MECH-420 Heat Transfer

STUDY AID

THERMAL CIRCUITS + HDE + ENERGY BALANCE

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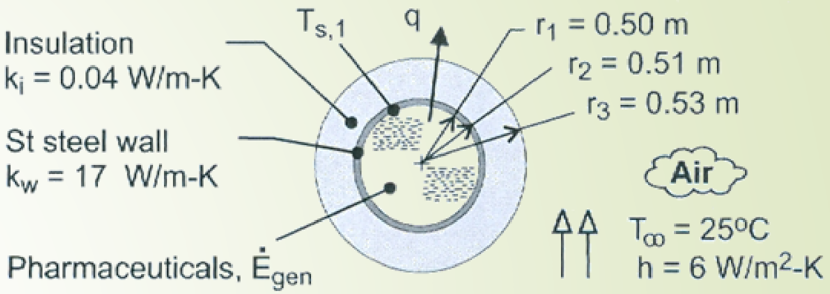
A "spherical" vessel used as a reactor for producing pharmaceuticals has a 10 mm thick stainless-steel wall and an inner diameter of 1m, as shown below with insulation. The exterior surface (r3) of the vessel is exposed to ambient air with a convective heat transfer coefficient and the thermal conductivity of the pharmaceuticals is 8.55 W/m2-K., with an internal heat generation rate of

$$S_{gen}(r) = S_o(2.5[\ ]r^2 - \left(\frac{r}{r_1}\right)^{2.5});0 \le r \le r_1; V = \frac{4}{3}\pi r^3, S_o = 50{,}000\left[\frac{W}{m^3}\right]$$



a) What is roadmap to determine resistances for thermal circuit.

$$R_{th} = \frac{\frac{1}{r_1}+\frac{1}{r_2}}{4\pi k_{steel}} + \frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}} + \frac{1}{h\frac{4}{3}\pi r_3^2}, r_1 = 0.5m, r_2 = 0.51m, r_3 = 0.53m$$

$$\overset{\leftrightarrow}{q} = \frac{\Delta T}{\sum R_t} \rightarrow \text{DETERMINE } \pm \dot{E}_{gen} \text{ to determine direction of } \overset{\leftrightarrow}{q}$$

3 What is the STEADY heat loss from the vessel [W] AND VERIFY UNITS?

$$\dot{E}_{in}[W] - \dot{E}_{out}[W] + \dot{E}_{gen}[W] = \dot{E}_{st}[W] = \rho \forall c \frac{dT}{dt} = 0$$

$$\dot{E}_{gen} \rightarrow \pm \rightarrow \text{SOURCE or SINK}$$

$$S_{gen}(r)\left[\frac{W}{m^3}\right] = S_o\left[\frac{W}{m^3}\right](2.5\left[\frac{1}{m^2}\right]r^2 - \left(\frac{r}{r_1}\right)^{2.5}); V = \frac{4}{3}\pi r^3; dV[m^3] = 4\pi r^2 dr[m^3]$$

$$\dot{E}_{gen}[W] = \int_V S_{gen}(r)dV = 4\pi S_o \int_0^{r_1}(2.5\left[\frac{1}{m^2}\right]r^4 - \left(\frac{r^{4.5}}{r_1^{2.5}}\right))dr$$

$$\dot{E}_{gen}[W](r_1) = 4\pi S_o\left[\frac{W}{m^3}\right]\left[\frac{2.5\left[\frac{1}{m^2}\right]r^5}{5} - \left(\frac{r^{5.5}}{5.5 r_1^{2.5}}\right)\right]_{0-r_1}$$

$$\dot{E}_{gen}[W](r_1) = 4\pi S_o\left[\frac{W}{m^3}\right]\left\{\left[\frac{2.5\left[\frac{1}{m^2}\right]r_1^5}{5}\right][m^3] - \left[\left(\frac{r_1^{5.5}}{5.5 r_1^{2.5}}\right)\right][m^3]\right\}[W]$$

EXTERIOR "STEADY" SURFACE TEMPERATURE

METHOD #1 → Energy Conservation

$$\dot{E}_{in}[W]-\dot{E}_{out}[W]+\dot{E}_{gen}[W]=\dot{E}_{st}[W]=\rho\forall c\frac{dT}{dt}=0$$

$$-\dot{E}_{out}[W]+\dot{E}_{gen}[W]=0$$

$$-hA_s(T_s-T_\infty)=-\dot{E}_{gen}[W]$$

$$T_s=\frac{\dot{E}_{gen}[W]}{hA_s}+T_\infty$$

$$T_s(r_1,r_3)[C]=\frac{4\pi S_o\left[\frac{W}{m^3}\right]\left\{\left[\frac{2.5\left[\frac{1}{m^2}\right]r_1^5}{5}\right][m^3]-\left[\left(\frac{r_1^{5.5}}{5.5r_1^{2.5}}\right)\right][m^3]\right\}[W]}{hA_s(=4\pi r_3^2)}+T_\infty[C]$$

2nd METHOD → THERMAL CIRCUITS

$$q[W]=\frac{\Delta T}{\sum R_{th}}$$

$$R_{th}=\frac{\frac{1}{r_1}+\frac{1}{r_2}}{4\pi k_{steel}}+\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}+\frac{1}{h\frac{4}{3}\pi r_3^2}$$

$$q[W]=\frac{T_s-T_\infty}{\frac{1}{h\frac{4}{3}\pi r_3^2}}=\dot{E}_{gen}[W](r_1)$$

$$T_s(r_1,r_3)[C]=q[W]\bullet\frac{1}{h\left[\frac{W}{m^2-K}\right]\frac{4}{3}\pi r_3^2[m^2]}+T_\infty[C]$$

EXTERIOR "INSULATION" SURFACE TEMPERATURE

METHOD → THERMAL CIRCUITS

$$q[W]=\frac{\Delta T}{\sum R_{th}}$$

$$R_{th}=\frac{\frac{1}{r_1}+\frac{1}{r_2}}{4\pi k_{steel}}+\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}+\frac{1}{h\frac{4}{3}\pi r_3^2}$$

$$q[W]=\frac{T_{INS}-T_\infty}{\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}+\frac{1}{h\frac{4}{3}\pi r_3^2}}=\dot{E}_{gen}[W](r_1)$$

$$T_{INS}(r_1,r_3)[C]=\dot{E}_{gen}[W](r_1)\bullet\frac{1}{\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}+h\left[\frac{W}{m^2-K}\right]\frac{4}{3}\pi r_3^2[m^2]}+T_\infty[C]$$

2ndMETHOD → THERMAL CIRCUITS

$$q[W]=\frac{\Delta T}{\sum R_{th}}$$

$$R_{th}=\frac{\frac{1}{r_1}+\frac{1}{r_2}}{4\pi k_{steel}}+\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}+\frac{1}{h\frac{4}{3}\pi r_3^2}$$

$$q[W]=\frac{T_{INS}-T_s}{\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}}=\dot{E}_{gen}[W](r_1)$$

$$T_{INS}(r_1,r_3)[C]=\dot{E}_{gen}[W](r_1)\bullet\frac{1}{\frac{\frac{1}{r_2}+\frac{1}{r_3}}{4\pi k_{ins}}}+T_s[C]$$

SPHERE CENTER TEMPERAURE

$$S_{gen}(r)\left[\frac{W}{m^3}\right]=S_o\left[\frac{W}{m^3}\right](2.5\left[\frac{1}{m^2}\right]r^2-\left(\frac{r}{r_1}\right)^{2.5})$$

$1D, Steady, Homogeneous$

$$\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{dT}{dr}\right)=-\frac{S_{gen}}{k}=-\frac{S_o}{k}(2.5\left[\frac{1}{m^2}\right]r^2-\left(\frac{r}{r_1}\right)^{2.5})$$

$BC\#1, @r=0, T(r=0)\rightarrow FINITE, BC\#2@r=r_1, T(r=r_1)=T_s$

multiply by r^2

$$\left(\frac{d}{dr}r^2\frac{dT}{dr}\right)=-\frac{S_o}{k}(2.5\left[\frac{1}{m^2}\right]r^4-\left(\frac{r^{4.5}}{r_1^{2.5}}\right))$$

integrate

$$r^2\frac{dT}{dr}=-\frac{S_o}{k}(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^5-\frac{1}{5.5}\left(\frac{r^{5.5}}{r_1^{2.5}}\right))+C_1$$

divide by r^2

$$\frac{dT}{dr}=-\frac{S_o}{k}(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^3-\frac{1}{5.5}\left(\frac{r^{3.5}}{r_1^{2.5}}\right))+\frac{C_1}{r^2}$$

integrate $\rightarrow$ MOST GENERAL SOLUTION

$$T(r)=-\frac{S_o}{k}(\frac{2.5}{20}\left[\frac{1}{m^2}\right]r^4-\frac{1}{5.5\bullet 4.5}\left(\frac{r^{4.5}}{r_1^{2.5}}\right))-\frac{C_1}{r}+C_2$$

$$T(r)=-\frac{S_o}{k}(\frac{2.5}{20}\left[\frac{1}{m^2}\right]r^4-\frac{1}{5.5\bullet 4.5}\left(\frac{r^{4.5}}{r_1^{2.5}}\right))-\frac{C_1}{r}+C_2$$

APPLY BC's

#1: $@r=0, T(r=0)=\text{FINITE}, C_1=0$

$\#2: @r=r_1, T(r=r_1)=T_{INS}$

$$T_{INS}=-\frac{S_o}{k}(\frac{2.5}{20}\left[\frac{1}{m^2}\right]r_1^4-\frac{1}{5.5\bullet 4.5}\left(\frac{r_1^{4.5}}{r_1^{2.5}}\right))+C_2$$

$$T_{INS}+\frac{S_o}{k}(\frac{2.5}{20}\left[\frac{1}{m^2}\right]r_1^4-\frac{1}{5.5\bullet 4.5}\left(r_1^2\right))=C_2$$

EXACT SOLUTION

$0\le r\le r_1$

$$T(r)=T_{INS}-\frac{S_o}{k}\left[\frac{2.5}{20}\left[\frac{1}{m^2}\right](r^4-r_1^4)-\frac{1}{5.5\bullet 4.5}(\left(\frac{r^{4.5}}{r_1^{2.5}}\right)-r_1^2)\right]$$

CENTER TEMPERATURE

EXACT SOLUTION

$$T(r)=T_{INS}-\frac{S_o}{k}\left[\frac{2.5}{20}\left[\frac{1}{m^2}\right](r^4-r_1^4)-\frac{1}{5.5\bullet 4.5}\left(\left(\frac{r^{4.5}}{r_1^{2.5}}\right)-r_1^2\right)\right]$$

$$T_{center}=T(r=0)$$

$$T_{center}=T_{INS}[K]-\frac{S_o\left[\frac{W}{m^3}\right]}{k\left[\frac{W}{m-K}\right]}\left[\frac{K}{m^2}\right]\left[\left\{\frac{2.5}{20}\left[\frac{1}{m^2}\right](r_1^4)\right\}[m^2]+\left\{\frac{1}{5.5\bullet 4.5}(r_1^2)\right\}[m^2]\right]$$

HEAT FLUX

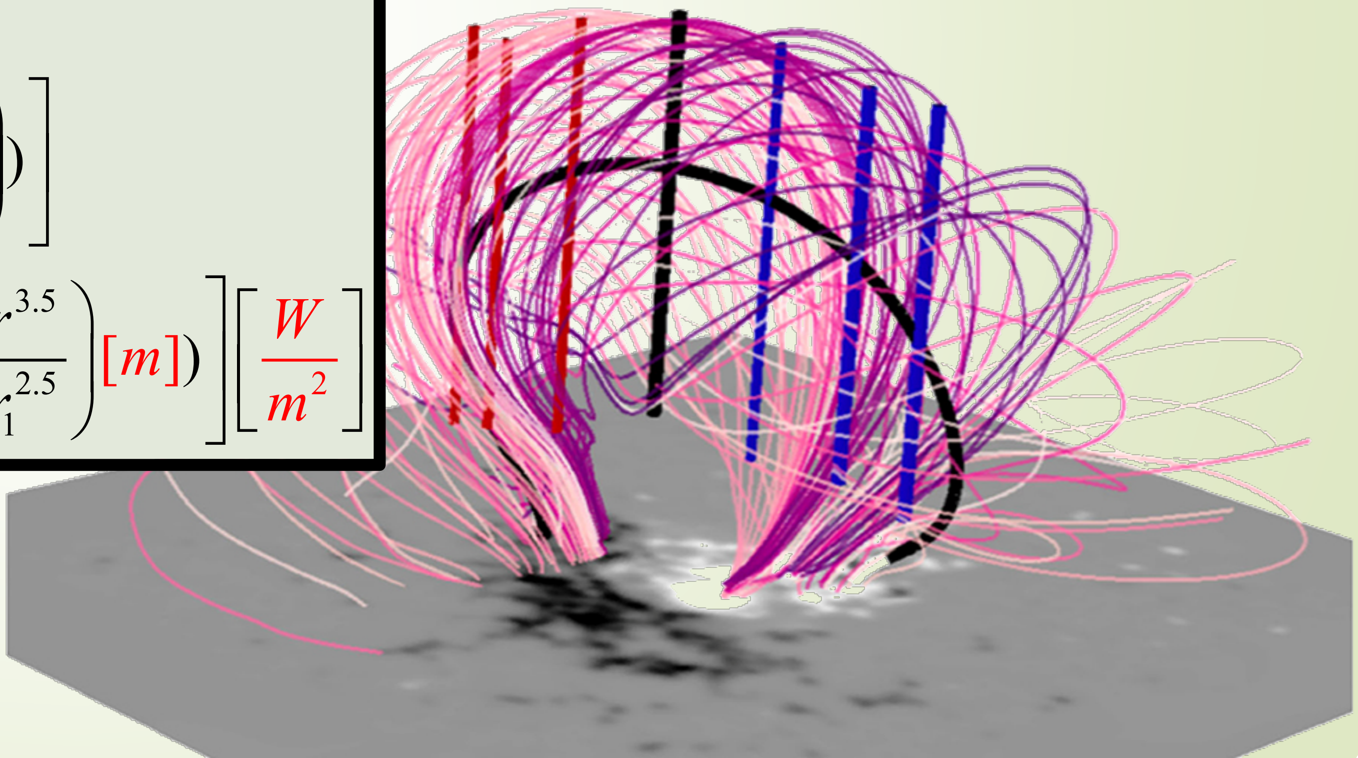
$$\frac{dT}{dr} = -\frac{S_o}{k}\left(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^3 - \frac{1}{5.5}\left(\frac{r^{3.5}}{r_1^{2.5}}\right)\right)$$

$$0 \le r \le r_1$$

$$q''\left[\frac{W}{m^2}\right](r) = -k\frac{dT}{dr}$$

$$= -k\left[-\frac{S_o}{k}\left(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^3 - \frac{1}{5.5}\left(\frac{r^{3.5}}{r_1^{2.5}}\right)\right)\right]$$

$$= \left[S_o\left[\frac{W}{m^3}\right]\left(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^3[m] - \frac{1}{5.5}\left(\frac{r^{3.5}}{r_1^{2.5}}\right)[m]\right)\right]\left[\frac{W}{m^2}\right]$$



HEAT RATE and Egen

HEAT FLUX

$$q''\left[\frac{W}{m^2}\right](r) = \left[S_o\left[\frac{W}{m^3}\right]\left(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^3[m] - \frac{1}{5.5}\left(\frac{r^{3.5}}{r_1^{2.5}}\right)[m]\right)\right]\left[\frac{W}{m^2}\right]$$

HEAT RATE

$$q[W](r) = q''\left[\frac{W}{m^2}\right](r)\bullet A_r = 4\pi r^2\left[m^2\right]\left[S_o\left[\frac{W}{m^3}\right]\left(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^3[m] - \frac{1}{5.5}\left(\frac{r^{3.5}}{r_1^{2.5}}\right)[m]\right)\right]\left[\frac{W}{m^2}\right]$$

$$= 4\pi S_o\left[\frac{W}{m^3}\right]\left[\left(\frac{2.5}{5}\left[\frac{1}{m^2}\right]r^5[m^3] - \frac{1}{5.5}\left(\frac{r^{5.5}}{r_1^{2.5}}\right)[m^3]\right)\right][W]$$

$\dot{E}_{gen} \rightarrow$ PAGE #3

$$\dot{E}_{gen}[W](r_1) = 4\pi S_o\left[\frac{W}{m^3}\right]\left\{\left[\frac{2.5\left[\frac{1}{m^2}\right]r_1^5}{5}\right][m^3] - \left[\left(\frac{r_1^{5.5}}{5.5 r_1^{2.5}}\right)\right][m^3]\right\}[W] \equiv q[W](r)$$

YES!!!!

BABY MATH & ENERGY CONSERVATION

WORK TOGETHER

Consider a Seagram's Wine Cooler refrigerated at temperature Tm = 5C after a long Heat Exam. The kitchen temperature on a hot summer day is 30C. The rectangular sides have a height of 200 mm and a width of 100 mm. As the carton is taken from the refrigerator, it experiences a convective heat transfer coefficient of 10 W/m2-K and has a surface emissivity of 0.90. The density is 2700 kg/m3, and the specific heat is 900 J/kg-K, and assume the surrounding temperature is 30C.

a) Applying the 1st Law, what is the parametric road map expression to determine the initial rate of change of temperature.

$$\dot{E}_{in}[W] - \dot{E}_{out}[W] + \dot{E}_{gen}[W] = \dot{E}_{st}[W] = \rho \forall c \frac{dT}{dt}$$

$$\frac{dT}{dt}\left\{\frac{K}{s}\right\} = \frac{\cancel{\dot{E}_{in}[W]} - \dot{E}_{out}[W] + \cancel{\dot{E}_{gen}[W]}}{\rho \forall c}$$

$$= \frac{-\dot{E}_{out}[W]}{\rho \forall c}$$

$$= \frac{-\left\{hA_s(T_i[K] - T_\infty[K]) + \varepsilon\sigma(T_i^4[K]^4 - T_{surr}^4[K]^4)\right\}}{\rho \forall c}$$