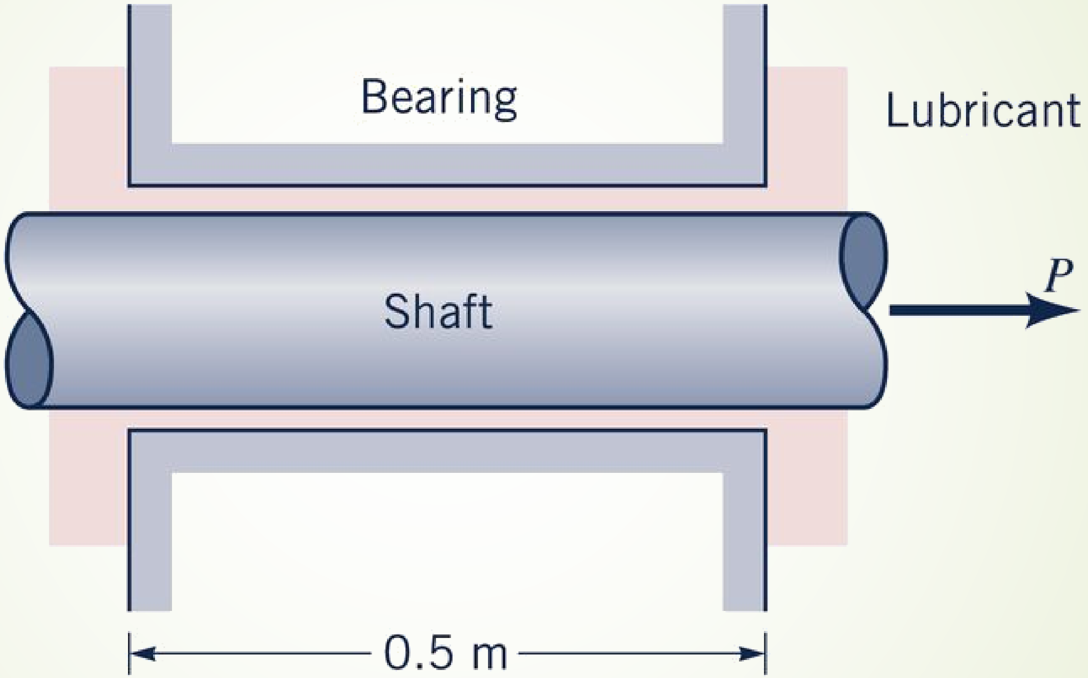
PARAMETRIC MODELS STUDY AIDS

MECH-322 Fluid Mechanics

A 50mm Diameter shaft of length 0.2m, is pulled through a bearing of Diameter 52mm. The oil in the gap has a kinematic viscosity of and specific gravity of 0.85. If the POWER to "pull" the shaft through the bearing is 450 Watts, assume a linear velocity distribution in the gap and find the equilibrium pull velocity of the shaft?



FUNDAMENTALS: Force to pull shaft MUST balance opposing SHEARING FORCE at the exterior walls of cylinder

PARAMETRIC ROAD MAP

LINEAR GAP VELOCITY PROFILE

$$u(y) = \frac{\Delta V}{\Delta GAP} \bullet y$$

$$\frac{\partial u}{\partial y} \equiv \text{velocity gradient} = \frac{\Delta V}{\Delta GAP}$$

$$F_s \equiv \text{shear FORCE=shear STRESS} \bullet \text{AREA}_{contact}$$

$$F_s = (\tau_s \bullet A_c) = \mu \frac{\partial u}{\partial y} \bullet A_c = \mu \frac{\Delta V}{\Delta GAP} \bullet A_c$$

POWER=FORCE • VELOCITY

$$\text{VELOCITY} = \frac{POWER}{FORCE} = \frac{POWER}{\mu \frac{\partial u}{\partial y} A_c} = \frac{POWER}{\mu \frac{\Delta V}{GAP} A_c}$$

$$\Delta V^2 = \frac{POWER\left[\frac{J = \cancel{N} - m}{s}\right] \bullet GAP[m]}{\mu\left[\frac{\cancel{N} - s}{\cancel{m^2}}\right] A\left[\cancel{m^2}\right]} = \frac{m^2}{s^2}$$

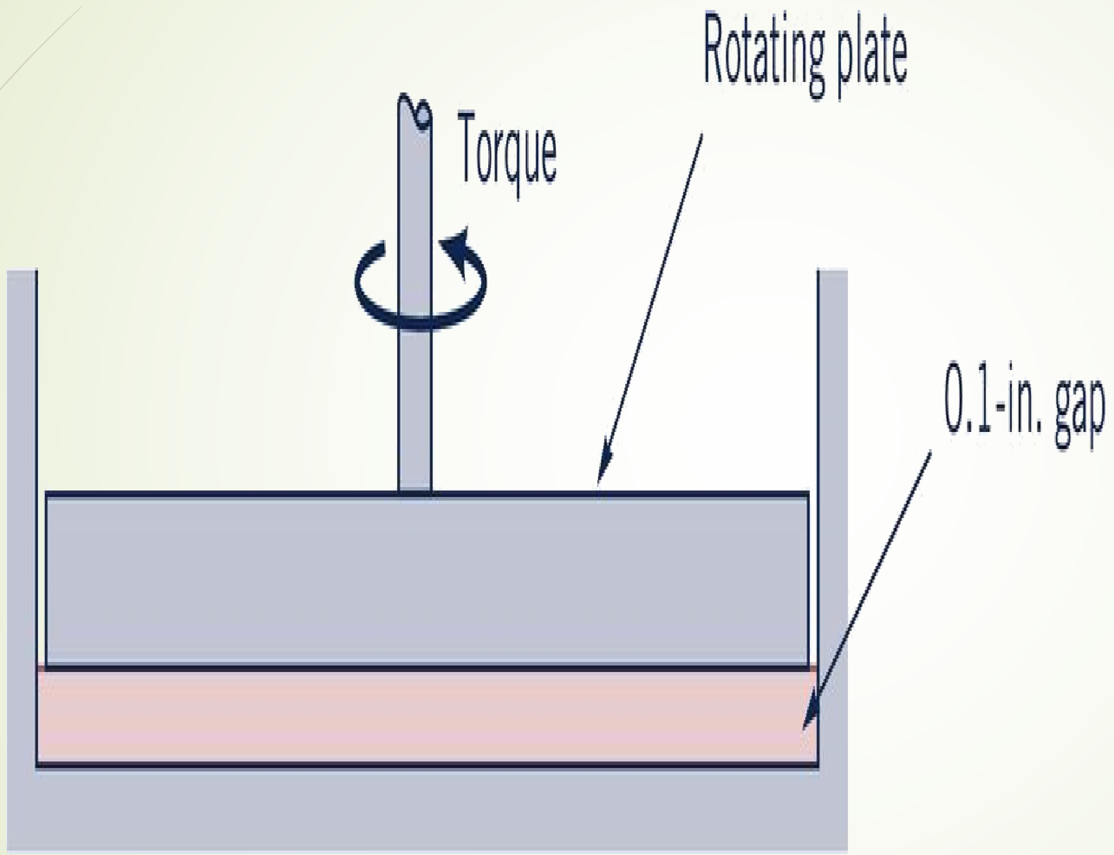
$$\Delta V = \sqrt{\frac{POWER \bullet GAP}{\mu A}}$$

$$\text{GAP} = \frac{52}{2}\text{mm} - \frac{50}{2}\text{mm} = 26\text{mm} - 25\text{mm} = 1\text{mm}$$

$A_c = \pi DL \rightarrow$ Contact Area Touching Fluid

Parametric Model: Rotating Disk

Find Parametric Expression for Torque and Power



Assume small gap and linear velocity distribution in gap.

Neglect edge effects.

FUNDAMENTALS: TORQUE must balance counter acting SHEAR FORCE on bottom of disk.

Parametric Model

$$T_q[N-m] = F_{shear} \bullet r = \left(\mu \frac{\partial V}{\partial y} \bullet A_c \right) \bullet r_0 \rightarrow Torque$$

$V \equiv$ LINEAR EDGE VELOCITY

$$= r_0[\text{m}] \bullet \omega_0 \left[\frac{radians}{\text{sec}} \right] = \left[\frac{m}{s} \right]$$

A_c = AREA IN CONTACT WITH FLUID$=\pi r_0^2$

$$T_q = \mu \left(\frac{r_0 \omega_0}{\Delta_{gap}} \right) \bullet \pi r_0^2 \bullet r_0$$

$$T_q(r_0, \mu, \omega_0, \Delta_{gap}) = \mu \left[\frac{N-s}{m^2} \right] \left(\frac{r_0^4 [m^4] \pi \omega_0 \left[\frac{rad}{s} \right]}{\Delta_{gap}[m]} \right) \rightarrow N-m = Joule$$

$$P = F_s \bullet V = F_s \bullet r\omega_0 = T_q \omega_0 \rightarrow Power$$

$$= \frac{N-m}{s} = \frac{J}{s} = Watt$$

$$\omega_0 = \omega \frac{rev}{\min} \bullet \frac{2\pi \; radians}{revolution} \bullet \frac{1 \min}{60 \sec}$$

$$= \frac{radians}{\sec}$$

VISC	GAP	Rotation	
lbf-s/ft2	ft	RPM	rad/s
0.0313	0.008333333	10	1.046667
Dia (ft)	Torq (ft-lbs)	Power (HP)	
0.1	7.71514E-05	1.46821E-07	
0.2	0.001234422	2.34914E-06	
0.3	0.006249261	1.18925E-05	
0.4	0.019750751	3.75863E-05	
0.5	0.048219606	9.17634E-05	
0.6	0.099988176	0.000190281	
0.7	0.185240439	0.000352518	
0.8	0.316012012	0.00060138	
0.9	0.506190139	0.000963295	
1	0.7715137	0.001468214	
1.1	1.129573208	0.002149612	
1.2	1.599810808	0.003044488	
1.3	2.203520279	0.004193366	
1.4	2.96384703	0.005640291	
1.5	3.905788106	0.007432833	
1.6	5.056192184	0.009622087	
1.7	6.443759574	0.01226267	
1.8	8.099042217	0.015412723	
1.9	10.05444369	0.019133911	
2	12.3442192	0.023491423