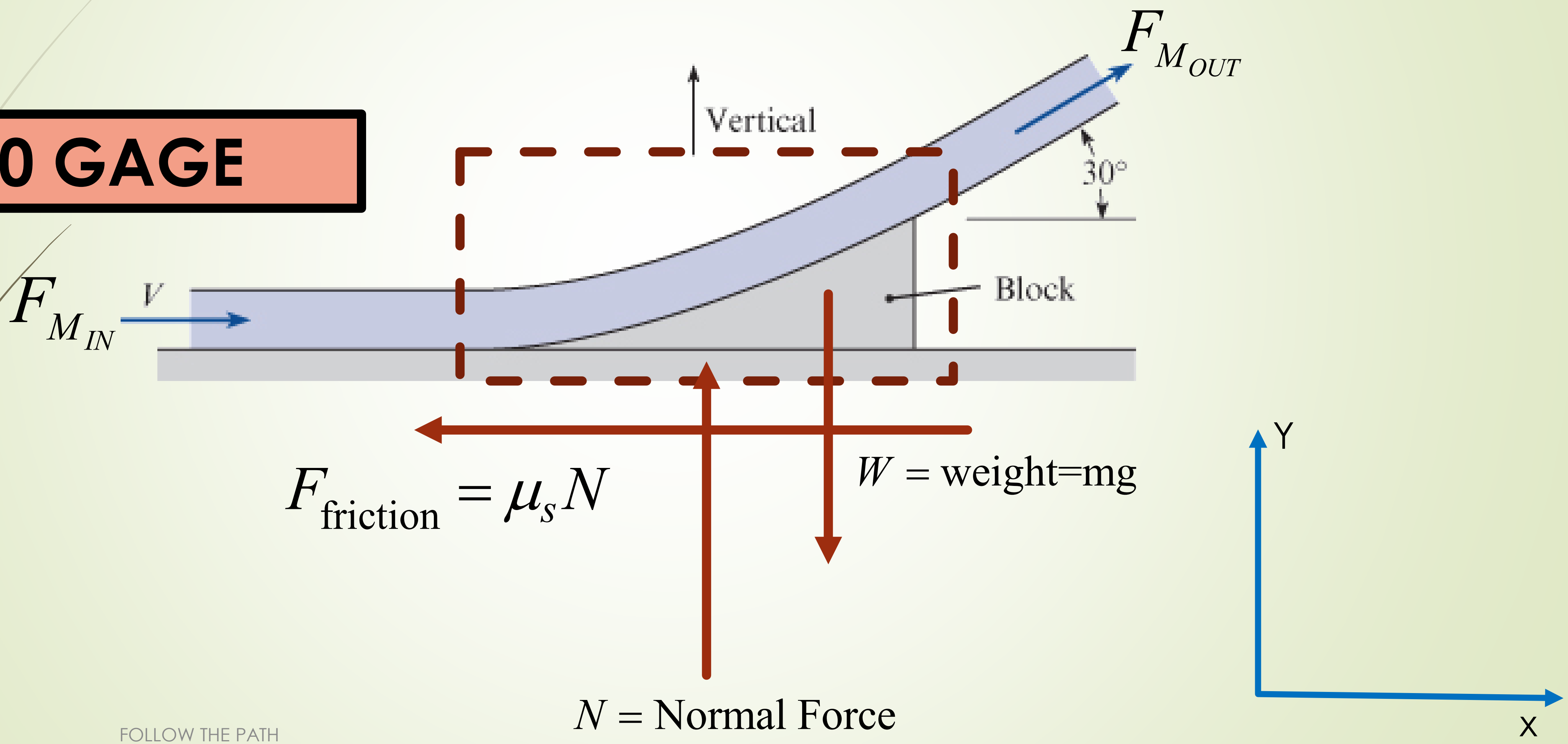
Water strikes a block as shown with a mass flow of 5 kg/s with a velocity of 25 m/s. If the coefficient of static friction between the block and the horizontal surface is 0.25, and is you neglect the shear force between the water and the block and the weight of the water, provide:

1. The minimum block weight to restrict block motion as shown.
2. Plot the minimum block weight (Left Y axis) and Friction Force (Right Y Axis) vs the exit angle as it varies from 20^0-45^0 (X AXIS) in increments of 1^0.



FREE BODY DIAGRAM (FLUID and BLOCK)

P=0 GAGE



X: MOMENTUM

$$\rightarrow \sum F_x = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out} (u_{out} \pm)\dot{m} - \sum_{in} (u_{in} \pm)\dot{m}$$

$$-\mu_s N = 0 + (V_{out} \cos\theta +)\dot{m}_{out} - (V_{in} +)\dot{m}_{in}$$

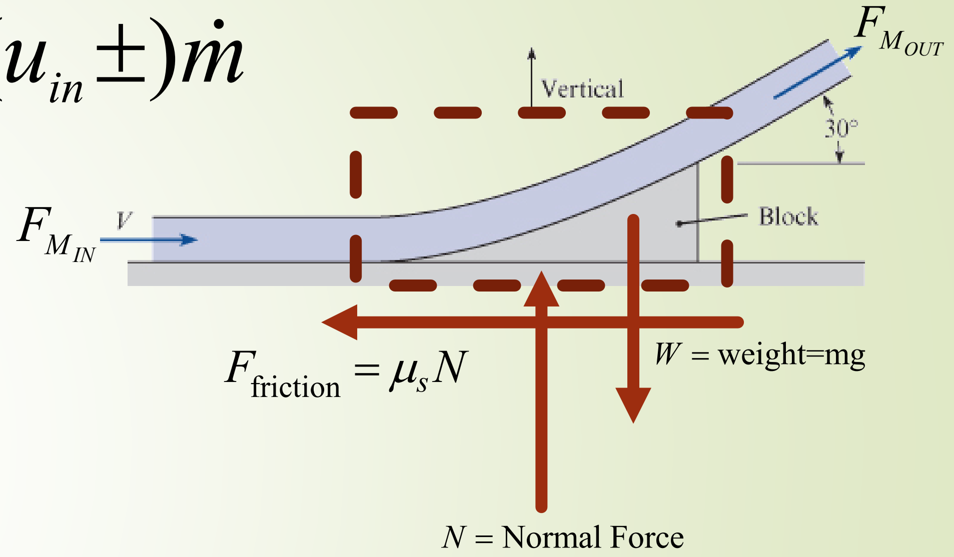
NO FRICTION $\rightarrow$

$$V_{in} = V_{out} = V$$

MASS CONSERVATION $\rightarrow$

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

$$N = \frac{\dot{m}V(1+\cos\theta)}{\mu_s} = 933 Newtons$$

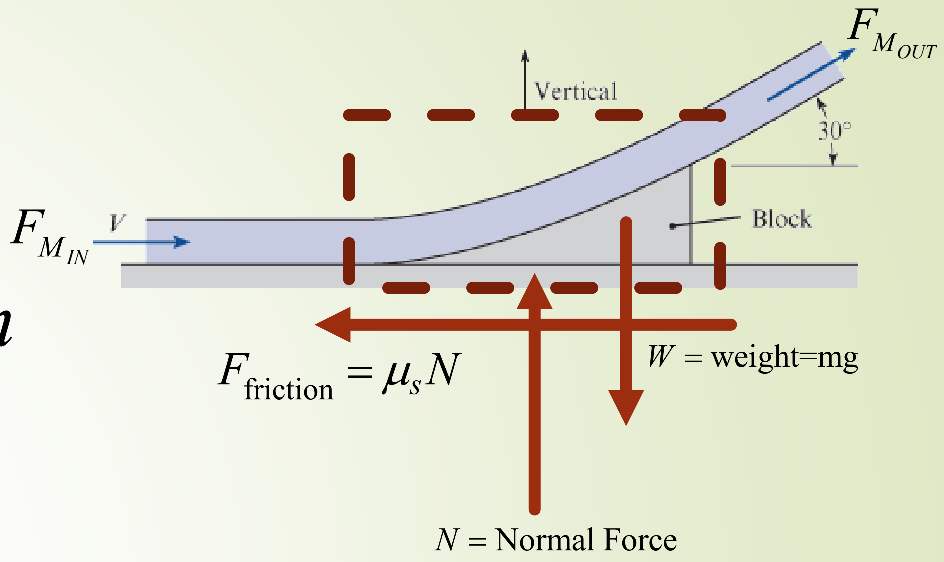


$$\dot{m} = 5kg/s$$

$$V = 25m/s$$

$$\mu_s = 0.25$$

Y:MOMENTUM



$$\uparrow \sum F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out} \pm)\dot{m} - \sum_{in}(v_{in} \pm)\dot{m}$$

$$N - W = 0 + (V_{out}\sin\theta +)\dot{m}_{out} - 0$$

$$W = N - (V_{out}\sin\theta +)\dot{m}_{out}$$

$$N = \frac{\dot{m}V(1+\cos\theta)}{\mu_s} = 933 Newtons$$

$$W = \frac{\dot{m}V(1+\cos\theta)}{\mu_s} - (V\sin\theta)\dot{m}$$

$$W = \dot{m}V(\frac{1+\cos\theta}{\mu_s} - \sin\theta)$$

$$\dot{m} = 5kg/s$$
$$V = 25m/s$$
$$\mu_s = 0.25$$

NOTE:

$$W = \dot{m}V(\frac{1+\cos\theta}{\mu_s} - \sin\theta)$$

If $\theta = 90^0, W = \dot{m}V\left(\frac{1}{\mu_s} - 1\right)$

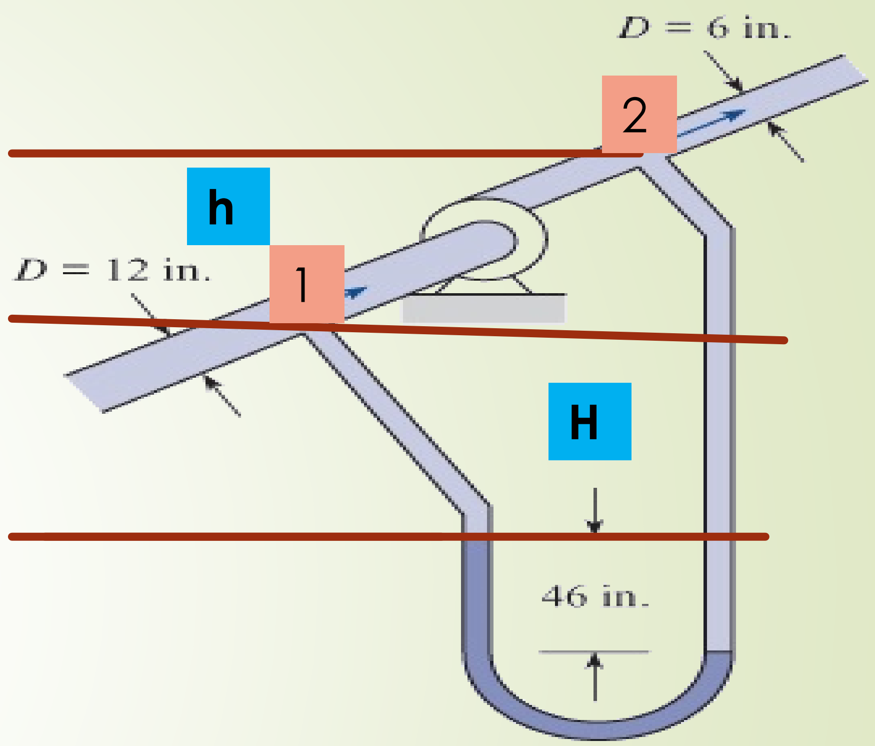
Oil (SG = 0.86) flows through the pump and pipe system as shown with the mercury manometer deflection as shown. The energy flow loss through the pipes is defined as: . If the pump volume flow rate is Q and efficiency is, ,

$$h_L = 5.2\frac{V^2}{2g}(V \rightarrow 6''\, pipe)$$

A. Determine the parametric expression for pump POWER with your other variable names that you choose to represent problem. You should avoid plugging in numbers until you can validate solution units.

B. If pump efficiency is 80%, 85%, 90%, and 95% plot volume flow rate vs pump work with flow varying from **1 CFS to 20 CFS** in increments of 0.5 CFS. (i.e. 4 curves on one plot)

C. If pump efficiency is 80%, flow is 10 CFS, allow smaller pipe diameter to vary from 3" to 10" in steps of 0.5 in and plot pipe diameter vs pump work.



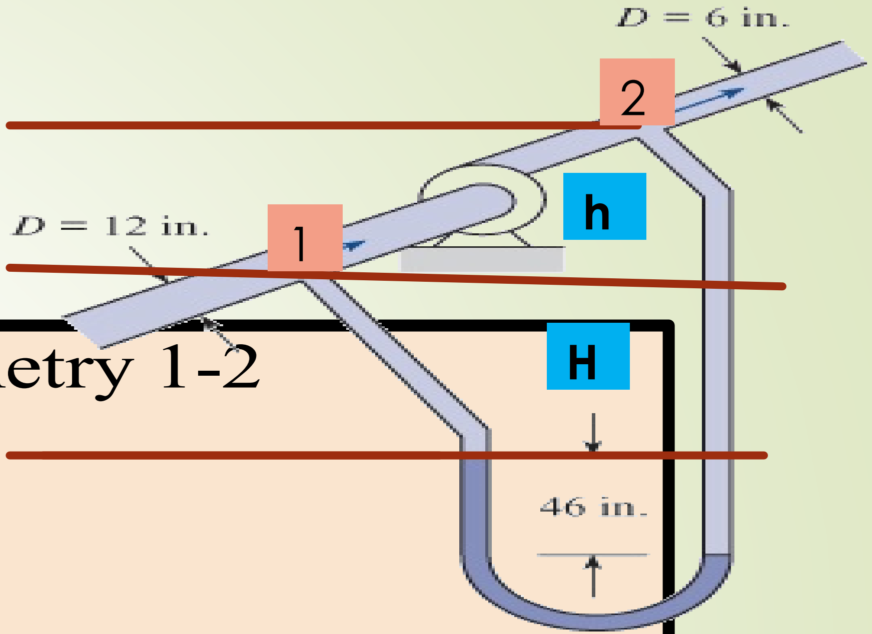
Important Principals

Manometry → Pressure Difference

Chang in D → Mass Conservation

Pump → Conservation of Energy/STREAMLINE

MANOMETRY



Manometry 1-2

$$P_1 + \gamma_{oil} H + \gamma_m \left(\frac{46}{12}\right) - \gamma_{oil}(H + h) = P_2$$

$$P_1 + \not{\gamma_{oil} H} + \gamma_m \left(\frac{46}{12}\right) - \gamma_{oil}\left(\not{H} + h + \frac{46}{12}\right) = P_2$$

$$\frac{P_1 - P_2}{\gamma_{oil}} = h - \frac{\gamma_m}{\gamma_{oil}}\left(\frac{46}{12}\right) + \frac{46}{12} =$$

$$= h - \frac{13.55\gamma_{water}}{SG_{oil} \bullet \gamma_{water}}\left(\frac{46}{12}\right) + \frac{46}{12}$$

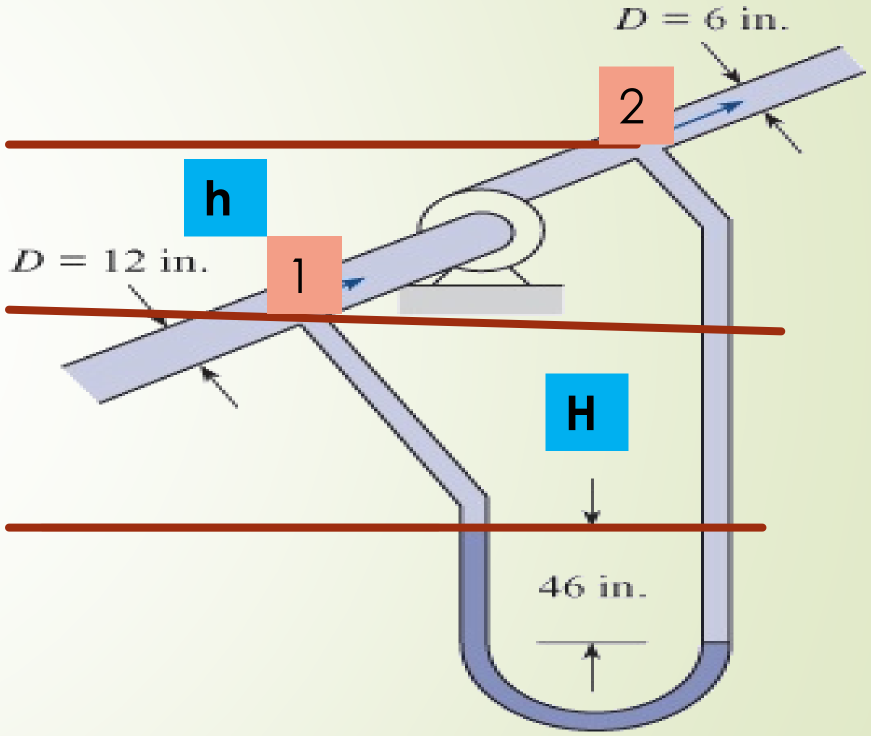
$$\frac{P_1 - P_2}{\gamma_{oil}} = h - \left(\frac{46}{12}\right)\left(1 - \frac{13.55}{SG_{oil}}\right)$$

MASS CONSERVATION

Mass Conservation

$$V_{12} = \frac{Q}{A_{12}} = \frac{Q}{\frac{\pi D_{12}}{4}}$$

$$V_6 = \frac{Q}{A_6} = \frac{Q}{\frac{\pi D_6}{4}}$$

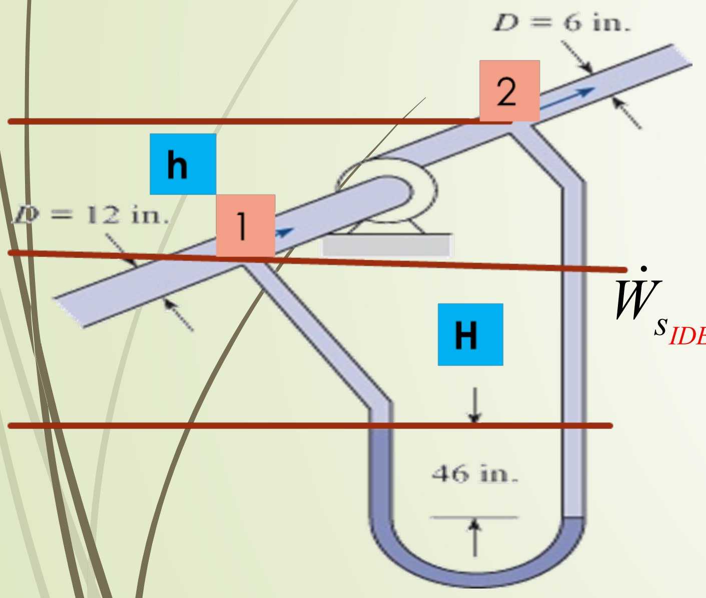


CONSERVATION of ENERGY

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in}\left(\dot{m}g\left(\frac{p_1}{\gamma}+\frac{u_1}{g}+\frac{V_1^2}{2g}+z_1\right)\right) = \sum_{out}\left(\dot{m}g\left(\frac{p_2}{\gamma}+\frac{u_2}{g}+\frac{V_2^2}{2g}+z_2\right)\right) + \sum H_L$$

$$H_q = \sum_{out}\left(\dot{m}g\frac{u_2}{g}\right) - \sum_{in}\left(\dot{m}g\frac{u_1}{g}\right) - \dot{Q}_{cs} = 0$$

$$\dot{W}_{s_{IDEAL}} = \cancel{\dot{W}_{s_{IDEALTURBIN}}} - \dot{W}_{s_{IDEALPUMP}}$$



$$\dot{W}_{s_{IDEALPUMP}}[W] = \sum_{out}\left(\dot{m}g\left(\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2\right)\right) - \sum_{in}\left(\dot{m}g\left(\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1\right)\right) + \cancel{H_q} + H_L$$

$$H_L = \dot{m}gh_l[W] = \dot{m}g\frac{5.2V^2}{2g},$$

$$\frac{P_1 - P_2}{\gamma_{oil}} = h - \left(\frac{46}{12}\right)\left(1 - \frac{13.55}{SG_{oil}}\right)$$

$$\dot{W}_{s_{IDEALPUMP}} = \dot{m}g\left(\frac{p_2 - p_1}{\gamma_{oil}} + (z_2 - z_1) + \frac{5.2V^2}{2g}\right)$$

PUMP PARAMETRIC WORK

$$\dot{W}_{s_{IDEAL\,PUMP}} = \dot{m}g\left(\frac{P_2 - P_1}{\gamma_{oil}} + (z_2 - z_1) + \frac{5.2V^2}{2g}\right)$$

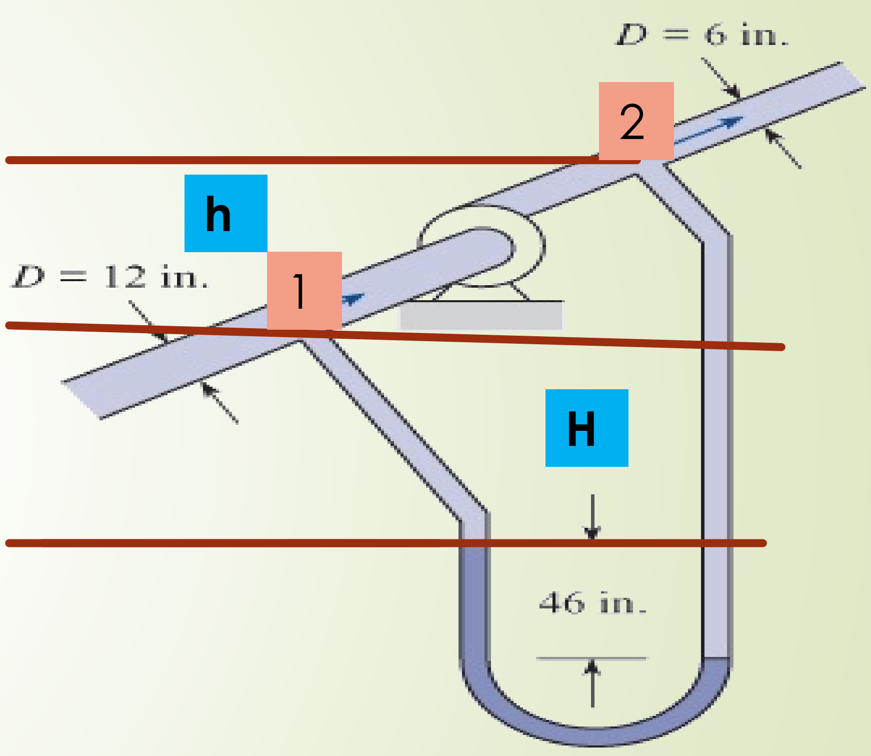
$$\frac{P_2 - P_1}{\gamma_{oil}} = \frac{13.55}{SG_{oil}}\left(\frac{46}{12}\right) - h$$

$$\dot{W}_{s_{IDEAL\,PUMP}} = \dot{m}g\left[\frac{13.55}{SG_{oil}}\left(\frac{46}{12}\right) - \cancel{h + (z_2 - z_1)} + \frac{5.2V^2}{2g}\right]$$

$$= \dot{m}g\left[\frac{13.55}{SG_{oil}}\left(\frac{46}{12}\right) + \frac{5.2V^2}{2g}\right]$$

$$= \gamma_{oil}Q\left[\frac{13.55}{SG_{oil}}\left(\frac{46}{12}\right) + \frac{5.2\left(\frac{Q}{A_6}\right)^2}{2g}\right]; \frac{lbf}{ft^3}\,\frac{ft^3}{s}\,ft = \frac{lbf - ft}{s}$$

FOLLOW THE PATH

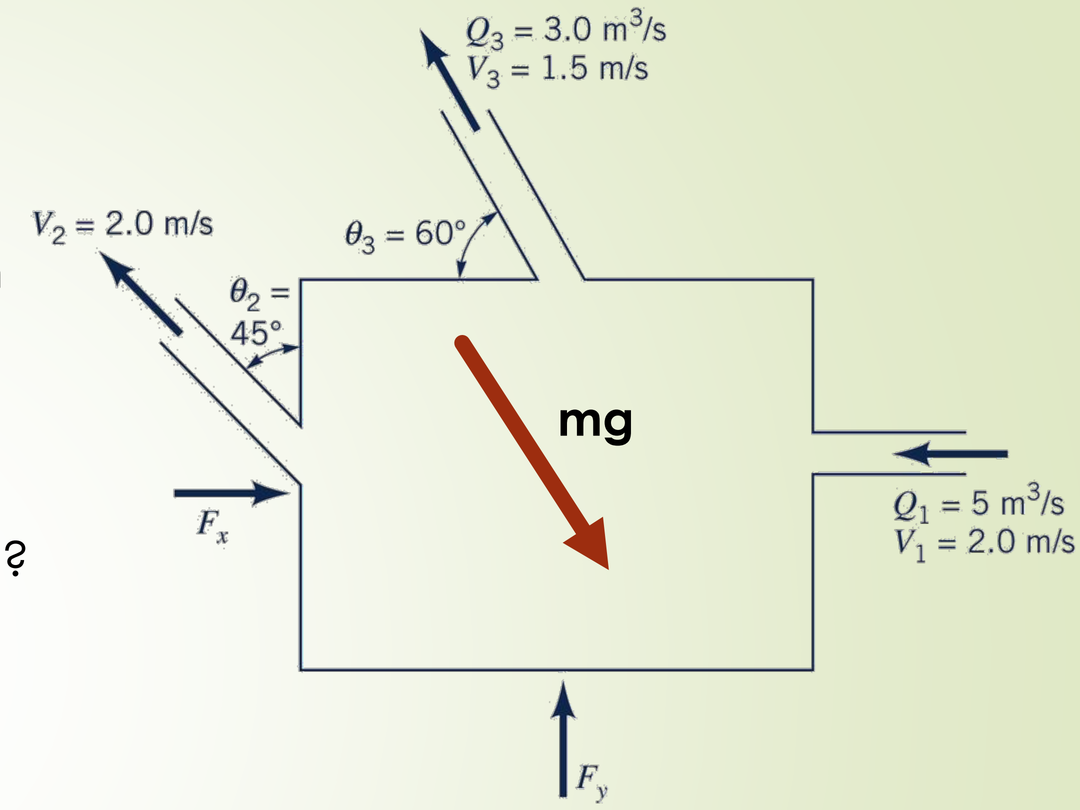


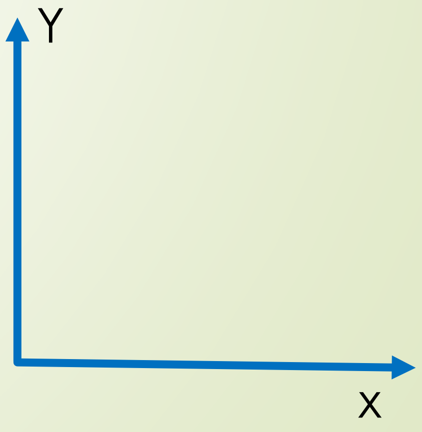
$$\dot{W}_{pump\,ACTUAL_{IN}} = \frac{\dot{W}_{pump\,IDEAL_{IN}}}{\eta_{pump}}$$

Selium 32 (S.G. 0.67) flows as shown in the plane with the gravitational acceleration vector defined as:

$$\vec{g} = g_0(\cos 42\hat{i} - \sin 42\hat{j})$$

caused by the passing comet SARIUS 4, with a volume of 0.2 m3 and discharges into an unknown environment.





Determine:

a. Anchoring forces if the inlet pressure is 150kPa?

$$h_{L_{1-3}} = 1.7 J/kg,$$

$$h_{L_{1-2}} = 0.2m$$

b. Angle of resultant anchoring force makes with the X and Y axis.

c. Overall Total System head loss (W).

d. Plot Fx and Fy vs θ_3 from 0 to 90 in steps of 50.

Important Principals
Mass Conservation
Momentum Conservation

Energy Conservation
ASSUME STEADY/INCOMPRESSIBLE

MASS CONSERVATION

$$\sum Q_{in} - \sum Q_{out} = 0$$

$$Q_1 - Q_2 - Q_3 = 0$$

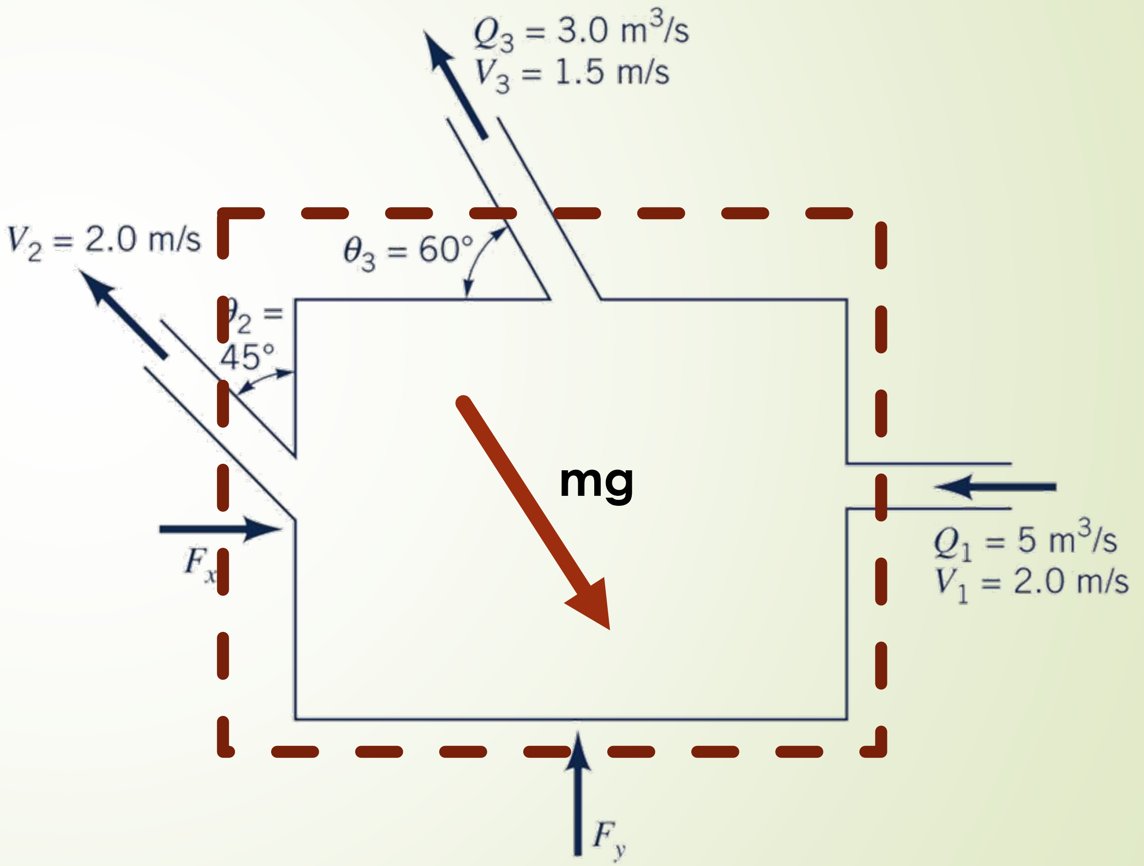
$$Q_2 = Q_1 - Q_3 = (5-3)\frac{m^3}{s} = 2\frac{m^3}{s}$$

AREAS

$$A_1 = \frac{Q_1}{V_1} = \frac{5}{2}m^2 = 2.5m^2$$

$$A_2 = \frac{Q_2}{V_2} = \frac{2}{2}m^2 = 1m^2$$

$$A_3 = \frac{Q_3}{V_3} = \frac{3}{1.5}m^2 = 2m^2$$



ENERGY CONSERVATION

1-3

$$\dot{Q}_{cs} - \dot{W}_{s_{IDEAL}} + \sum_{in}(\dot{m}g(\frac{p_1}{\gamma} + \frac{u_1}{g} + \frac{V_1^2}{2g} + z_1)) = \sum_{out}(\dot{m}g(\frac{p_2}{\gamma} + \frac{u_2}{g} + \frac{V_2^2}{2g} + z_2)) + \sum H_L$$

$$H_q = \sum_{out}(\dot{m}g\frac{u_2}{g}) - \sum_{in}(\dot{m}g\frac{u_1}{g}) - \dot{Q}_{cs} = 0$$

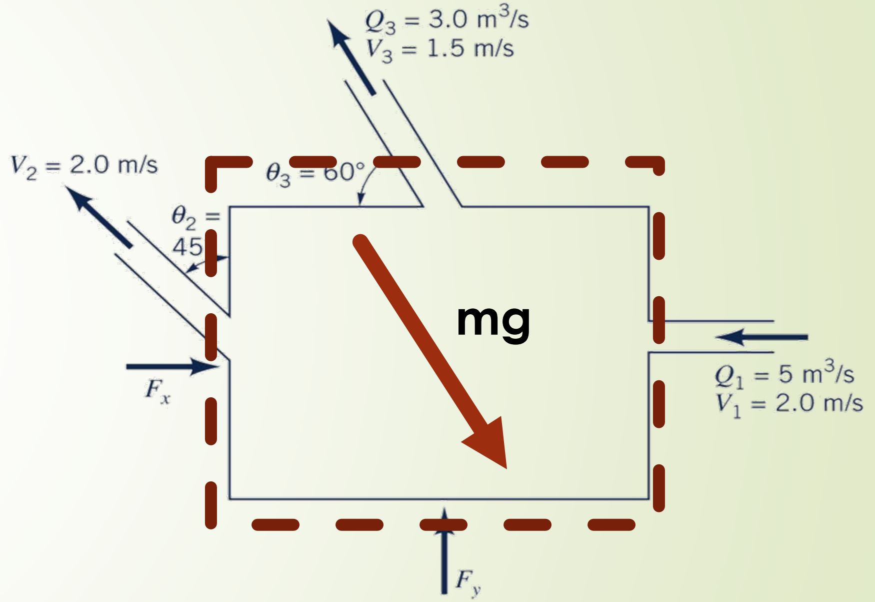
$$1-3, z_1 = z_3$$

$$\dot{m}_1 g(\frac{p_1}{\gamma} + \frac{V_1^2}{2g}) = \dot{m}_3 g(\frac{p_3}{\gamma} + \frac{V_3^2}{2g}) + \dot{m}_3 g h_{L_{1-3}}$$

$$\left[\frac{\dot{m}_1 g}{\dot{m}_3 g}(\frac{p_1}{\gamma} + \frac{V_1^2}{2g}) - \frac{\dot{m}_3 g h_{L_{1-3}}}{\dot{m}_3 g} - \frac{V_3^2}{2g}\right]\gamma = p_3$$

$$\left[\frac{Q_1}{Q_3}(\frac{p_1}{\gamma} + \frac{V_1^2}{2g}) - \frac{\dot{m}_3 g\, h_{L_{1-3}}}{\dot{m}_3 g} - \frac{V_3^2}{2g}\right]\gamma = p_3$$

FOLLOW THE PATH



$$h_{L_{1-3}} = \frac{1.7J/kg}{9.81m/s^2} = \frac{\left(\frac{N = \frac{kg-m}{s^2}}{kg}\right)m}{\frac{m}{s^2}} = 0.1733m, (J = N-m)$$

$$p_1 = 150{,}000PA$$

ENERGY CONSERVATION 1-2

$$\cancel{\dot{Q}_{cs}-\dot{W}_{s_{IDEAL}}}+\sum_{in}(\dot{m}g(\frac{p_1}{\gamma}+\frac{u_1}{g}+\frac{V_1^2}{2g}+z_1))=\sum_{out}(\dot{m}g(\frac{p_2}{\gamma}+\frac{u_2}{g}+\frac{V_2^2}{2g}+z_2))+\sum H_L$$

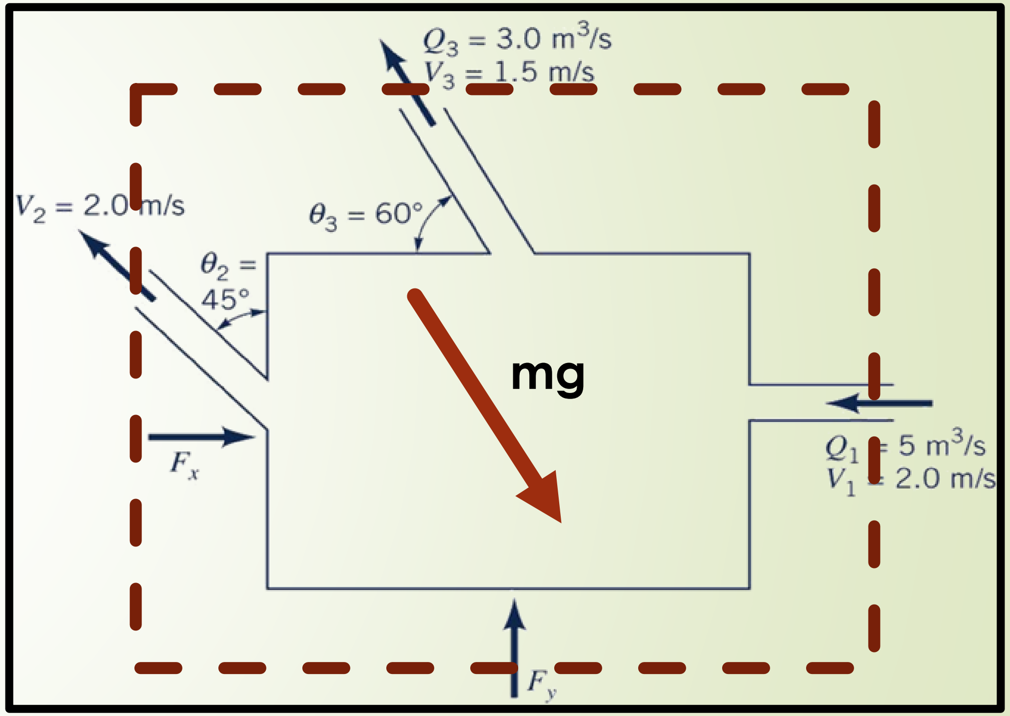
$$H_q=\sum_{out}(\dot{m}g\frac{u_2}{g})-\sum_{in}(\dot{m}g\frac{u_1}{g})-\dot{Q}_{cs}=0$$

$$1-2, z_1=z_2$$

$$\dot{m}_1g(\frac{p_1}{\gamma}+\frac{V_1^2}{2g})=\dot{m}_2g(\frac{p_2}{\gamma}+\frac{V_2^2}{2g})+\dot{m}_2gh_{L_{1-2}}$$

$$\left[\frac{\dot{m}_1g}{\dot{m}_2g}(\frac{p_1}{\gamma}+\frac{V_1^2}{2g})-\frac{\dot{m}_2gh_{L_{1-2}}}{\dot{m}_2g}-\frac{V_2^2}{2g}\right]\gamma=p_2$$

$$\left[\frac{Q_1}{Q_2}(\frac{p_1}{\gamma}+\frac{V_1^2}{2g})-\frac{\cancel{\dot{m}_2g}\,h_{L_{1-2}}}{\cancel{\dot{m}_2g}}-\frac{V_2^2}{2g}\right]\gamma=p_2$$



$$h_{L_{1-2}}=0.2m$$

$$p_1=150,000PA$$

$$\gamma=SG\bullet\gamma_{water}$$

X: MOMENTUM CONSERVATION

$$\vec{g} = g_0(\cos 42\hat{i} - \sin 42\hat{j})$$

$$\to \sum F_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out}\pm)\dot{m} - \sum_{in}(u_{in}\pm)\dot{m}$$

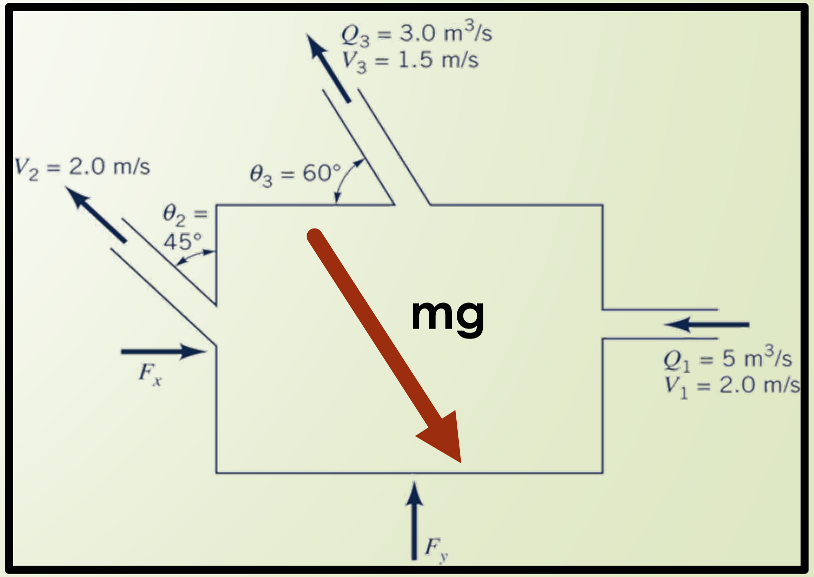
$$+F_x + \{W_x = mg_x = g_0(\cos 42)\rho\forall\} - P_1A_1 + P_2A_2\sin 45 + P_3A_3\cos 60$$

$$= 0 + \{(V_2\sin 45-)\dot{m}_2 + (V_3\cos 60-)\dot{m}_3\} - (V_1-)\dot{m}_1$$

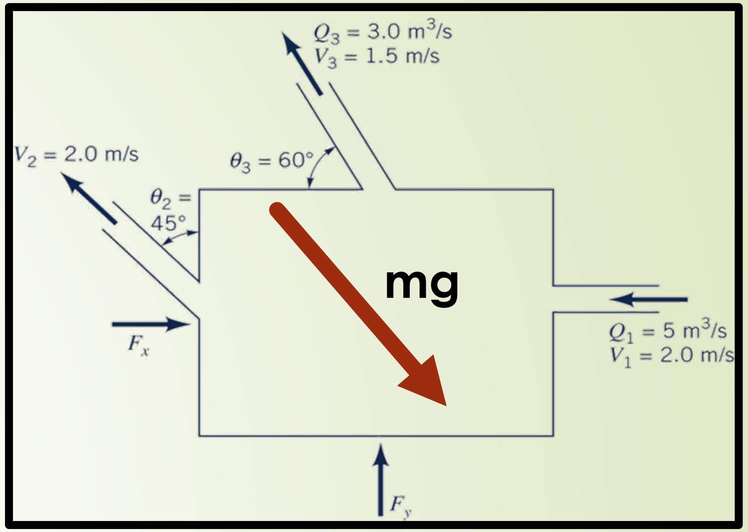
$$F_x = P_1A_1 - P_2A_2\sin 45 - P_3A_3\cos 60 - g_0(\cos 42)\rho\forall$$

$$+$$

$$\{(V_2\sin 45-)\dot{m}_2 + (V_3\cos 60-)\dot{m}_3\} - (V_1-)\dot{m}_1$$



Y: CONSERVATION OF MOMENTUM



$$\vec{g} = g_0(\cos 42\hat{i} - \sin 42\hat{j})$$

$$\uparrow \sum F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out} \pm)\dot{m} - \sum_{in}(v_{in} \pm)\dot{m}$$

$$F_y - g_0 \sin 42 \rho \forall - P_2 A_2 \cos 45 - P_3 A_3 \sin 60$$

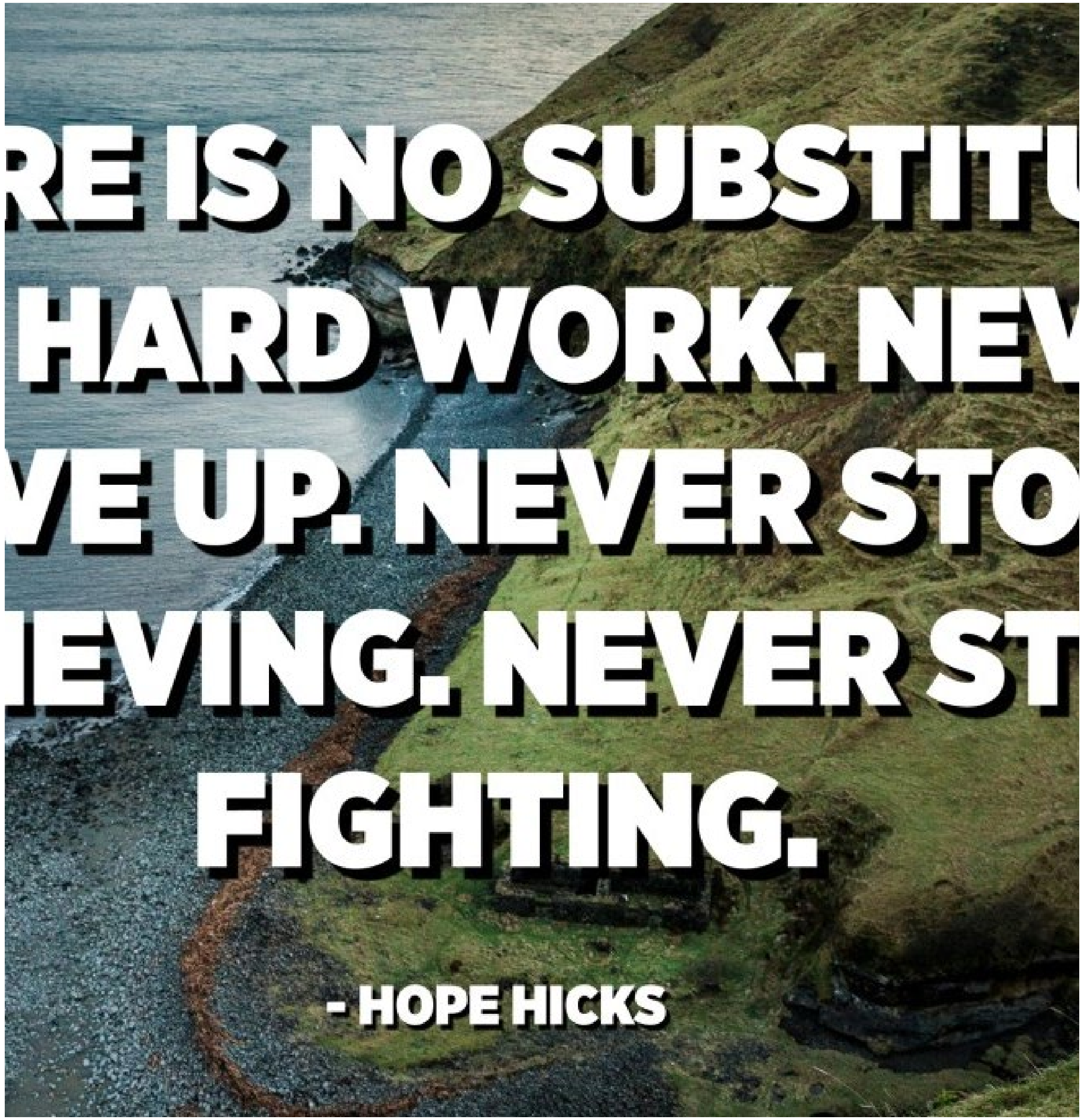
$$= 0 + \{(V_2 \cos 45+)\dot{m}_2 + (V_3 \cos 60+)\dot{m}_3\} - 0$$

$$F_y = P_2 A_2 \cos 45 + P_3 A_3 \sin 60 + g_0 \sin 42 \rho \forall + \{(V_2 \cos 45+)\dot{m}_2 + (V_3 \cos 60+)\dot{m}_3\}$$

$$\dot{m} = \rho_f Q = SG \bullet \rho_{H20} \bullet Q$$

TOTAL POWER LOSS

$$P_{loss}[W] = \dot{m}_2 g \left\{ h_{L_{1-2}}[m] \right\} + \dot{m}_3 g \left\{ h_{L_{1-3}}[m] \right\}$$


RE IS NO SUBSTITU
HARD WORK. NEV
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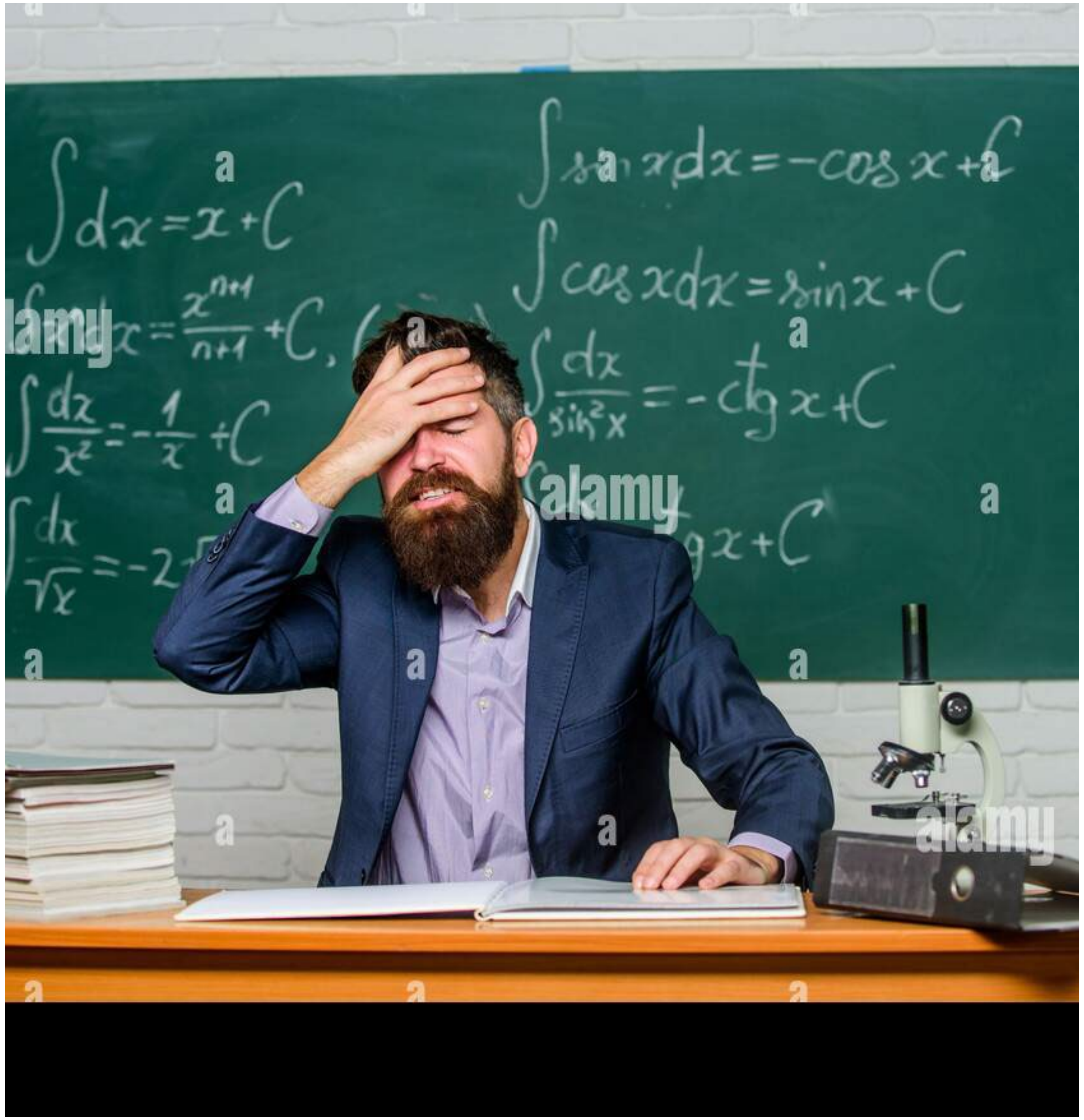
FIGHTING.
- HOPE HICKS


$\int dx = x + C$
$\int \sin x\,dx = -\cos x + C$
$\int \cos x\,dx = \sin x + C$