



THERMAL RADIATION

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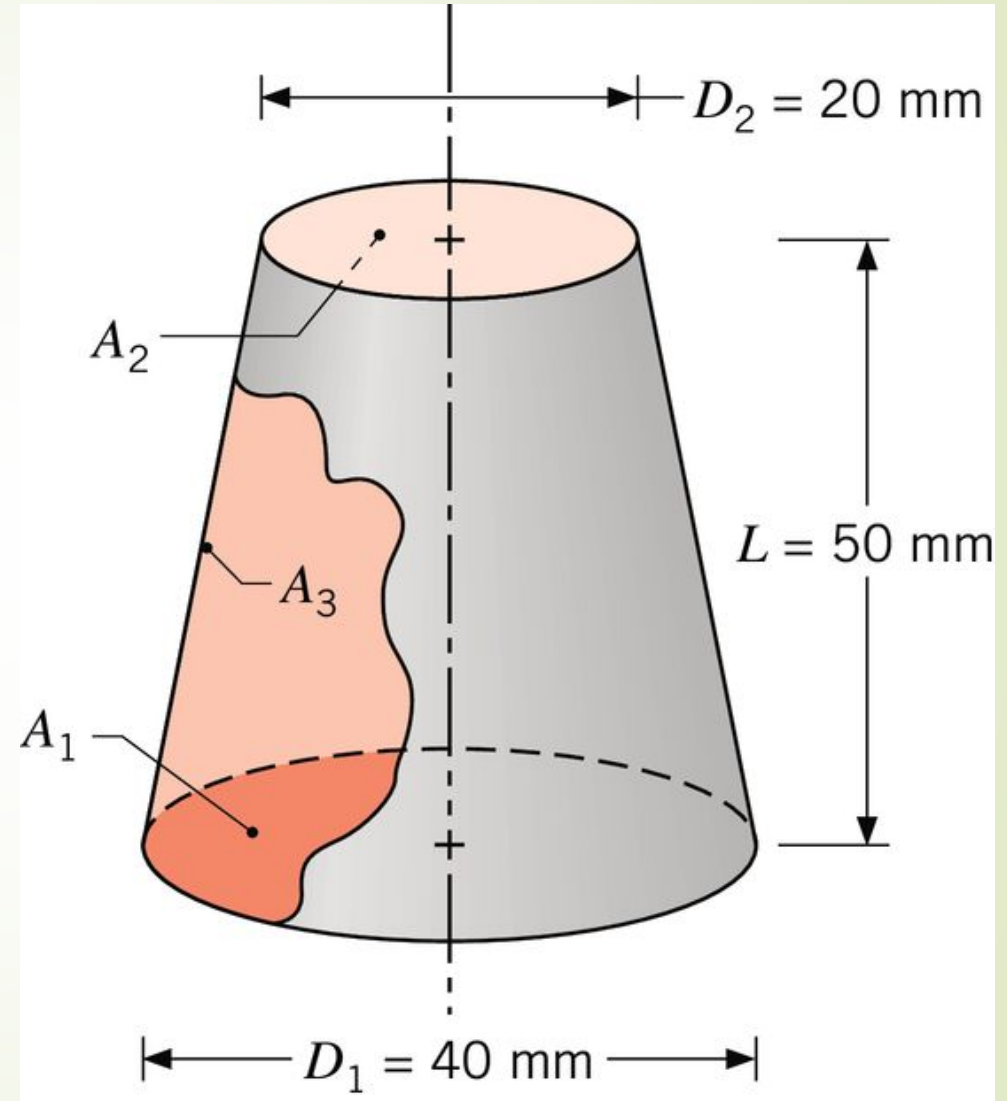
Study Aid #2

MECH-420 Heat Transfer

High Temperature
Furnace with 1000K floor
& emissivity 1.0.

INSULATING WALLS. Top
surface 500k at $e=?$

Plot wall temperature vs
emissivity of top surface
and vs q (input)
required at floor.



Shape Factors (Figure 13.5)

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$$R_1 = \frac{r_1}{L} = 2/5 = 0.4$$

$$R_2 = \frac{r_2}{L} = 1/5 = 0.2$$

$$S_{12} = 1 + \frac{1 + R_2^2}{R_1^2} = 7.5$$

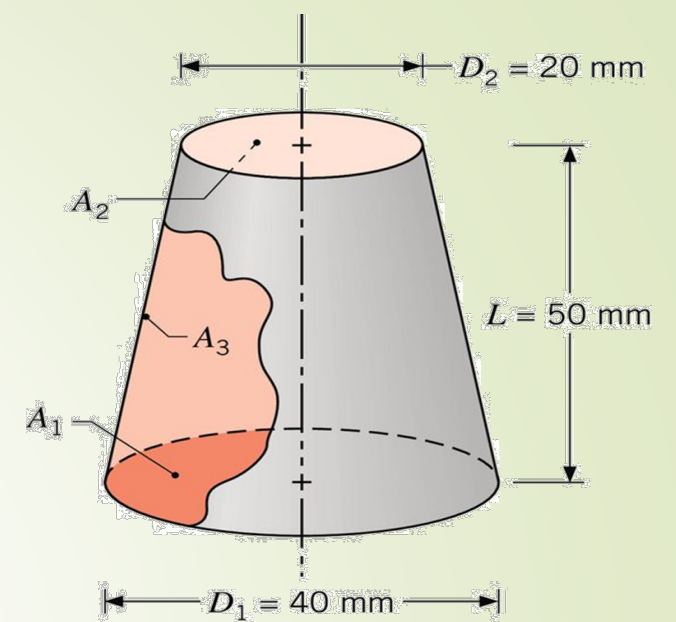
$$S_{21} = 1 + \frac{1 + R_1^2}{R_2^2} = 30$$

$$F_{ij} = 1/2(S - (S^2 - 4(r_j / r_i)^2)^{1/2})$$

$$F_{21} = 0.1339$$

$$F_{12} = 0.03348$$

MEEC-420 Heat Transfer Radiation Networks



CONSERVATION

$$\sum_i F_{ij} = 0$$

$$i = 2$$

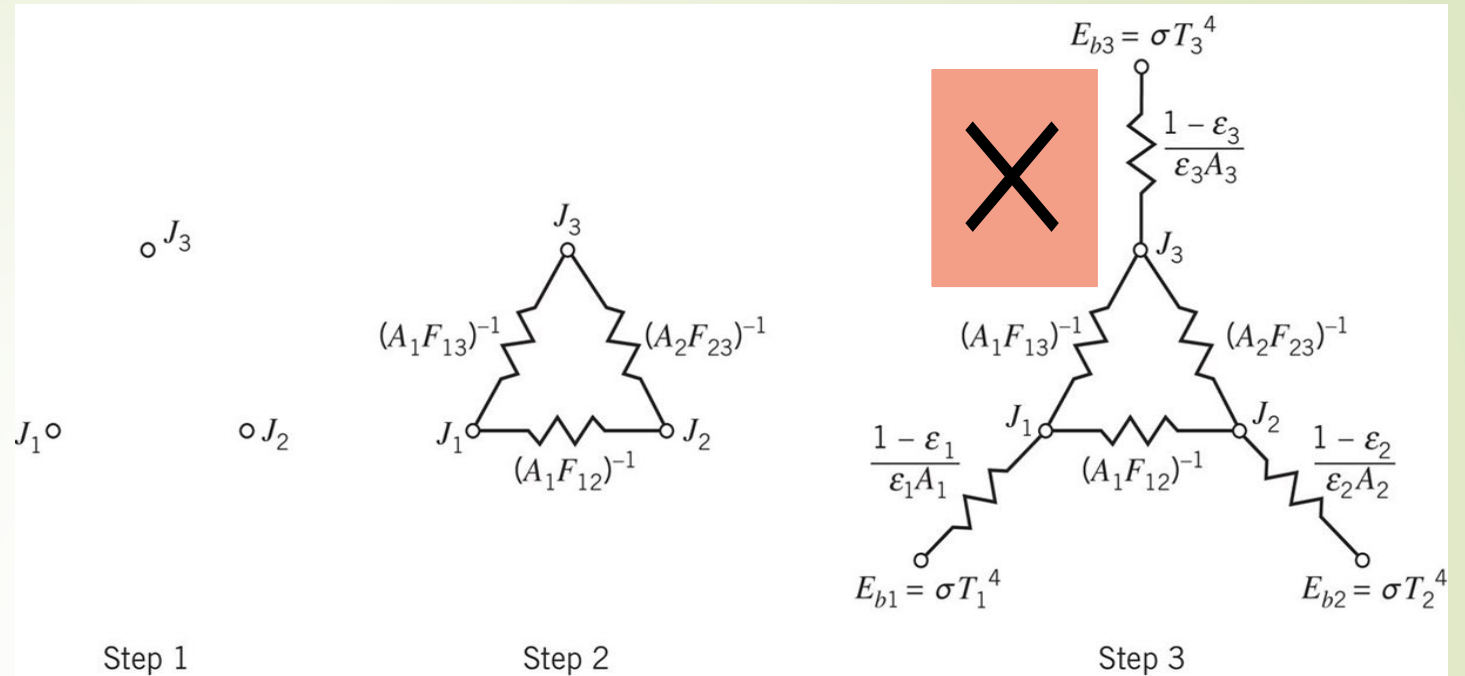
$$F_{21} + \cancel{F_{22}} + F_{23} = 1 \rightarrow F_{23} = 1 - F_{21} = 0.8661$$

$$i = 1$$

$$\cancel{F_{11}} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{12} = 0.9665$$

6/11/2021

3-Surface Radiation Circuit



$$E_{b_1} = \sigma T_1^4$$

$$\frac{E_{b_2} - J_2}{\frac{1 - \epsilon_2}{\epsilon_2 A_2}} = \frac{J_2 - J_1}{\frac{1}{A_2 F_{21}}} + \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}}$$

$$J_3 = \sigma T_3^4 \rightarrow q_{wall} = 0 \rightarrow RE - RADIATING$$

NODAL BALANCE

$$q_i[\text{Watts}] = \frac{E_{b_i} - J_i}{\frac{1 - \varepsilon_i}{\varepsilon_i A_i}} = \sum_j \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

Surface 1

$$q_1 = \frac{E_{b_1} - J_1}{\frac{1 - \varepsilon_1}{\varepsilon_1 A_1}} \rightarrow \varepsilon = 1$$

$$J_1 = E_{b_1} = \sigma T_1^4 = 56,7000 \text{ W} / \text{m}^2$$

$$A_1 = \pi D_1^2 / 4 = 0.001257 \text{ m}^2; \frac{1 - \varepsilon_1}{\varepsilon_1 A_1} = 341.05 \frac{1}{\text{m}^2}$$

$$q_i[\text{Watts}] = \frac{E_{b_i} - J_i}{\frac{1 - \varepsilon_i}{\varepsilon_i A_i}} = \sum_j \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

Surface 2

$$\frac{E_{b_2} - J_2}{\frac{1 - \varepsilon_2}{\varepsilon_2 A_2}} = \frac{J_2 - J_1}{\frac{1}{A_2 F_{21}}} + \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}}$$

Surface 3 ($E_{b_3} = J_3 = \sigma T_3^4$)

$$q_3 = 0 = \frac{J_3 - J_1}{\frac{1}{A_3 F_{31}}} + \frac{J_3 - J_2}{\frac{1}{A_3 F_{32}}}$$

RADIATION SYSTEM EQUATIONS

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Surface 2

$$0 = \frac{J_2 - J_1}{A_2 F_{21}} + \frac{J_2 - J_3}{A_2 F_{23}} - \left(\frac{E_{b_2} - J_2}{1 - \epsilon_2} \right) \frac{1}{\epsilon_2 A_2}$$

$$\frac{E_{b_2}}{1 - \epsilon_2} + \frac{J_1}{\epsilon_2 A_2} = J_2 \left[\frac{1}{A_2 F_{21}} + \frac{1}{A_2 F_{23}} + \frac{1}{\epsilon_2 A_2} \right] - J_3 \left[\frac{1}{A_2 F_{23}} \right]$$

$$F_1 = aJ_2 + bJ_3$$

Surface 3

$$q_3 = 0 = \frac{J_3 - J_1}{A_3 F_{31}} + \frac{J_3 - J_2}{A_3 F_{32}}$$

$$\frac{J_1}{A_3 F_{31}} = -J_2 \left[\frac{1}{A_3 F_{32}} \right] + J_3 \left[\frac{1}{A_3 F_{31}} + \frac{1}{A_3 F_{32}} \right]$$

$$F_2 = cJ_2 + dJ_3$$

Radiation Equation Matrix

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} J_2 \\ J_3 \end{Bmatrix} \rightarrow T_3 = \left(\frac{J_3}{\sigma} \right)^{\frac{1}{4}}$$

TWO EQUATIONS AND TWO UNKNOWNNS

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$$(1): F_1 = aJ_2 + bJ_3 \quad (2): F_2 = cJ_2 + dJ_3$$

$$(1): J_2 = \frac{F_1 - bJ_3}{a}$$

$$(2): J_3 = \frac{F_2 - cJ_2}{d} = \frac{F_2 - c\left(\frac{F_1 - bJ_3}{a}\right)}{d}$$

$$J_3\left(1 - \frac{cb}{ad}\right) = \frac{F_2}{d} - \frac{cF_1}{ad}$$

NOW, (3) \rightarrow (1)

$$(3): J_3 = \frac{\frac{F_2}{d} - \frac{cF_1}{ad}}{\left(1 - \frac{cb}{ad}\right)}$$

$$J_2 = \frac{F_1 - bJ_3}{a} = \frac{F_1}{a} - \frac{b}{a} \left\{ \frac{\frac{F_2}{d} - \frac{cF_1}{ad}}{\left(1 - \frac{cb}{ad}\right)} \right\}$$

SUMMARY

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Radiation Equation Matrix

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} J_2 \\ J_3 \end{Bmatrix} \rightarrow T_3 = \left(\frac{J_3}{\sigma} \right)^{\frac{1}{4}}$$

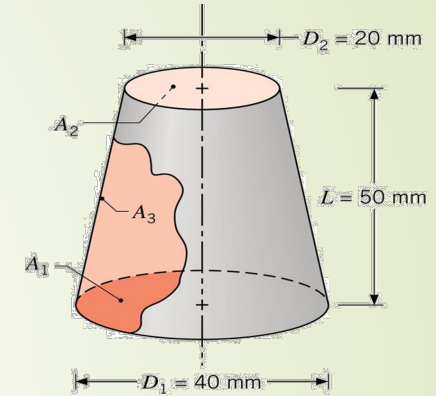
$$(F_1) = \frac{E_{b_2}}{1 - \epsilon_2} + \frac{J_1}{1} \frac{J_1}{A_2 F_{21}}, (F_2) = \frac{J_1}{1} \frac{J_1}{A_3 F_{31}}$$

$$a = \begin{bmatrix} \frac{1}{1} + \frac{1}{1} + \frac{1}{1 - \epsilon_2} \\ \frac{1}{A_2 F_{21}} & \frac{1}{A_2 F_{23}} & \frac{1}{\epsilon_2 A_2} \end{bmatrix}, b = \begin{bmatrix} -1 \\ 1 \\ \frac{1}{A_2 F_{23}} \end{bmatrix}$$

$$c = \begin{bmatrix} -1 \\ 1 \\ \frac{1}{A_3 F_{32}} \end{bmatrix}, d = \begin{bmatrix} \frac{1}{1} + \frac{1}{1} \\ \frac{1}{A_3 F_{31}} & \frac{1}{A_3 F_{32}} \end{bmatrix}$$

$$J_1 = E_{b_1} \rightarrow \epsilon = 1 \rightarrow \text{Blackbody}$$

$$J_3 = \frac{F_2 - cF_1}{\left(1 - \frac{cb}{ad}\right)}$$



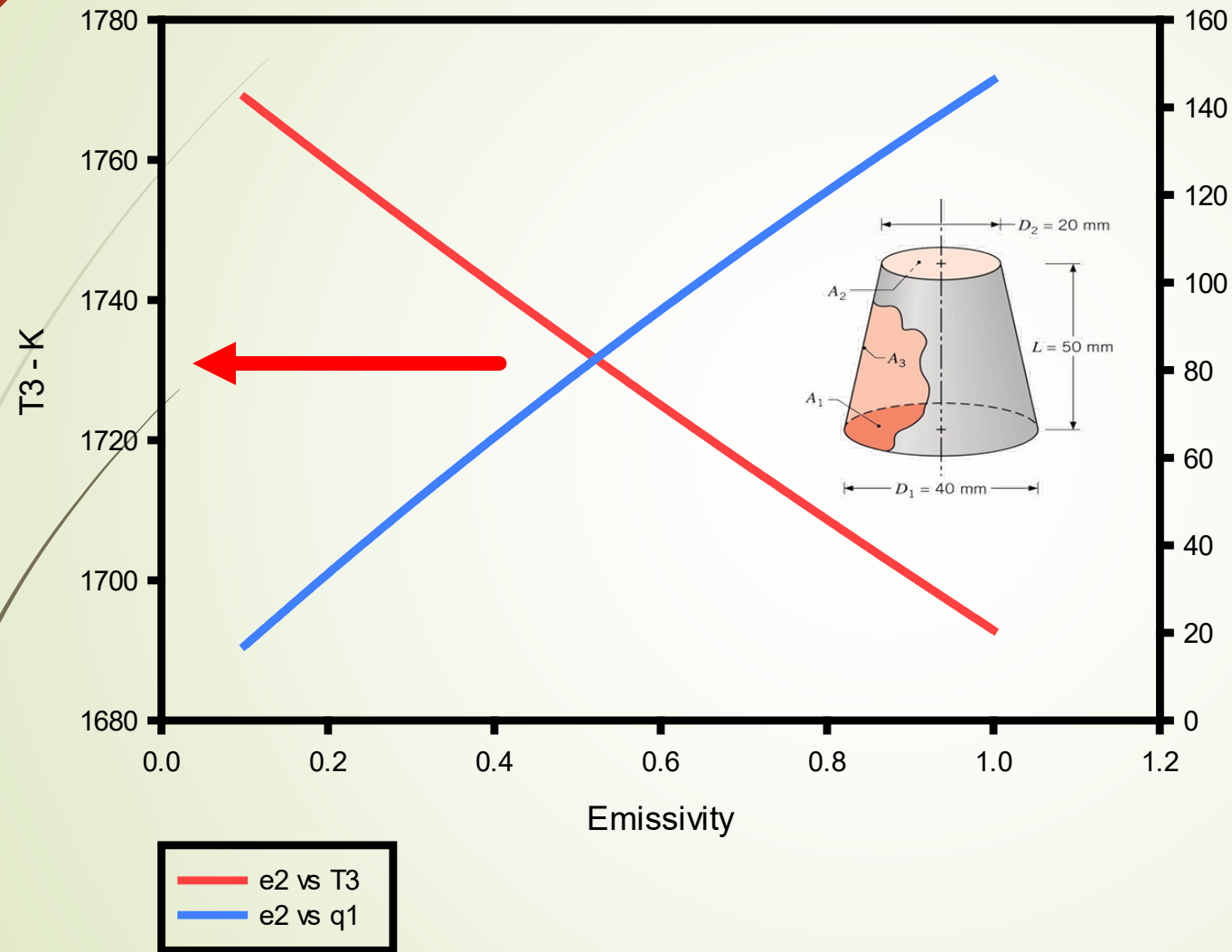
$$J_2 = \frac{F_1 - bJ_3}{a} = \frac{F_1}{a} - \frac{b}{a} \begin{Bmatrix} \frac{F_2 - cF_1}{d - ad} \\ \left(1 - \frac{cb}{ad}\right) \end{Bmatrix}$$

SPREADSHEET

			W	W/m2	K	W/m2	W	W	W	W	W	W
e2	$1-e2/e2A2$	a	F1	J3	T3	J2	q12	q13	q12+q13	q2	Surface 3	q1
0.1	28647.914	0.000349066	2.42E+01	5.55E+05	1.77E+03	5.02E+05	2.71E+00	1.44E+01	1.71E+01	-1.71E+01	0.00E+00	1.71E+01
0.2	12732.406	0.000392699	2.48E+01	5.44E+05	1.76E+03	4.40E+05	5.31E+00	2.82E+01	3.35E+01	-3.35E+01	0.00E+00	3.35E+01
0.3	7427.237	0.000448799	2.56E+01	5.33E+05	1.75E+03	3.80E+05	7.83E+00	4.15E+01	4.94E+01	-4.94E+01	0.00E+00	4.94E+01
0.4	4774.6523	0.000523598	2.66E+01	5.22E+05	1.74E+03	3.22E+05	1.03E+01	5.44E+01	6.47E+01	-6.47E+01	0.00E+00	6.47E+01
0.5	3183.1016	0.000628318	2.80E+01	5.12E+05	1.73E+03	2.66E+05	1.26E+01	6.68E+01	7.94E+01	-7.94E+01	1.28E-13	7.94E+01
0.6	2122.0677	0.000785398	3.02E+01	5.02E+05	1.73E+03	2.12E+05	1.49E+01	7.88E+01	9.37E+01	-9.37E+01	0.00E+00	9.37E+01
0.7	1364.1864	0.001047197	3.37E+01	4.93E+05	1.72E+03	1.60E+05	1.70E+01	9.05E+01	1.08E+02	-1.08E+02	0.00E+00	1.08E+02
0.8	795.77539	0.001570795	4.09E+01	4.83E+05	1.71E+03	1.10E+05	1.92E+01	1.02E+02	1.21E+02	-1.21E+02	0.00E+00	1.21E+02
0.9	353.67795	0.00314159	6.23E+01	4.74E+05	1.70E+03	6.09E+04	2.12E+01	1.13E+02	1.34E+02	-1.34E+02	0.00E+00	1.34E+02
1	3.534E-13	2.82969E+12	3.85E+16	4.66E+05	1.69E+03	1.36E+04	2.32E+01	1.23E+02	1.46E+02	-1.46E+02	0.00E+00	1.46E+02

Furnance Emissivity vs Temperature and Heat Input

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As e_2 increases:

1. Insulated Wall Temp Decreases.
2. Required Heat Input Increases to Maintain Floor Temperature.