Mole Dispersion Unit Analysis & TIME RATE of CHANGE

MECH-322 Fluid Mechanics

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Considering exit stream #3, the chemical molar dispersion [moles/sec] is measured as: ,

$$\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5e^{-5t} + 20.2x^2 - 0.3y^3$$

$$\vec{\mathrm{V}}_3 = -V_3\cos(\theta_3)\hat{i} + V_3\sin(\theta_3)\hat{j}$$

What is the parametric form of the TIME RATE OF CHANGE of the chemical molar dispersion and VERIFY UNITS:

TIME RATE of CHANGE or TOTAL/MATERIAL DERIVATIVE

$$\frac{D\psi(t,x,y)\left[\frac{moles}{s}\right]}{Dt[s]}\left[\frac{moles}{s^2}\right] = \frac{\partial\varphi}{\partial t} + V_x\frac{\partial\varphi}{\partial x} + V_y\frac{\partial\varphi}{\partial y}$$

UNIT ANALYSIS

$$\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5[]e^{-5[]t} + 20.2[]x^2 - 0.3[]y^3$$

"3.5"

$$\left[\frac{moles}{s}\right] = 3.5[] \rightarrow [] = \frac{moles}{s}$$

"– 5"

$$[1] = -5[][s] \rightarrow [] = \frac{1}{s}$$

"20.2"

$$\left[\frac{moles}{s}\right] = 20.2[][m^2] \rightarrow [] = \frac{\frac{moles}{s}}{m^2} = \frac{moles}{s-m^2}$$

"0.3"

$$\left[\frac{moles}{s}\right] = -0.3[][m^3] \rightarrow [] = \frac{\frac{moles}{s}}{m^3} = \frac{moles}{s-m^3}$$

$$\frac{D\psi(t,x,y)\left[\frac{moles}{s}\right]}{Dt[s]}\left[\frac{moles}{s^2}\right] = \overbrace{\frac{\partial\varphi}{\partial t}}^{(-5)3.5e^{-5t}} + \overbrace{V_x}^{-V_3\cos\theta_3}\overbrace{\frac{\partial\varphi}{\partial x}}^{20.2\bullet 2x} + \overbrace{V_y}^{V_3\sin\theta_3}\overbrace{\frac{\partial\varphi}{\partial y}}^{-0.3\bullet 3y^2}$$

$$\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5[\,]e^{-5[\,]t} + 20.2[\,]x^2 - 0.3[\,]y^3$$

"3.5"

$$\left[\frac{moles}{s}\right] = 3.5[\,] \rightarrow [\,] = \frac{moles}{s}$$

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$$\left[\frac{moles}{s}\right] = 20.2[\,][m^2] \rightarrow [\,] = \frac{\frac{moles}{s}}{m^2} = \frac{moles}{s-m^2}$$

"0.3"

$$\left[\frac{moles}{s}\right] = -0.3[\,][m^3] \rightarrow [\,] = \frac{\frac{moles}{s}}{m^3} = \frac{moles}{s-m^3}$$

$$= \overbrace{(-5\left[\frac{1}{s}\right])3.5\left[\frac{moles}{s}\right]e^{-5t}}^{\left[\frac{moles}{s^2}\right]}$$

$$+ \overbrace{(-V_3\left[\frac{m}{s}\right]\cos\theta_3)(20.2\left[\frac{moles}{s-m^2}\right]\bullet 2x[m])}^{\left[\frac{moles}{s^2}\right]}$$

$$+ \overbrace{(V_3\left[\frac{m}{s}\right]\sin\theta_3)(-0.3\left[\frac{moles}{s-m^3}\right]\bullet 3y^2[m^2])}^{\left[\frac{moles}{s^2}\right]}$$