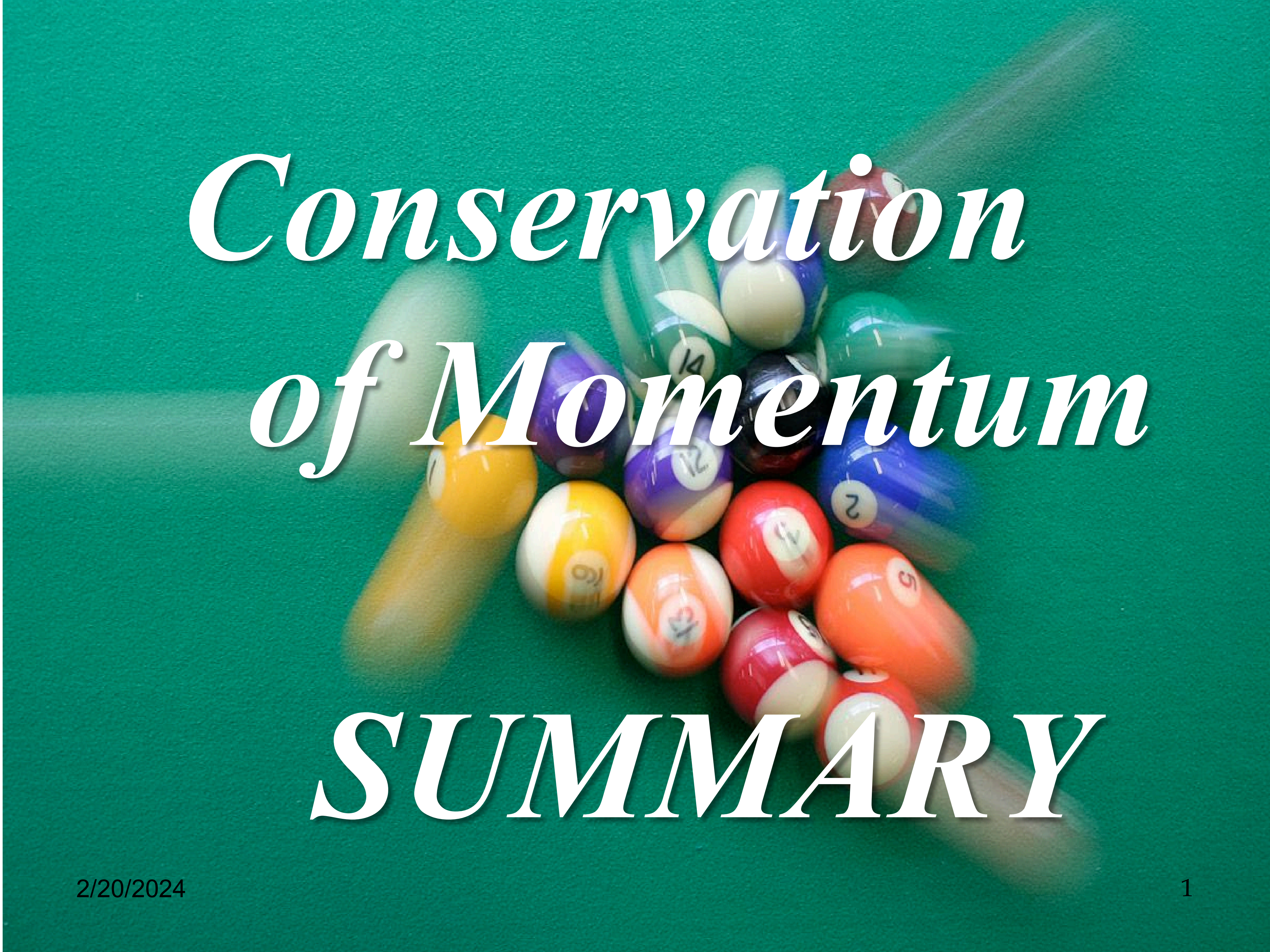


Conservation of Momentum

SUMMARY

Challenging

WHAT PATH WILL YOU FOLLOW?

Class 12: Conservation of Momentum

Newton's 2nd law of motion:

$$\sum \vec{F} = m\vec{a}$$

$$\Rightarrow \sum \vec{F} = m\frac{d\vec{V}}{dt} = \frac{d\left(m\vec{V}\right)}{dt}; \text{ if m=constant in time.}$$

$$\Rightarrow \sum \vec{F} = \frac{d\left(m\vec{V}\right)}{dt} = \frac{d\left(Momentum\right)_{System}}{dt}$$

Class 12: Conservation of Momentum

Recall Mass Conservation:

$$\frac{dM_{sys}}{dt} = 0$$

Reynolds Transport Theorem:

$$\frac{dM_{sys}}{dt} = \frac{d}{dt}\int_{CV} \boxed{\rho}\, d\forall + \int_{CS} \boxed{\rho}\vec{V} \cdot d\vec{A}$$

Mass was carried away by each fluid particle

Class 12: Conservation of Momentum

Recall Mass Conservation:

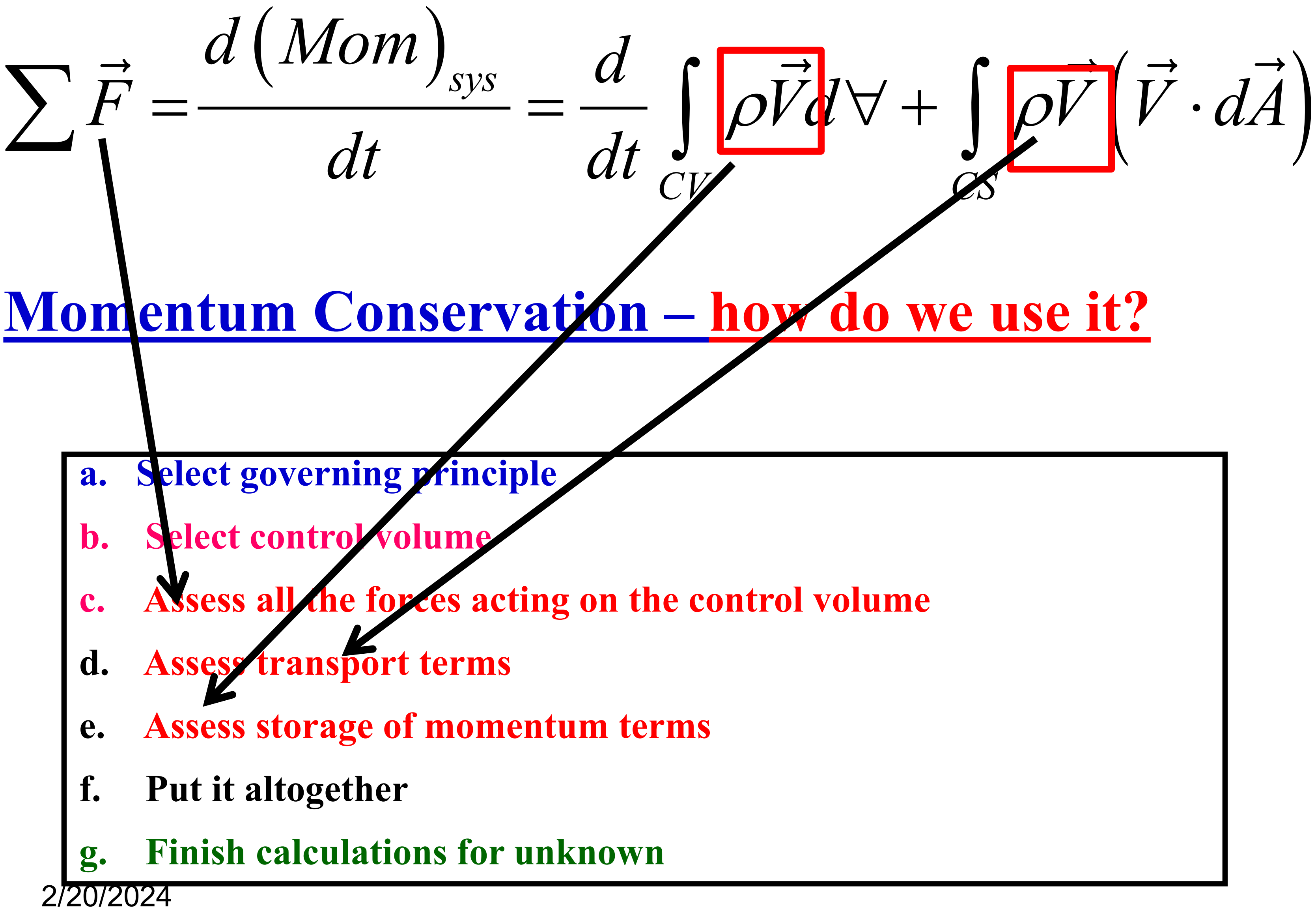
$$\frac{dM_{sys}}{dt} = 0 \Rightarrow \frac{dM_{sys}}{dt} = \frac{d}{dt}\int_{CV} \boxed{\rho}\, d\forall + \int_{CS} \boxed{\rho}\, \vec{V} \cdot d\vec{A}$$

Momentum Conservation:

$$\sum \vec{F} = \frac{d\left(Mom\right)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \boxed{\rho\vec{V}}\, d\forall + \int_{CS} \boxed{\rho\vec{V}}\left(\vec{V} \cdot d\vec{A}\right)$$

Momentum is carried away by each fluid particle

Class 12: Conservation of Momentum



$$\sum \vec{F} = \frac{d\left(Mom\right)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \boxed{\rho\vec{V}}\, d\forall + \int_{CS} \boxed{\rho\vec{V}}\left(\vec{V}\cdot d\vec{A}\right)$$

Momentum Conservation – how do we use it?

a. Select governing principle

b. Select control volume

c. Assess all the forces acting on the control volume

d. Assess transport terms

e. Assess storage of momentum terms

f. Put it altogether

g. Finish calculations for unknown

Class 12: Momentum Conservation – how it works?

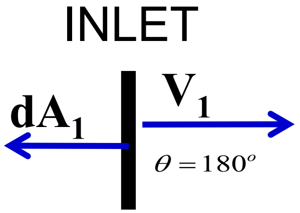
$$\sum \vec{F} = \frac{d\left(Mom\right)_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho \vec{V} d\forall + \int_{CS} \rho \vec{V}\left(\vec{V}\cdot\hat{n}\right)dA$$

$$\sum\left(F_x\hat{i}+F_y\hat{j}+F_z\hat{k}\right) = \frac{d}{dt}\int_{CV}\rho\left(u\hat{i}+v\hat{j}+w\hat{k}\right)d\forall + \int_{CS}\rho\left(u\hat{i}+v\hat{j}+w\hat{k}\right)\left(\vec{V}\cdot d\vec{A}\right)$$

Equate x, y and z components:

x-component:

$$\sum F_x = \frac{dM_{CV_x}}{dt} + \int_{CS}\rho u\left(\vec{V}\cdot\hat{n}\right)dA$$



y-component:

$$\sum F_y = \frac{dM_{CV_y}}{dt} + \int_{CS}\rho v\left(\vec{V}\cdot\hat{n}\right)dA$$

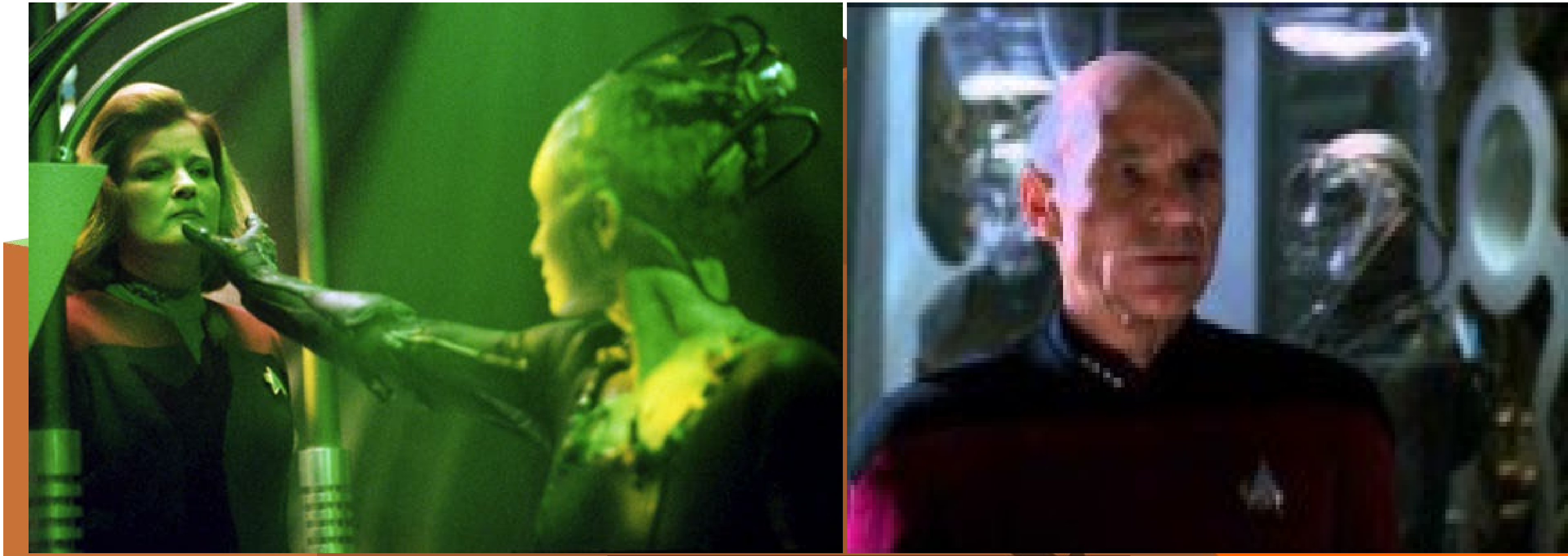
Dot Product

$\vec{A}\bullet\vec{B}=\|A\|\|B\|\cos\theta$

z-component:

$$\sum F_z = \frac{dM_{CV_z}}{dt} + \int_{CS}\rho w\left(\vec{V}\cdot\hat{n}\right)dA$$

WHEN is MOMENTUM IMPORTANT ?



RESISTANCE IS FUTILE

When fluid boundaries forced:1)INLET/EXIT DIRECTION CHANGE:2) AREA CHANGE:) PRESURE CHANGE,

Fluid FORCES resist, and external REACTION forces must be applied to force fluid to COMPLY.

WHY/WHEN?

- ✓ **WHEN** ***do we need Manometers and Fluid Statics?***
- ✓ **WHEN** ***do we need Mass Conservation ?***
- ✓ **WHEN** ***do we need Mass Continuity ?***
- ✓ **WHEN** ***do we need Bernoulli & Energy Conservation***
- ✓ **WHEN** ***do we need Momentum Conservation***

Constant Properties

$$\underbrace{\overset{\rightarrow +}{\sum} F_x}_{\text{NET FORCES}} = \overbrace{\underbrace{\frac{dM_{CV_x}}{dt}}_{\text{CONTROL VOLUME}}}^{\text{Momentum STORAGE}} + \underbrace{\overbrace{\sum_{out}(u_{out}\pm)\dot{m}_{out}}^{\text{Momentum OUT}} - \overbrace{\sum_{in}(u_{in}\pm)\dot{m}_{in}}^{\text{Momentum IN}}}_{\text{@ SURFACE}}$$

$$\uparrow \sum F_y = \frac{dM_{CV_y}}{dt} + \sum_{out}(v_{out}\pm)\dot{m}_{out} - \sum_{in}(v_{in}\pm)\dot{m}_{in}$$

$$\nearrow \sum F_z = \frac{dM_{CV_z}}{dt} + \sum_{out}(w_{out}\pm)\dot{m}_{out} - \sum_{in}(w_{in}\pm)\dot{m}_{in}$$

$$\int_{CS} \rho u\left(\vec{V}\cdot\hat{n}\right)dA = \rho A u\int_{CS}\vec{V}\bullet\hat{n}\,dA = \dot{m}\int_{CS}\vec{V}\bullet\hat{n}\,dA$$

‘u+/-’,’v+/-’,’w+/-’ are velocity VECTOR quantities

Class 12: Momentum conservation - Example

Problem #1: **Water flows steady through the elbow at 14kg/s and exits to atmosphere. Determine the magnitude and direction of the anchoring force (Rx) needed to hold the horizontal elbow in place as shown in Figure below.**

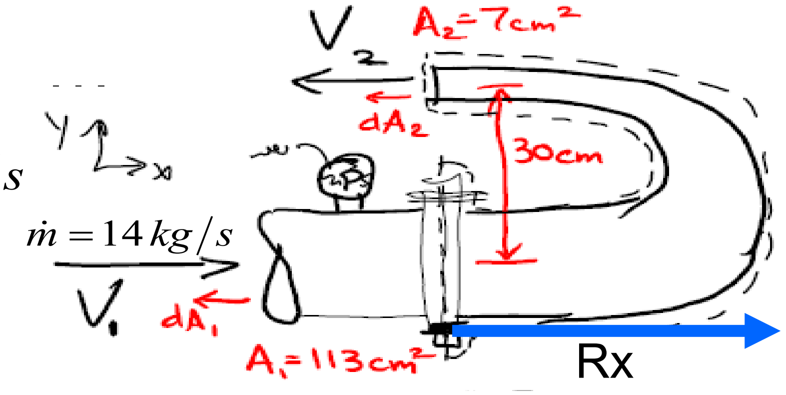
$$\rho_{H_2O} = 1000\,kg/m^3$$

Solution:

$$\dot{m} = 14\,kg/s$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14\,kg/s}{\left(1000\,kg/m^3\right)\left(113cm^2\right)} = 1.24\,m/s$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = 20\,m/s$$



Momentum Conservation (ROAD MAP):

$$\sum F_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out}\pm)\dot{m} - \sum_{in}(u_{in}\pm)\dot{m} \rightarrow \text{ROAD MAP}$$

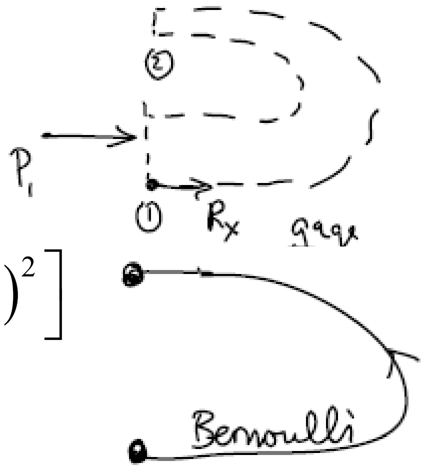
$$\sum \vec{F}_x = P_1A_1 + R_x = \cancel{\frac{dM_{CV}}{dt}} + (V_2\pm)\dot{m}_2 - (V_1\pm)\dot{m}_1$$

$$\sum \vec{F}_x = P_1A_1 + R_x = (V_2-)\dot{m}_2 - (V_1+)\dot{m}_1 = -\dot{m}(V_1+V_2)\,;\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho Q$$

$$\sum \vec{F}_x = P_1A_1 + R_x = -\dot{m}(V_1+V_2)$$

Class 12: Momentum conservation

Apply Bernoulli:



$$p_1 + \gamma z_1 + 1/2\rho V_1^2 = p_2 + \gamma z_2 + 1/2\rho V_2^2$$

$$\Rightarrow p_1 = \gamma\left(z_2 - z_1\right) + 1/2\rho\left(V_2^2 - V_1^2\right)$$

$$\Rightarrow p_1 = \left(9810\,N/m^3\right)\left(0.3m\right) + 1/2\left(1000\,kg/m^3\right)\left[\left(20\,m/s\right)^2 - \left(1.24\,m/s\right)^2\right]$$

$$\Rightarrow p_1 = 203kPa$$

$$\dot{m} = \rho AV$$

$$R_x = -P_1A_1 - \rho(V_1^2A_1 + V_2^2A_2)$$

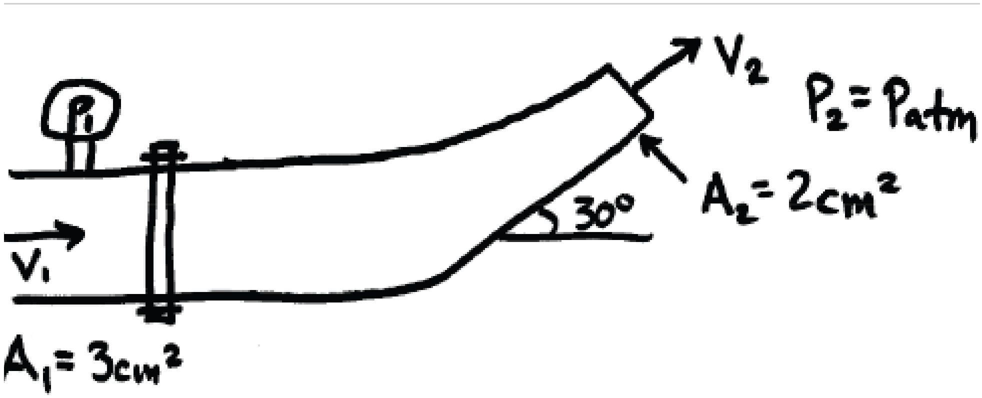
$$\Rightarrow R_x = -\left(203kPa\right)\left(113cm^2\right)\left(1m^2/\left(100cm\right)^2\right) - \left(14\,kg/s\right)\left[\left(1.24\,m/s\right) + \left(20\,m/s\right)\right]$$

$$\Rightarrow R_x = -2.3kN - 297.4N$$

$$\Rightarrow R_x = -2.6kN$$

Class 12: Momentum Conservation - Example

Problem #2: **Water flows steady through elbow as shown in the Figure below. The pressure just upstream of the flange is 10kPa (gage) and the velocity is 3m/s. The jet exits to the atmosphere and the velocity is known to be 4m/s. Determine the forces on the flange.**



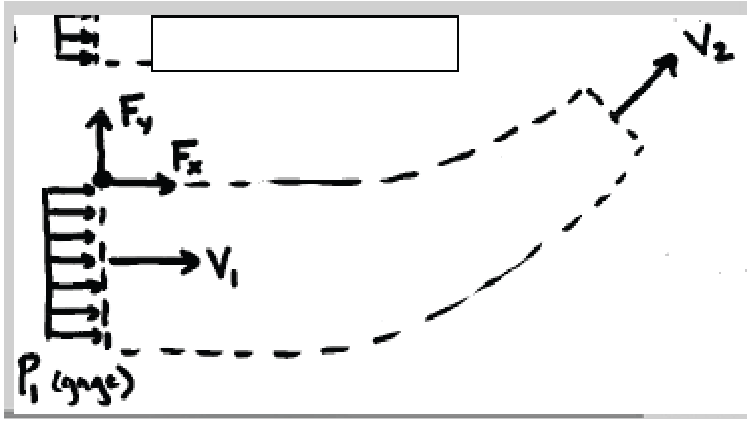
Solution:

Momentum conservation:

No storage

$$\rightarrow \sum F_x = \cancelto{0}{\frac{dM_{CV}}{dt}} + \sum_{out}(u_{out}\pm)\dot{m}_{out} - \sum_{in}(u_{in}\pm)\dot{m}_{in}$$

$$\uparrow \sum F_y = \xcancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out}\pm)\dot{m}_{out} - \sum_{in}(v_{in}\pm)\dot{m}_{in}$$

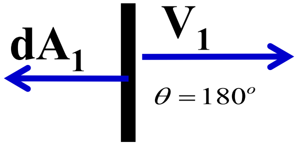


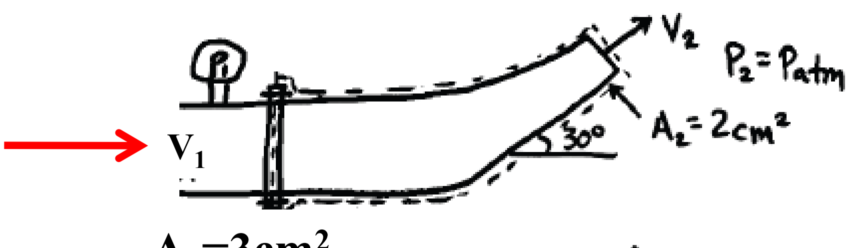
$$\sum_{\rightarrow} F_x = F_x + p_1 A_1$$

$$\sum_{\uparrow} F_y = F_y$$

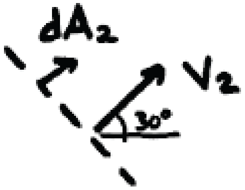
Class 12: Momentum Conservation - Example

Inflow:





A_1=3cm^2



x-direction:

$$\sum \vec{F}_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out}\pm)\dot{m}_{out} - \sum_{in}(u_{in}\pm)\dot{m}_{in}$$

$$F_x + p_1A_1 = 0 + (V_2\cos 30\pm)\dot{m}_2 - (V_1\pm)\dot{m}_1\ ;\ \dot{m}_1 = \dot{m}_2 = \dot{m} = \rho AV = \rho Q$$

$$F_x + p_1A_1 = \dot{m}\left[(V_2\cos 30+) - (V_1+)\right]$$

$$\dot{m} = \rho AV = 1000\frac{kg}{m^3}\bullet(0.0003)m^2\bullet 3.0\frac{m}{s} = 0.9\frac{kg}{s}$$

$$F_x + p_1A_1 = \rho Q\left[(V_2\cos 30+) - (V_1+)\right]$$

$$F_x + p_1A_1 = 0.9\frac{kg}{s}(4.0\frac{m}{s}\cos 30 - 3.0\frac{m}{s}) = 0.42N$$

$$F_x = 0.42N - 10,000Pa\bullet 0.0003m^2 = -2.58N(\text{fluid X reaction force on bolt})$$

y-direction:

$$\sum\uparrow F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out}\pm)\dot{m}_{out} - \cancel{\sum_{in}(v_{in}\pm)\dot{m}_{in}}$$

$$F_y = 0 + \dot{m}_{out}V_2\sin 30 - 0 = 1.6N(\text{fluid Y reaction force on bolt})$$

Putting It All Together

$$\vec{F} = \left(-2.58\hat{i} + 1.6\hat{j}\right)N$$

A nozzle is attached to a vertical pipe and discharges water steady at ?kPa at 0.1m3/s as shown. Determine the anchoring reaction forces required to hold nozzle in place.

The nozzle has a weight of 200N and the water volume is 0.012m3.

$DATA$

$A_1 = 0.02 m^2$

$A_2 = 0.01 m^2$

$P_1 = 40,000 Pa$

$P_2 = ?$

$W_{nozzle} = 200 N$

$V_{water} = 0.012 m^3$

$Q = 0.1 m^3 / s$

$\gamma_{H20} = 9800 N / m^3$

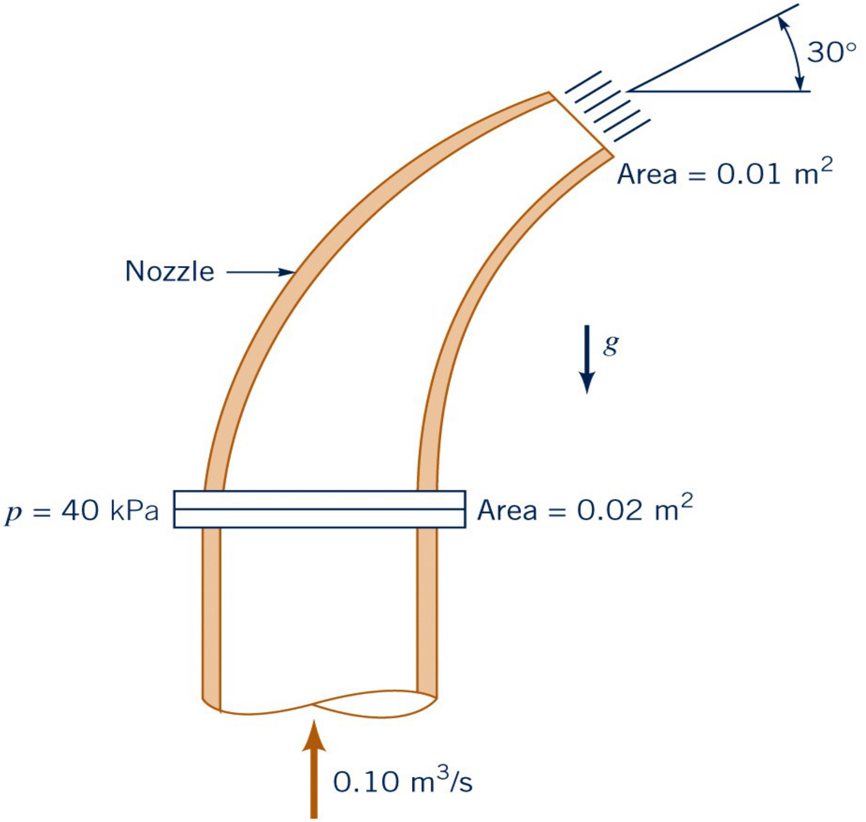


Figure P5.50
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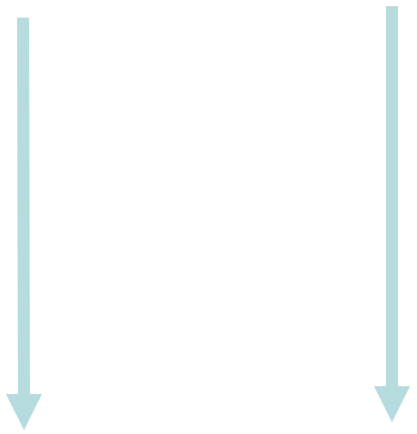
FREE BODY DIAGRAM



Water Weight=$\gamma \frac{N}{m^3} \bullet \forall m^3$

Nozzle Weight = 200N

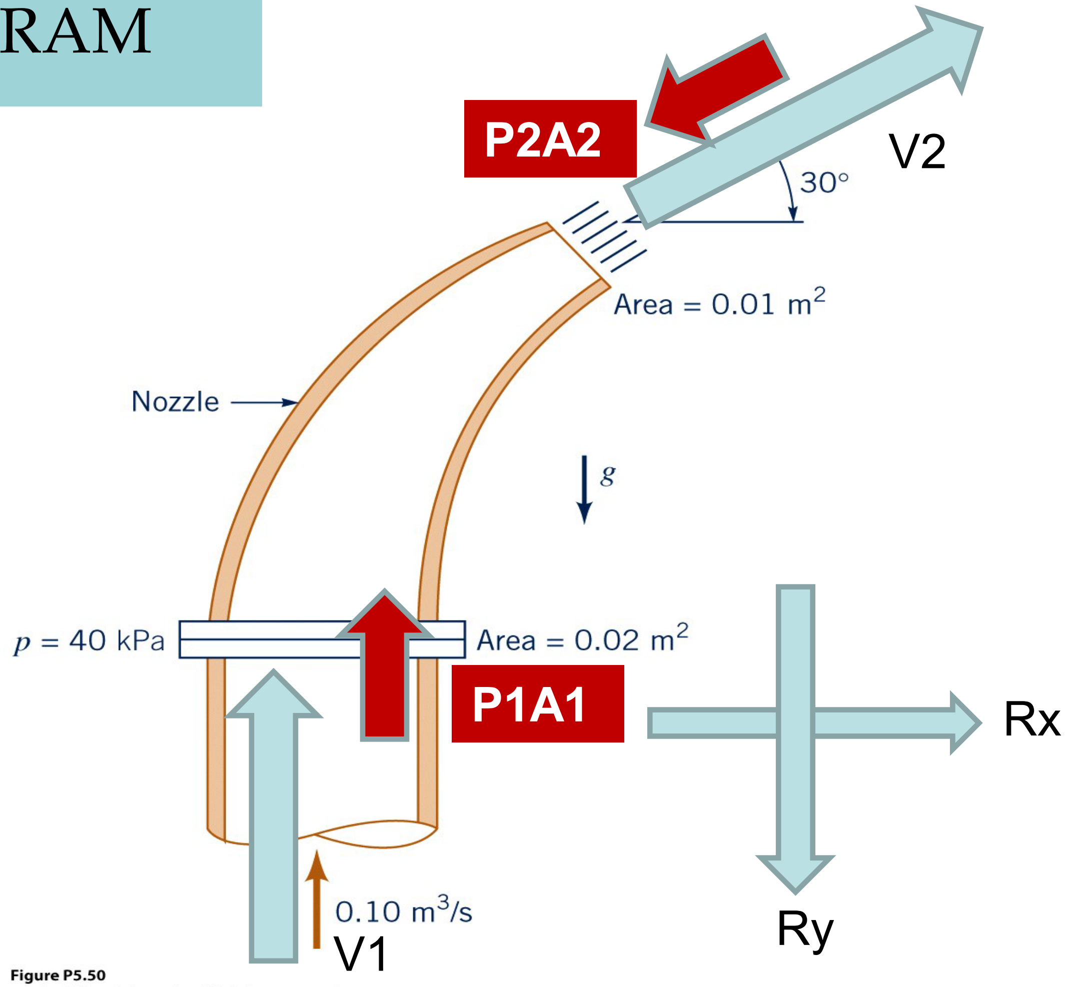


Figure P5.50

ROAD MAP

Mass Conservation → Change in Diameter

$$Q = A_1 V_1 = A_2 V_2 = 0.1 \frac{m^3}{s}$$

$$V_1 = \frac{Q}{A_1} = 5 m / s$$

$$V_2 = \frac{Q}{A_2} = 10 m / s$$

Bernoulli → Change in pressure and velocity along streamline

assume Δz small

$$\dot{m}_1 \left(\frac{P_1}{\gamma_{H20}} + \frac{V_1^2}{2g} \right) = \dot{m}_2 \left(\frac{P_2}{\gamma_{H20}} + \frac{V_2^2}{2g} \right)$$

$$P_2 = \overbrace{\left(\frac{\dot{m}_1}{\dot{m}_2} \right)}^{\text{important for multi-branch flows}} \left(\frac{P_1}{\gamma_{H20}} + \frac{V_1^2 - V_2^2}{2g} \right) \gamma_{H20}$$

$$= 40,000 Pa - 37,462 Pa$$

$$= 2,538 Pag$$

Y MOMENTUM

$$\uparrow \sum F_y = \frac{dM_{CV}}{dt} + \sum_{out} (v_{out} \pm)\dot{m}_{out} - \sum_{in} (v_{in} \pm)\dot{m}_{in}$$

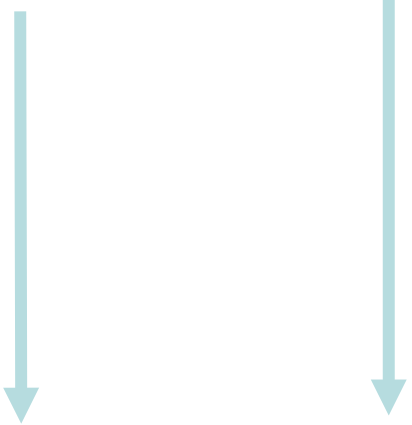
$$-R_y + P_1A_1 - P_2A_2 \sin\theta - W_w - W_N = 0 + \left(V_2 \sin\theta \pm\right)\dot{m}_2 - \left(V_1 \pm\right)\dot{m}_1$$

$$R_y = P_1A_1 - P_2A_2 \sin\theta - W_w - W_N - \left(V_2 \sin\theta \pm\right)\dot{m}_2 + \left(V_1 \pm\right)\dot{m}_1$$

$$R_y = P_1A_1 - P_2A_2 \sin\theta - W_w - W_N - \left(V_2 \sin\theta +\right)\dot{m}_2 + \left(V_1 +\right)\dot{m}_1$$

$$\dot{m}_2 = \dot{m}_1 = \dot{m} = \rho Q$$

$$R_y = P_1A_1 - P_2A_2 \sin\theta - W_w - W_N - \dot{m}(V_2 \sin\theta - V_1)$$



Water Weight=$\gamma \frac{N}{m^3} \bullet \forall m^3$ Nozzle Weight = 200N

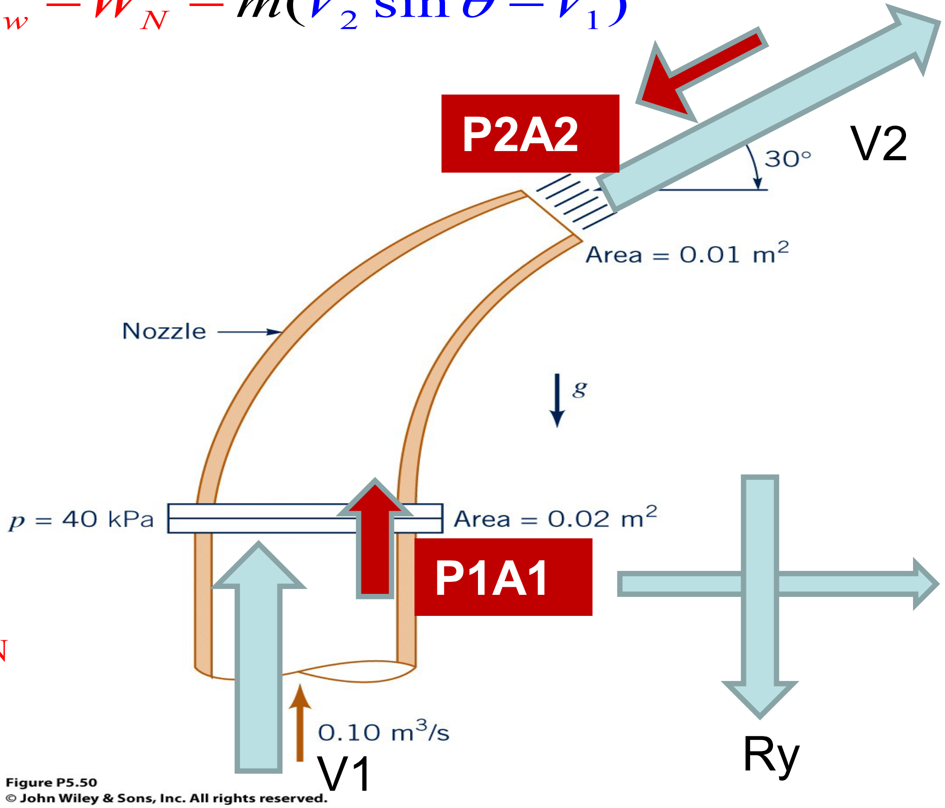


Figure P5.50
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X MOMENTUM

$$\overset{\rightarrow +}{\sum} F_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out} \pm)\dot{m}_{out} - \sum_{in}(u_{in} \pm)\dot{m}_{in}$$

$$+R_x - P_2 A_2 \cos\theta = 0 + (V_2 \cos\theta \pm)\dot{m}_2 - 0$$

$$R_x = +(V_2 \cos\theta +)\dot{m}_2 + P_2 A_2 \cos\theta$$

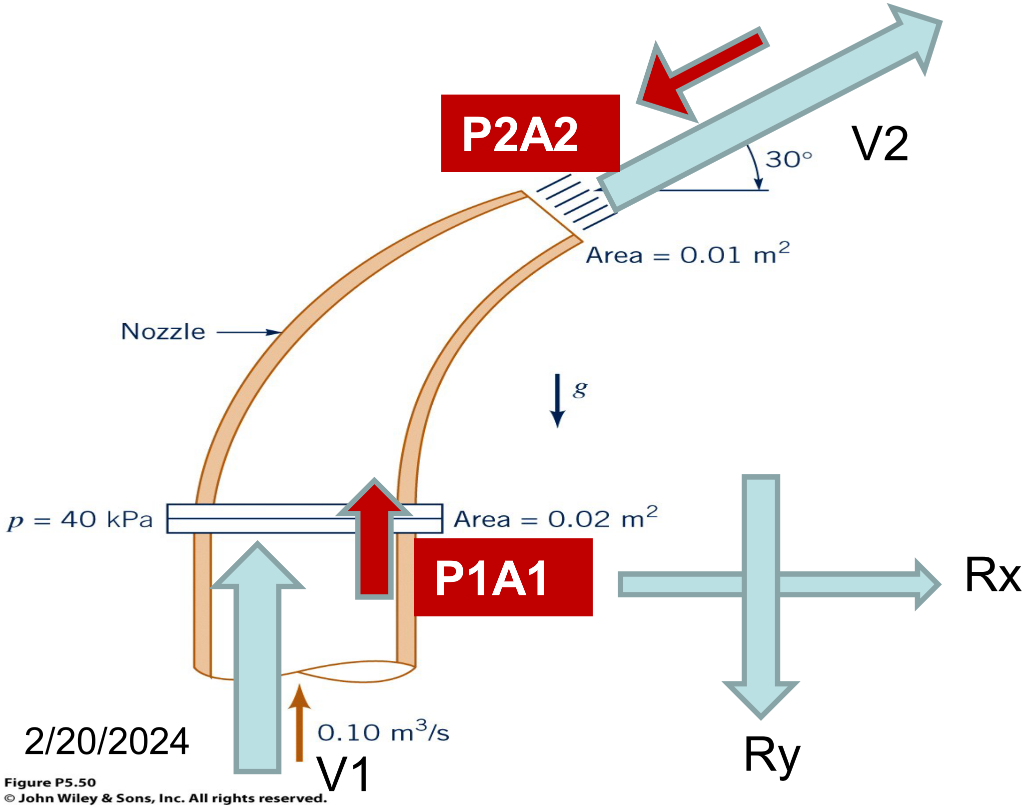

Figure P5.50
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2/20/2024

SYSTEM MODELING

3 Equations/3 Unknowns

$$1.R_x = +(V_2 \cos\theta +)\dot{m}_2 + P_2 A_2 \cos\theta$$

$$R_x - P_2 A_2 \cos\theta = +(V_2 \cos\theta +)\dot{m}_2$$

$$2.R_y = P_1 A_1 - P_2 A_2 \sin\theta - W_w - W_N - \dot{m}(V_2 \sin\theta - V_1)$$

$$R_y + P_2 A_2 \sin\theta = P_1 A_1 - W_w - W_N - \dot{m}(V_2 \sin\theta - V_1)$$

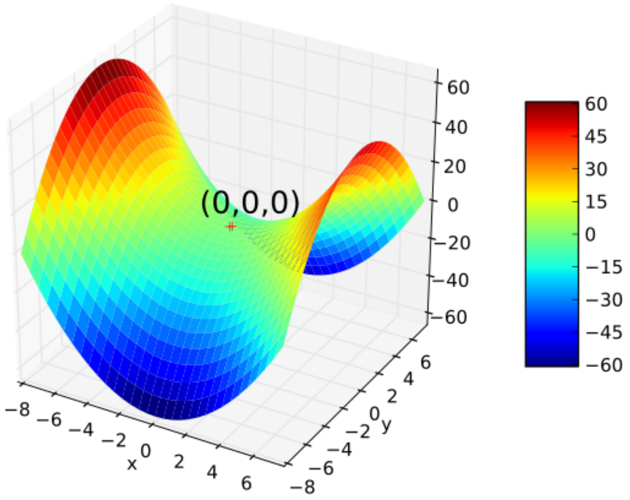
$$3.P_2 = \left(\frac{P_1}{\gamma_{H20}} + \frac{V_1^2 - V_2^2}{2g} \right) \gamma_{H20}$$

Matrix Equation

$$[A]\{x\} = \{B\}$$

$$\{x\} = [A]^{-1}\{B\}$$

MATLAB/MATHCAD



System Modeling

Matrix Equation

$[A]\{x\} = \{B\}$

$\{x\} = [A]^{-1}\{B\}$

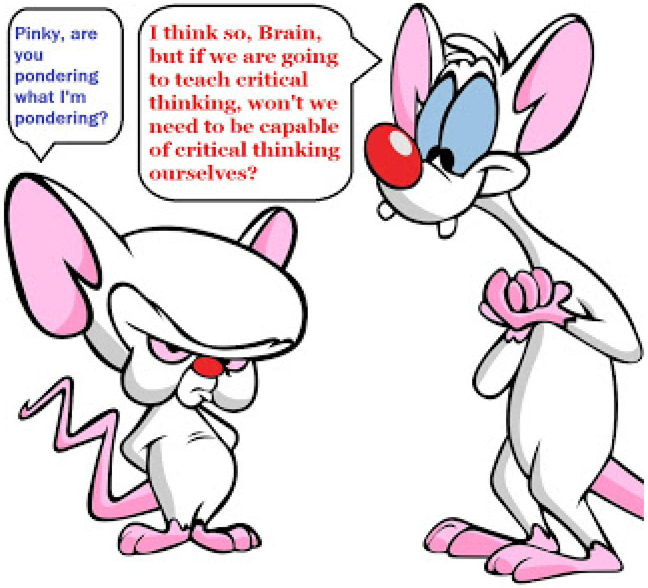
MATLAB/MATHCAD

$$\boxed{\begin{gathered}1R_x + 0R_y - P_2\left[A_2 \cos\theta\right] = +(V_2 \cos\theta)\dot{m}_2 \\ 0R_x + 1R_y + P_2\left[A_2 \sin\theta\right] = P_1A_1 - W_w - W_N - \dot{m}(V_2 \sin\theta - V_1) \\ 0R_x + 0R_y + 1P_2 = \left(\frac{P_1}{\gamma_{H20}} + \frac{V_1^2 - V_2^2}{2g}\right)\gamma_{H20}\end{gathered}}$$

$$\begin{bmatrix} 1 & 0 & -A_2\cos\theta \\ 0 & 1 & A_2\sin\theta \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} R_x \\ R_y \\ P_2 \end{Bmatrix} = \begin{Bmatrix} (V_2\cos\theta)\dot{m}_2 \\ P_1A_1 - W_w - W_N - \dot{m}(V_2\sin\theta - V_1) \\ \left(\dfrac{P_1}{\gamma_{H20}} + \dfrac{V_1^2 - V_2^2}{2g}\right)\gamma_{H20} \end{Bmatrix}$$

A vertical jet of water with a nozzle exit velocity of 15 ft/s and a diameter of 1” suspends a hollow hemisphere as shown. Determine the WEIGHT.

DEFINE CONTROL VOLME



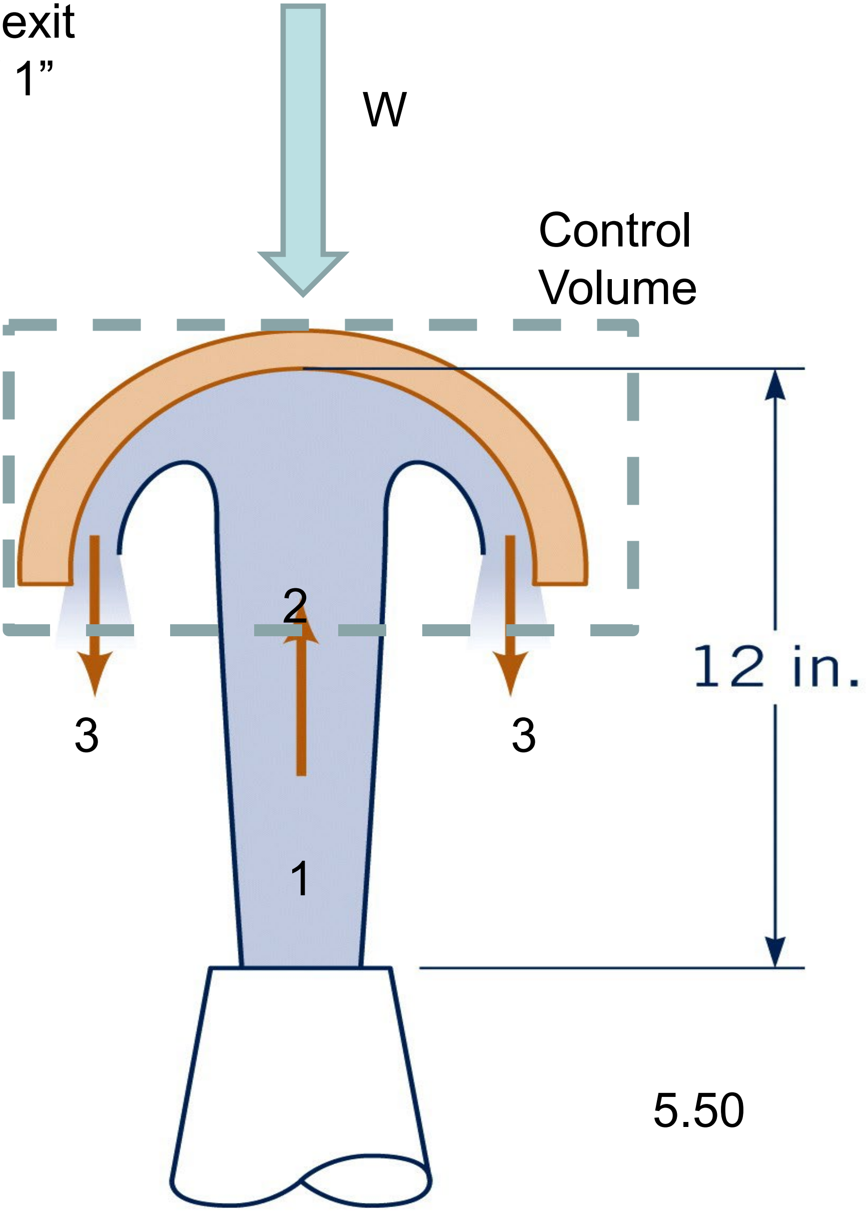


Figure P5.56

ROAD MAP

Mass Conservation → Change in Area

$\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

$A_1V_1 = A_2V_2 = A_3V_3 = Q$

Bernoulli → Change in flow/pressure/height along streamline

1-3

$P_1 = P_2 = P_3 = 0$

$$\cancel{\frac{P_1}{\gamma}} + z_1 + \frac{V_1^2}{2g} = \cancel{\frac{P_3}{\gamma}} + z_3 + \frac{V_3^2}{2g}$$

$$V_3(V_1, \Delta z) = \sqrt{\left(\frac{V_1^2}{2g} - \Delta z\right)2g} = V_2 \text{(no friction)}$$

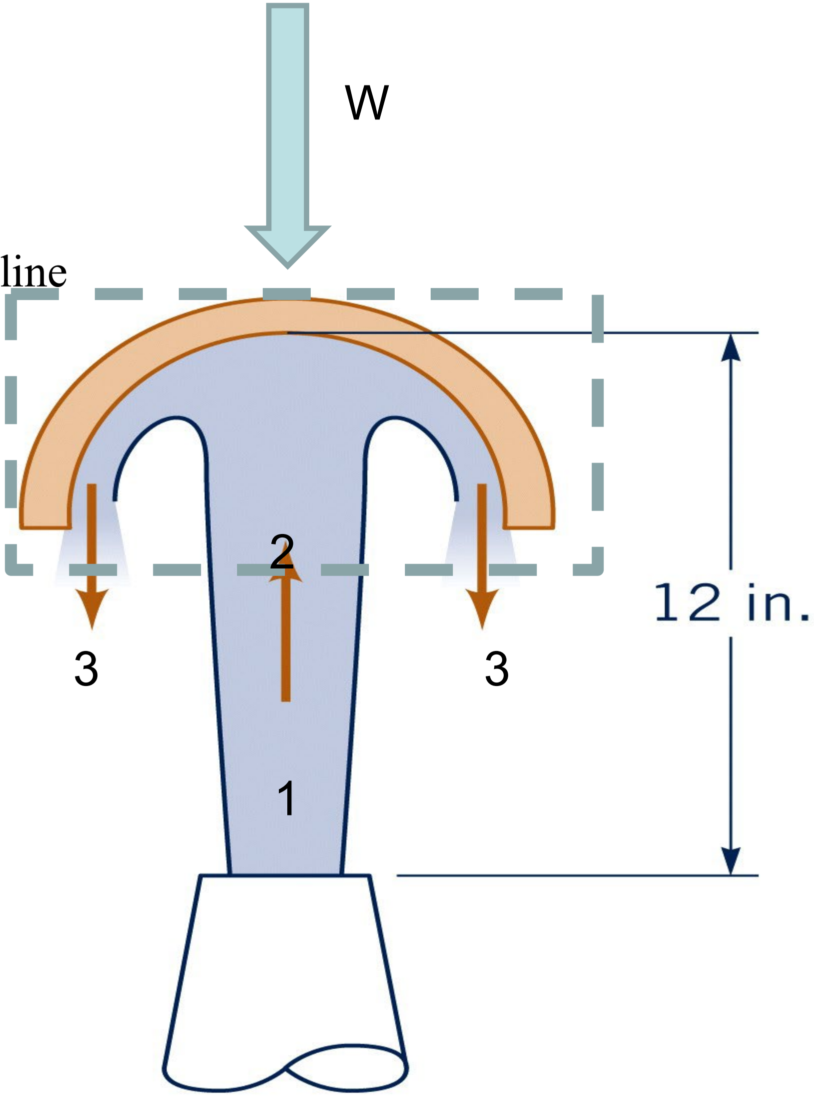


Figure P5.56

Road Map

Mass Conservation

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3$$

BERNOULLI

$$V_3 = \sqrt{\left(\frac{V_1^2}{2g} - \Delta z\right)2g} = V_2 \text{(no friction)}$$

Y Momentum (neglect weight of water)

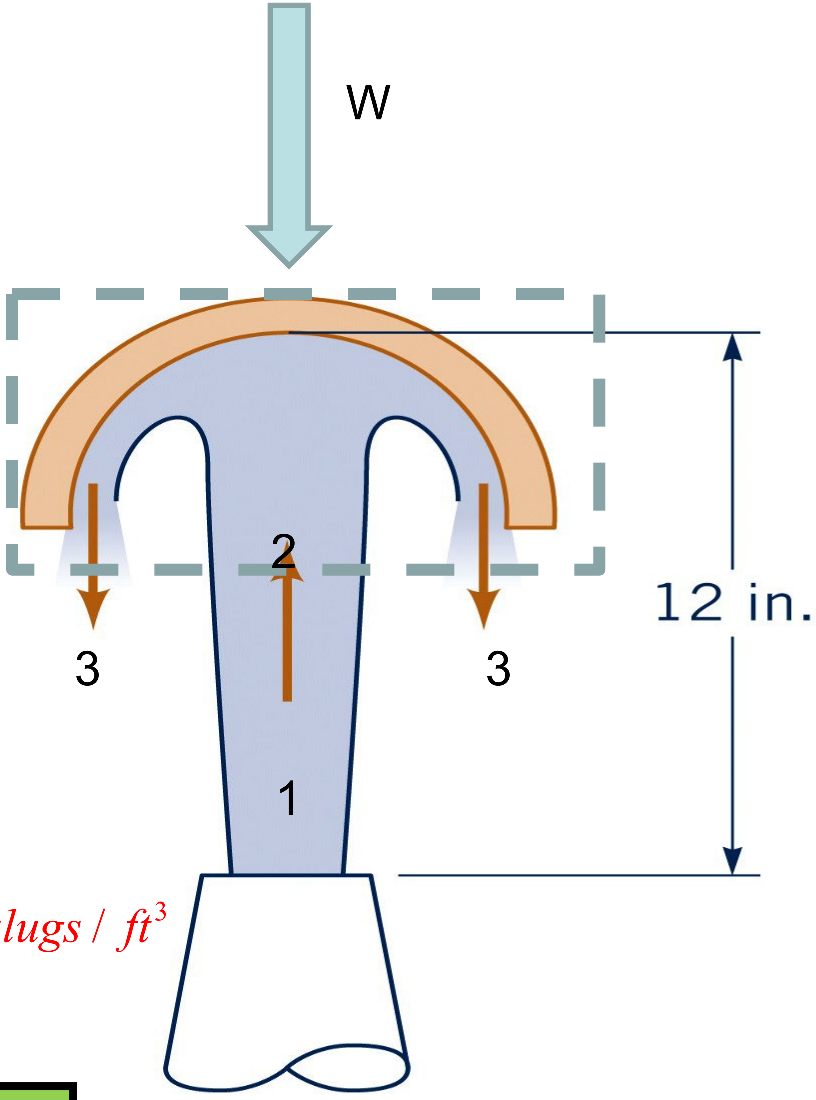
$$\uparrow \sum F_y = \frac{dM_{CV}}{dt} + \sum_{out}(v_{out}\pm)\dot{m}_{out} - \sum_{in}(v_{in}\pm)\dot{m}_{in}$$

$$-W = 0 + (V_3 -)\dot{m}_3 - (V_2 +)\dot{m}_2 ; V_3 = V_2$$

$$-W = 0 - 2\dot{m}V_2$$

$$W(A_1, V_1, \Delta z) = 2(\rho A_1 V_1)\sqrt{\left(\frac{V_1^2}{2g} - \Delta z\right)2g} \rightarrow \rho = 1.94 slugs / ft^3$$

$$W = 4.02 lb$$



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How would problem change if we included weight of water?

Two steady water jets strike each other as shown in space. Neglect gravity and determine V, theta, exit diameter.

Mass Conservation → Multiple inlet/exit streams

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \rightarrow \dot{m} = \rho A V$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Three Equations, $M_x, M_y, \sum \dot{m} = 0$

$Unknowns \rightarrow V_3, D_3, \theta$

Y Momentum

$$\uparrow \sum F_y = \frac{dM_{CV}}{dt} + \sum_{out} (v_{out} \pm)\dot{m}_{out} - \sum_{in} (v_{in} \pm)\dot{m}_{in}$$

$$0 = 0 + (V_3 \sin\theta +)\dot{m}_3 - (V_1 +)\dot{m}_1$$

$$\sin\theta = \frac{V_1 \dot{m}_1}{V_3 \dot{m}_3}$$

X Momentum

$$\overset{\rightarrow +}{\sum} F_x = \frac{dM_{CV}}{dt} + \sum_{out} (u_{out} \pm)\dot{m}_{out} - \sum_{in} (u_{in} \pm)\dot{m}_{in}$$

$$0 = 0 + (V_3 \cos\theta +)\dot{m}_3 - (V_2 +)\dot{m}_2$$

$$\cos\theta = \frac{V_2 \dot{m}_2}{V_3 \dot{m}_3}$$

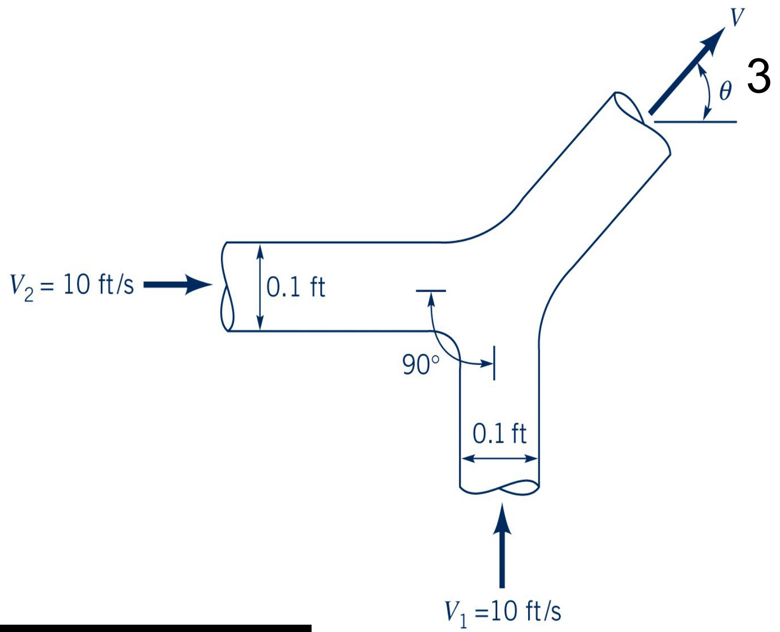

Figure P5.59
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COMBINE

$$\tan\theta = \frac{\sin\theta = \frac{V_1 \dot{m}_1}{V_3 \dot{m}_3}}{\cos\theta = \frac{V_2 \dot{m}_2}{V_3 \dot{m}_3}} = \frac{V_1 \dot{m}_1}{V_2 \dot{m}_2}$$

$$\rightarrow \theta = \tan^{-1}\left(\frac{V_1 \dot{m}_1}{V_2 \dot{m}_2}\right) = KNOWN$$

5-57

COMBINE

COMBINE MASS + Y MOMENTUM

$$\uparrow \sum F_y = \frac{dM_{CV}}{dt} + \sum_{out} (v_{out} \pm) \dot{m}_{out} - \sum_{in} (v_{in} \pm) \dot{m}_{in}$$

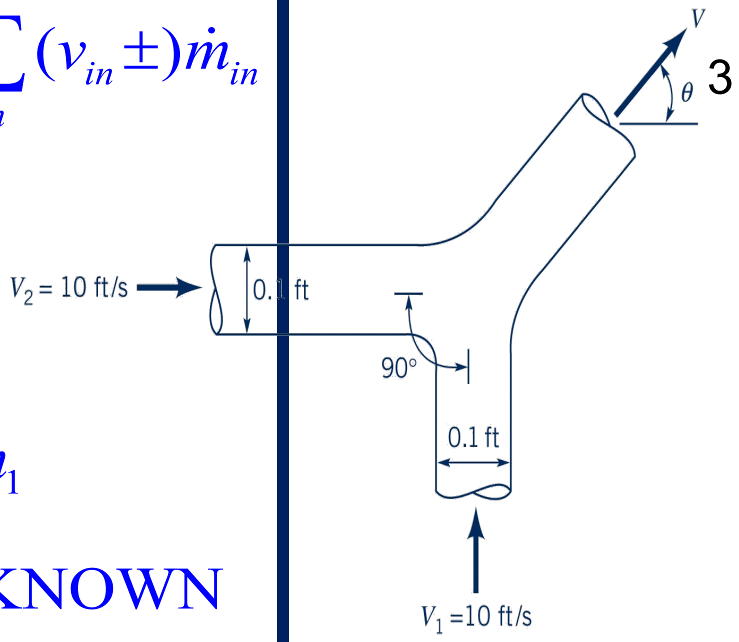
$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Y MOMENTUM

$$0 = 0 + (V_3 \sin\theta +)\dot{m}_3 - (V_1 +)\dot{m}_1$$

$$0 = 0 + (V_3 \sin\theta +)(\dot{m}_1 + \dot{m}_2) - (V_1 +)\dot{m}_1$$

$$V_3(V_1, \dot{m}_1, \dot{m}_2, \theta) = \frac{(V_1)\dot{m}_1}{\sin\theta(\dot{m}_1 + \dot{m}_2)} = \text{KNOWN}$$



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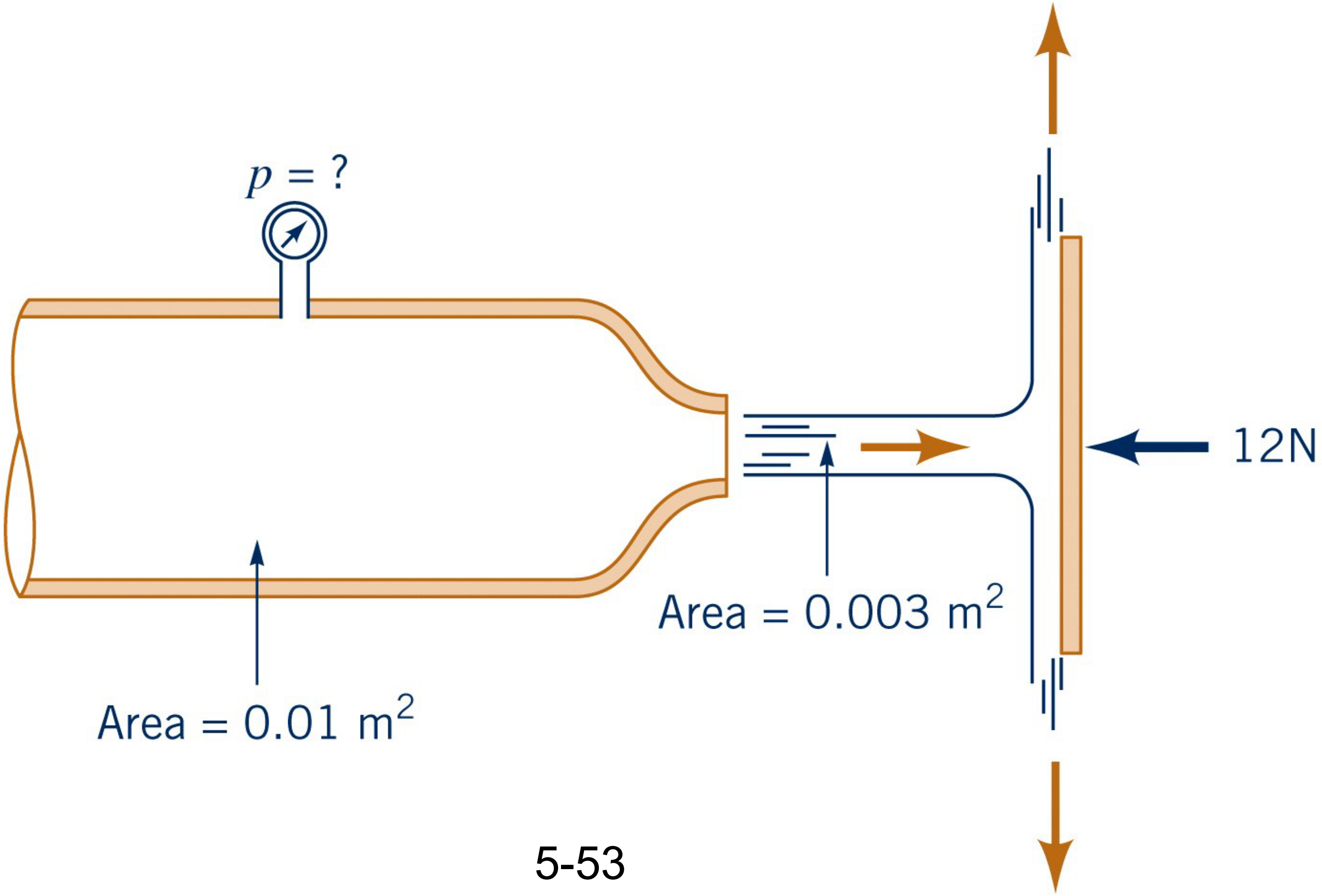
MASS CONSERVATION

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \rho A_3 V_3$$

$$A_3 = \frac{\pi D_3^2}{4} = \frac{\dot{m}_1 + \dot{m}_2}{\rho V_3} = \text{KNOWN}$$

$$\mathrm{A}_3 = \frac{A_1 V_1 + A_2 V_2}{V_3}, D_3 = \sqrt{\frac{4}{\pi} \frac{A_1 V_1 + A_2 V_2}{V_3}}$$

Air flows as shown. A force of 12N is required to hold plate in place. Find gauge pressure reading



5-53

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Fundamantals Driving Problem Road Map

1. Conservation of Mass: Change in Diameter
2. Bernouli: Steady Inviscid Flow/Change in Velocity/Pressure Along Streamline
3. Conservation of Momentum: Exit/Inlet Flow to CV, External Applied CV Force

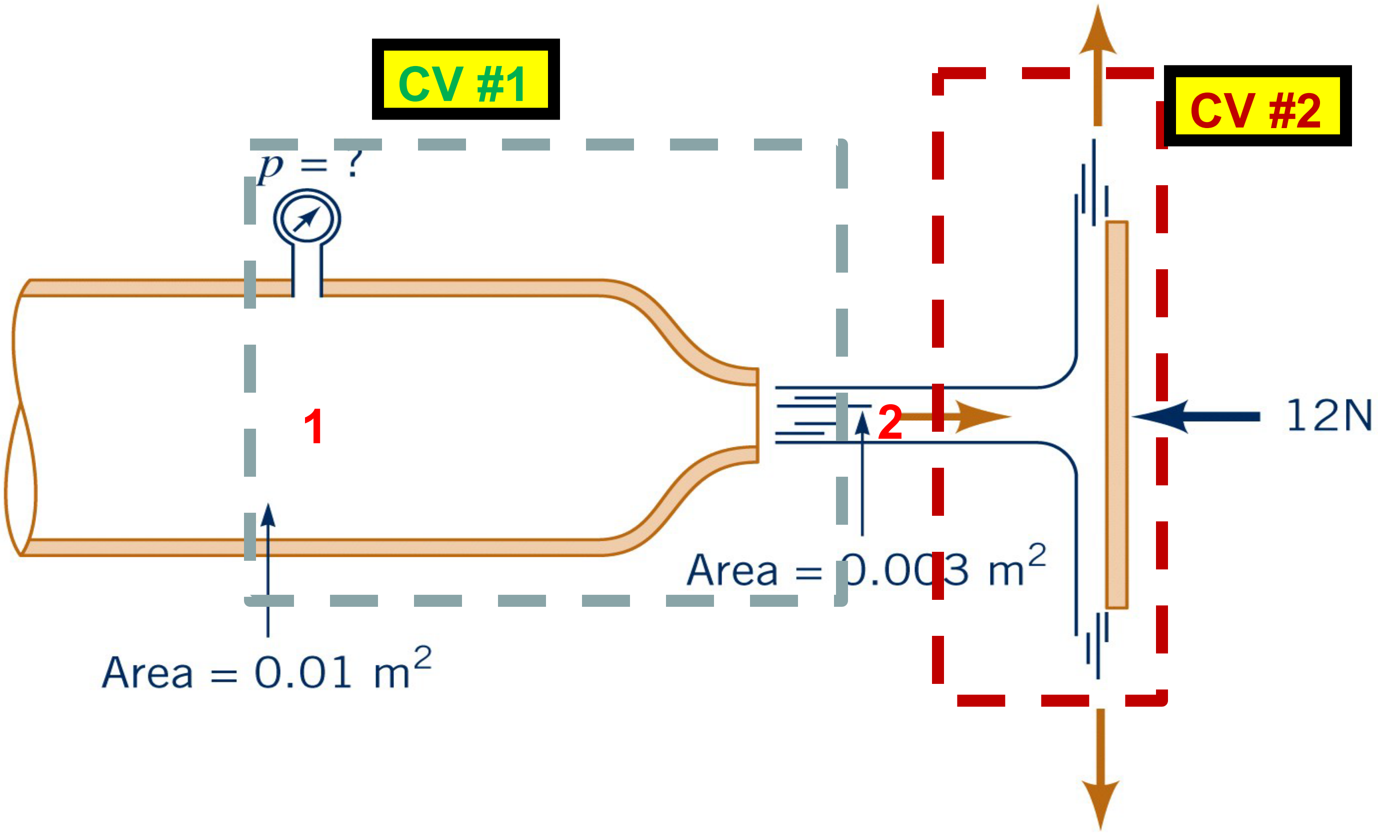


Figure P5.57

CONTROL VOLUME#1

Mass Conservation + Bernoulli

$$\dot{m}_1 = \dot{m}_2$$

$$\cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2$$

MASS CONSERVATION

$$V_1 = \frac{A_2 V_2}{A_1}$$

BERNOULLI

$$\frac{P_1}{\gamma} + \cancel{z_1} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \cancel{z_2} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\gamma} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} - \frac{\left[\frac{A_2 V_2}{A_1}\right]^2}{2g}$$

$P_2 = 0 \rightarrow$ FREE JET

$$\frac{P_1}{\gamma} = \frac{V_2^2}{2g} - \frac{\left[\frac{A_2 V_2}{A_1}\right]^2}{2g}$$

$$\frac{P_1}{\gamma} = \frac{V_2^2}{2g}(1 - \left(\frac{A_2}{A_1}\right)^2)$$

Control Volume #2

X Momentum

$$\sum^{\rightarrow +} F_x = \frac{dM_{CV}}{dt} + \sum_{out} (u_{out} \pm)\dot{m}_{out} - \sum_{in} (u_{in} \pm)\dot{m}_{in}$$

$$-14N = 0 + 0 - (V_2 \pm)\dot{m}_2$$

$$= 0 + 0 - (V_2 +)\rho_2 A_2 V_2$$

$$= -\rho_2 A_2 V_2^2$$

$$\sqrt{\frac{12N\left[\frac{kg-m}{s^2}\right]}{\rho_2\left[\frac{kg}{m^3}\right]A_2\left[m^2\right]}} = V_2, \rho_{air} = 1.23\frac{kg}{m^3}$$

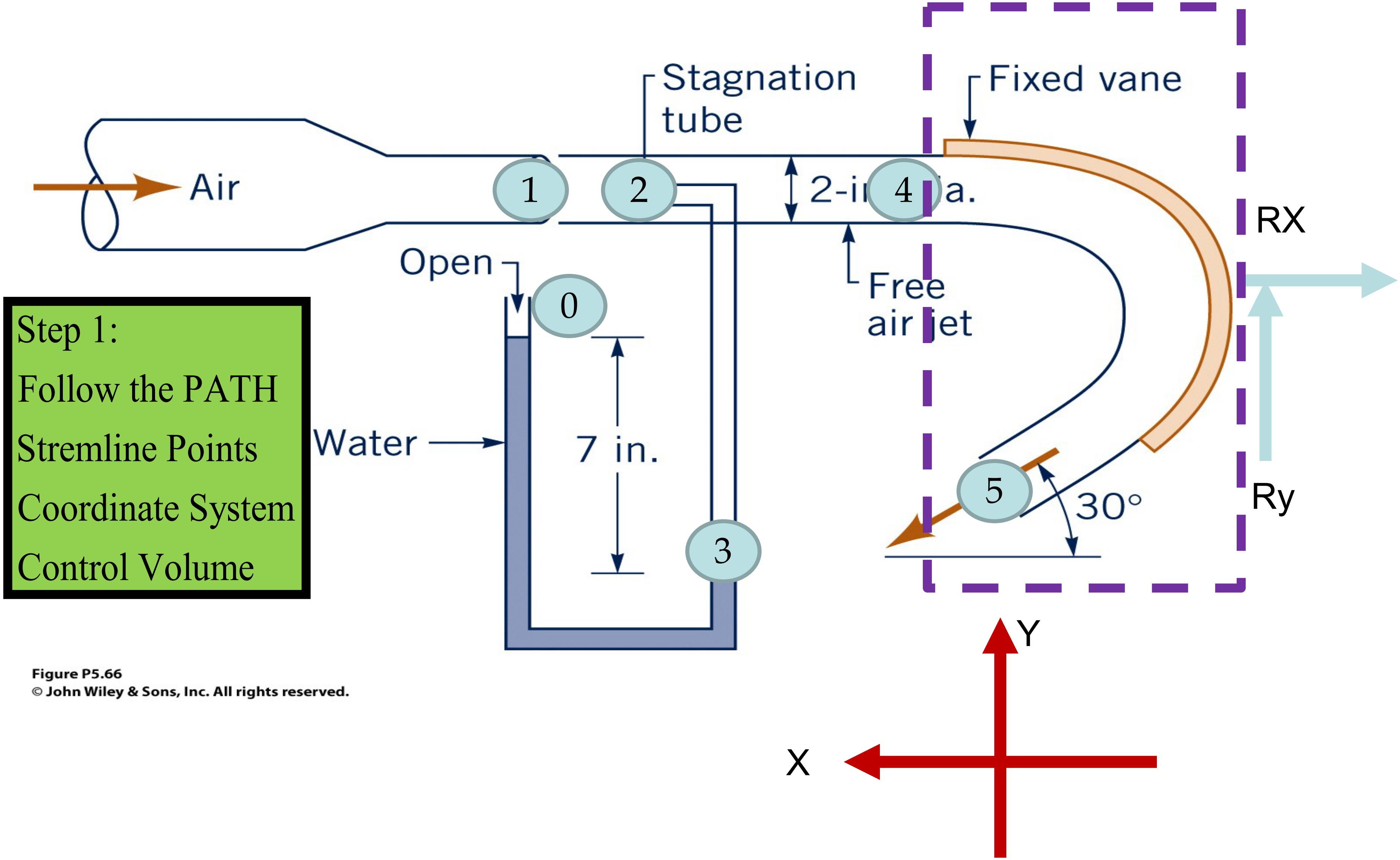
$$57.96\frac{m}{s} = V_2$$

$$\boxed{\begin{gathered}\frac{P_1}{\gamma} = \frac{V_2^2}{2g} - \frac{\left[\frac{\mathrm{A}_2 \mathrm{V}_2}{\mathrm{A}_1}\right]^2}{2g} \\ \frac{P_1}{\gamma} = \frac{V_2^2}{2g}(1 - \left(\frac{A_2}{A_1}\right)^2)\end{gathered}}$$

“…Learning within a **PROCESS** results in deeper and a more enriched learning and understanding….”

Dr. K. J. Berry

CONTROL
VOLUME



Step 1:

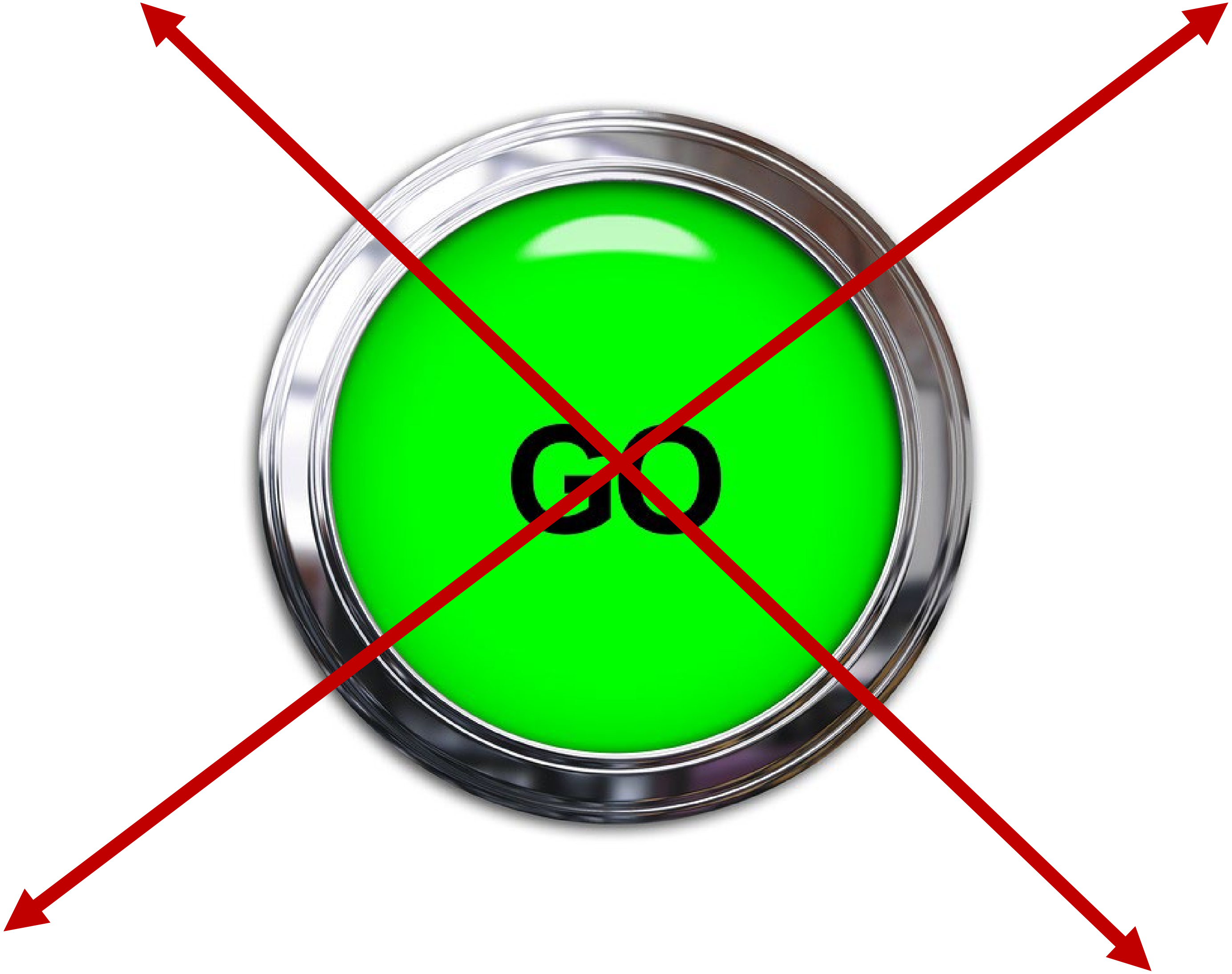
Follow the PATH

Stremline Points

Coordinate System

Control Volume

Figure P5.66


GO

FUNDAMENTAL PRINCIPALS DRIVING ROAD MAP

Manometry (ΔP)

Conservation of Mass

Conservation of Energy--Bernoulii

Conservation of Momentum

Manometry

$$2-0$$

$$P_2 = P_3 \rightarrow \text{AIR ABOVE LIQUID}$$

$$P_2 +_2 \Delta_0 P = P_0$$

$$P_3 - \gamma_{H20} \frac{7}{12} = P_0 = 0$$

$$P_3 = P_2 = \gamma_{H20} \frac{x}{12} \rightarrow \text{Stagnation Pressure}$$

$$P_3 = P_2 = \gamma_{H20}\frac{x}{12} \rightarrow \text{Stagnation Pressure}$$

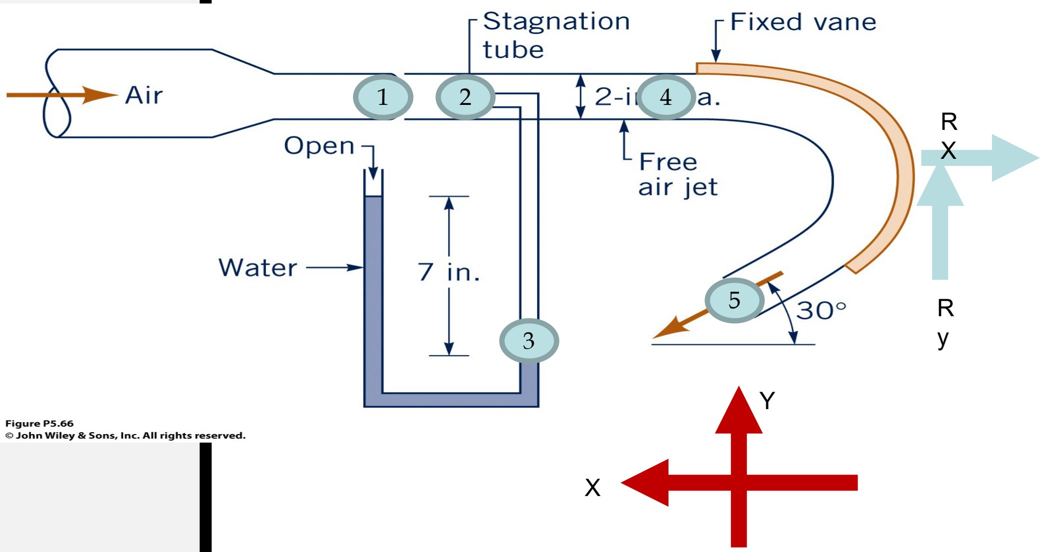
Energy

$$1-2, z_1 = z_2$$

$$\frac{\cancel{P_1}}{\cancel{\gamma_{air}}} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_{air}} + \frac{\cancel{V_2^2}}{\cancel{2g}}$$

$$\frac{V_1^2}{2g} = \frac{P_2}{\gamma_{air}} = \frac{\gamma_{H20}\frac{x}{12}}{\gamma_{air}}$$

$$V_1\left[\frac{ft}{s}\right] = \sqrt{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\frac{x}{12}[ft]}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]} \bullet 2g\left[\frac{ft}{s^2}\right]}$$



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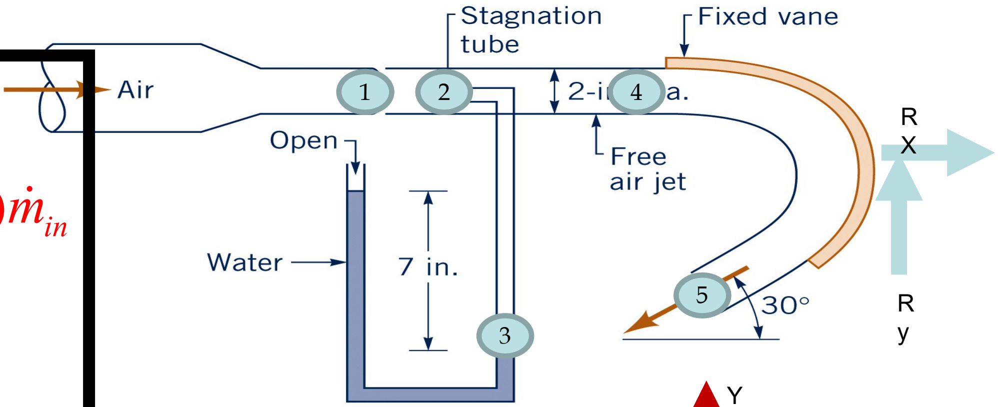

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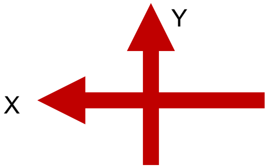


X Momentum

$$\overset{\leftarrow +}{\sum} F_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out}\pm)\dot{m}_{out} - \sum_{in}(u_{in}\pm)\dot{m}_{in}$$

$$-Rx = 0 + (V_5\cos\theta +)\dot{m}_5 - (V_4 -)\dot{m}_4$$

MASS CONSERVATION

$$\dot{m}_5 = \dot{m}_4 = \dot{m} = \rho_{air} A_4 V$$

$$-Rx = 0 + \dot{m}(V_5\cos\theta + V_4)$$

$$V_5 = V_4 = V \rightarrow \text{NO FRICTION}$$

$$-Rx = 0 + \dot{m}V(\cos\theta + 1)$$

$$Rx = -\dot{m}V(\cos\theta + 1)$$

$$Rx = \dot{m}V(\cos\theta + 1) \leftarrow \text{TO THE LEFT}$$

$$V_1\left[\frac{ft}{s}\right] = \sqrt{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\frac{x}{12}[ft]}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]} \bullet 2g}\left[\frac{ft}{s^2}\right]$$

$$\uparrow \sum F_y = \frac{dM_{CV}}{dt} + \sum_{out}(v_{out}\pm)\dot{m}_{out} - \sum_{in}(v_{in}\pm)\dot{m}_{in}$$

$$+R_y = 0 + (V_5\sin\theta -)\dot{m}_5 - 0$$

$$= 0 - \rho_{air}A_5V_5^2\sin\theta$$

$$= -\rho_{air}A_5V_5^2\sin\theta$$

$$+R_y = \rho_{air}A_5V_5^2\sin\theta \rightarrow DOWN$$

$$\left|\vec{R}\right| = \left[\rho_{air}A_5V_5^2(\cos\theta + 1)\right]i + \left[\rho_{air}A_5V_5^2\sin\theta\right]j$$

MATRIX EQUATIONS

$$EQUATION\ 1$$

$$V_1\left[\frac{ft}{s}\right]=\sqrt{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\frac{x}{12}[ft]}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]}\bullet 2g\left[\frac{ft}{s^2}\right]}$$

$$\overbrace{\{1\}\left\{V_1^2\right\}}^{UNKNOWN\,1}\left[\frac{ft}{s}\right]^2+\overbrace{\{0]\left\{R_x\right\}}^{UNKNOWN\,2}+\overbrace{\{0]\left\{R_y\right\}}^{UNKNOWN\,3}=\overbrace{\left\{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\bullet\frac{2g}{12}}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]}\right\}\left\{\overset{\text{CONTROLLING INPUT}}{\overbrace{x}}\right\}}^{FORCING\ FUNCTION\,1}$$

$$a_{11}=1$$

$$a_{12}=0$$

$$a_{13}=0$$

$$F_1=\left\{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\bullet\frac{2g}{12}}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]}\right\}\{x\}$$

MATRIX EQUATIONS

$$EQUATION\ 2$$

$$Rx = -\dot{m}V(\cos\theta + 1)$$

$$= -\rho_{air}AV^2(\cos\theta + 1)$$

$$= -\rho_{air}A(\cos\theta + 1)\left\{V^2\right\}$$

$$\rho_{air}A(\cos\theta + 1)\overbrace{\left\{V^2\right\}}^{UNKNOWN\,1} + \overbrace{\{1\}\left\{Rx\right\}}^{UNKNOWN\,2} + \overbrace{\{0\}\left\{Ry\right\}}^{UNKNOWN\,3} = 0$$

$$a_{21} = \rho_{air}A(\cos\theta + 1)$$

$$a_{22} = 1$$

$$a_{23} = 0$$

$$F_2 = 0$$

MATRIX EQUATIONS

$$EQUATION\ 3$$

$$+R_y = \rho_{air} A_5 V^2 \sin\theta$$

$$R_y - \rho_{air} A_5 V^2 \sin\theta = 0$$

$$-\rho_{air} A_5 \sin\theta \overbrace{\left\{V^2\right\}}^{UNKNOWN1} + \overbrace{\{0\}\left\{R_x\right\}}^{UNKNOWN2} + \overbrace{\{1\}\left\{R_y\right\}}^{UNKNOWN3} = 0$$

$$a_{31} = -\rho_{air} A_5 \sin\theta$$

$$a_{32} = 0$$

$$a_{33} = 1$$

$$F_3 = 0$$

Matrix Equation

$$[A]\{x\} = \{B\}$$

$$\{x\} = [A]^{-1}\{B\}$$

MATLAB/MATHCAD

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

“FINAL MATRIX SYSTEM EQUATIONS”

$$\begin{bmatrix} 1 & 0 & 0 \\ \rho_{air} A(\cos\theta + 1) & 1 & 0 \\ -\rho_{air} A \sin\theta & 0 & 1 \end{bmatrix} \begin{Bmatrix} \phi_1 = V^2 \\ \phi_2 = R_x \\ \phi_3 = R_y \end{Bmatrix} = \begin{Bmatrix} \left\{ \dfrac{\gamma_{H20} \left[\dfrac{lbf}{ft^3} \right] \bullet \dfrac{2g}{12}}{\gamma_{air} \left[\dfrac{lbf}{ft^3} \right]} \right\} \{x\} \\ 0 \\ 0 \end{Bmatrix}$$

Water flows steadily from a 1m Dia. tank as shown and strikes the cart that weighs 100N. If the fluid is inviscid and if the coefficient of friction for the wheels is 0.3, determine V1, V2, and the tenson in Rope B first, then Rope A.

Show and execute Road Map.

Control Volumes

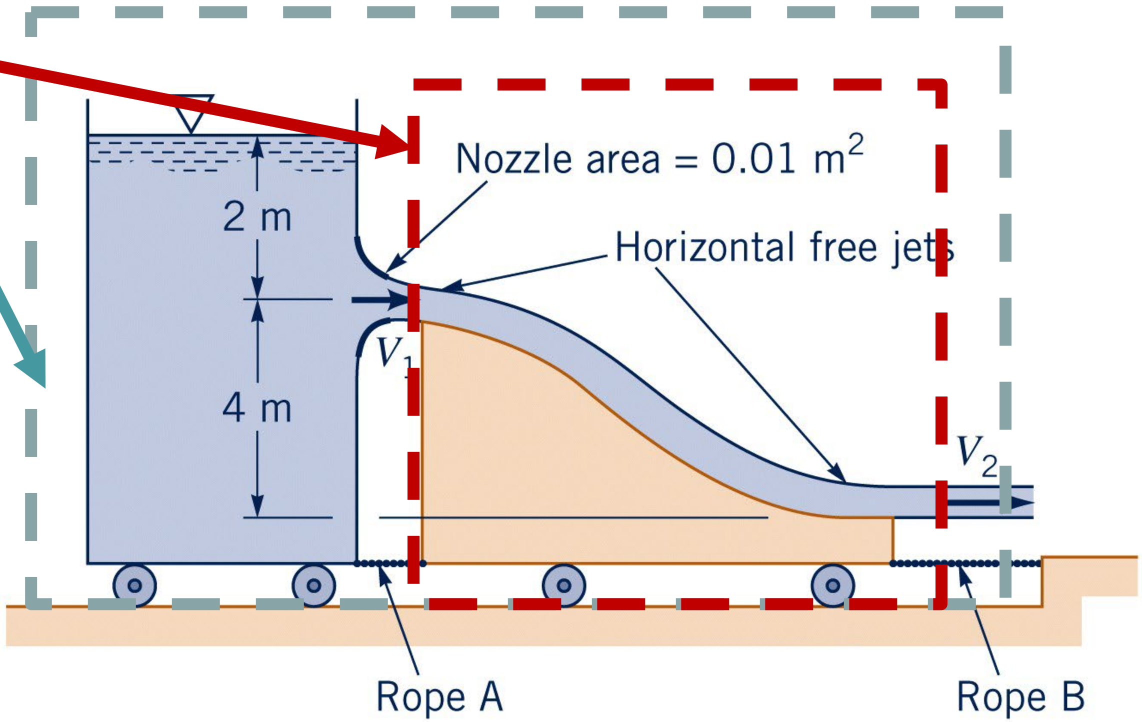
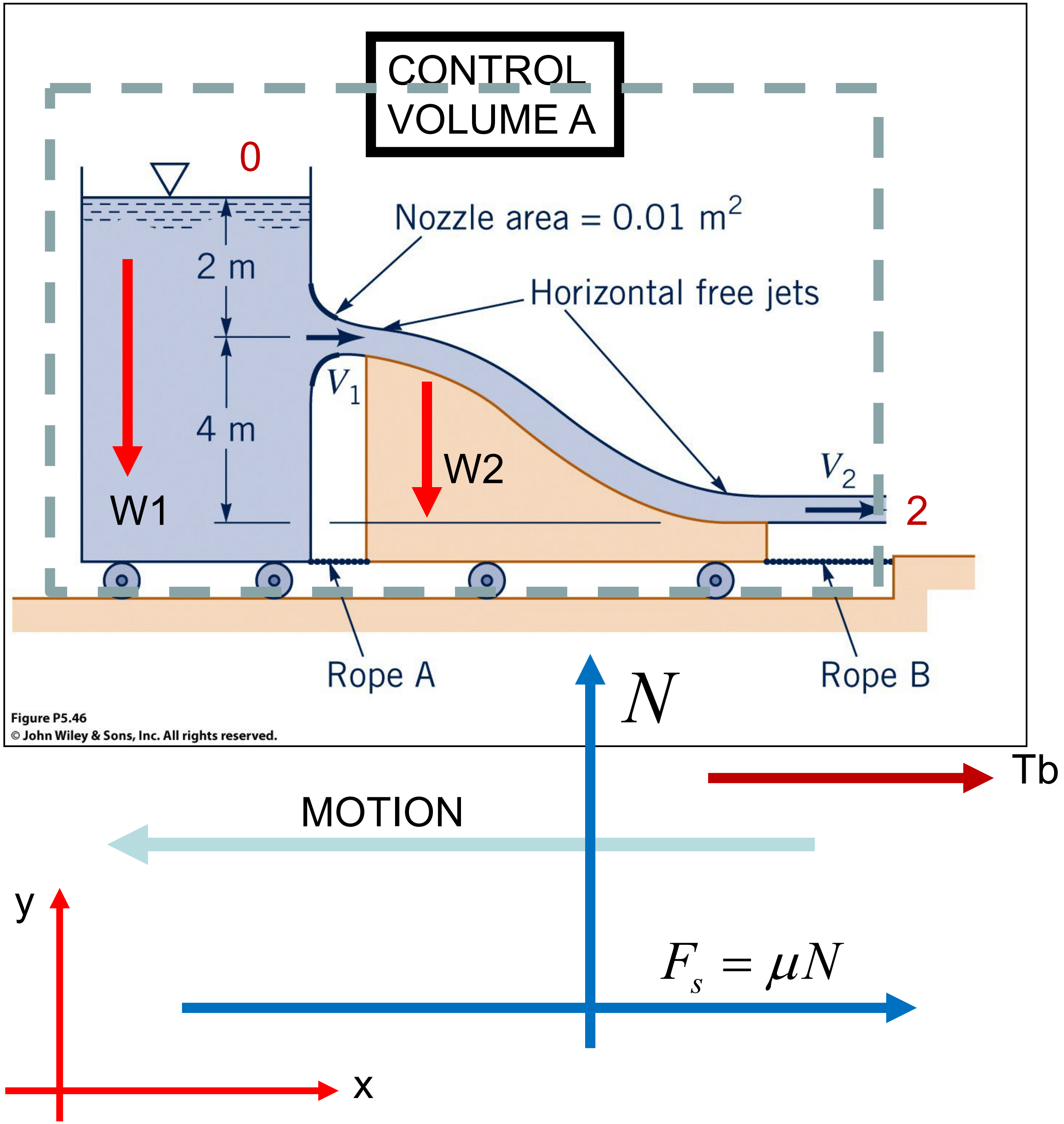


Figure P5.46

5.44

Step 1:
Follow the PATH
Stremline Points
Coordinate System
Control Volume
FREE BODY DIAGRAM



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$$\overset{\rightarrow +}{\sum} F_x = \frac{dM_{CV}}{dt} + \sum_{out} (u_{out} \pm) \dot{m}_{out} - \sum_{in} (u_{in} \pm) \dot{m}_{in}$$

$$T_b + F_s = 0 + (V_2 +) \dot{m}_2$$

$$T_b = (V_2) \dot{m}_2 - F_s$$

$$= (V_2) \dot{m}_2 - \mu N$$

ENERGY

$$\frac{P_0}{\gamma} + z_0 = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\left(z_0 - z_2\right) 2g} = 10.85 m / s$$

$$\dot{m}_2 = \rho A V_2 = 108.5 kg / s$$

$$N = W_1 + W_2 = \gamma_{H20} \forall + W_2$$

$$T_b = (V_2) \dot{m}_2 - \mu N$$

CONTROL VOLUME B

$$\overset{\rightarrow +}{\sum} F_x = \frac{dM_{CV}}{dt} + \sum_{out} (u_{out} \pm) \dot{m}_{out} - \sum_{in} (u_{in} \pm) \dot{m}_{in}$$

$$T_B - T_A + F_s = 0 + \dot{m}(V_2 - V_1)$$

$$T_A = \dot{m}(V_1 - V_2) + T_B + F_s$$

$$= \dot{m}(V_1 - V_2) + T_B + \mu(W_2)$$

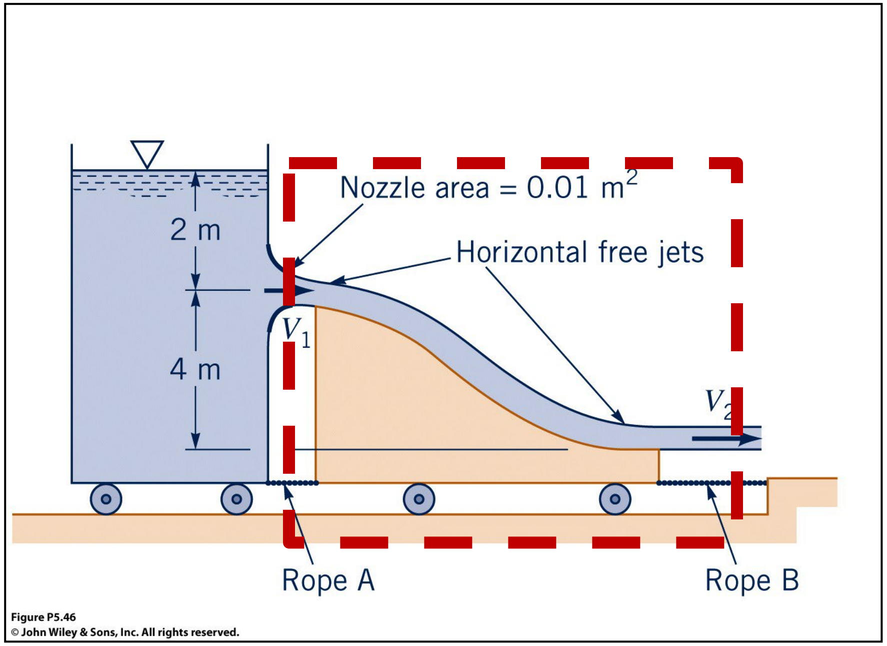

Figure P5.46