MECH-322 FLUID MECHANICS--SOLUTION

ASSESMENT 1

Name__________________________ HOUR______________________

BE ABLE TO PROVIDE DEFINITIONS OF CORE FLUID MECHANICS PARAMETERS.

1. Definitions 10 Points:

a) What is Fluid Mechanics?

- Fluid Mechanics is the study of fluid behavior (liquids, gases, blood, and plasmas) at **REST** and in **MOTION**, and the forces they produce.

b) What is the difference between an incompressible and a compressible fluid?

- Incompressible – Density is constant.
- Compressible – Density varies with pressure.

c) What is fluid viscosity?

- It is a measure of the resistance of a fluid to deform under shear stress. It is denoted by m. Also called as dynamic viscosity or absolute viscosity.

d) What is the time rate of strain **and** mathematical expression for shear stress with units?

- $\dot{\gamma} = \frac{dU}{dy} \rightarrow \text{Velocity Gradient}$

e) What is Pascal's Law?

- Pressure at any depth is not a function of direction.

f) What is the Law of Hydrostatics **and** when can it be applied?

- $\frac{dP}{dz} = -\gamma \rightarrow \text{NO SHEAR STRESS, INCOMPRESSIBLE}$

g) What is a manometer?

- A device to pressure differential within a fluid network.

h) What is the definition **and** units for Torque?

- $T_q = Force[N \; or \; lbf] \bullet \text{Momemnt Arm}[\text{m } or \text{ ft}]$

i) What is the definition of **and** units for Specific Gravity?

- $S_g = \dfrac{\rho_f}{\rho_{H20}} = \dfrac{\gamma_f}{\gamma_{H20}} \rightarrow NO\ UNITS$

j) What is the definition of **and** units for Specific Weight?

- $$\gamma = \rho\left[\frac{kg}{m^3}\right] \bullet g\left[\frac{m}{s^2}\right] = \frac{N}{m^3}$$
  $$= \rho\left[\frac{slugs}{ft^3}\right] \bullet g\left[\frac{ft}{s^2}\right] = \frac{lbs}{ft^3}$$

BE ABLE TO COMPUTE UNKNOWN PROPERTY AT A KNOWN TEMPERATURE OF A FLUID IF PROVIDED TWO DATA POINTS OF TEMPERATURES AND PROPERTY VALUES:

2. The **ANDRAD'S** equation for any temperature dependent property (p) for liquids is provided as follows (10 Points):

$$p(T) = De^{\frac{B}{T}}$$

$$\rho_1 = 2.21x10^{-2}[\frac{moles}{kg}], T_1 = 500K,$$
$$\rho_2 = 1.0x10^{-4}[\frac{moles}{kg}], T_2 = 1000K$$

$$\text{B}=\frac{\ln(p_1)-\ln(p_2)}{\left(\dfrac{1}{T_1[K]}-\dfrac{1}{T_2[K]}\right)} = \frac{5.398}{0.001} = 5398.16[K]$$

$$\ln(D) = \left\{\ln(p_1) - \ln\left(e^{\frac{B}{T_1}}\right)\right\} = -10.7966$$

$$D = e^{-10.7966} = 2.04699x10^{-5}\left[\frac{moles}{kg}\right]$$

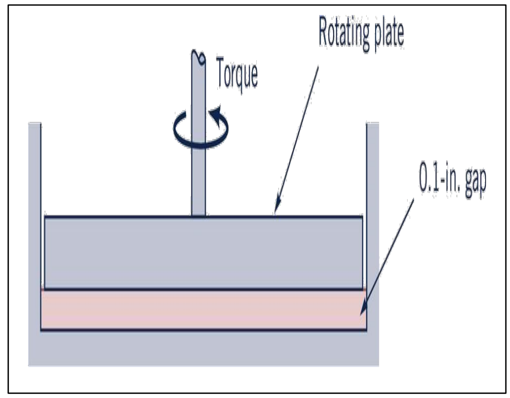
$$\rho(T) = 2.04699x10^{-5}\left[\frac{moles}{kg}\right]e^{\frac{5398.16[K]}{T}}$$

FIND

$$\rho(T = 800K) = 1.744x10^{-2}\left[\frac{moles}{kg}\right]$$

UNDERSTAND THE RELATION BETWEEN FORCE AND FLUID SHEAR STRESS AND BE ABLE TO APPLY CONCEPTS TO SOLVE PROBLEMS INVOLVING FLUIDS FLOWING OVER STATIONARY SURFACES.

3. A D = 12” diameter circular plate is placed over a fixed bottom plate with a $\Delta = 0.1"$ gap between two plates filled with an unknown fluid, μ. If the TORQUE required to rotate the plate slowly at $\omega = 10\ \mathrm{RPM}$ (revolutions per minute) is 3.5 ft-lbf and if the velocity in the gap is LINEAR (ignore edge effects) (10 points) (Homework 1.89):



   a. In “words” what principal is driving the fluid fundamentals?
      i. Torque must overcome shear stress or (frictional drag) on plate bottom.
   b. What is the definition for Torque?
      i. Torque is a Force acting at a point causing a rotation (or moment) about an axis.
   c. What is the parametric expression for the linear velocity profile in the gap?
      i. $V(y) = \dfrac{\frac{D}{2}[ft]\omega_0\left[\frac{rad}{s}\right]}{\Delta_{gap}[ft]} \bullet y[ft]$

ii. What is the “parametric expression” for the fluid viscosity in terms of problems variables, $\mu(T_q, D, \Delta, \omega_0)[\frac{lbf-s}{ft^2}] = ?$

$$V(y) = \frac{\frac{D}{2}[ft]\omega_0\left[\frac{rad}{s}\right]}{\Delta_{gap}[ft]} \bullet y[ft]$$

$$\frac{\partial V}{\partial y} = \frac{\frac{D}{2}[ft]\omega_0\left[\frac{rad}{s}\right]}{\Delta_{gap}[ft]}$$

$$T_q = F_{DRAG} \bullet \frac{D}{2} = F_{SHEAR} \bullet \frac{D}{2}$$

$$= \left(F_{SHEAR} = \mu\frac{\partial V}{\partial y} \bullet A_{contact}\right) \bullet \frac{D}{2}$$

$$T_q = \mu \bullet \frac{\frac{D}{2}[ft]\omega_0\left[\frac{rad}{s}\right]}{\Delta_{gap}[ft]} \bullet \pi\left(\frac{D}{2}\right)^2 \bullet \frac{D}{2}$$

$$\mu = \frac{T_q[ft-lbf]}{\frac{\frac{D}{2}[ft]\omega_0\left[\frac{rad}{s}\right]}{\Delta_{gap}[ft]} \bullet \pi\left(\frac{D}{2}\right)^2 \bullet \frac{D}{2}}$$

$$\mu[\frac{lbf-s}{ft^2}](T_q, D, \Delta_{gap}, \omega_0) = \frac{T_q[ft-lbf]}{\frac{\frac{D^4}{16}[ft^4]\omega_0\left[\frac{rad}{s}\right]}{\Delta_{gap}[ft]} \bullet \pi} = 0.14854[\frac{lbf}{ft^2}] / \omega_0\left[\frac{rad}{s}\right]$$

$$\omega_0\left[\frac{rad}{s}\right] = 10\frac{rev}{\min} \bullet \frac{2\pi rad}{rev} \bullet \frac{1\min}{60\sec} = 1.047 rad/s$$

d. Validate that your units are correct.

e. What is the POWER in [ft-lbs/s)], and in [hp], to overcome the rotational drag shear?

$$\text{POWER} = \text{FORCE} \bullet \text{VELOCITY}$$

$$\text{POWER}[\frac{ft-lbf}{s}] = \text{FORCE}_{SHEAR}[lbf] \bullet \frac{D}{2}[ft]\omega_0[\frac{rad}{s}]$$

$$= T_q \bullet \omega_0[\frac{rad}{s}]$$

i. $$= 3.5\, ft-lbf \bullet 1.047\, rad/s$$

$$= 3.665\left[\frac{ft-lbf}{s}\right]$$

$$= 3.665\left[\frac{ft-lbf}{s}\right] \bullet \frac{1hp}{550\,\frac{ft-lbf}{s}}$$

$$= 6.664x10^{-3}\, hp$$

- Be able to determine correct dimensional units for constants within SI and BKS measurement systems if given any arbitrary function. (10 Points) (Homework 1.28, 1.29, 1.31, 1.32, 1.44, 1.52)

4. Considering the velocity stream of $\vec{V}(x,y)[\frac{m}{s}] = \left\{10.2[]x^{\frac{1}{3}} - 20.4[]e^{-\frac{y}{3[]}}\right\}[\frac{m}{s}]$, the chemical molar dispersion [moles/sec] is measured as:

   $\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5[]e^{-5[]t} + 20.2[]x^2 - 0.3[]y^3$, where x and y are in meters.

   What are the units in brackets [] for both $\vec{V}$ and ψ, and, determine $\frac{\partial\psi}{\partial x}[\frac{\frac{moles}{\sec}}{m}]$ and show that units are correct?

$$\vec{V}(x,y)[\frac{m}{s}] = \left\{10.2[]x^{\frac{1}{3}} - 20.4[]e^{-\frac{y}{3[]}}\right\}[\frac{m}{s}]$$

$10.2[]x^{\frac{1}{3}}$

$$\frac{m}{s} = []m^{\frac{1}{3}} \rightarrow [] = \frac{\frac{m}{s}}{m^{\frac{1}{3}}} = \frac{m^{2/3}}{s}$$

$e^{-\frac{y[m]}{3[]}}$

$$m = 3[] \rightarrow [] = m$$

$20.4[]$

$$\frac{m}{s} = []$$

$$\vec{V}(x,y)[\frac{m}{s}] = \left\{10.2[\frac{m^{2/3}}{s}]x^{\frac{1}{3}} - 20.4[\frac{m}{s}]e^{-\frac{y}{3[m]}}\right\}[\frac{m}{s}]$$

$$\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5[]e^{-5[]t} + 20.2[]x^2 - 0.3[]y^3$$

$e^{-5[]t} \rightarrow^{-5[]t}$ NO UNITS

$$[] = \frac{1}{s}$$

$3.5[]$

$$\frac{moles}{s} = 3.5[] \rightarrow [] = \left[\frac{moles}{s}\right]$$

$20.2[]x^2$

$$\frac{moles}{s} = []m^2 \rightarrow [] = \frac{\frac{moles}{s}}{m^2}$$

$0.3[]y^3$

$$\frac{moles}{s} = []m^3 \rightarrow [] = \frac{\frac{moles}{s}}{m^3}$$

$$\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5[\frac{moles}{s}]e^{-5[]t} + 20.2[\frac{\frac{moles}{s}}{m^2}]x^2 - 0.3[\frac{\frac{moles}{s}}{m^3}]y^3$$

$$\psi(t,x,y)\left[\frac{moles}{s}\right] = 3.5[\frac{moles}{s}]e^{-5[]t} + 20.2[\frac{\frac{moles}{s}}{m^2}]x^2 - 0.3[\frac{\frac{moles}{s}}{m^3}]y^3$$

$$\frac{\partial\psi}{\partial x}\left[\frac{\frac{moles}{s}}{m}\right] = 40.4[\frac{\frac{moles}{s}}{m^2}]x[m] = 40.4 \bullet x\left[\frac{\frac{moles}{s}}{m}\right]$$

- Be able to apply the ***"BERRY" POINT-TO-POINT METHOD*** for Manometry to determine the pressure differential for any two points within a fluid network. This primary concept method is used ***THROUGHOUT*** the entire semester, and although seems "simple" in the concept it is these simple concepts and building blocks that allow us to solve more complicated problems. (Homework 2.34, 2-35, 2-37,2-47, 2-49,2-53, 2-54, 2-57, 2-73)

5. The inverted U-Tube manometer contains oil (SG = 0.9) and water as shown. If pressure difference $\Delta P_{AB} = P_A - P_B = 3223.2 PA$, determine manometer difference "h".

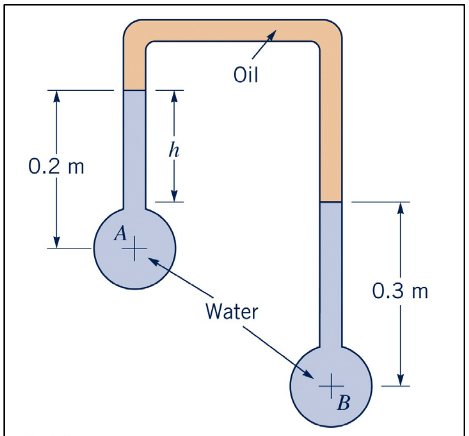


Figure P2.53

$$P_B - \gamma_{H20} \bullet 0.3 - h \bullet \gamma_{oil} + 0.2 \bullet \gamma_{oil} = P_A$$

$$h[m] = \frac{(P_B - P_A)\left[\frac{N}{m^2}\right] - (\gamma_{H20} \bullet 0.3)\left[\frac{N}{m^2}\right] + (0.2 \bullet \gamma_{oil})\left[\frac{N}{m^2}\right]}{\gamma_{oil}\left[\frac{N}{m^3}\right]}$$

$$h[m] = \frac{(P_A - P_B)\left[\frac{N}{m^2}\right] - (\gamma_{H20} \bullet 0.3)\left[\frac{N}{m^2}\right] + (0.2 \bullet 0.9 \bullet \gamma_{H20})\left[\frac{N}{m^2}\right]}{\gamma_{oil}\left[\frac{N}{m^3}\right]}$$