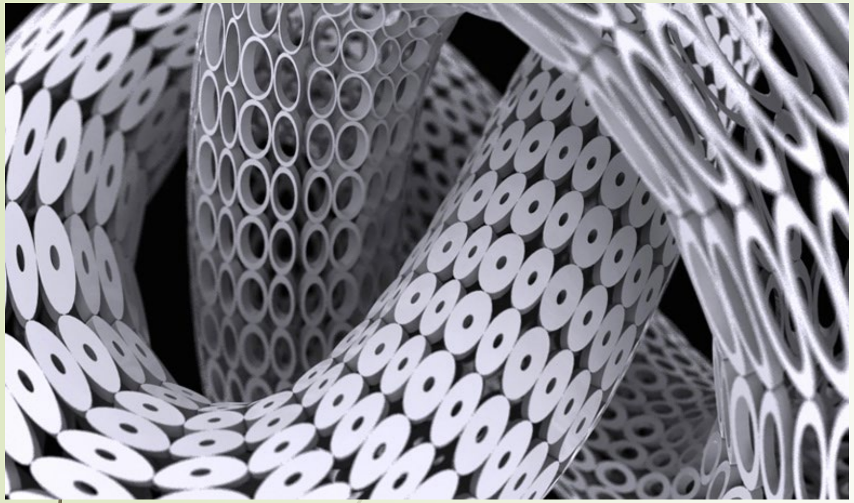
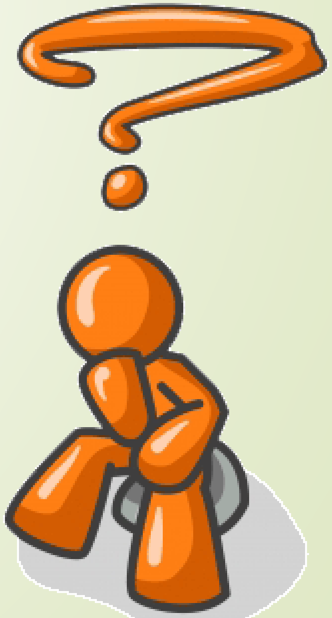


PARAMETRIC THINKING?

The Language of Engineers…

Dr. K. J. Berry
ASME FELLOW

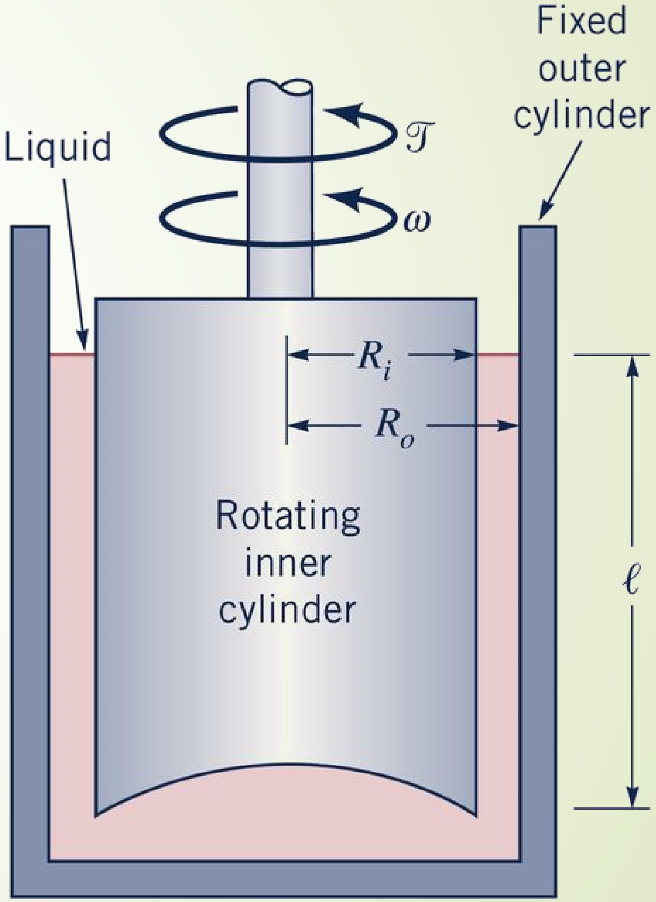


I do think that MECH 322 has enhanced my skills and abilities as a student and engineer because it has taught me to step back and see the bigger picture through following and understanding universal and parametric concepts.

Fluids Student, Fall 2019

PROBLEM DEFINITION

- Determine the ***"PARAMETRIC EQUATION***" to model the ***Torque*** and the ***Power*** for the rotating cylinder in terms of the relevant problem variables.
- Cylinder rotates at "w" REVOLUTIONS PER MINUTES (RPM)
- **The TORQUE of course has to overcome the "SHEAR STRESS" or "DRAG" in the "small gap" due to fluid viscosity.**
- Assume linear velocity in the gap and neglect end effects.



TORQUE DEFINITION

$$\text{TORQUE}_{\mathit{SHAFT}} = \text{FORCE x DISTANCE} \rightarrow \text{ROAD MAP}$$

$$= F_{\text{shear force}} \text{ x DISTANCE}$$

$$= \tau[N/m^2] \bullet A_c[m^2] \bullet R_i[m]$$

$$= \mu \frac{dV_t}{dr} \bullet A_c[m^2] \bullet R_i[m]$$

$$A_c[m^2] \rightarrow \text{Fluid Contact Area}$$

$$= 2\pi R_i L$$

$$\text{TORQUE}_{\mathit{SHAFT}} = \mu \frac{dV_t}{dr} \bullet 2\pi R_i^2 L$$

FIND TANGENTIAL VELOCITY

$$\mathrm{V}_t \rightarrow \text{Tangential Velocity=radius} \bullet \omega_0$$

$$\mathrm{V}_t\text{=r(ft or m)} \bullet \omega_0 \left(\frac{radians}{\text{sec}}\right) = ft/s, m/s$$

$$\omega_0\left(\frac{radians}{\text{sec}}\right) = w\frac{\cancel{REVOLUTIONS}}{\cancel{\text{min}}} \bullet 2\pi \frac{Radians}{\cancel{REVOLUTIONS}} \bullet \frac{1\,\cancel{\text{min}}}{60\,\text{sec}}$$

$$= w\left[\frac{REVOLUTIONS}{\text{min}}\right]\frac{2\pi}{60}\text{(Radians Have NO UNITS)}$$

NOTE: REVOLUTION is NOT a RADIUS is NOT a RADIAN.

LINEAR GAP VELOCITY

Velocity Distribution in the Gap (Δ)

$$\Delta = R_o - R_i, R_i \le r \le R_o$$

$$\text{V}_t\text{(r)}=\frac{V_t}{\Delta}(R_o - r) \rightarrow \text{Velocity Profile Distribution in the Gap}$$

$$@\text{r} = \text{R}_0 \rightarrow \text{V}_\text{t}(\text{r=R}_0)\text{=0}$$

$$@r = R_i \rightarrow \text{V}_t(\text{r=R}_i) = \frac{V_t(R_o - R_i)}{\Delta} = \frac{V_t(R_o - R_i)}{R_o - R_i} = V_t$$

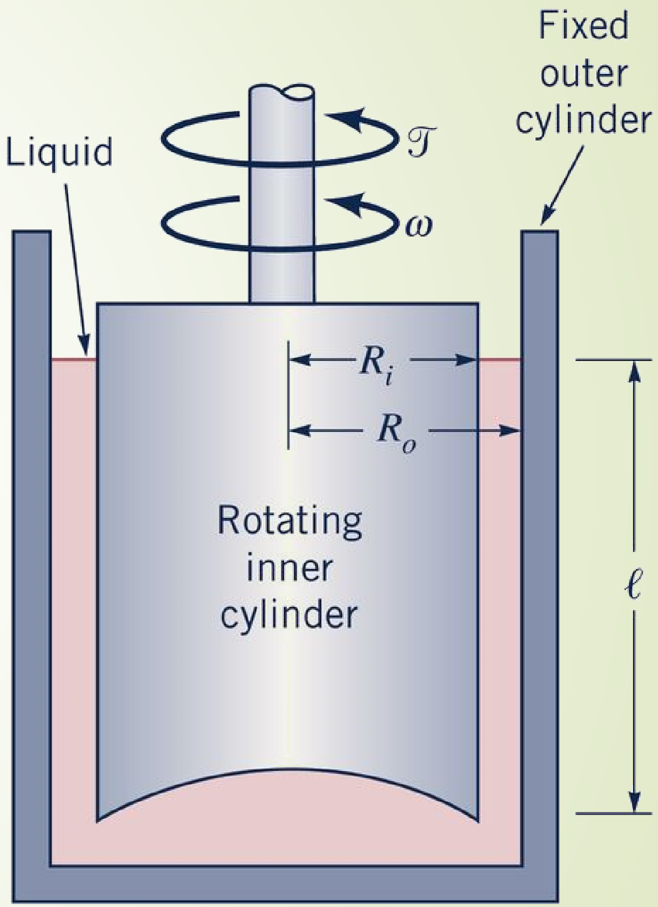
$$V_t = \text{Tangential Velocity=radius } \bullet \text{angular velocity}$$

$$=\text{R}_i(m)\bullet\omega_o\frac{radians}{\text{sec}}$$

$$\text{V}_t\text{(r)=}\frac{\text{R}_i(m)\bullet\omega_o\left(\frac{radians}{\text{sec}}\right)}{(R_o - R_i)(m)}\bullet(R_0 - r)(m)$$

$$\frac{dV_t}{dr} = -\frac{\text{R}_i\bullet\omega_o}{(R_o - R_i)}$$

$- \rightarrow$ implies Rotation oposses DRAG (or Shear Stress)



TORQUE FINAL PARAMETRIC EQUATION

- Note Torque varies as cube of Radius. 2X of radius increase results in 8X Torque increase. In fact its even greater since as Ri increases with a fixed Ro, (R0-Ri) "deceases" resulting in a smaller "gap", and pushing Torque higher. **Smaller gap means higher shear stress and higher drag forces, and thus higher Torque.**
- Parametric Equation allows for engineer to study impact on Torque of all system variables.
- More useful than obtaining a single value at single operating point and not understanding "how" each variable will impact system operations.
- Engineering today is about "*systems*" engineering and "*systems*" thinking.
- Engineers can now plot one variable against another, and can consider trade-offs and system optimizations.

$$\text{TORQUE}_{SHAFT} = \mu \frac{dV_t}{dr} \bullet 2\pi R_i^2 L$$

$$\frac{dV_t}{dr} = -\frac{R_i \bullet \omega_o}{(R_o - R_i)}$$

$$\text{TORQUE}_{SHAFT} = \mu \frac{R_i \bullet \omega_o}{(R_o - R_i)} \bullet 2\pi R_i^2 L \rightarrow \text{ROAD MAP}$$

$$T_q(\mu, R_i, R_o, \omega_0, L) = \mu \frac{R_i^3 \bullet \omega_0}{(R_o - R_i)} 2\pi L$$

$$\mu \rightarrow \frac{N-s}{m^2}$$

$$\omega_o = \omega(RPM)\frac{2\pi}{60}$$

$$POWER[HP] = Force\ x\ Velocity$$

$$= Force\ xR_i\omega$$

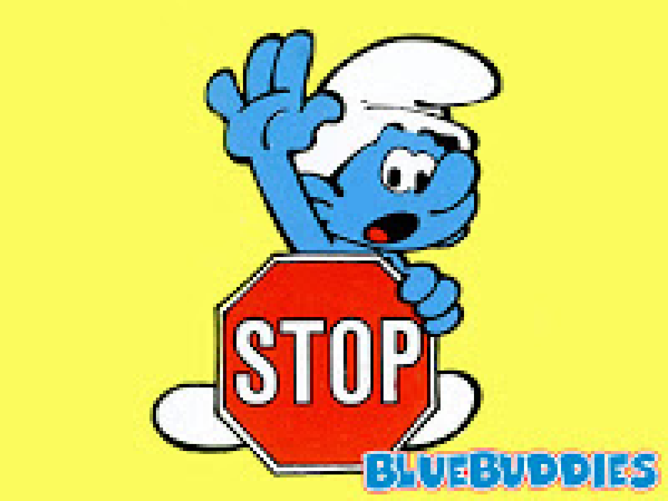
$$= T_{orque}(\cancel{ft-lbs})\omega\frac{rad}{\cancel{sec}}\, x\, \frac{1HP}{550\cancel{\frac{ft-lbs}{sec}}}$$

UNIT CHECK

$$T_q(\mu, R_i, R_o, \omega_0, L) = \mu \frac{\mathrm{R}_i^3 \bullet \omega_0}{(R_o - R_i)} 2\pi L \rightarrow \text{ROAD MAP}$$

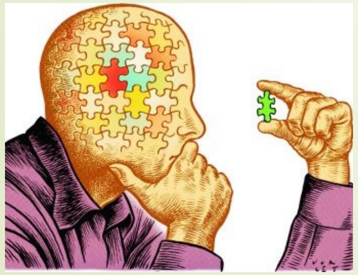
$$= \left(\frac{lb_f - \sec}{ft^2} \right) \frac{ft^3 \bullet \left(\frac{rad}{\sec} \right)}{ft} ft$$

$$= ft - lb_f$$



Power Final Parametric Equation

- Note the power of the required motor varies by cube of the radius, and by the square of the angular velocity.
- It is very important to know the power vs speed, which governs the system energy consumption and the overall system electric efficiency.
- ***Increased power consumption vs speed may also be important in considering thermal heating with speed, as the engineer may need to dissipate the excess heat due to motor inefficiencies.***
- ***Engineering today is about "systems" thinking, which can be enhanced by parametric thinking—considering how the "one" impacts the "many".***
- **NOTE: *REVOLUTION* is NOT a *RADIUS* is NOT a *RADIAN*.**



POWER=FORCE x VELOCITY

$$=\text{N}\bullet\text{m/s}=\text{J/s}=\text{WATT;or}$$

$$=\frac{\text{ft-lbf}}{s}\rightarrow\text{Convert to HP}\rightarrow\frac{\text{ft-lbf}}{s}\bullet\frac{1HP}{550\,\frac{\text{ft-lbf}}{s}}$$

POWER=$\text{F}_{\text{shear force}}$ x VELOCITY

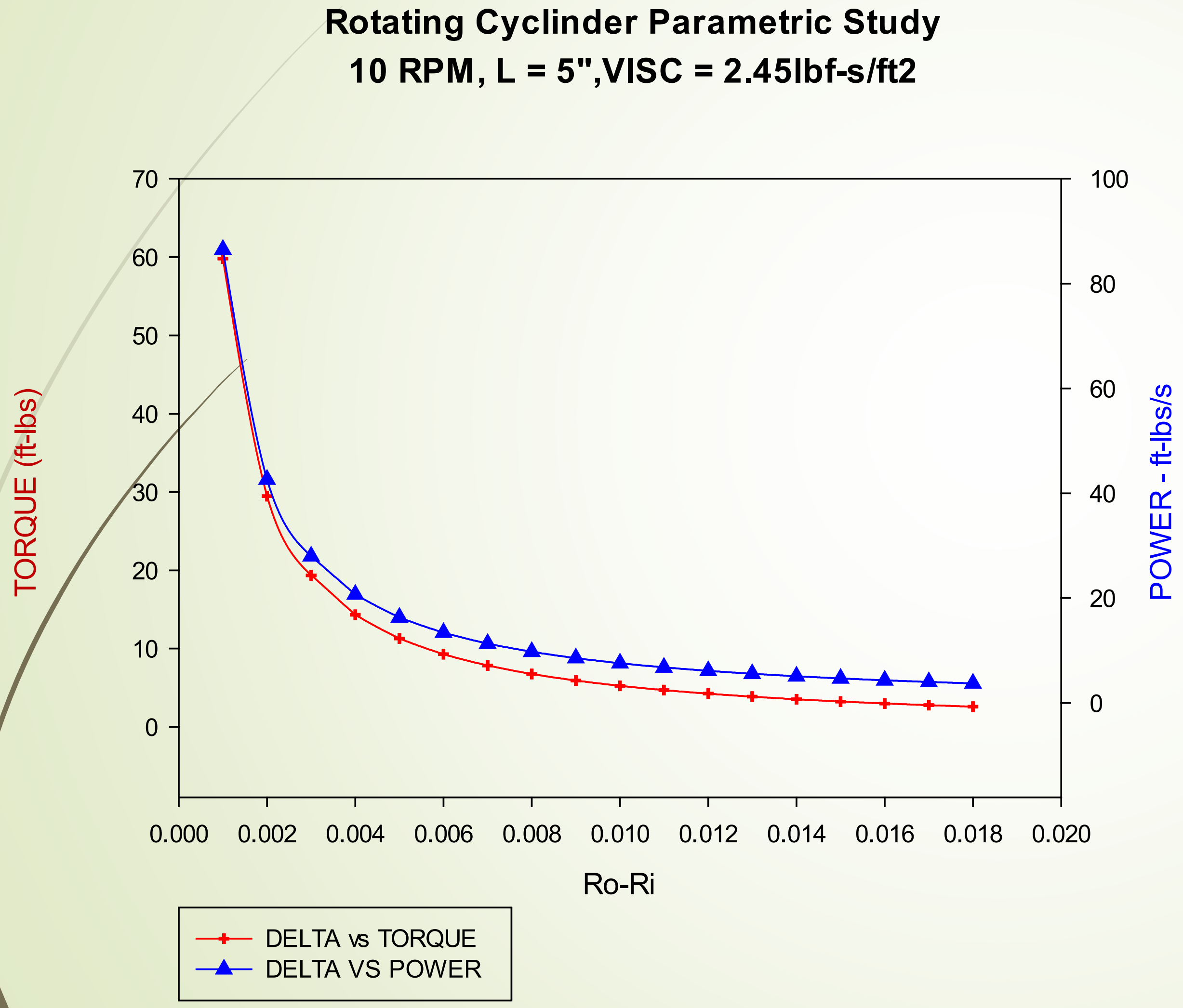
$$=\tau[N/m^2]\bullet A_c[m^2]\bullet R_i\omega_0$$

$$=\mu\frac{dV_t}{dr}\bullet A_c[m^2]\bullet R_i\omega_0$$

$$=\mu\frac{\text{R}_i\bullet\omega_0}{(R_o-R_i)}\bullet 2\pi R_i L\bullet R_i\omega_0$$

$$\dot{P}(\mu,R_i,R_o,\omega_0,L)=\mu\frac{\text{R}_i^3\bullet\omega_0^2}{(R_o-R_i)}\bullet 2\pi L=\text{TORQUE(ft-lbf)}\bullet\omega_0(rad/s)$$

Parametric Study



	Rotating Cyclinder Parametric Study				
	Viscosity	R0	w	wo	L
	lbf-s/ft2	ft	RPM	rad/sec	ft
	2.45	0.21	10	1.05	0.42
DELTA	RI	TORQUE	POWER	POWER	POWER
	ft	ft-lbs	ft-lbs/s	HP	W
0.001	0.2073	59.80	62.59	0.11381	86.49
0.002	0.2063	29.47	30.85	0.05609	42.62
0.003	0.2053	19.36	20.27	0.03685	28.01
0.004	0.2043	14.31	14.98	0.02723	20.70
0.005	0.2033	11.28	11.81	0.02147	16.32
0.006	0.2023	9.26	9.70	0.01763	13.40
0.007	0.2013	7.82	8.19	0.01489	11.31
0.008	0.2003	6.74	7.06	0.01283	9.75
0.009	0.1993	5.91	6.18	0.01124	8.54
0.01	0.1983	5.23	5.48	0.00996	7.57
0.011	0.1973	4.69	4.91	0.00892	6.78
0.012	0.1963	4.23	4.43	0.00805	6.12
0.013	0.1953	3.85	4.03	0.00732	5.56
0.014	0.1943	3.52	3.68	0.00669	5.09
0.015	0.1933	3.23	3.38	0.00615	4.68
0.016	0.1923	2.98	3.12	0.00568	4.32
0.017	0.1913	2.76	2.89	0.00526	4.00
0.018	0.1903	2.57	2.69	0.00489	3.72