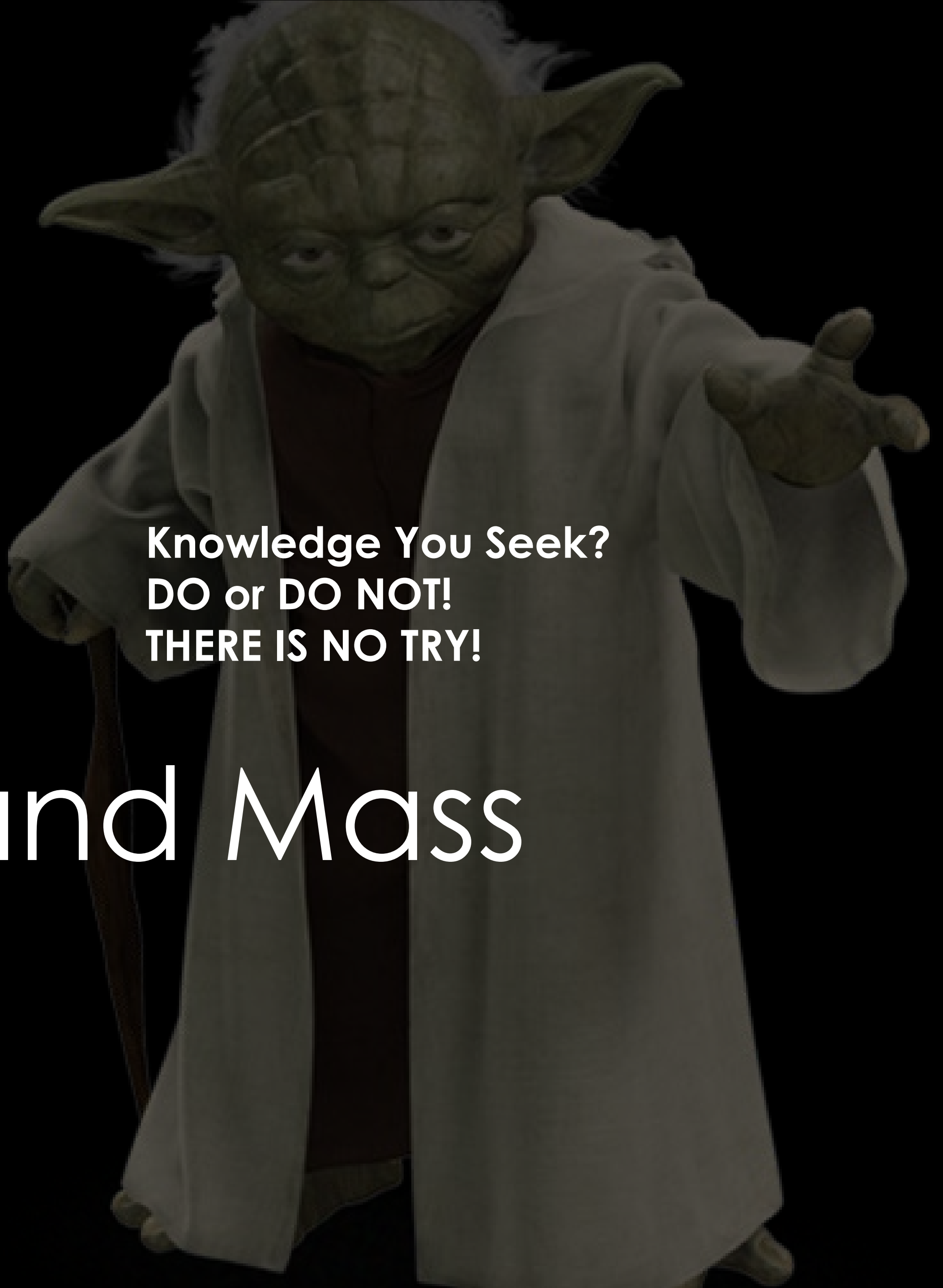
Study Aid
Momentum and Mass

Dr. K. J. Berry

ASME FELLOW



Knowledge You Seek?
DO or DO NOT!
THERE IS NO TRY!

HOMEWORK

1. A nozzle is attached to a vertical pipe $(A_1 = 0.02m^2)$ as shown discharges water $(\rho = 1000\frac{kg}{m^3})$ at $P_2 = ??\ kPa$ (NOT FREE JET) $and\ A_2 = 0.01m^2$. The discharge is $0.10\frac{m^3}{s}$, the flange gage pressure is $40kPa$, the nozzle weight is 200N, and the water volume is $0.01m^3$.

a. Draw Free Body Diagram and discuss fundamental principles governing your solution, and WHY?
b. Determine the OUTLET pressure (kPa).
c. Determine the magnitude AND direction of the anchoring forces, $F_x\ and\ F_z$.

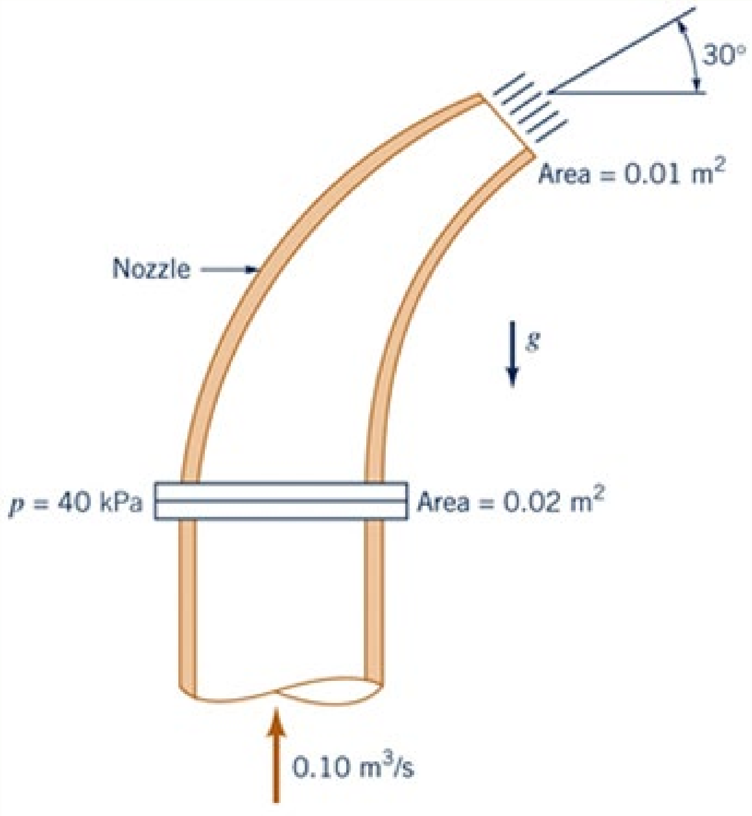
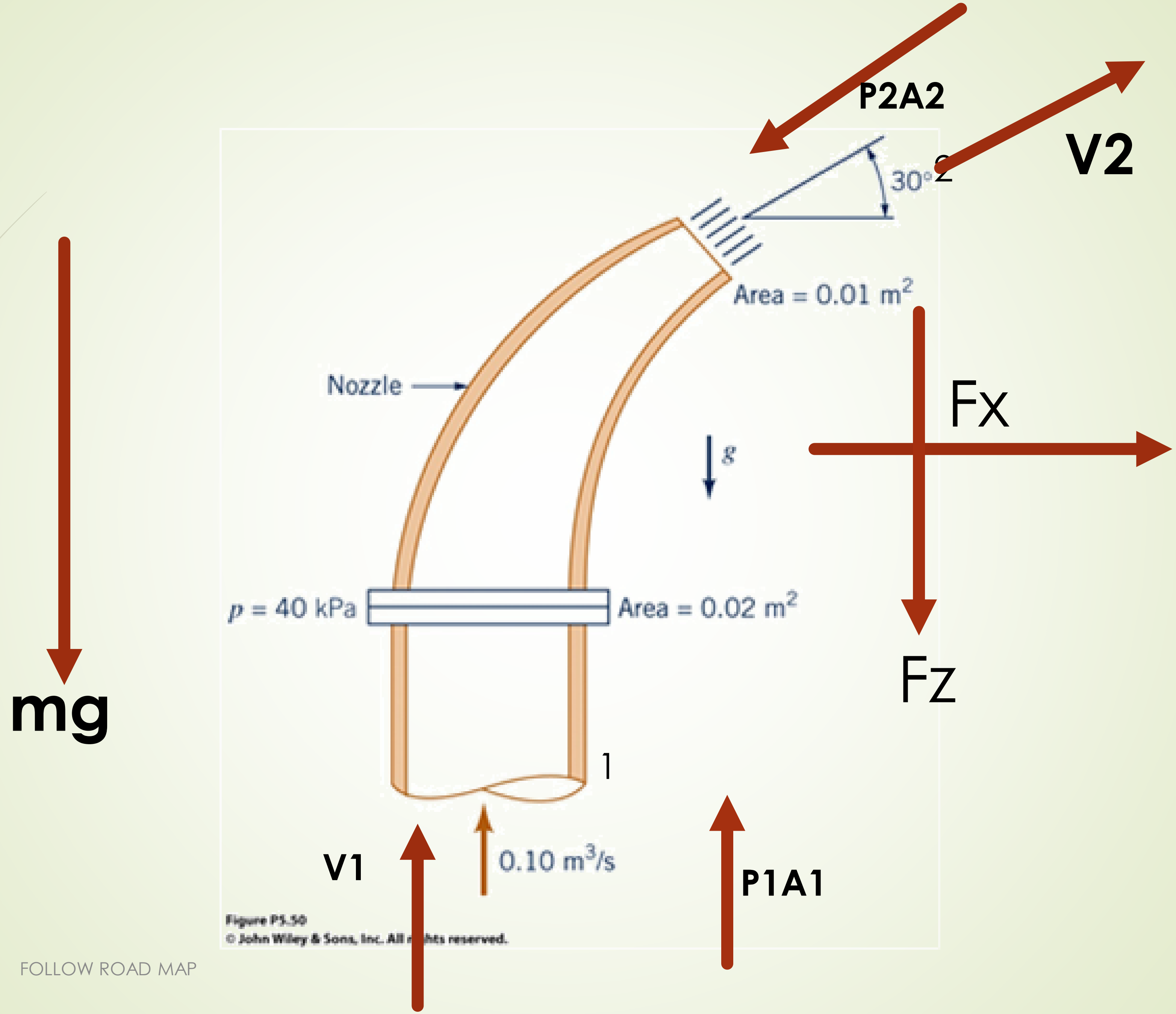


Figure P5.50



P2A2
V2
30°
2
Area = 0.01 m²
Nozzle
g
Fx
Fz
mg
p = 40 kPa
Area = 0.02 m²
1
V1
0.10 m³/s
P1A1
Figure P5.50
© John Wiley & Sons, Inc. All rights reserved.

FUNDAMENTALS #1

- Mass Conservation (Change in D)
- Bernoulli (Change of V and P along Streamline)
- Momentum (External Applied Forces)

#1: MASS CONSERVATION

$$\sum Q_{in} = \sum Q_{out} = 0.10\frac{m^3}{s}; \rho = \text{constant}$$

$$\mathrm{A_1 V_1} = A_2 V_2 = Q$$

$$\mathrm{V_1} = \frac{Q}{A_1} = 5\frac{m}{s}$$

$$\mathrm{V_2} = \frac{Q}{A_2} = 10\frac{m}{s}$$

#2 Bernoulli (1-2)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}=\frac{P_2}{\gamma}+\frac{V_2^2}{2g};z_1=z_2$$

$$P_2=\left(\frac{P_1}{\gamma}+\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)\gamma;\gamma=9810\frac{N}{m^3}$$

$$=\left(\frac{40,000Pa}{\gamma\frac{N}{m^3}}+\frac{5^2\frac{m^2}{s^s}}{2g}-\frac{10^2\frac{m^2}{s^s}}{2g}\right)\gamma\frac{N}{m^3}$$

$$=2492Pa$$

#3: Momentum X

$$\overrightarrow{+\sum F_x} = F_x - P_2 A_2 \cos\Theta = \cancel{\frac{dM}{dt}}_x + \sum u_{out}(\pm)\dot{m}_{out} - \sum u_{in}(\pm)\dot{m}_{in}$$

$$F_x = P_2 A_2 \cos\Theta + \sum u_{out}(\pm)\dot{m}_{out} - \cancel{\sum u_{in}(\pm)\dot{m}_{in}}$$

$$= P_2 A_2 \cos\Theta + V_2 \cos\Theta(+)\dot{m}_2$$

$$= P_2 A_2 \cos\Theta + \frac{\rho Q^2 \cos\Theta}{A_2}$$

$$= 21.5814N + 866.03N$$

$$= 887.6N \rightarrow$$

#3 Momentum Y

$$\uparrow \sum F_y = F_y + P_1 A_1 - P_2 A_2 \sin\Theta - W = \cancel{\frac{dM}{dt}}_y + \sum v_{out}(\pm)\dot{m}_{out} - \sum v_{in}(\pm)\dot{m}_{in}$$

$$F_y = P_2 A_2 \sin\Theta - P_1 A_1 + W + \sum v_{out}(\pm)\dot{m}_{out} - \sum v_{in}(\pm)\dot{m}_{in}$$

$$= P_2 A_2 \sin\Theta - P_1 A_1 + W + V_2 \sin\Theta \dot{m}_2(+) - V_1(+)\dot{m}_1$$

$$\dot{m}_2 = \dot{m}_1 = \dot{m} = \rho Q \rightarrow \text{MASS CONSERVATION}$$

$$W = W_{pipe} + W_{fluid}$$

$$= 200N + \gamma_{fluid} \bullet Volume$$

$$= 298.1N$$

$$F_y = P_2 A_2 \sin\Theta - P_1 A_1 + W + \dot{m}(V_2 \sin\Theta - V_1)$$

$$= -489.44N \downarrow$$

$$\vec{F} = 888\hat{i} - 489.44\hat{j}$$

QUIZ IV Problem--REPEAT

Consider a box 4.8ft by 3.2ft and 50ft tall. Little Johnny is 4.2ft tall is trapped in the box with glycerin filling box at a rate of

$$Q(t) = 3.4e^{-0.8t}\,\frac{ft^3}{s}$$

through 6" Dia hole. A small 4" diameter hole in the bottom of the box allows glycerin at

$$V_{exit}(t) = 0.06t\,\frac{ft}{s}$$

to escape. State and use appropriate conservation law to solve.

a. Draw Free Body Diagram and discuss fundamental principles governing your solution, and WHY?
b. If box is ½ filled at the beginning, what is the HEIGHT of glycerin in the box after 10 sec, and after 200 sec.

MASS CONTINUITY

$$\frac{dM_{sys}}{dt} = \frac{d}{dt}\int \rho dV + \int \rho \vec{V} \bullet \hat{n} dA = 0$$

$$\rho = \text{constant}$$

$$\frac{d}{dt}\int dV + \sum_{out} Q(t) - \sum_{in} Q(t) = 0;$$

$$V(t) = A \bullet h(t), \frac{dV}{dt} = A\frac{dh}{dt}$$

$$A\frac{dh}{dt} = \sum_{in} Q(t) - \sum_{out} Q(t)$$

$$\int_{h_0}^{h(t)} dh = \frac{\int_o^{t^*} (3.4e^{-0.8t} - A_{exit}0.06t)dt}{Area}$$

$$h(t) = h_0 + \left[\frac{\frac{-3.4e^{-0.8t}}{0.8} - \frac{A_{exit}0.06t^2}{2}}{Area}\right]_{0-t^*}$$

$$h(t) = h_0 + \frac{-4.25(e^{-0.8t^*} - 1) - A_{exit}0.03t^{*2}}{L \bullet W}; h_0 = 25\,ft, A_e = 0.0873\,ft^2$$

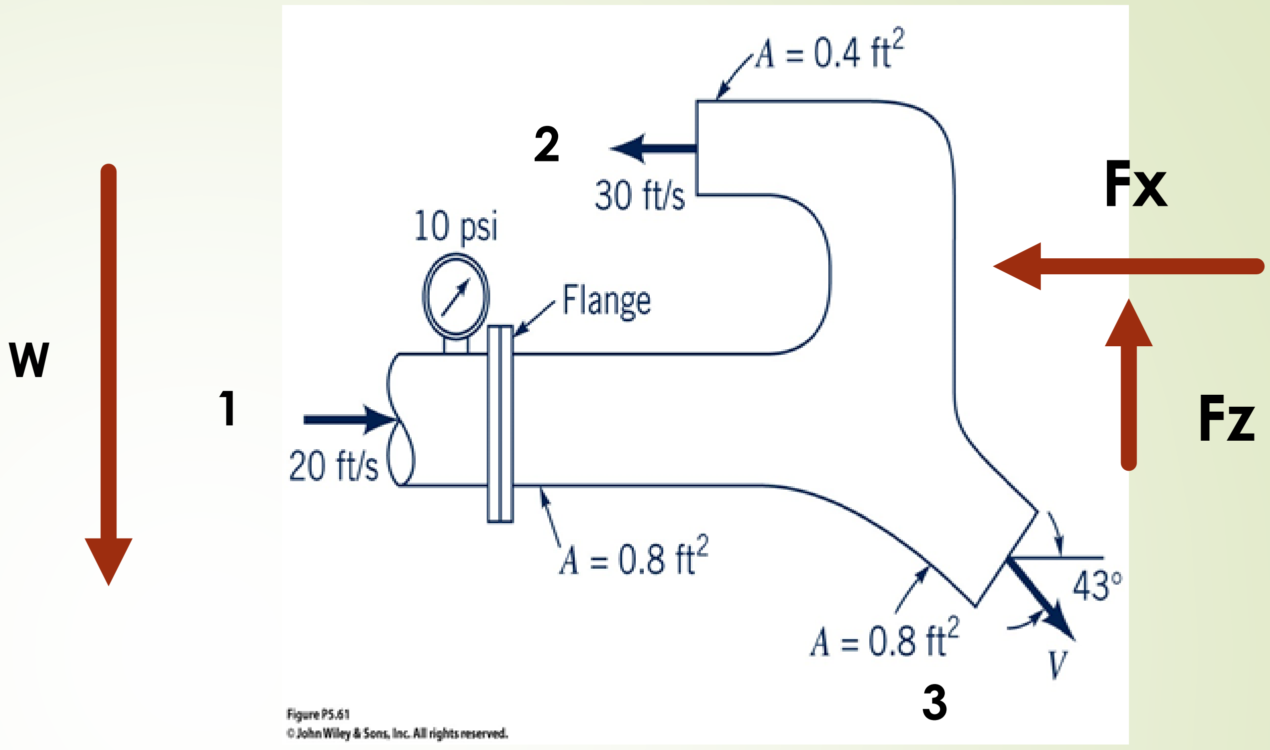
$$h(t) = h_0 + \frac{-4.25(e^{-0.8t^*} - 1) - A_{exit}0.03t^{*2}}{L \bullet W}; h_0 = 25\,ft, A_e = 0.0873\,ft^2$$

$$h(t = 10) = 24.7\,ft$$

$$h(t = 200) = 18.5\,ft$$

FUNDAMENTALS HOMEWORK

Mass Bernoulli Momentum



1. Water discharges through the device as shown in the plane. The flange gage pressure $10PSIG$, the nozzle weight is 50lb, and the water volume is $0.1ft^3$.

a. Draw Free Body Diagram and discuss fundamental principles governing your solution, and WHY?
b. Determine exit velocity "V".
c. Determine both exit output pressures, PSIG.
d. Determine the magnitude AND direction of the anchoring forces, F_x *and* F_z.

#1: MASS CONSERVATION

$$\sum Q_{in} = \sum Q_{out} ; \rho = \text{CONSTANT}$$

$$\mathrm{Q}_1 = Q_2 + Q_3$$

$$A_1V_1 = A_2V_2 + A_3V_3$$

$$V_3 = \frac{A_1V_1 - A_2V_2}{A_3} = 5\frac{ft}{s}$$

#2 Bernoulli

$$1-2$$

$$\dot{m}_1\left(\frac{P_1}{\gamma}+\frac{V_1^2}{2g}\right)=\dot{m}_2\left(\frac{P_2}{\gamma}+\frac{V_2^2}{2g}\right);z_1=z_2$$

$$P_2=\left(\frac{\dot{m}_1}{\dot{m}_2}\right)\left(\frac{P_1}{\gamma}+\frac{V_1^2}{2g}\right)\gamma-\frac{V_2^2}{2g}\gamma$$

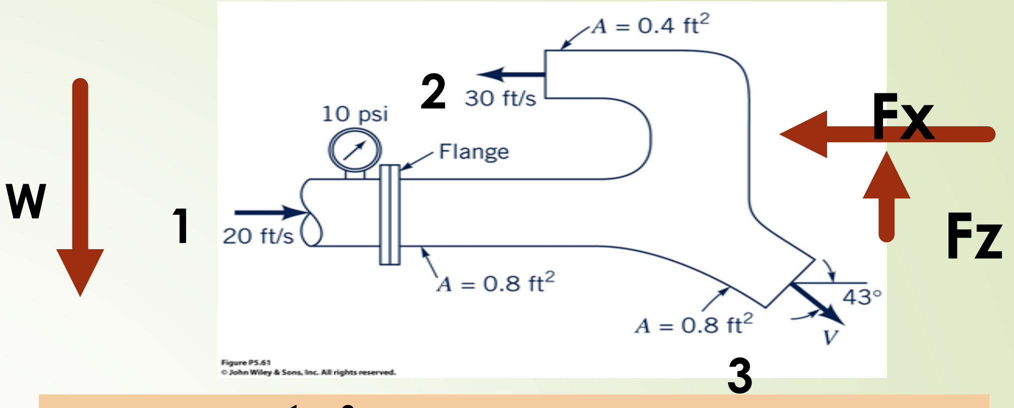
$$P_2=\left(\frac{\dot{m}_1}{\dot{m}_2}\right)\left(\frac{10\,psig\bullet\frac{144in^2}{ft^2}}{\gamma}+\frac{20^2}{2g}\right)\gamma$$

$$-\frac{30^2}{2g}\gamma$$

$$P_2=\left(\frac{\dot{m}_1}{\dot{m}_2}\right)1828\frac{lb}{ft^2}-872\frac{lb}{ft^2}$$

$$P_2=\left(\frac{\dot{m}_1}{\dot{m}_2}\right)12.7\,psig-6.1\,psig$$

FOLLOW ROAD MAP



$$1-3$$

$$\dot{m}_1\left(\frac{P_1}{\gamma}+\frac{V_1^2}{2g}\right)=\dot{m}_3\left(\frac{P_3}{\gamma}+\frac{V_3^2}{2g}\right);z_1=z_3$$

$$P_3=\left(\frac{\dot{m}_1}{\dot{m}_3}\right)\left(\frac{P_1}{\gamma}+\frac{V_1^2}{2g}\right)\gamma-\frac{V_3^2}{2g}$$

$$P_3=\left(\frac{\dot{m}_1}{\dot{m}_3}\right)\left(\frac{10\,psig\bullet\frac{144in^2}{ft^2}}{\gamma}+\frac{20^2}{2g}\right)\gamma-\frac{5^2}{2g}\gamma$$

$$\left(\frac{\dot{m}_1}{\dot{m}_3}\right)1872.6\frac{lb}{ft^2}-24.2\frac{lb}{ft^2}$$

$$P_3=\left(\frac{\dot{m}_1}{\dot{m}_3}\right)13\,psig-0.1681\,psig$$

#3 Momentum X

$$\overrightarrow{+\sum F_x} = P_1A_1 + P_2A_2 - P_3A_3\cos\theta - F_x = \cancel{\frac{dM}{dt}}_x + \sum u_{out}(\pm)\dot{m}_{out} - \sum u_{in}(\pm)\dot{m}_{in}$$

$$\overrightarrow{+\sum F_x} = P_1A_1 + P_2A_2 - P_3A_3\cos\theta - F_x = \cancel{\frac{dM}{dt}}_x + V_3\cos\theta(+)\dot{m}_3 + V_2(-)\dot{m}_2 - V_1(+)\dot{m}_1$$

$$F_x = P_1A_1 + P_2A_2 - P_3A_3\cos\theta - V_3\cos\theta\dot{m}_3 + V_2\dot{m}_2 + V_1\dot{m}_1$$

$$\dot{m}_1 = \rho A_1V_1$$

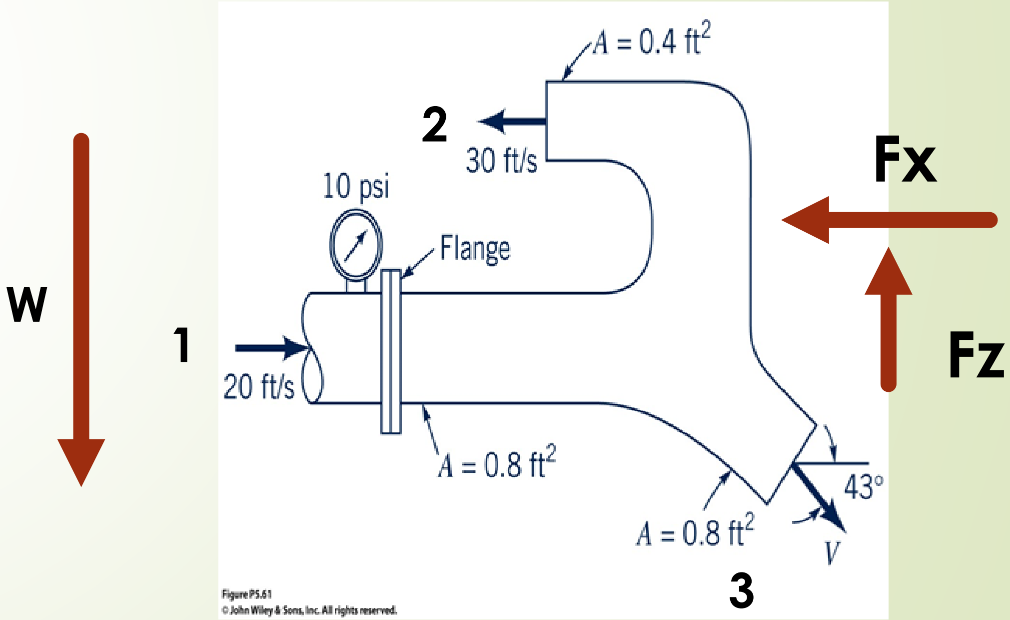
$$\dot{m}_2 = \rho A_2V_2$$

$$\dot{m}_3 = \rho A_3V_3$$

$$F_x = P_1A_1 + P_2A_2 - P_3A_3\cos\theta - V_3\cos\theta\dot{m}_3 + V_2\dot{m}_2 + V_1\dot{m}_1$$

$$F_x = -637lb$$

$$F_x = 637lb \rightarrow$$



Momentum: Z

$$\uparrow \sum F_z = P_3 A_3 \sin\theta + F_z - W_z = \frac{dM}{dt}_x + \sum w_{out}(\pm)\dot{m}_{out} - \sum w_{in}(\pm)\dot{m}_{in}$$

$$F_z = W_z - P_3 A_3 \sin\theta + V_3 \sin\theta(+)\dot{m}_3 \quad ; \quad \dot{m}_3 = \rho A_3 V_3$$

$$W_z = W_{pipe} + W_{fluid}$$

$$= 50lb + \gamma_{fluid} \bullet Volume$$

$$= 56.24lb$$

$$F_z = -902lb$$

$$= 902lb \downarrow$$

$$\vec{F} = 637\hat{i} - 902\hat{j}$$

$$\left\| \vec{F} \right\| = \sqrt{637^2 + 902^2} = 1104lb$$

$$\Theta = \tan^{-1}\frac{F_z}{F_x} = 55^0$$