MOMENTUM AND SYSTEMS MODELLING STUDY AID

MECH-322 Fluid Mechanics

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Step 1:

Follow the PATH

Stremline Points

Coordinate System

Control Volume

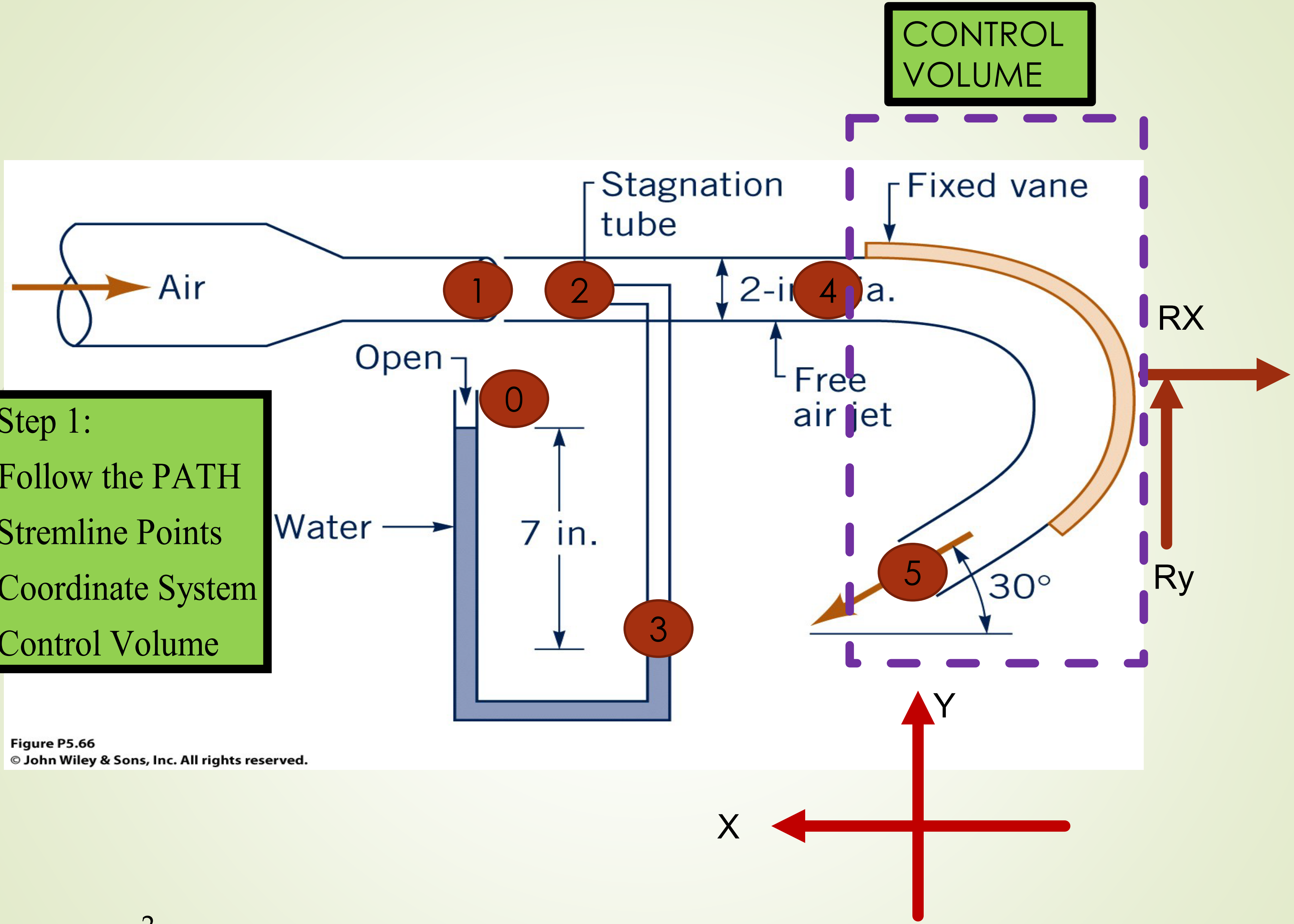
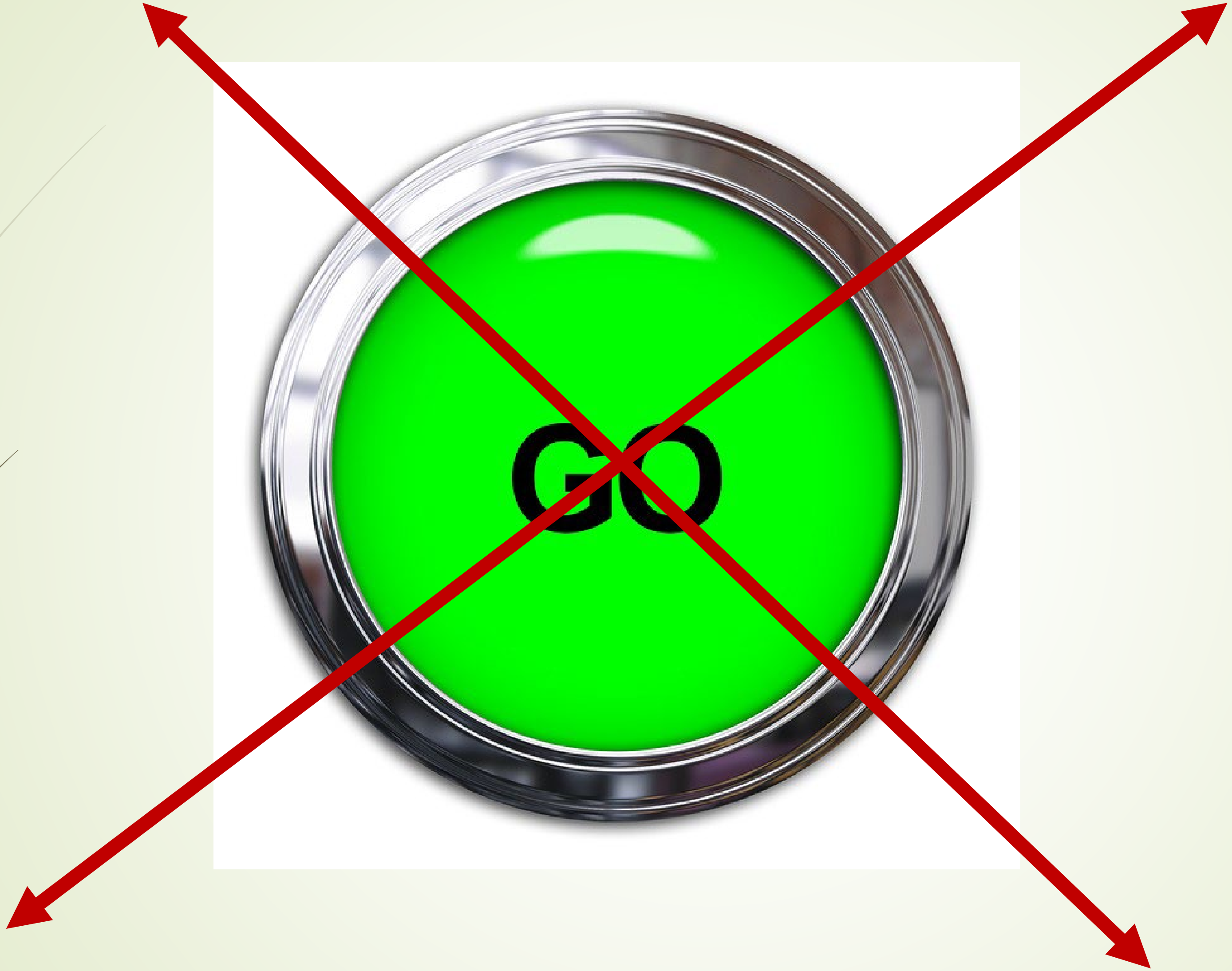


Figure P5.66


GO

FUNDAMENTAL PRINCIPALS DRIVING ROAD MAP

Manometry (ΔP)

Conservation of Mass

Conservation of Energy--Bernoulii

Conservation of Momentum

Manometry

$2-0$

$P_2 = P_3 \rightarrow$ AIR ABOVE LIQUID

$P_2 +_2 \Delta_0 P = P_0$

$$P_3 - \gamma_{H20}\frac{7}{12} = P_0 = 0$$

$$P_3 = P_2 = \gamma_{H20}\frac{x}{12} \rightarrow \text{Stagnation Pressure}$$

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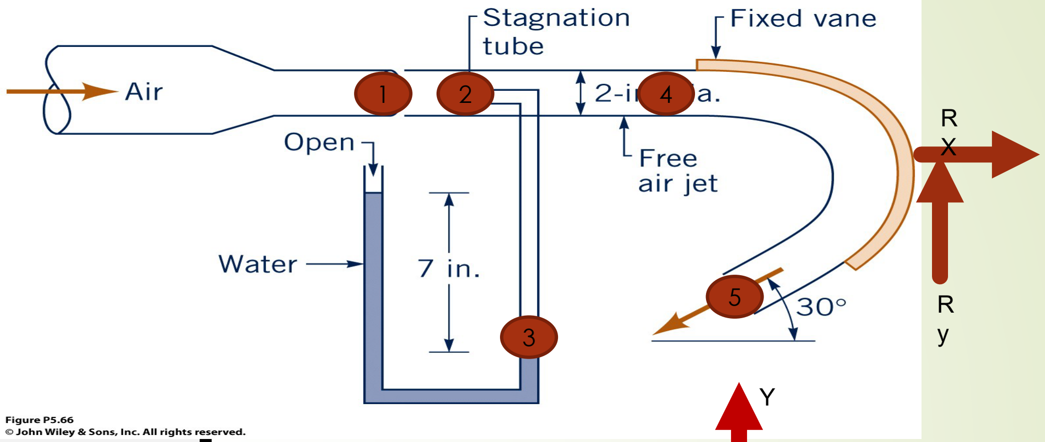
Energy

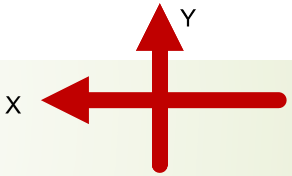
$$1-2, z_1 = z_2$$

$$\frac{P_1}{\gamma_{air}} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_{air}} + \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{2g} = \frac{P_2}{\gamma_{air}} = \frac{\gamma_{H20}\frac{x}{12}}{\gamma_{air}}$$

$$V_1\left[\frac{ft}{s}\right] = \sqrt{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\frac{x}{12}[ft]}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]} \bullet 2g\left[\frac{ft}{s^2}\right]}$$





X Momentum

$$\overset{\leftarrow +}{\sum} F_x = \frac{dM_{CV}}{dt} + \sum_{out}(u_{out} \pm)\dot{m}_{out} - \sum_{in}(u_{in} \pm)\dot{m}_{in}$$

$$-Rx = 0 + (V_5 \cos\theta +)\dot{m}_5 - (V_4 -)\dot{m}_4$$

MASS CONSERVATION

$$\dot{m}_5 = \dot{m}_4 = \dot{m} = \rho_{air} A_4 V$$

$$-Rx = 0 + \dot{m}(V_5 \cos\theta + V_4)$$

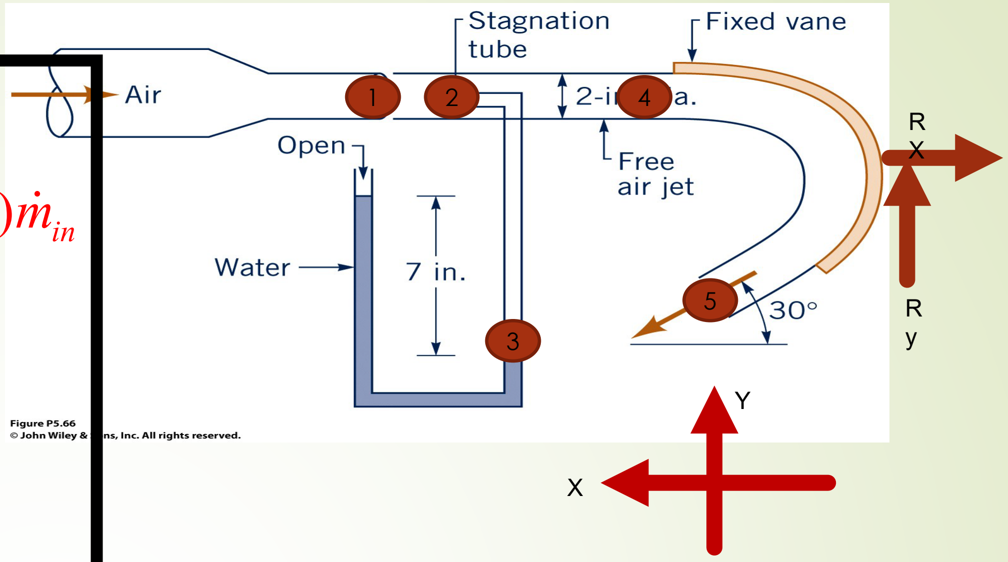
$$V_5 = V_4 = V \rightarrow \text{NO FRICTION}$$

$$-Rx = 0 + \dot{m}V(\cos\theta + 1)$$

$$Rx = -\dot{m}V(\cos\theta + 1)$$

$$Rx = \dot{m}V(\cos\theta + 1) \leftarrow \text{TO THE LEFT}$$

$$V_1\left[\frac{ft}{s}\right] = \sqrt{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\frac{x}{12}[ft]}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]} \bullet 2g\left[\frac{ft}{s^2}\right]}$$



$$\uparrow \sum F_y = \frac{dM_{CV}}{dt} + \sum_{out}(v_{out} \pm)\dot{m}_{out} - \sum_{in}(v_{in} \pm)\dot{m}_{in}$$

$$+R_y = 0 + (V_5 \sin\theta -)\dot{m}_5 - 0$$

$$= 0 - \rho_{air} A_5 V_5^2 \sin\theta$$

$$= -\rho_{air} A_5 V_5^2 \sin\theta$$

$$+R_y = \rho_{air} A_5 V_5^2 \sin\theta \rightarrow DOWN$$

$$\left|\vec{R}\right| = \left[\rho_{air} A_5 V_5^2 (\cos\theta + 1)\right] i + \left[\rho_{air} A_5 V_5^2 \sin\theta\right] j$$

MATRIX EQUATIONS

EQUATION 1

$$V_1\left[\frac{ft}{s}\right]=\sqrt{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\frac{x}{12}[ft]}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]}\bullet 2g\left[\frac{ft}{s^2}\right]}$$

$$\overbrace{\{1\}\left\{V_1^2\right\}}^{UNKNOWN1}\left[\frac{ft}{s}\right]^2+\overbrace{\{0\}\left\{R_x\right\}}^{UNKNOWN2}+\overbrace{\{0\}\left\{R_y\right\}}^{UNKNOWN3}=\overbrace{\left\{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\bullet\frac{2g}{12}}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]}\right\}\{\overbrace{x}^{\text{CONTROLLING INPUT}}\}}^{FORCING\ FUNCTION1}$$

$$a_{11}=1$$

$$a_{12}=0$$

$$a_{13}=0$$

$$F_1=\left\{\frac{\gamma_{H20}\left[\frac{lbf}{ft^3}\right]\bullet\frac{2g}{12}}{\gamma_{air}\left[\frac{lbf}{ft^3}\right]}\right\}\{x\}$$

MATRIX EQUATIONS

$$EQUATION\ 2$$

$$Rx = -\dot{m}V(\cos\theta + 1)$$

$$= -\rho_{air} A V^2 (\cos\theta + 1)$$

$$= -\rho_{air} A(\cos\theta + 1)\left\{V^2\right\}$$

$$\rho_{air} A(\cos\theta + 1) \overbrace{\left\{V^2\right\}}^{UNKNOWN1} + \overbrace{\{1\}\left\{Rx\right\}}^{UNKNOWN2} + \overbrace{\{0\}\left\{Ry\right\}}^{UNKNOWN3} = 0$$

$$a_{21} = \rho_{air} A(\cos\theta + 1)$$

$$a_{22} = 1$$

$$a_{23} = 0$$

$$F_2 = 0$$

MATRIX EQUATIONS

$$EQUATION\ 3$$

$$+R_y = \rho_{air} A_5 V^2 \sin\theta$$

$$R_y - \rho_{air} A_5 V^2 \sin\theta = 0$$

$$-\rho_{air} A_5 \sin\theta \overbrace{\left\{V^2\right\}}^{UNKNOWN1} + \overbrace{\{0\}\left\{R_x\right\}}^{UNKNOWN2} + \overbrace{\{1\}\left\{R_y\right\}}^{UNKNOWN3} = 0$$

$$a_{31} = -\rho_{air} A_5 \sin\theta$$

$$a_{32} = 0$$

$$a_{33} = 1$$

$$F_3 = 0$$

Matrix Equation

$$[A]\{x\} = \{B\}$$

$$\{x\} = [A]^{-1}\{B\}$$

MATLAB/MATHCAD

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

"FINAL MATRIX SYSTEM EQUATIONS"

$$\begin{bmatrix} 1 & 0 & 0 \\ \rho_{air} A(\cos\theta + 1) & 1 & 0 \\ -\rho_{air} A \sin\theta & 0 & 1 \end{bmatrix} \begin{Bmatrix} \phi_1 = V^2 \\ \phi_2 = R_x \\ \phi_3 = R_y \end{Bmatrix} = \begin{Bmatrix} \left\{ \dfrac{\gamma_{H20} \left[\dfrac{lbf}{ft^3} \right] \bullet \dfrac{2g}{12}}{\gamma_{air} \left[\dfrac{lbf}{ft^3} \right]} \right\} \{x\} \\ 0 \\ 0 \end{Bmatrix}$$