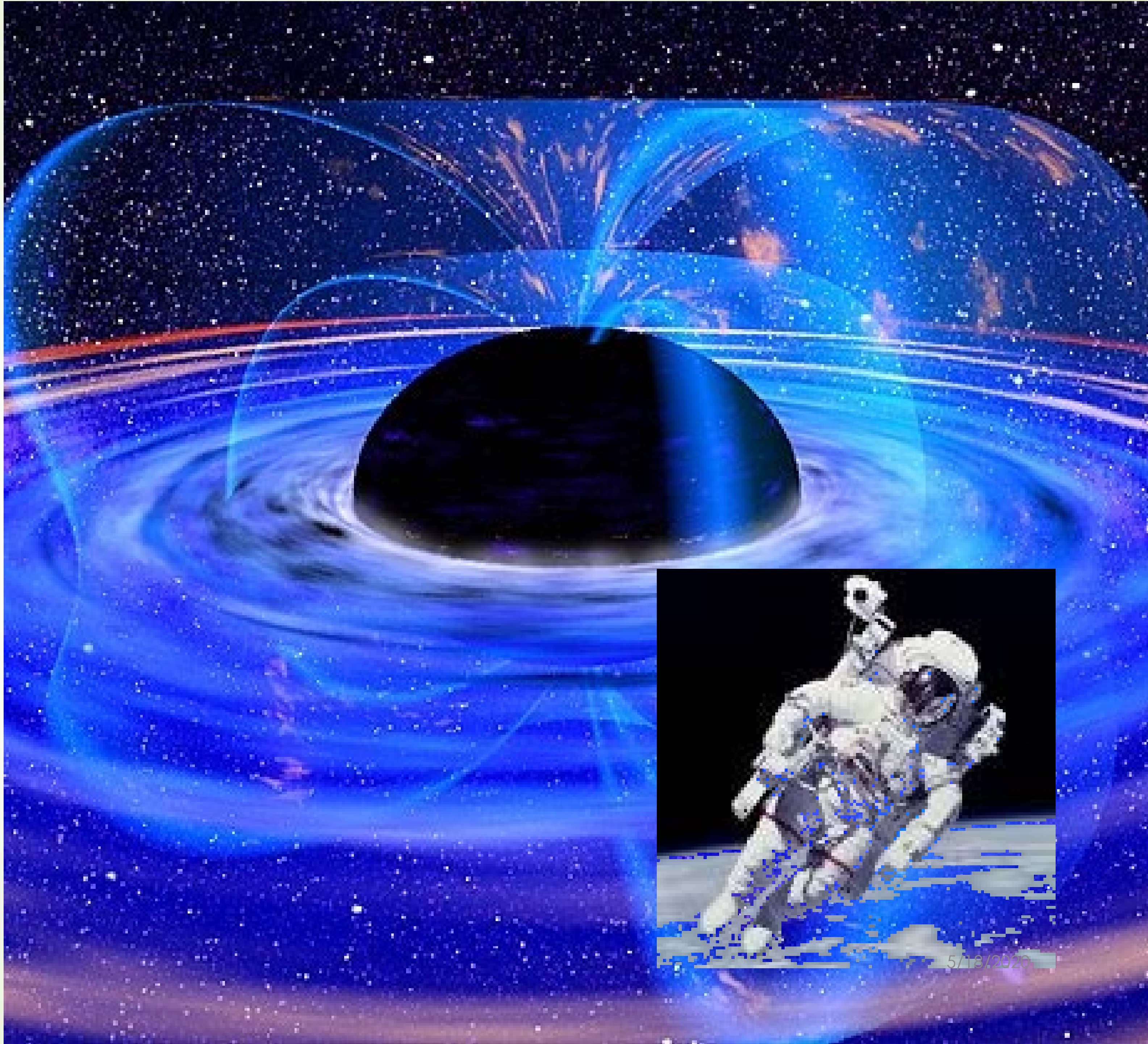
STUDY AID
Reactor Core Melt Down

MECH-420 HEAT TRANSFER

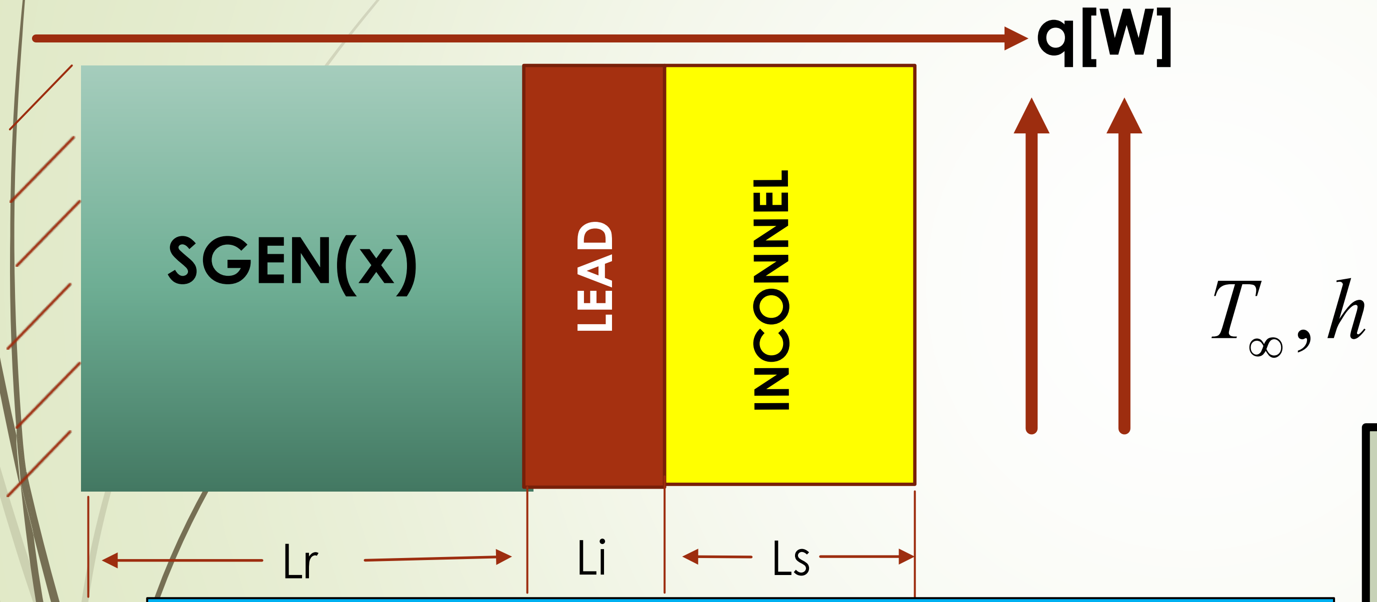
As Chief Engineer on research science space station Jarvis 7 circling the event horizon of IC-1101, you are tasked to ensure the station is protected from deep space thermal anomalies

On day 443 of the 4-year deployment the shielding protecting the super thermal hydrogen reactor 3D cubical core and has been bombarded by neutron isotopes GAMMA I-45 and initiated a chain heat reaction within the core.

You must react quickly to determine the convective cooling requirements.



You Quickly Realized the Following Geometry and Conditions



The 5m Cubical Reactor core is protected by an outer 0.8 cm thickness of lead followed by a 2.0 cm thick layer of high strength Inconel 718. You realized that 5-sides are insulated, and that one side is exposed to a convective fluid to maintain adequate core cooling.

$$k_{reactor} = 45 \frac{W}{m-K}$$

$$k_{lead} = 34.7 \frac{W}{m-K}, T_{melt} = 327.5C(600K)$$

$$k_{inconnel} = 12.7 \frac{W}{m-K}, T_{melt} = 1370C$$

$$T_\infty = 300K$$

$$\dot{S}_{gen}(x) = S_0(1+\cos\frac{\pi x}{L})\frac{W}{m^3}, S_0 = 12\frac{kW}{m^3}$$

To maintain Fuel Cell Temperatures, minimum reactor temperature is 450K. You quickly realize that you must determine the MIN and MAX values of "h" to ensure adequate reactor temperatures while not violating material safety concerns.

As a result of that intense experience with Berry, you realize that the HEAT absorbed by the fluid, is governed by the magnitude of HEAT GENERATED within the reactor core.

$$\dot{S}_{gen}(x) = S_0(1+\cos\frac{\pi x}{L_r})\frac{W}{m^3}, S_0 = 12\frac{kW}{m^3}; 0 \le x \le L_r$$

$$\dot{E}_{gen}[W] = \int_0^{L_r} \dot{S}_{gen}(x) d\forall \frac{W}{m^3} \rightarrow d\forall = L_r^2 dx$$

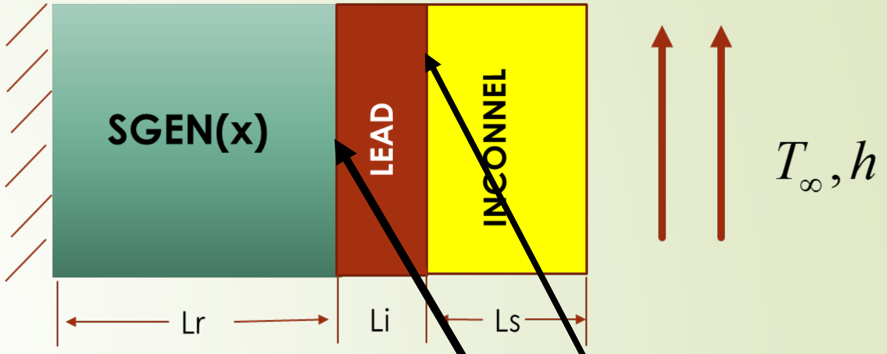
$$\dot{E}_{gen}[W] = S_0 L_r^2 \int_0^{L_r} (1+\cos\frac{\pi x}{L_r}) dx = S_0 L_r^2 \left[x + \frac{L}{\pi}\sin\frac{\pi x}{L_r} \right]_{0-L_r}$$

$$= S_0 L_r^2 \left[L_r + \frac{L}{\pi}(\sin\pi - \sin 0) \right]$$

$$= S_0 L_r^3 [W]$$

$$= 1500 kW \rightarrow \text{MUST BE DISSIPATED BY CONVECTIVE FLUID}$$

To find "h", you recall that 1D, SS Heat Transfer with NO Sgen in portion of problem is ideal to apply thermal circuits.



$$q[W] = \frac{\Delta T}{\sum R_{th}} = \frac{T_r - T_\infty}{\left[\frac{L_{Lead}}{k_{Lead}A} + \frac{L_{Inconnel}}{k_{Lnconnel}A} + \frac{1}{hA_s}\right]} = \frac{T_r - T_{LEAD}}{\frac{L_{Lead}}{k_{Lead}A}}$$

solving for "h"

$$q[W] \bullet \left[\left[\frac{L_{Lead}}{k_{Lead}A} + \frac{L_{Inconnel}}{k_{Lnconnel}A} + \frac{1}{hA_s}\right]\right] = T_r - T_\infty$$

$$\frac{1}{hA_s} = \frac{T_r - T_\infty}{q[W]} - \frac{L_{Lead}}{k_{Lead}A} - \frac{L_{Inconnel}}{k_{Inconnel}A}$$

$$h = \frac{1}{A_s}\left[\frac{T_r - T_\infty}{q[W]} - \frac{L_{Lead}}{k_{Lead}A} - \frac{L_{Inconnel}}{k_{Inconnel}A}\right]^{-1} \frac{W}{m^2 - K}; A = L_r^2 = A_s$$

$$q[W] = \frac{\Delta T}{\sum R_{th}} = \frac{T_r - T_\infty}{\left[\frac{L_{Lead}}{k_{Lead}A} + \frac{L_{Inconnel}}{k_{Lnconnel}A} + \frac{1}{hA}\right]} = \frac{T_r - T_{LEAD}}{\frac{L_{Lead}}{k_{Lead}A}}$$

$$T_{LEAD} = T_r - q[W] \bullet \frac{L_{Lead}}{k_{Lead}A} \rightarrow \text{(LEAD/INCONNEL INTERFACE)}$$

Plotting Solution

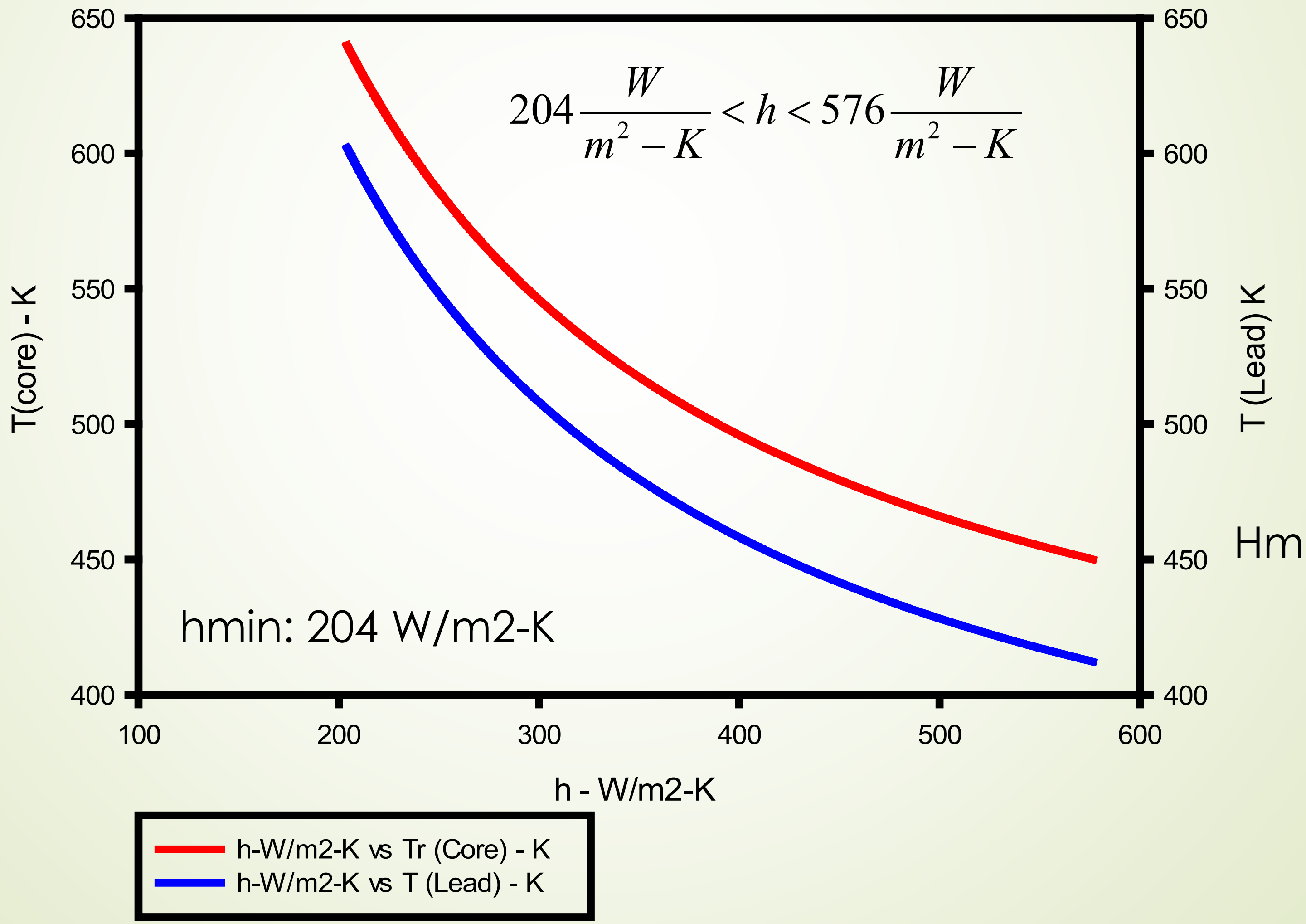
Now by varying the value of the Reactor temperature (min = 450K) we can compute "h" and Lead temperature to check limts.

$$h(T_r) = \frac{1}{A_s}\left[\frac{T_r - T_\infty}{q[W]} - \frac{L_{Lead}}{k_{Lead}A} - \frac{L_{Inconnel}}{k_{Inconnel}A}\right]^{-1} \frac{W}{m^2 - K}; A = L_r^2 = A_s$$

$$T_{LEAD} = T_r(h) - q[W] \bullet \frac{L_{Lead}}{k_{Lead}A}$$

W	m	K	K	W/m-K	W/m-K	W/m-K	m	m	m2	W/m2-K
q	Lr	Tr	Tf	kr	klead	kinconnel	Llead	Linconnel	A	h
1500000	5	450	300	45	12.7	34	0.008	0.02	25	576.7056

Reactor Core Temp. vs. HT Coefficient



$$204\frac{W}{m^2 - K} < h < 576\frac{W}{m^2 - K}$$

hmin: 204 W/m2-K

Hmax: 576 W/m2-K

Find Maximum Reactor Temperature

$$\dot{S}_{gen}(x) = S_0(1 + \cos\frac{\pi x}{L_r})$$

$HDE:$

$$\frac{d^2T}{dx^2} = -\frac{\dot{S}_{gen}(x)}{k_r} = -\frac{S_0(1 + \cos\frac{\pi x}{L_r})}{k_r}$$

$BC\#1$

$$@\, x = 0, \frac{dT}{dx} = 0$$

$$@\, x = L_r, T(x = L_r) = T_r$$

HDE DEFINITION:

2nd Order ODE that when solved along with BC's provides the temperature, T(x) and heat transfer rate q(x), EVERYWHERE.

def·i·ni·tion

\dĕ-fə′-nĭ-shən\

meaning of a word; can be subjective

INTEGRATE HDE, APPLY BC's and OBTAIN EXACT SOLUTION

$$\frac{d^2T}{dx^2} = -\frac{S_0(1+\cos\frac{\pi x}{L_r})}{k_r} \rightarrow INTEGRATE$$

$$\frac{dT}{dx} = -\frac{S_0}{k_r}(x + \frac{L_r}{\pi}\sin\frac{\pi x}{L_r}) + C_1 \rightarrow INTEGRATE$$

$$BC\#1: @\, x=0, dT/dx = 0 \rightarrow C_1 = 0 \rightarrow \text{APPLY BC}$$

$$T(x) = -\frac{S_0}{k_r}(\frac{x^2}{2} - \left(\frac{L_r}{\pi}\right)^2 \cos\frac{\pi x}{L_r}) + C_2$$

$BC\#2$

$$T(x=L_r) = T_r(h) = -\frac{S_0}{k_r}(\frac{L_r^{\ 2}}{2} - \left(\frac{L_r}{\pi}\right)^2 \cos\frac{\pi L_r}{L_r}) + C_2$$

$$T_r(h) + \frac{S_0}{k_r}\frac{L_r^{\ 2}}{2} + \left(\frac{L_r}{\pi}\right)^2 = C_2 \rightarrow \text{APPLY BC}$$

EXACT SOLUTION

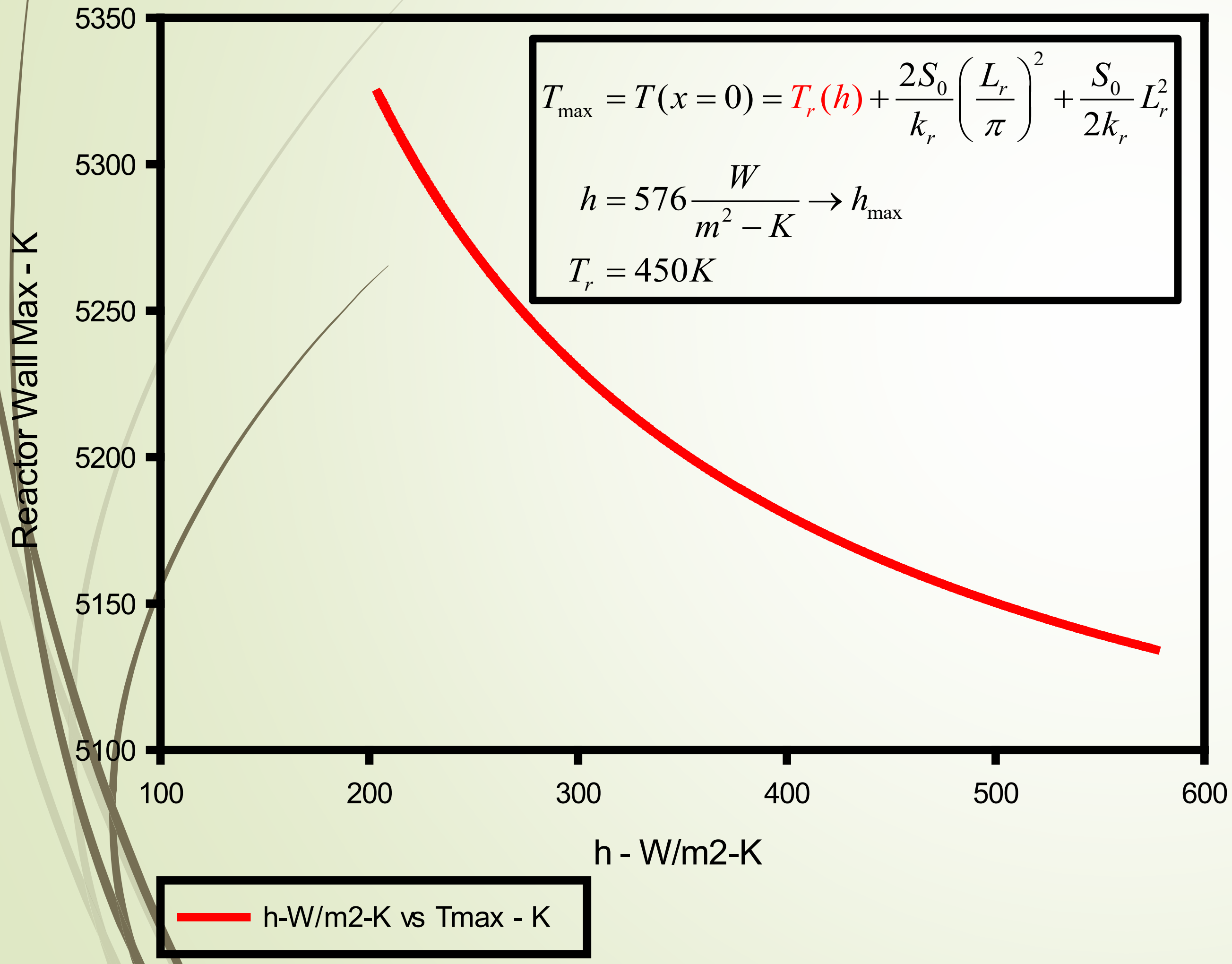
$$T(x) = -\frac{S_0}{k_r}(\frac{x^2}{2} - \left(\frac{L_r}{\pi}\right)^2 \cos\frac{\pi x}{L_r}) + T_r + \frac{S_0 L_r^{\ 2}}{k_r}(\frac{1}{2} + \frac{1}{\pi^2})$$

$$T(x) = T_r(h) + \frac{S_0}{k_r}\left(\frac{L_r}{\pi}\right)^2 (\cos\frac{\pi x}{L_r} + 1) + \frac{S_0}{2k_r}(L_r^2 - x^2)$$

$$T_{\max} = T(x=0) = T_r(h) + \frac{2S_0}{k_r}\left(\frac{L_r}{\pi}\right)^2 \frac{\frac{\frac{W}{m^3}m^2}{W}}{m-K} + \frac{S_0}{2k_r}L_r^2$$

Reactor Wall Temperature--MAX

HT Coefficient vs Reactor Wall Temperatutre Max

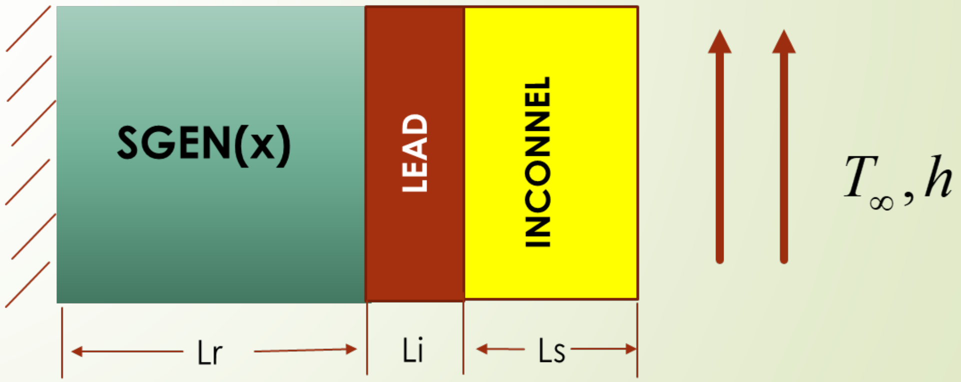


$$T_{\max} = T(x=0) = T_r(h) + \frac{2S_0}{k_r}\left(\frac{L_r}{\pi}\right)^2 + \frac{S_0}{2k_r}L_r^2$$

$$h = 576\frac{W}{m^2-K} \rightarrow h_{\max}$$

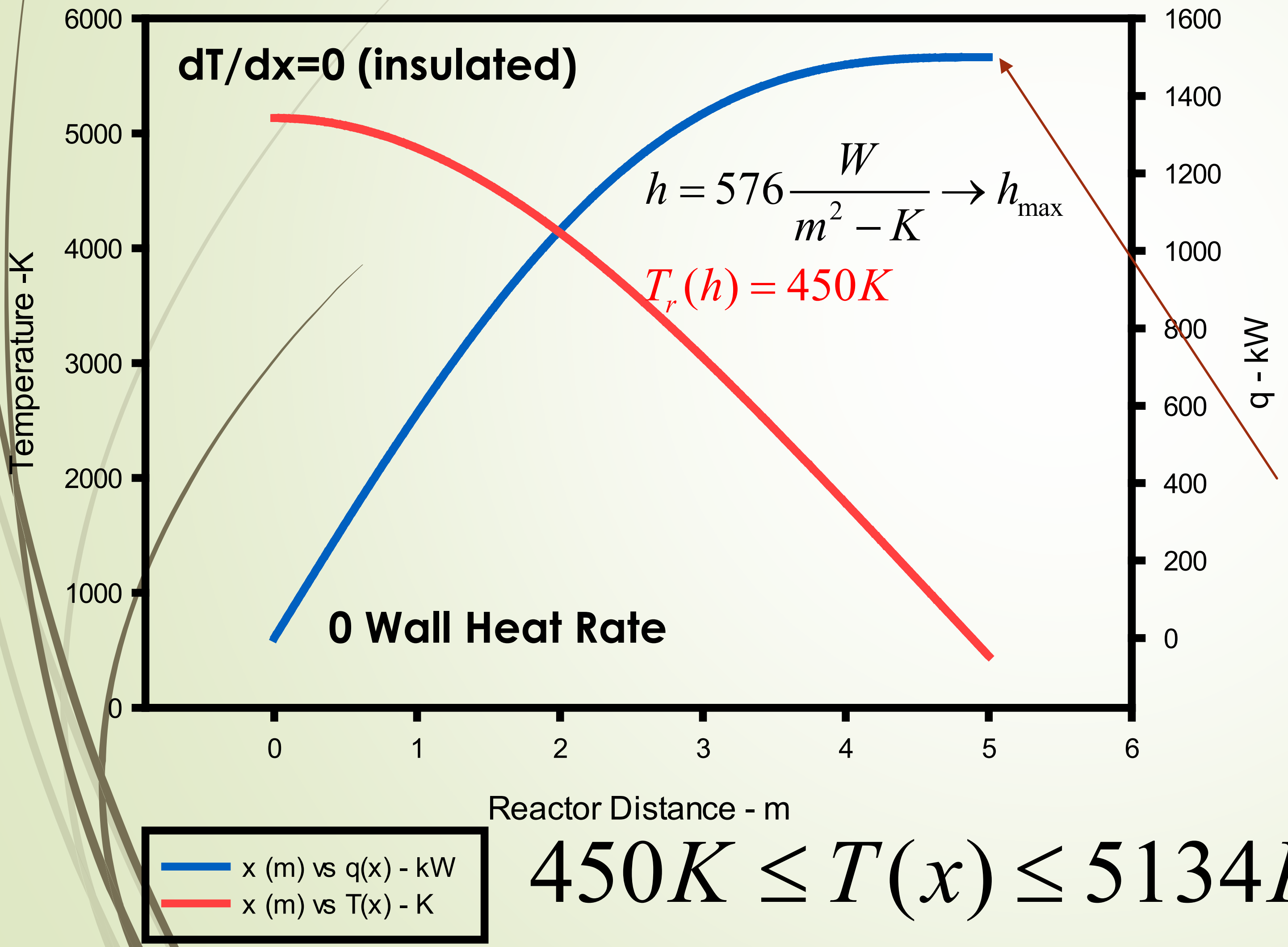
$$T_r = 450K$$

MAX REACTOR TEMPERATURE "MUST" be at right INSULATED boundary. Heat will be dissipated by conduction through reactor and be absorber by COLDER CONVECTIVE FLUID.



Reactor Distance vs Temperature and Heat Rate

$$q = -k_r A\frac{dT}{dx} = k_r A(\frac{S_0}{k_r}(x + \frac{L_r}{\pi}\sin\frac{\pi x}{L_r}))$$



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$$\dot{E}_{gen}[W] = \int_0^{L_r} \dot{S}_{gen}(x)d\forall\frac{W}{m^3} \to d\forall = L_r^2 dx$$

$$\dot{E}_{gen}[W] = S_0L_r^2\int_0^{L_r}(1+\cos\frac{\pi x}{L_r})dx = S_0L_r^2\left[x + \frac{L}{\pi}\sin\frac{\pi x}{L_r}\right]_{0-L_r}$$

$$= S_0L_r^2\left[L_r + \frac{L}{\pi}(\sin\pi - \sin 0)\right]$$

$$= S_0L_r^3W$$

$\dot{E}_{gen}[W] = 1500kW \to$ MUST BE DISSIPATED BY CONVECTIVE FLUID

$$450K \le T(x) \le 5134K$$