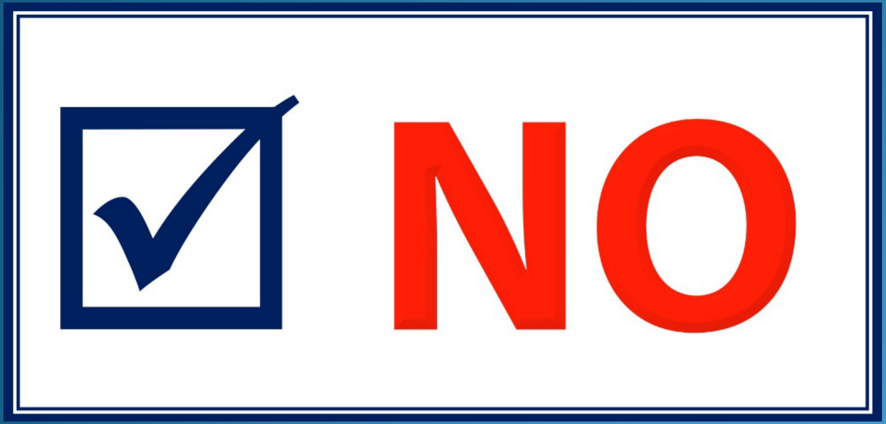
One-Dimensional, Steady-State Conduction

NO

INTERNAL *THERMAL ENERGY GENERATION*



Chapter Three

Sections 3.1 through 3.4

MECH-420 Spring 2020 Helpful Comments

I believe that this class has taught me new skills and provided me the knowledge to look at things in the world differently. **This class has given me the understanding of heat transfer and I will be able to apply this information in my field in the future**. Overall, I thought that the subject was interesting, and I didn't realize at first the importance of proper heat transfer.

I really enjoyed how Heat Transfer taught me to think outside of the box and look at the fundamentals of the problem in order to understand how to solve the problem. This has helped me be more objective in my problem solving by changing my view to look at the problem as a whole and evaluate what is being designed for. **I would recommend for future Mech-420 students to look at a problem from the elements of what are being asked, not the question itself**.

I believe Heat Transfer was one of the most important courses I've taken as it truly lays out the methodology and process in which a Mechanical Engineer must be able to think to be successful. The variety of problems encountered and ways of solving them helped build upon the instilled in me in Fluid Mechanics. It is so important to be able to identify exactly what the problem is that faces you because you cannot solve it unless you have a high-level understanding of the situation. There's only one suggestion that any future student of Heat Transfer needs to hear. **FOLLOW THE ROAD MAP!**

MECH-420 Spring 2020 HOMEWORK and ASSIGNMENTS

As this class most definitely would have been much different in person, it's hard to compare the style of lectures or how different the learning would have been. A suggestion for future classes would be to provide a grade for homework and assign relevant problems to students. Without a reasonable incentive of a grade, most students opt out of completing all of the recommended homework, including me at the first half of the semester. **Once I realized the importance and similarities between the homework and assignments, the class got much easier and my understanding of the material was much greater**. Otherwise, I thought the material for the class was reasonable and the lectures and the book was enough for me to grasp the concepts of the chapters.

Read through the equation packet provided early on. Get familiar with it because it does have all the formulas needed to solve all quiz and exam problems. **Do ALL homework assigned in the syllabus. The problems might be a little different than how the quizzes or exams might be, but the mindset or thought process required to solve them is very similar to the quizzes and exams**. Also, understand all the variables you're working with in a problem and the meaning of the variables. It will help what you're doing make a little more sense, or can tell you if the value you solved for seems wrong.

Methodology of a Conduction Analysis

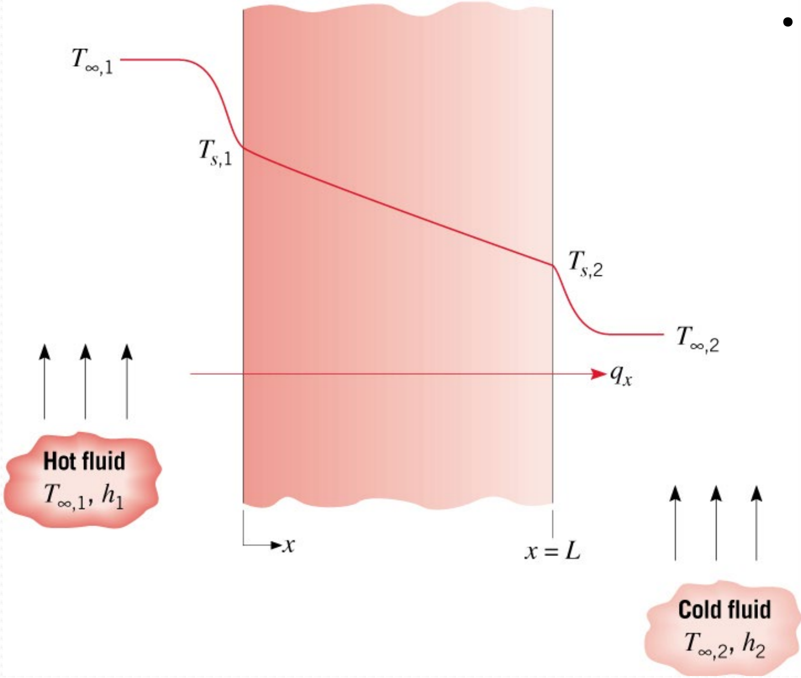
- Specify appropriate form of the heat equation.
- Solve for the temperature distribution.
- Apply Fourier's law to determine the heat flux.

Simplest Case: One-Dimensional, Steady-State Conduction with No Thermal Energy Generation.

- Common Geometries:
  - The Plane Wall: Described in rectangular (x) coordinate. Area perpendicular to direction of heat transfer is constant (independent of x).
  - The Tube Wall: Radial conduction through tube wall.
  - The Spherical Shell: Radial conduction through shell wall.

The Plane Wall

- Consider a plane wall between two fluids of different temperature:



- Heat Equation:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0 \qquad (3.1)$$

$$T(x)=C_1x+C_2$$

$$T(0)=T_{s,1},\quad T(L)=T_{s,2}$$

- Implications:

  Heat flux $\left(q''_x \frac{W}{m^2}\right)$ is independent of x.

  Heat rate $\left(q_x W = q''_x A_x\right)$ is independent of x.

- Boundary Conditions: $T(0)=T_{s,1},\quad T(L)=T_{s,2}$

- Temperature Distribution for Constant k :

$$T(x)=T_{s,1}+\left(T_{s,2}-T_{s,1}\right)\frac{x}{L} \qquad (3.3)$$

NOTE: Thermal Circuit is written in the DIRECTION of HEAT TRANSFER

- Heat Flux and Heat Rate:

$$q_x'' = -k\frac{dT}{dx} = \frac{k}{L}\left(T_{s,1} - T_{s,2}\right) \qquad (3.5)$$

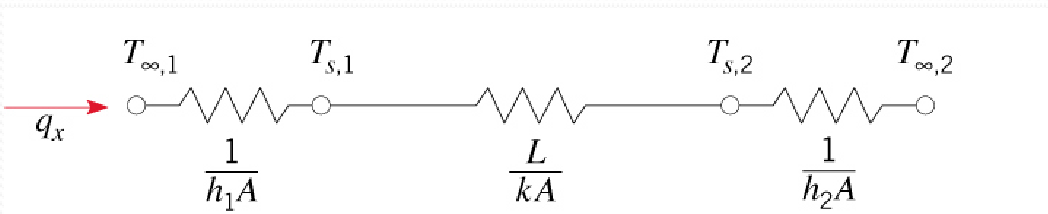
$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) \qquad (3.4)$$

- Thermal Resistances $\left(R_t = \dfrac{\Delta T}{q}\right)$ and Thermal Circuits:

Conduction in a plane wall: $R_{t,\text{cond}} = \dfrac{L}{kA}$ (3.6)

Convection: $R_{t,\text{conv}} = \dfrac{1}{hA}$ (3.9)

Thermal circuit for plane wall with adjoining fluids:



$$R_{\text{tot}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (3.12)$$

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{\text{tot}}} \qquad (3.11)$$

Sgen = 0.0
STEADY STATE
HOMEGENOUS

- Thermal Resistance for Unit Surface Area:

$$R''_{t,\text{cond}} = \frac{L}{k} \qquad R''_{t,\text{conv}} = \frac{1}{h}$$

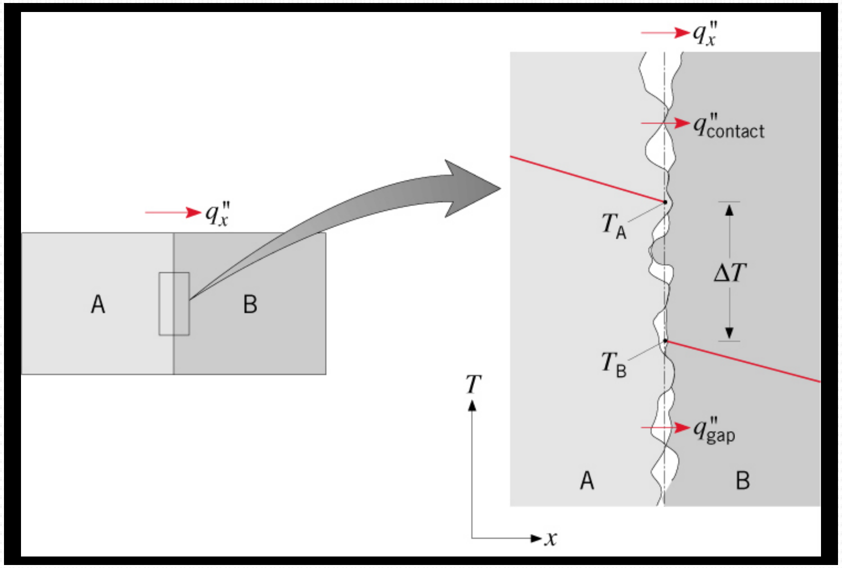
Units: $R_t \leftrightarrow \text{K/W} \qquad R''_t \leftrightarrow \text{m}^2 \cdot \text{K/W}$

- Radiation Resistance:

$$R_{t,\text{rad}} = \frac{1}{h_r A} \qquad R''_{t,\text{rad}} = \frac{1}{h_r}$$

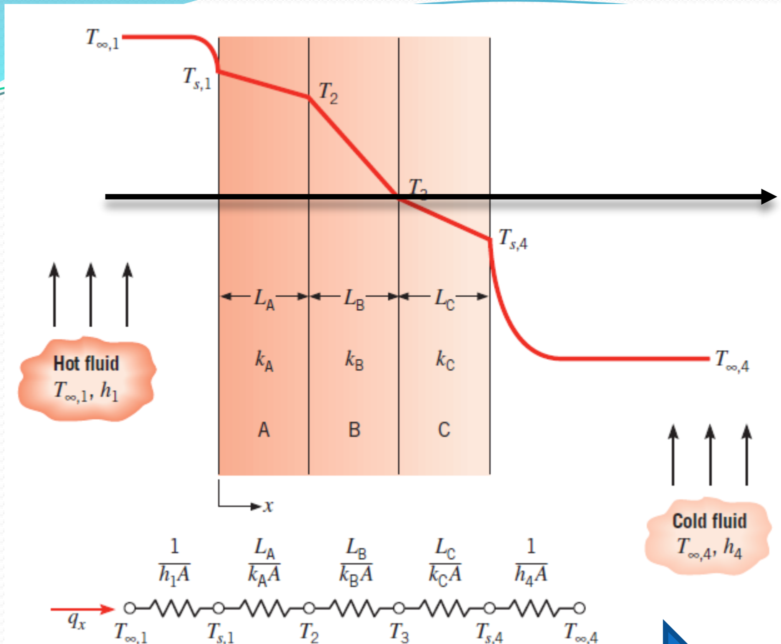
$$h_r = \varepsilon\sigma\left(T_s + T_{\text{sur}}\right)\left(T_s^2 + T_{\text{sur}}^2\right) \qquad (1.9)$$

- Contact Resistance:



$$R''_{t,c} = \frac{T_A - T_B}{q''_x} \qquad R_{t,c} = \frac{R''_{t,c}}{A_c}$$

Values depend on: Materials A and B, surface finishes, interstitial conditions, and contact pressure (Tables 3.1 and 3.2)



HEAT TRANSFER

- Composite Wall with Negligible Contact Resistance:

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R_t} \qquad (3.14)$$

NOTE: Thermal Circuit is written in the DIRECTION of HEAT TRANSFER

For the temperature distribution shown, $k_A > k_B < k_C$.

$$\sum R_t = R_{tot} = \frac{1}{A}\left[\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_4}\right] = \frac{R''_{tot}\left(\frac{m^2 - K}{W}\right)}{A\left(m^2\right)} = \frac{K}{W}$$

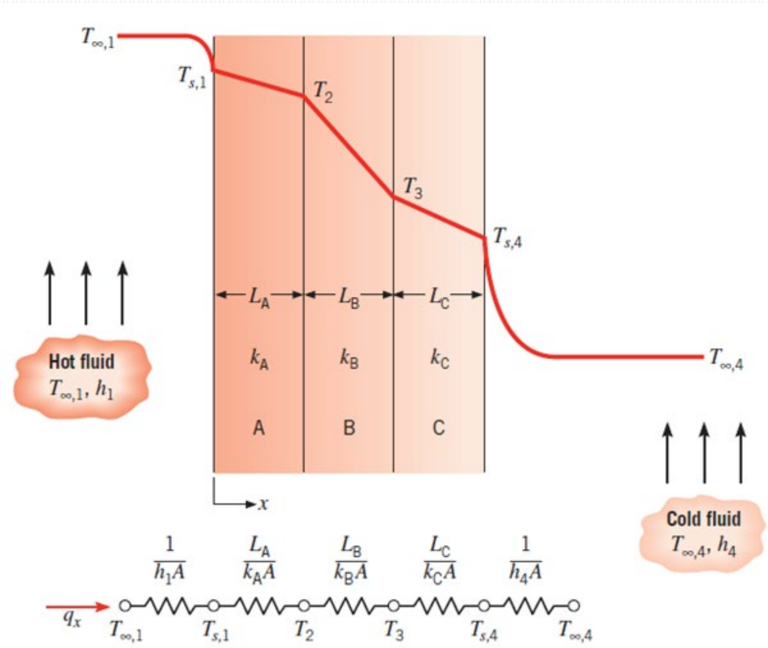
- Overall Heat Transfer Coefficient (U) :

A modified form of Newton's law of cooling to encompass multiple resistances to heat transfer.

$$q_x = UA\left[\frac{W}{K}\right]\Delta T_{overall}; \left\{U\left[\frac{W}{m^2 - K}\right]A[m^2]\right\} \rightarrow \text{OVERALL THERMAL RESISTANCE} \quad (3.17)$$

$$R_{tot} = \frac{1}{UA} \rightarrow \frac{K}{W} \qquad (3.19)$$

Composite Wall/Series Circuit



HEAT TRANSFER

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R_t} = \frac{\Delta T}{\sum R_t}$$

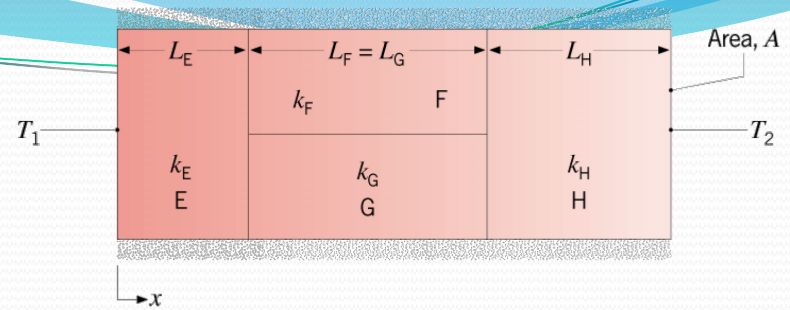
$$= \frac{T_{\infty,1} - T_{s1}}{\frac{1}{h_1 A}} = \frac{T_{s1} - T_2}{\frac{L_A}{k_A A}} = \frac{T_2 - T_3}{\frac{L_B}{k_B A}} = \frac{T_3 - T_{s4}}{\frac{L_C}{k_C A}}$$

$$= \frac{T_{s4} - T_{\infty,4}}{\frac{1}{h_1 A}}$$

$$= \frac{T_{s1} - T_3}{\frac{L_A}{k_A A} + \frac{L_B}{k_B A}} = \frac{\Delta T}{\sum R_t}$$

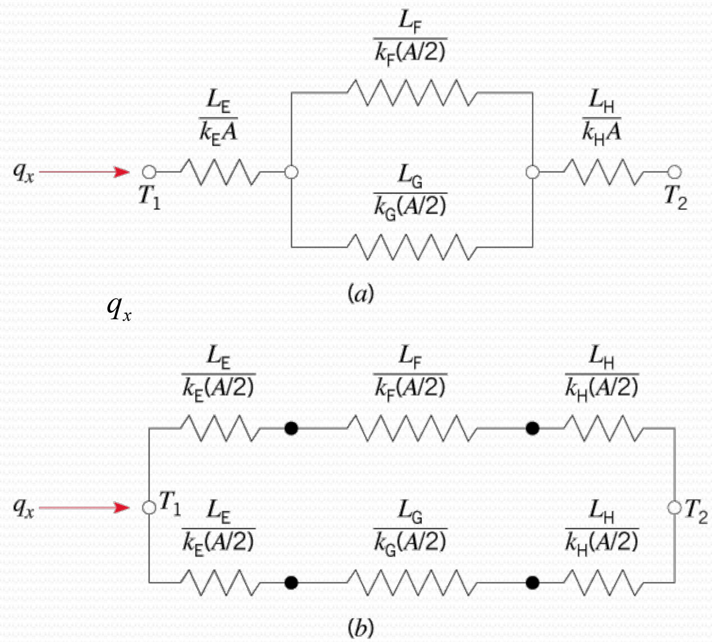
NOTE: Thermal Circuit is written in the DIRECTION of HEAT TRANSFER

- **Series – Parallel Composite Wall:**



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} (parallel\ circuit -- Berry)$$

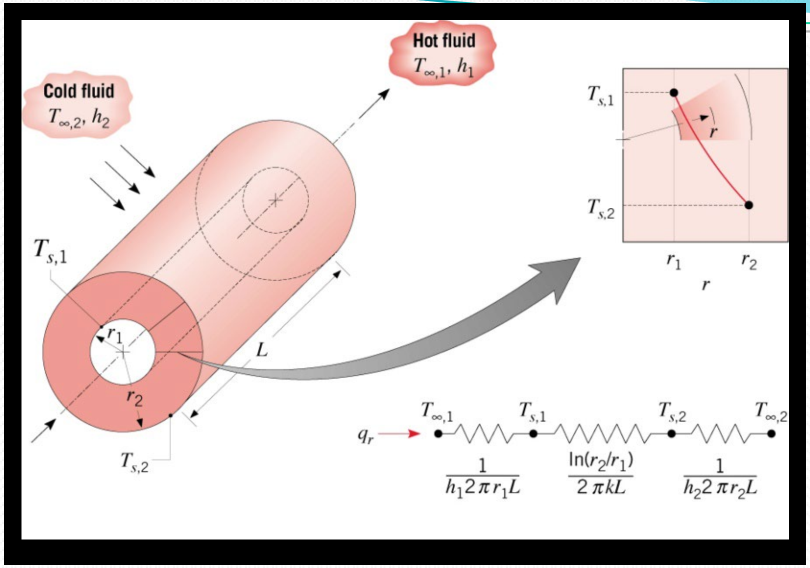
$$R_{eq}(F/G) = \frac{1}{\frac{1}{R_F} + \frac{1}{R_G}}$$



$$\boxed{q_x = \frac{\Delta T}{\sum R_{eq}} = \frac{\Delta T}{R_E + R_{fg} + R_H}}$$

- Note departure from one-dimensional conditions for $k_F \neq k_g$.
- Circuits based on assumption of **ISOTHERMAL SURFACES NORMAL** to *x* direction or **ADIABATIC SURFACES PARALLEL** to *x* direction provide approximations for qx.

The Tube Wall



Sgen = 0.0
STEADY STATE
HOMEGENOUS

- Heat Equation:

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0 \qquad (3.28)$$

What does the form of the heat equation tell us about the variation of q_r with r in the wall?

Is the foregoing conclusion consistent with the energy conservation requirement?

How does q_r'' vary with r ?

- Temperature Distribution for Constant :

$$T(r)=\frac{T_{s,1}-T_{s,2}}{\ln(r_1/r_2)}\ln\left(\frac{r}{r_2}\right)+T_{s,2} \qquad (3.31)$$

- Heat Flux and Heat Rate:

$$q_r'' = -k\frac{dT}{dr} = \frac{k}{r\ln(r_2/r_1)}\left(T_{s,1}-T_{s,2}\right) \qquad [\mathrm{W/m^2}]$$

$$q_r' = 2\pi r q_r'' = \frac{2\pi k}{\ln(r_2/r_1)}\left(T_{s,1}-T_{s,2}\right) \qquad [\mathrm{W/m}]$$

$$q_r = 2\pi r L q_r'' = \frac{2\pi L k}{\ln(r_2/r_1)}\left(T_{s,1}-T_{s,2}\right) \qquad [\mathrm{W}] \qquad (3.32)$$

- Conduction Resistance:

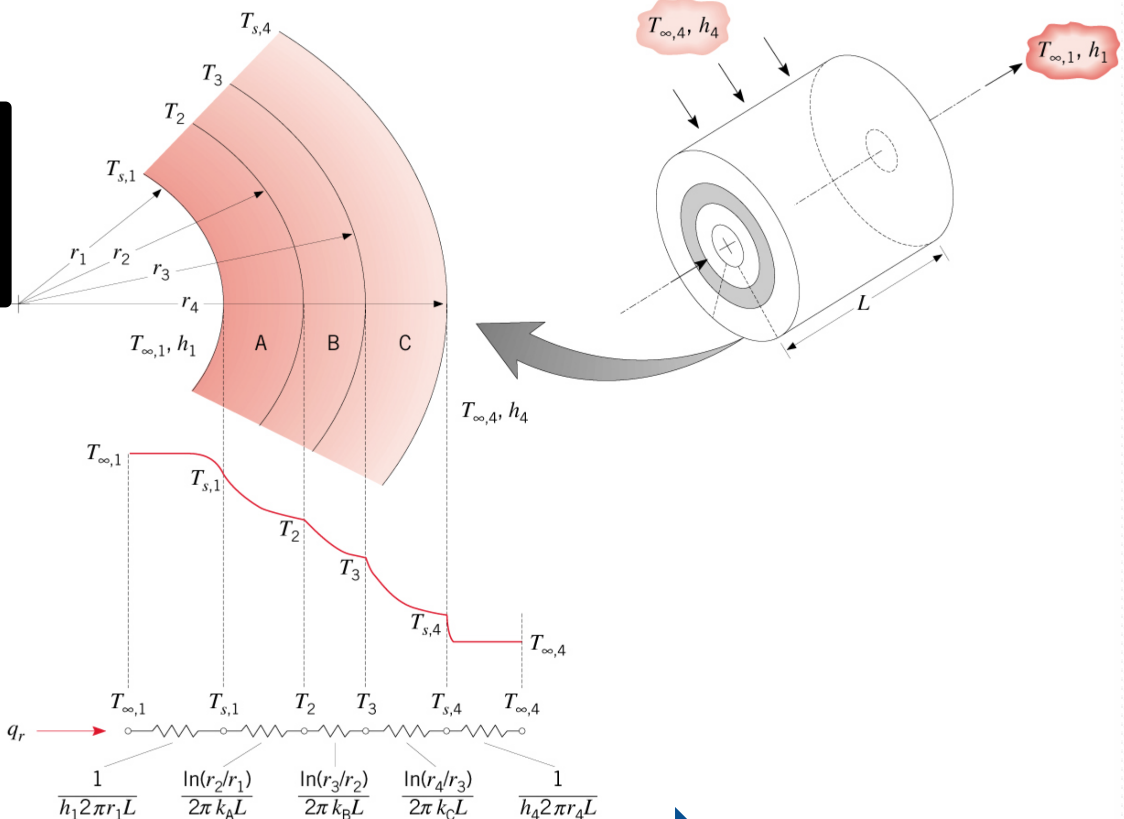
$$R_{t,\text{cond}} = \frac{\ln(r_2/r_1)}{2\pi L k} \rightarrow \text{ALWAYS POSITIVE} \rightarrow [\mathrm{K/W}] \qquad (3.33)$$

$$R'_{t,\text{cond}} = \frac{\ln(r_2/r_1)}{2\pi k} \rightarrow \text{ALWAYS POSITIVE} \rightarrow [\mathrm{m\cdot K/W}]$$

Why doesn't a surface area appear in the expressions for the thermal resistance?

- Composite Wall with Negligible Contact Resistance

NOTE: Thermal Circuit is written in the DIRECTION of HEAT TRANSFER



$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}}$$

$$= UA\left(T_{\infty,1} - T_{\infty,4}\right) \quad (3.35)$$

Note that

$$UA = R_{tot}^{-1}$$

is a constant independent of radius,

HEAT TRANSFER

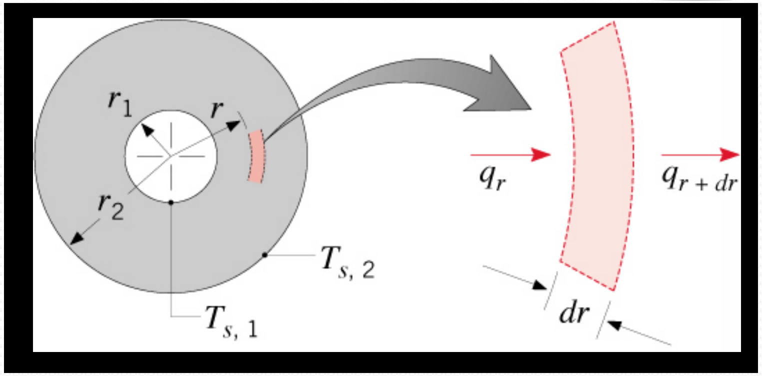
For the temperature distribution shown, $k_A > k_B > k_C$.

but U itself is tied to specification of an interface.

$$U_i = \frac{\left(R_{tot}\frac{K}{W}\right)^{-1}}{A_i}\left[\frac{W}{m^2 - K}\right] \qquad (3.37)$$

Spherical Shell

Sgen = 0.0
STEADY STATE
HOMEGENOUS



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

What does the form of the heat equation tell us about the variation of q_r with r. Is this result consistent with conservation of energy?

How does q''_r vary with r ?

- Temperature Distribution for Constant k :

$$T(r)=T_{s,1}-\left(T_{s,1}-T_{s,2}\right)\frac{1-\left(r_1/r\right)}{1-\left(r_1/r_2\right)}$$

- Heat flux, Heat Rate and Thermal Resistance:

$$q''_r = -k\frac{dT}{dr} = \frac{k}{r^2\left[(1/r_1)-(1/r_2)\right]}\left(T_{s,1}-T_{s,2}\right)\left[\frac{W}{m^2}\right]$$

$$q_r = (A_r = 4\pi r^2)q''_r = \frac{4\pi k}{(1/r_1)-(1/r_2)}\left(T_{s,1}-T_{s,2}\right)[W] \qquad (3.40)$$

$$R_{t,\text{cond}} = \frac{(1/r_1)-(1/r_2)}{4\pi k} \rightarrow \frac{K}{W} \rightarrow \text{Conduction Resistance} \qquad (3.41)$$

- Composite Shell:

$$q_r[W] = \frac{\Delta T_{\text{overall}}}{R_{\text{tot}}} = UA\left[\frac{W}{K}\right]\Delta T_{\text{overall}}$$

$$UA = R_{\text{tot}}^{-1}\left[\frac{K}{W}\right] \leftrightarrow \text{Constant} \rightarrow \text{OVERALL THERMAL RESISTANCE}$$

$$U_i = \frac{(R_{\text{tot}})^{-1}}{A_i}\frac{\left[\frac{K}{W}\right]^{-1}}{m^2} \leftrightarrow \text{Depends on } A_i$$

$$= \text{Overall Heat Transfer Coefficient}$$

SUMMARY

1D Heat Transfer Steady State Sgen = 0

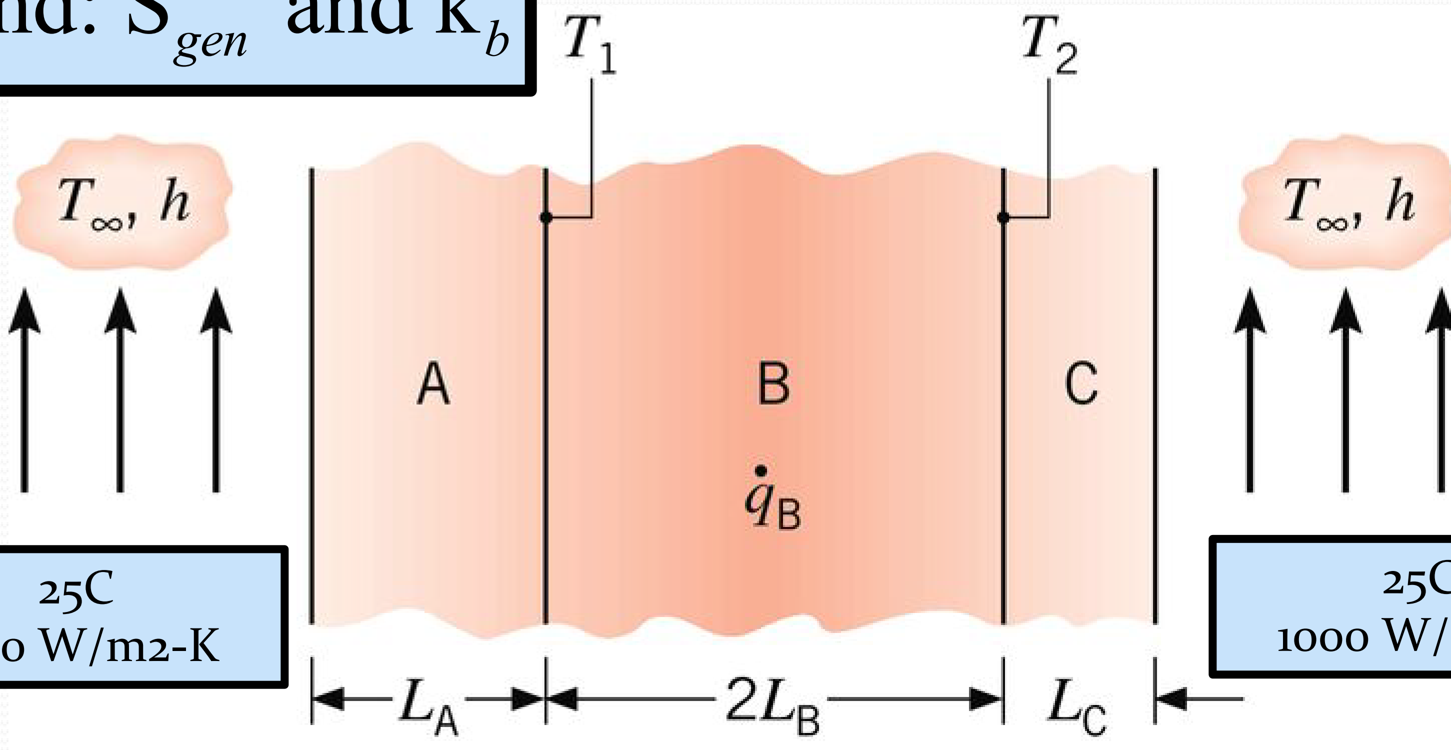
$$q[W] = \frac{\Delta T_{\text{overall}}[K]}{\sum R_{\text{tot}}\left[\frac{K}{W}\right]} = \sum U_i A_i \left[\frac{W}{K}\right] \Delta T_{\text{overall}}$$

$$UA = U_1 A_1 = U_2 A_2 = U_i A_i \rightarrow \text{OVERALL THERMAL RESISTANCE}$$

$$U_i = \frac{(R_{\text{tot}})^{-1}}{A_i} \frac{\left[\frac{K}{W}\right]^{-1}}{m^2} = \frac{W}{m^2 - K} \rightarrow \text{Overall HT Coefficient}$$

$\Delta T = T_2 - T_1$	Plane Wall	Cylindrical	Spherical
Heat Equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Profile T(x/r)	$T_1 - \Delta T\frac{x}{L}$	$T_2 + \Delta T\frac{\ln\left(\frac{r}{r_2}\right)}{\ln\left(\frac{r_1}{r_2}\right)}$	$T_1 - \Delta T\left[\frac{1-\frac{r_1}{r}}{1-\frac{r_1}{r_2}}\right]$
Flux (q" $\left[\frac{W}{m^2}\right]$)	$k\frac{\Delta T}{L}$	$\frac{k\Delta T}{r\ln\left(\frac{r_2}{r_1}\right)}$	$\frac{k\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Rate (q[W])	$kA_c\frac{\Delta T}{L}$	$\frac{(2\pi rL)k\Delta T}{r\ln\left(\frac{r_2}{r_1}\right)}$	$\frac{(4\pi r^2)k\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Resistance $\left[\frac{K}{W}\right]$	$\frac{L}{kA_c}$	$\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$	$\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi k}$

Find: $\dot{S}_{gen}$ and k_b



25C
1000 W/m2-K

25C
1000 W/m2-K

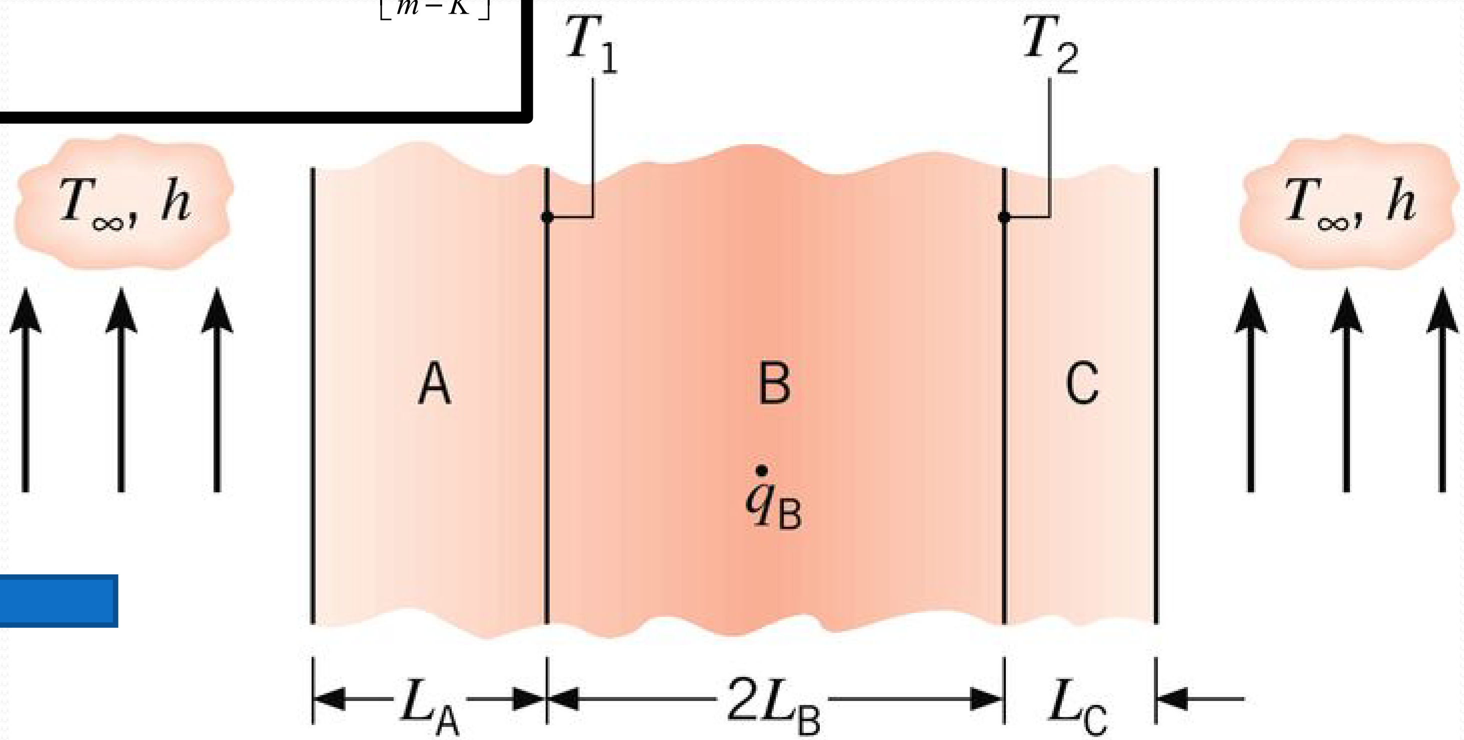
T1=261C
T2=211C

$k_A = 25$ W/m•K
$k_C = 50$ W/m•K

$L_A = 30$ mm
$L_B = 30$ mm
$L_C = 20$ mm

$$q''_A\left[\frac{W}{m^2}\right]=\frac{\Delta T}{\sum R_t}=\frac{T_1-T_\infty}{\frac{1}{h}+\frac{L_A}{k_A}}=\frac{(261-25)[K]}{\frac{1}{1000}\left[\frac{m^2-K}{W}\right]+\frac{(30/1000)[m]}{25\left[\frac{W}{m-K}\right]}}$$

$$q''_A=107.3\frac{kW}{m^2}$$



$k_A = 25$ W/m•K
$k_C = 50$ W/m•K

$L_A = 30$ mm
$L_B = 30$ mm
$L_C = 20$ mm

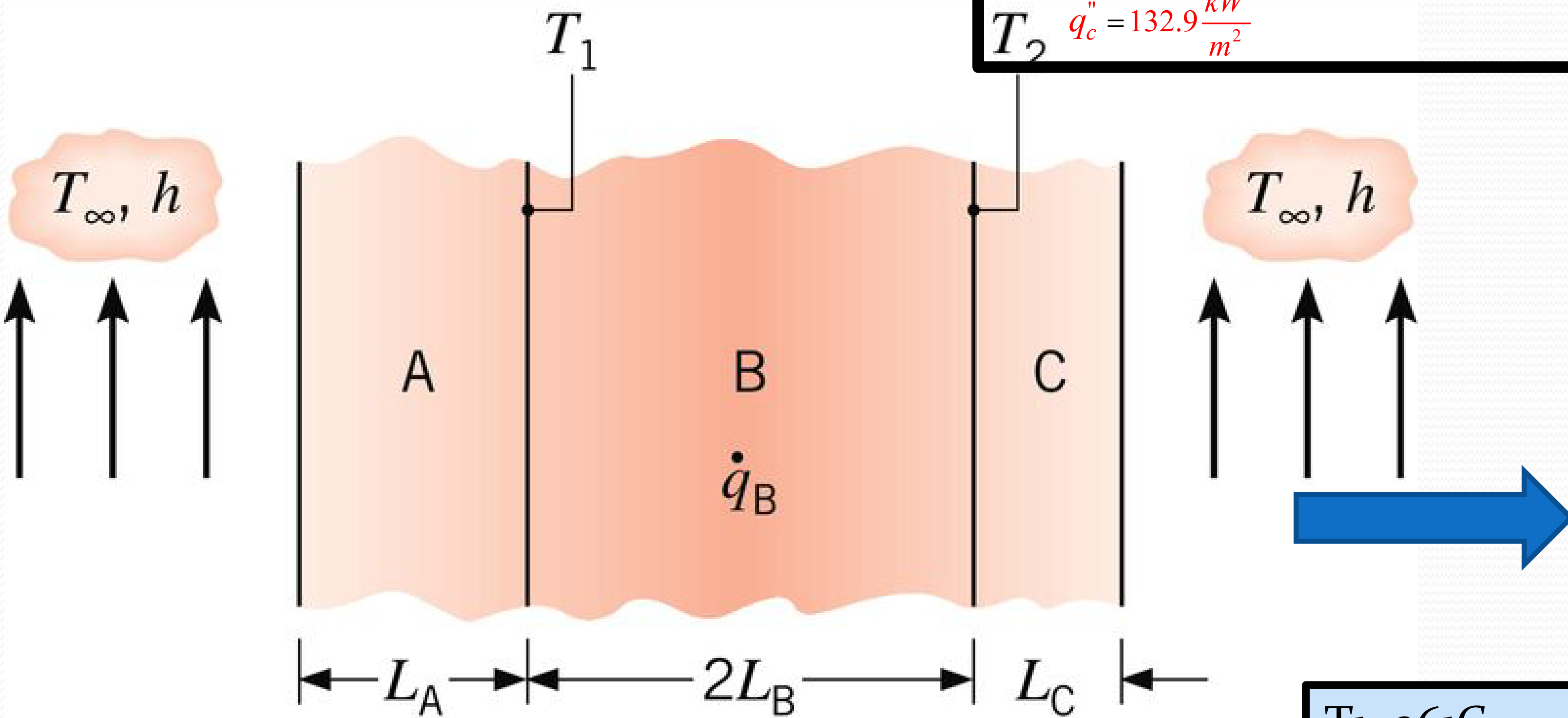
T1=261C
T2=211C

25C
1000 W/m2-K

NOTE: Thermal Circuit is written in the DIRECTION of HEAT TRANSFER

$$q_c''\left[\frac{W}{m^2}\right]=\frac{\Delta T}{\sum R_t}=\frac{T_2-T_\infty}{\frac{1}{h}+\frac{L_C}{k_C}}=\frac{(211-25)[K]}{\frac{1}{1000}\left[\frac{m^2-K}{W}\right]+\frac{(20/1000)[m]}{50\left[\frac{W}{m-K}\right]}}$$

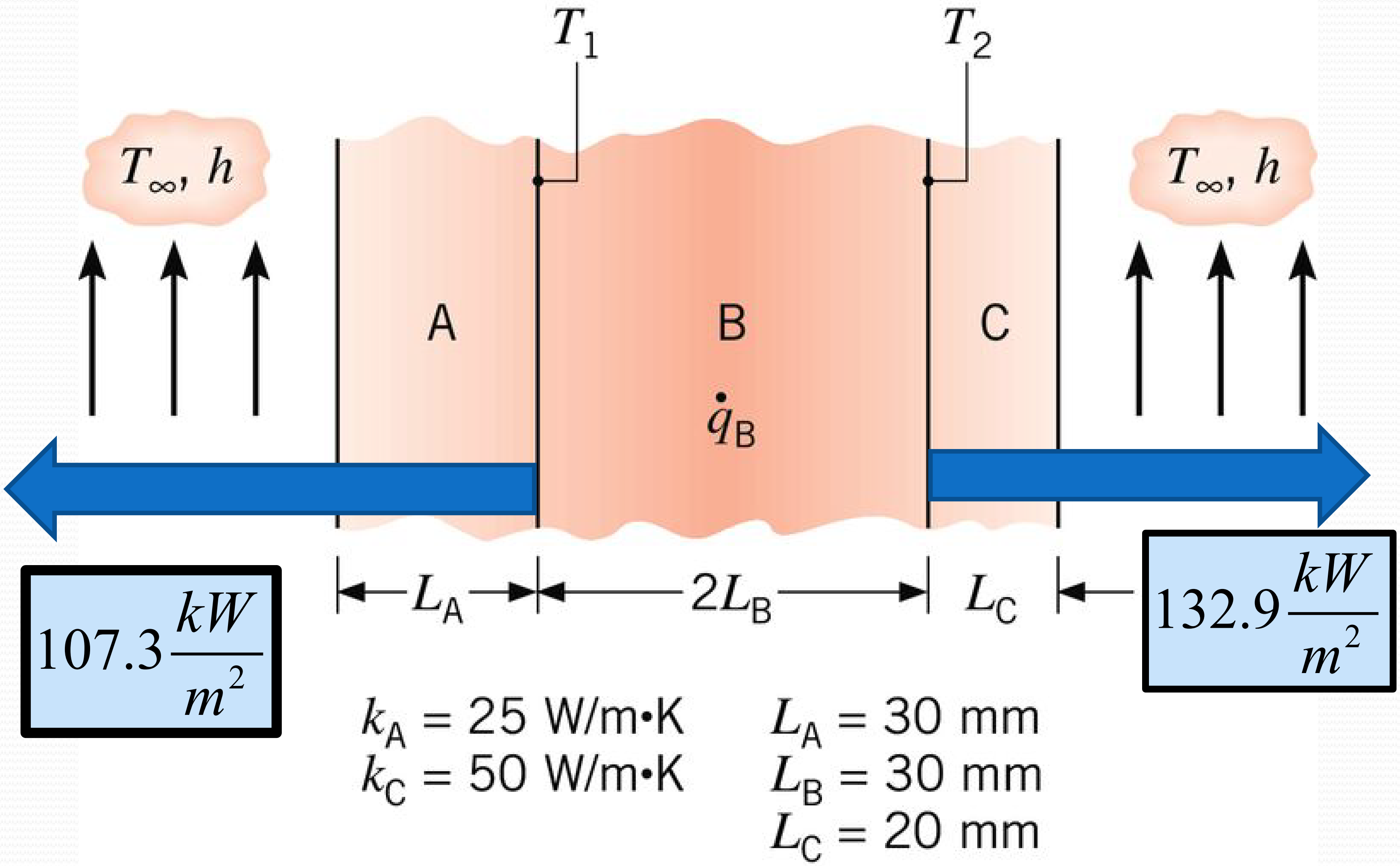
$$q_c''=132.9\frac{kW}{m^2}$$



$k_A = 25$ W/m•K $L_A = 30$ mm
$k_C = 50$ W/m•K $L_B = 30$ mm
$L_C = 20$ mm

T1=261C
T2=211C

25C
1000 W/m2-K



NOTE: Can't use THERMAL CIRCUIT within material B, because we have internal heat generation.

FIND Sgen ?

OVERALL CONTROL VOLUME

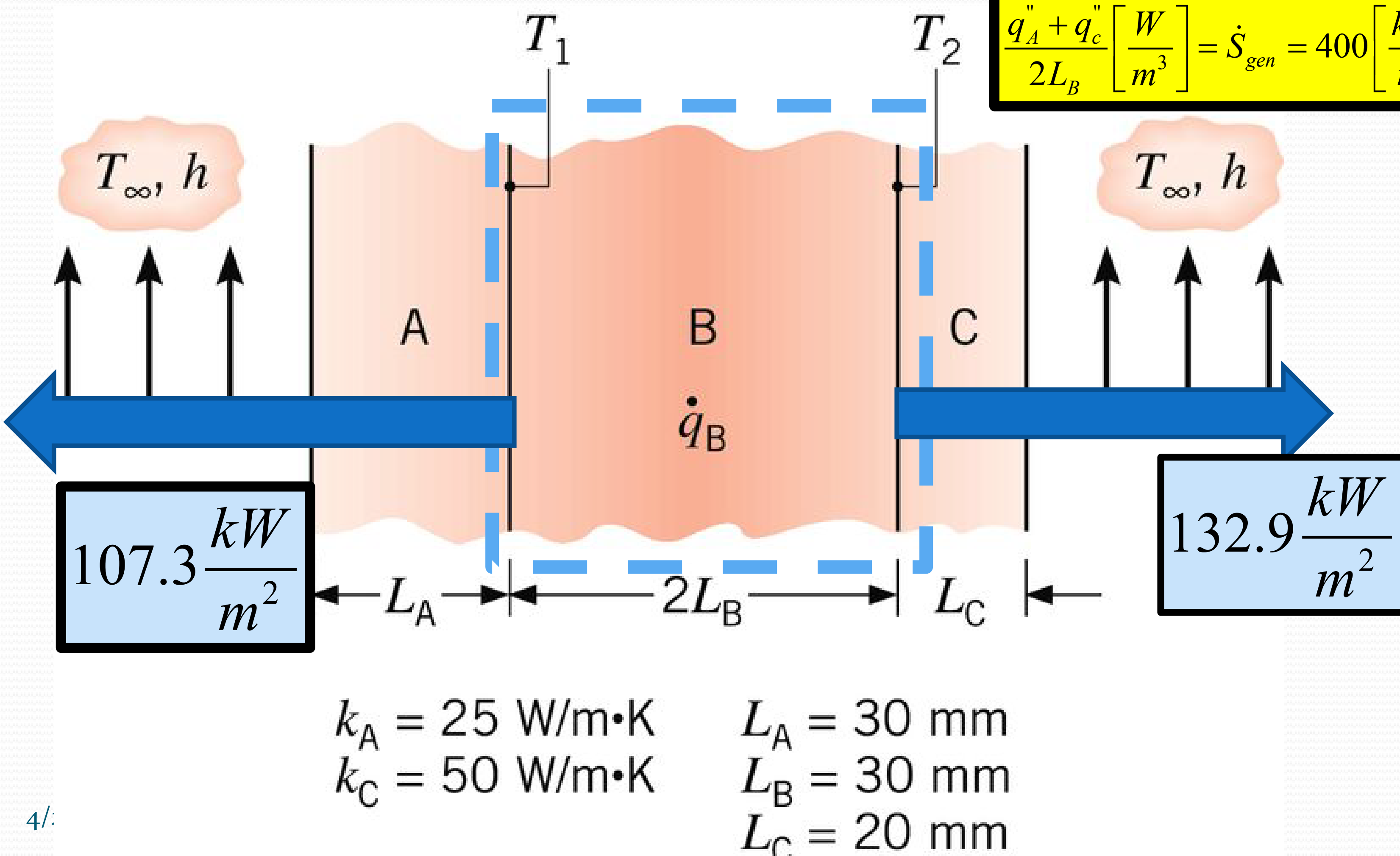
Steady State

$$\dot{E}_{out} = \dot{E}_{gen}$$

$$q''_A A_A + q''_C A_C = \dot{S}_{gen} \forall_B$$

$$\frac{q''_A A_A + q''_C A_C}{\forall = A \bullet 2L_B} = \dot{S}_{gen}, \quad A_A = A_C = A$$

$$\frac{q''_A + q''_c}{2L_B}\left[\frac{W}{m^3}\right] = \dot{S}_{gen} = 400\left[\frac{kW}{m^3}\right]$$



Find Kb=?

HDE

$$\frac{d^2T}{dx^2} = -\frac{\dot{S}_{gen}}{k_b}, 0 \le x \le 2L_B$$

3 Unknowns

3 Boundary Conditions (C_1, C_2, k_b)

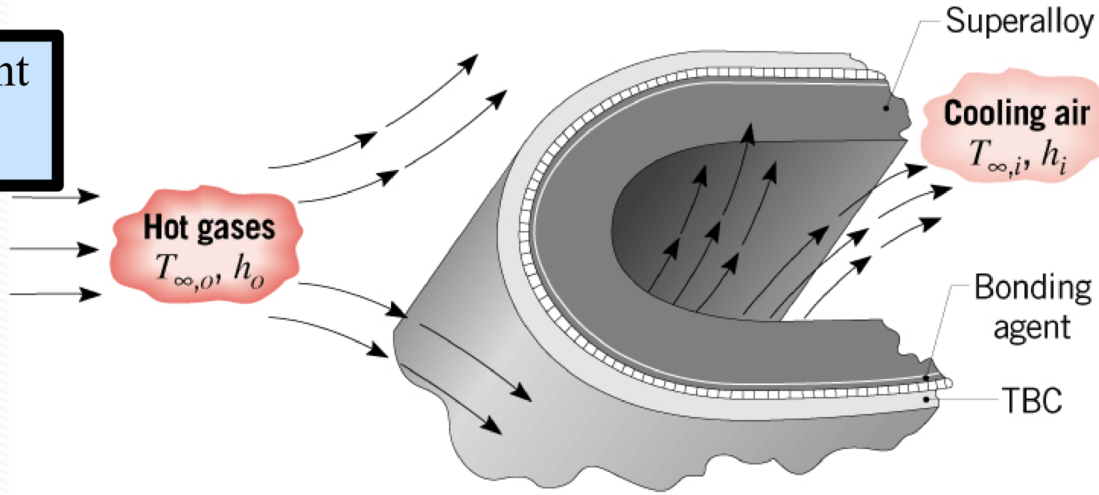
$T(x=0)=T_1$

$$T(x = 2L_B) = T_2$$

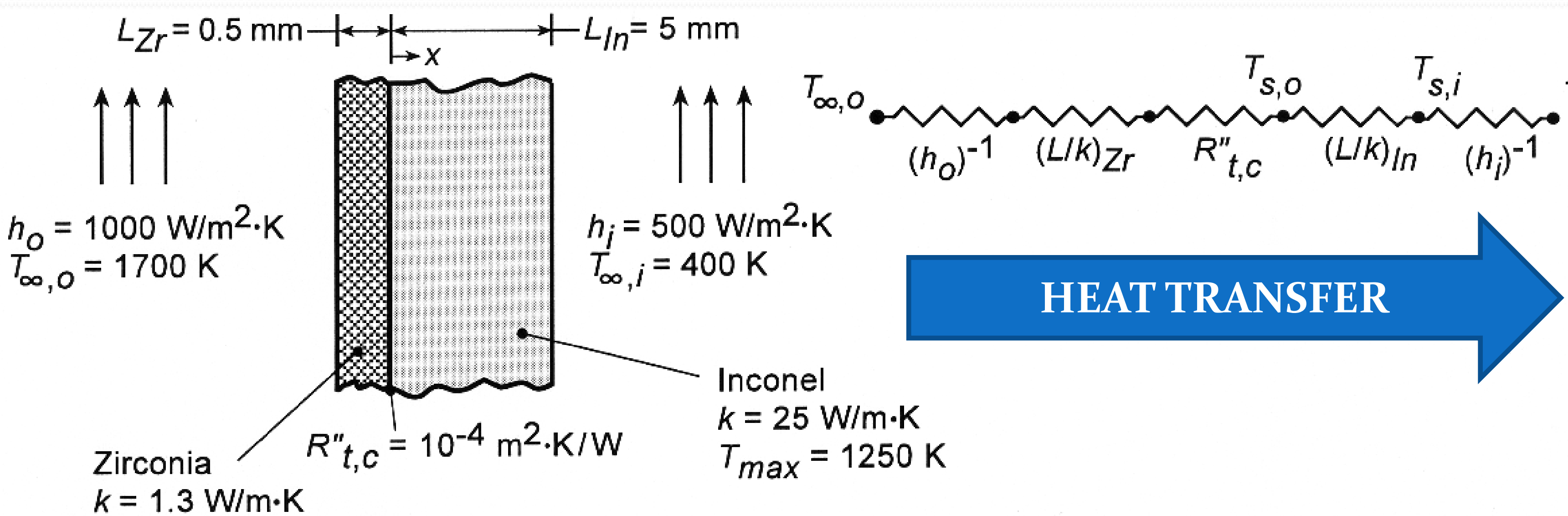
$$q''_A = -k_B \left.\frac{dT}{dx}\right|_{x=0} = 107,300W$$

Problem 3.30: Assessment of thermal barrier coating (TBC) for protection of turbine blades. Determine maximum blade temperature with and without TBC.

Assume: 1D, SS, Constant Properties, NO Radiation



Schematic:



ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$\boxed{R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1} \rightarrow \frac{K - m^2}{W}}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right)\mathrm{m^2\cdot K/W} = 3.69\times10^{-3}\ \mathrm{m^2\cdot K/W}$$

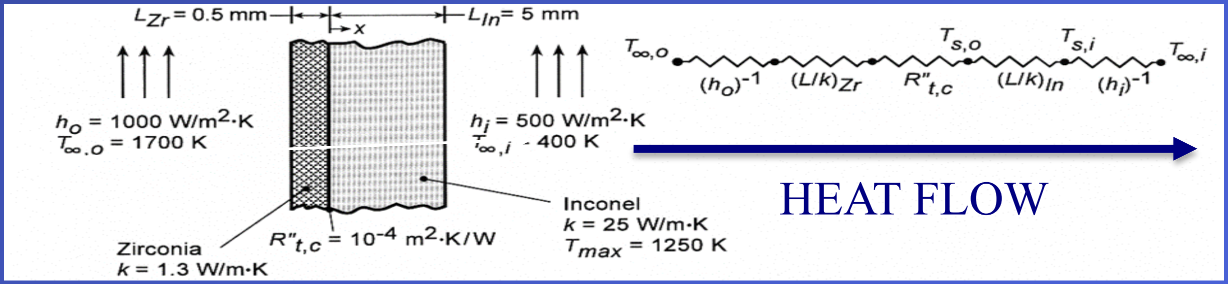
With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,\mathrm{K}}{3.69\times10^{-3}\ \mathrm{m^2\cdot K/W}} = 3.52\times10^{5}\ \mathrm{W/m^2}$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(\mathrm{w})} = T_{\infty,i} + (q''_w/h_i) = 400\,\mathrm{K} + \left(\frac{3.52\times10^{5}\ \mathrm{W/m^2}}{500\ \mathrm{W/m^2\cdot K/W}}\right) = 1104\,\mathrm{K}$$

$$T_{s,o(\mathrm{w})} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_w = 400\,\mathrm{K} + \left(2\times10^{-3} + 2\times10^{-4}\right)\mathrm{m^2\cdot K/W}\left(3.52\times10^{5}\ \mathrm{W/m^2}\right) = 1174\,\mathrm{K} \quad <$$



Without the TBC,

$$R''_{\mathrm{tot,wo}} = h_o^{-1} + (L/k)_{\mathrm{In}} + h_i^{-1} = 3.20 \times 10^{-3}\ \mathrm{m}^2 \cdot \mathrm{K/W}$$

$$q''_{\mathrm{wo}} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{\mathrm{tot,wo}} = 4.06\times10^5\ \mathrm{W/m^2}$$

The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(\mathrm{wo})} = T_{\infty,i} + \left(q''_{wo}/h_i\right) = 1212\ \mathrm{K}$$

$$T_{s,o(\mathrm{wo})} = T_{\infty,i} + \left[\left(1/h_i\right) + \left(L/k\right)_{\mathrm{In}}\right] q''_{wo} = 1293\ \mathrm{K}$$

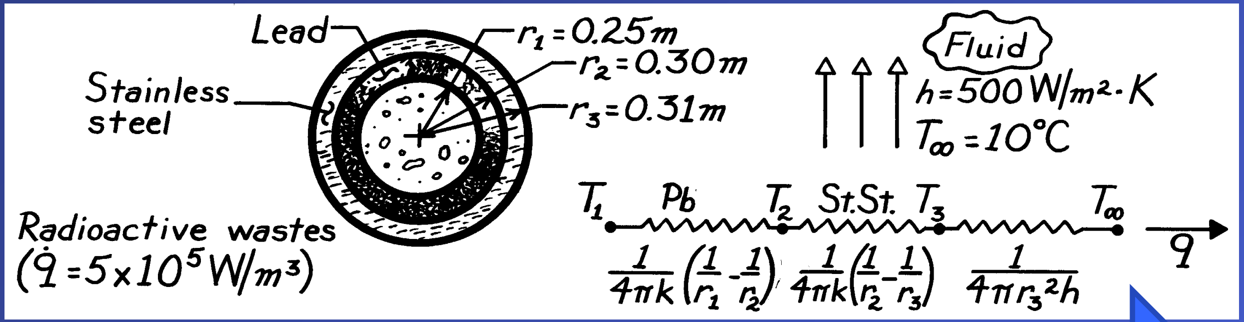
$<$

Use of the TBC facilitates operation of the Inconel below $T_{\max} = 1250$ K.

COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to its thickness are associated with reliability considerations.

Problem 3.72: Suitability of a composite spherical shell for storing radioactive wastes in oceanic waters.

SCHEMATIC:



HEAT TRANSFER

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300K, (4) Negligible contact resistance.

PROPERTIES: Table A-1, Lead: k = 35.3 W/m·K, MP = 601 K; St.St.: k = 15.1 W/m·K.

ANALYSIS: From the thermal circuit, it follows that

$$q = \frac{T_1 - T_\infty}{R_{tot}} = \dot{q}\left[\frac{4}{3}\pi\ r_1^3\right]$$

Sgen = 0.0
STEADY STATE
HOMEGENOUS

The thermal resistances are:

$$R_{\text{Pb}} = \left[1/\left(4\pi \times 35.3 \text{ W/m}\cdot\text{K}\right)\right] \left[\frac{1}{0.25\text{m}} - \frac{1}{0.30\text{m}}\right] = 0.00150 \text{ K/W}$$

$$R_{\text{St.St.}} = \left[1/\left(4\pi \times 15.1 \text{ W/m}\cdot\text{K}\right)\right] \left[\frac{1}{0.30\text{m}} - \frac{1}{0.31\text{m}}\right] = 0.000567 \text{ K/W}$$

$$R_{\text{conv}} = \left[1/\left(4\pi \times 0.31^2 \text{m}^2 \times 500 \text{ W/m}^2\cdot\text{K}\right)\right] = 0.00166 \text{ K/W}$$

$$R_{\text{tot}} = 0.00372 \text{ K/W}$$

The heat rate is then

$$q = 5\times10^5 \text{ W/m}^3 (4\pi/3)(0.25\text{m})^3 = 32,725 \text{ W}$$

and the inner surface temperature is

$$T_1 = T_\infty + R_{\text{tot}} q = 283 \text{ K} + 0.00372 \text{ K/W}(32,725 \text{ W})$$

$$= 405 \text{ K} < \text{MP} = 601 \text{ K}$$

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Hence, from the thermal standpoint, the proposal is adequate.

COMMENTS: In fabrication, attention should be given to maintaining a good thermal contact. A protective outer coating should be applied to prevent long term corrosion of the stainless steel.