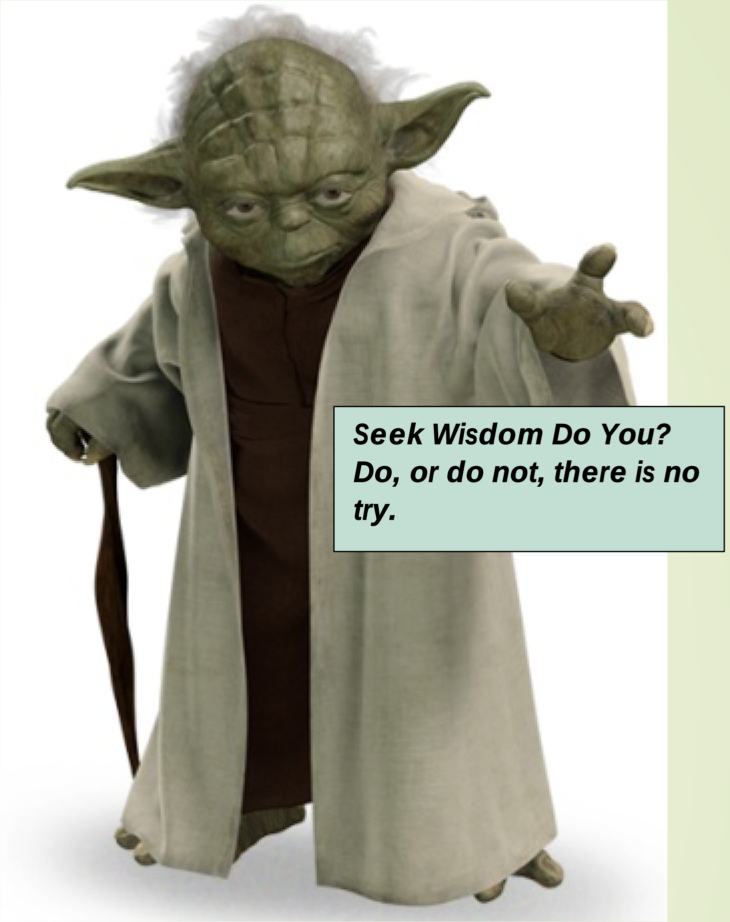

Study Aid

Energy and Flow #2

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Road Map

- Identify Points Along Streamline
- Manometry
- Conservation of Energy + Mass
- Conservation of Momentum
- Friction Loss
- Combine if Required

Build Parametric Model to Vary Angle and Friction Coefficient

A large water tank of diameter $D = 10'$ *and height* $H = 5'$ *is evacuated by a pump (efficiency of* $\eta_p = 80\%$*). The flow head loss through the pipe jet at the exit is, the spring force is 300 lbf, and the coefficient of friction between the wheels and the surface is 0.3.* Jet flow loss is $5.0\dfrac{V_j^2}{2g}$

Apply Conservation of Energy

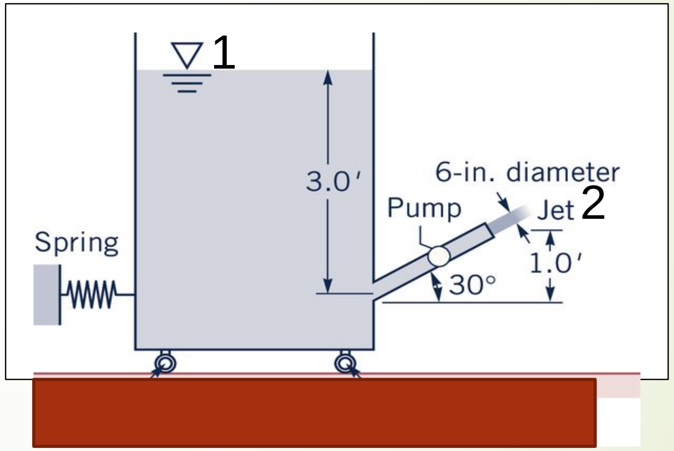
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_{p_{IDEAL}} = \cancel{h_{T_{IDEAL}}} + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

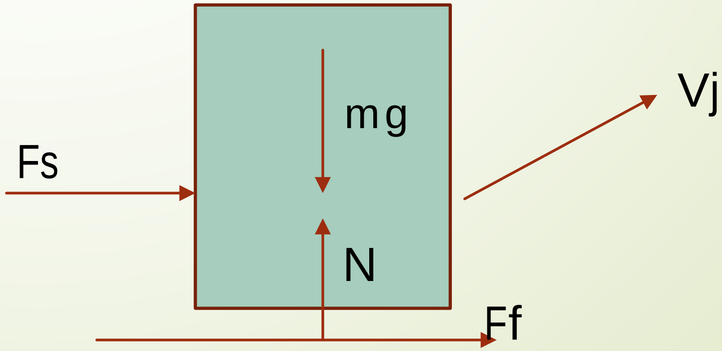
$$P_1 = P_2 = V_1 = 0$$

$$h_{p_{IDEAL}} = \left(\frac{V_2^2}{2g} + z_2 - z_1 + h_L \right)$$

$$h_{p_{IDEAL}} = \left(\frac{V_2^2}{2g} - 2.0 + 5.0\frac{V_2^2}{2g} \right) = \left(6.0\frac{V_2^2}{2g} - 2.0 \right)$$

$$\dot{W}_p = \frac{\gamma_f Q h_{p_{IDEAL}}}{\eta_p} = \frac{\gamma_f (V_2 A) h_{p_{IDEAL}}}{\eta_p}$$





Apply Momentum

$$\overrightarrow{\sum F_x} = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(u_{out} \pm)\dot{m}_{out} - \sum_{in}(u_{in} \pm)\dot{m}_{in}$$

$$\sum F_x = F_s + F_f = \rho A V_2^2 \cos\theta$$

$$\sum F_x = F_s + \mu_s N = \rho A V_2^2 \cos\theta$$

$$\uparrow \sum F_y = \cancel{\frac{dM_{CV}}{dt}} + \sum_{out}(v_{out} \pm)\dot{m}_{out} - \sum_{in}(v_{in} \pm)\dot{m}_{in}$$

$$\uparrow \sum F_y = N - mg = \rho A V_2^2 \sin\theta$$

$$N = mg + \rho A V_2^2 \sin\theta$$

COMBINE

$$\rightarrow \sum F_x = F_s + \mu_s N = \rho A V_2^2 \cos\theta : (1)$$

$$\uparrow \sum F_y = N - mg = \rho A V_2^2 \sin\theta : (2)$$

$$N = mg + \rho A V_2^2 \sin\theta \rightarrow (1)$$

$$F_s + \mu_s (mg + \rho A V_2^2 \sin\theta) = \rho A V_2^2 \cos\theta$$

$$F_s + \mu_s mg = \rho A V_2^2 (\cos\theta - \mu_s \sin\theta)$$

Solve for V_2

$$V_2 = \left[\frac{F_s + \mu_s mg}{\rho A(\cos\theta - \mu_s \sin\theta)} \right]^{1/2}$$

$$V_2 = \left[\frac{F_s + \mu_s \gamma_f \forall_{vol}}{\rho A(\cos\theta - \mu_s \sin\theta)} \right]^{1/2}$$

MU					
0.3					
	ft/s	ft3/s	ft	ft-lbf/s	HP
Theta	V2	Q	hp	Wp	Wp
30	167.49	32.89	2611.62	6,699,183	12,180
35	176.19	34.59	2890.10	7,798,486	14,179
40	187.20	36.76	3262.81	9,354,315	17,008
45	201.45	39.55	3778.83	11,658,484	21,197
50	220.54	43.30	4529.55	15,299,241	27,817
55	247.53	48.60	5706.47	21,633,101	39,333
60	289.18	56.78	7789.30	34,498,043	62,724
65	365.05	71.68	12413.94	69,405,046	126,191
70	578.05	113.50	31129.43	275,589,099	501,071

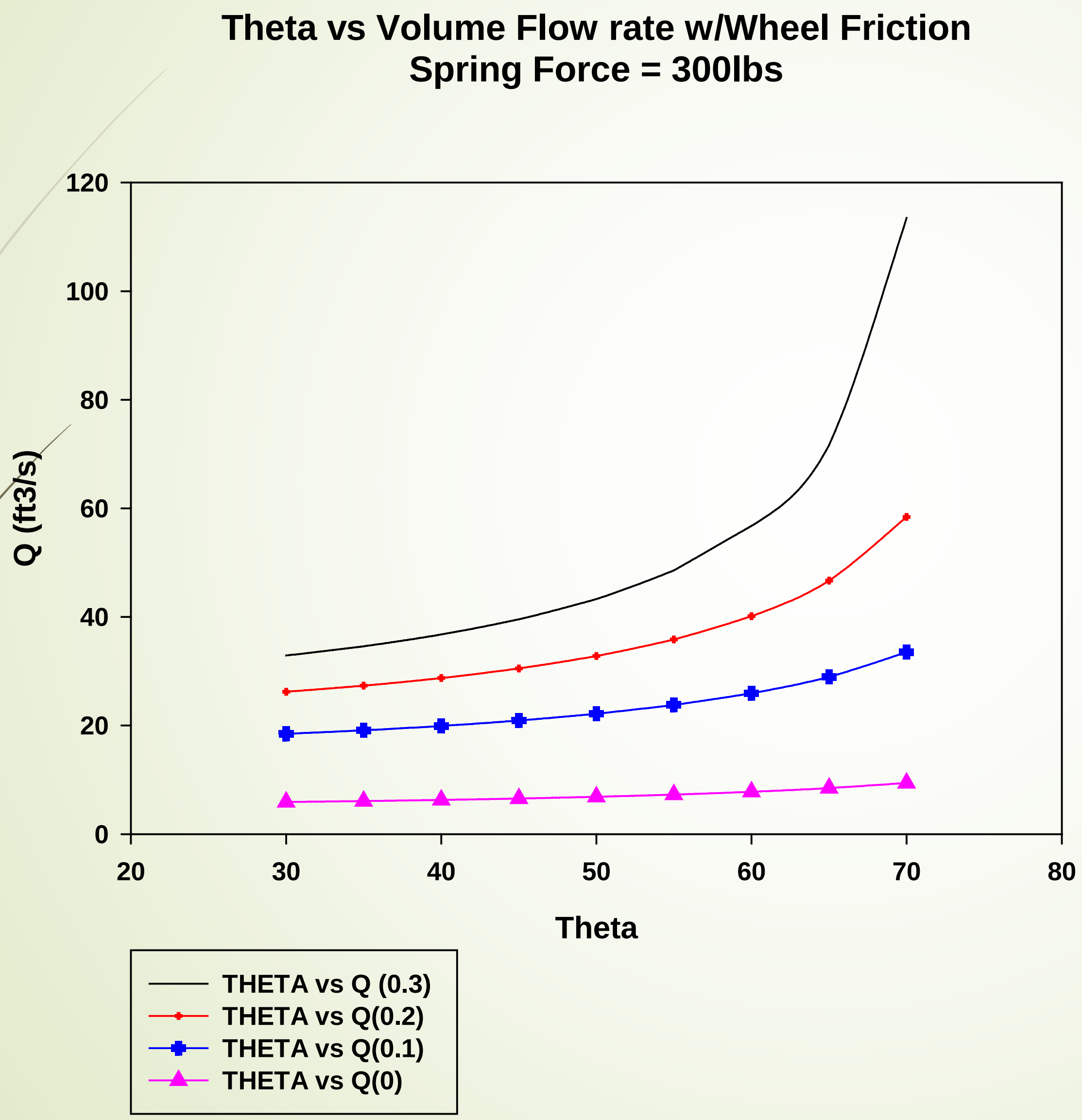
Analysis:

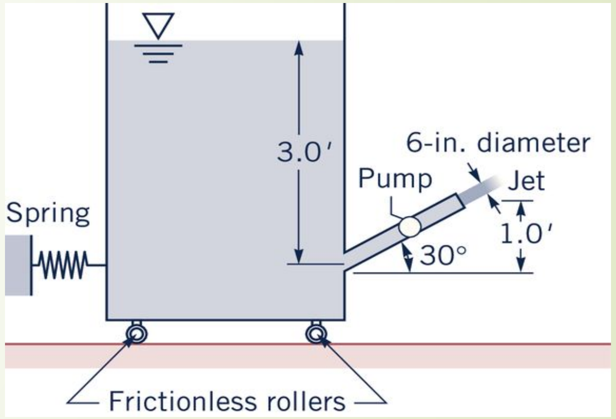
Large pumping power is required due to large external reaction force of 500lbs combined with the wheel friction force which is a function of coefficient of friction and water volume.

The X momentum jet exit force must balance the X spring and friction forces. As theta increases, the X momentum force must decrease (via $\cos\theta$), as such the jet velocity must increase to compensate (via V^2). And therefore the pumping power must also increase with theta.

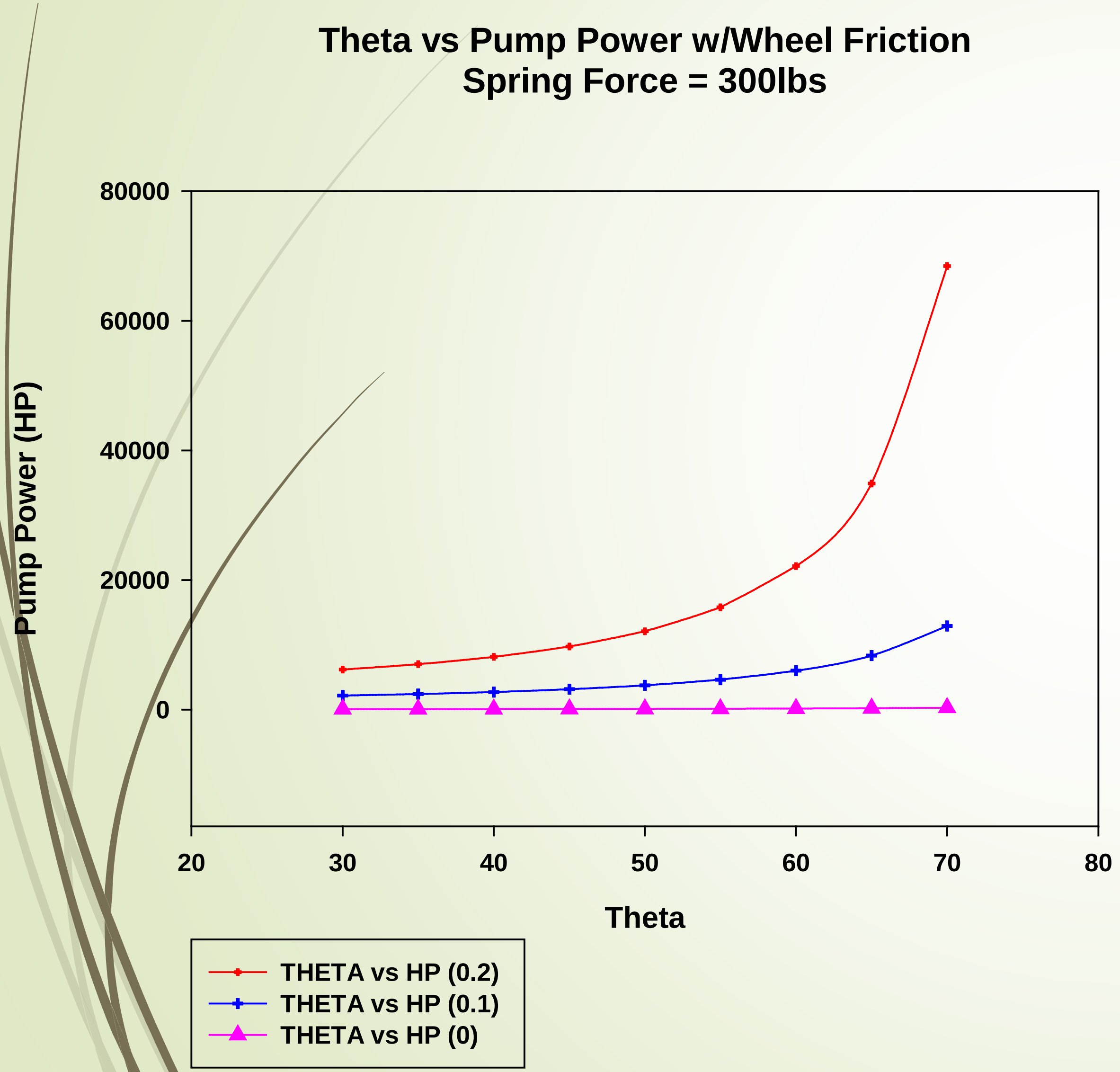
Learning points from parametric analysis and thought.

Parametric Model: Tank Jet





Parametric Model: Tank Jet



MU					
0.2					
	ft/s	ft3/s	ft	ft-lbf/s	HP
Theta	V2	Q	hp	Wp	Wp
30	133.51	26.21	1658.61	3,391,326	6,166
35	139.22	27.34	1803.80	3,846,036	6,993
40	146.35	28.74	1993.44	4,468,019	8,124
45	155.36	30.50	2246.72	5,345,747	9,720
50	167.00	32.79	2596.29	6,640,317	12,073
55	182.54	35.84	3102.53	8,673,707	15,770
60	204.40	40.13	3890.55	12,179,237	22,144
65	237.84	46.70	5268.48	19,191,171	34,893
70	297.68	58.45	8253.76	37,628,946	68,416

MU					
0					
	ft/s	ft3/s	ft	ft-lbf/s	HP
Theta	V2	Q	hp	Wp	Wp
30	30.16	5.92	82.73	38,207.97	69.47
35	31.01	6.09	87.58	41,588.28	75.62
40	32.06	6.30	93.79	46,055.26	83.74
45	33.37	6.55	101.77	52,016.90	94.58
50	35.00	6.87	112.15	60,123.78	109.32
55	37.06	7.28	125.93	71,465.01	129.94
60	39.69	7.79	144.75	87,985.09	159.97
65	43.17	8.48	171.62	113,466.85	206.30
70	47.99	9.42	212.54	156,198.72	284.00