



THERMAL RADIATION

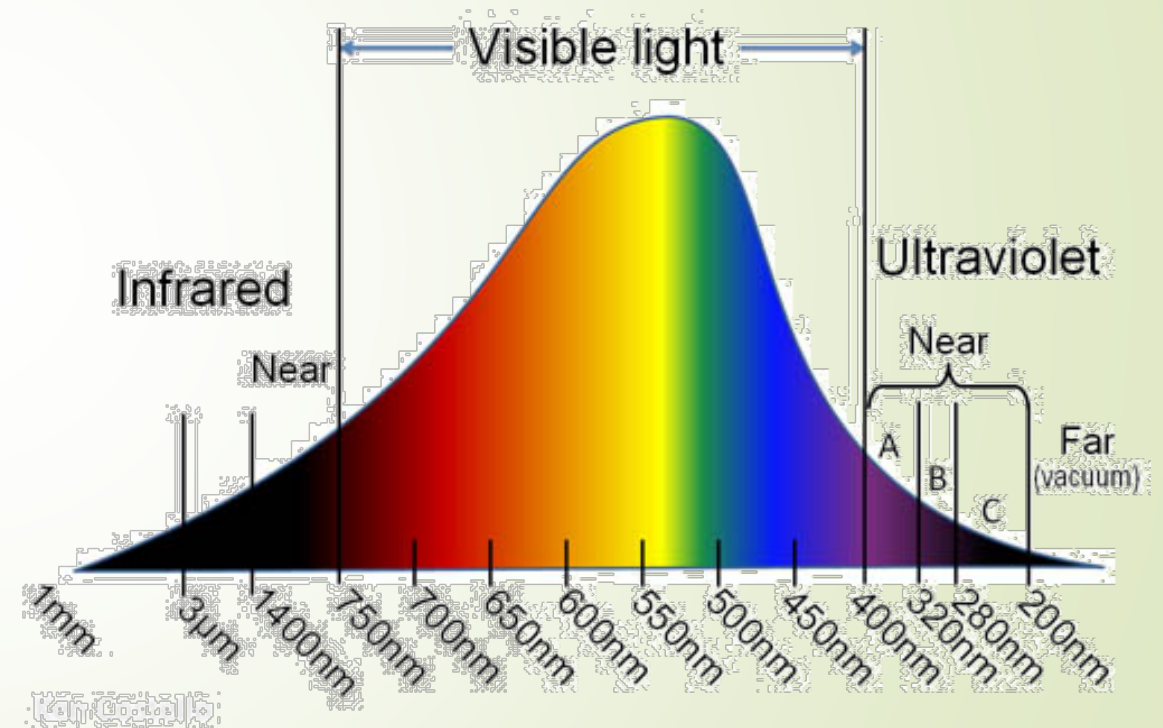
MECH-420 Heat Transfer

Dr. K. J. Berry



THERMAL RADIATION

- ▶ We associate thermal RADIATION WITH THE RATE AT WHICH **ENERGY IS EMITTED BY MATTER AS A RESULT OF ITS FINITE TEMPERATURE**. Thermal radiation is emitted by all matters that surrounds you: by the room walls, by the ground, buildings, and even the atmosphere and the sun.
- ▶ The **MECHANISM OF EMISSION** is related to energy released as a result of oscillations or translations of the many electrons that constitute matter. These oscillations are, in turn, sustained by the internal energy, and therefore the temperature, of the matter.
- ▶ Hence, we associate the emission of thermal radiation with thermally excited conditions within the matter.



Electromagnetic Wave

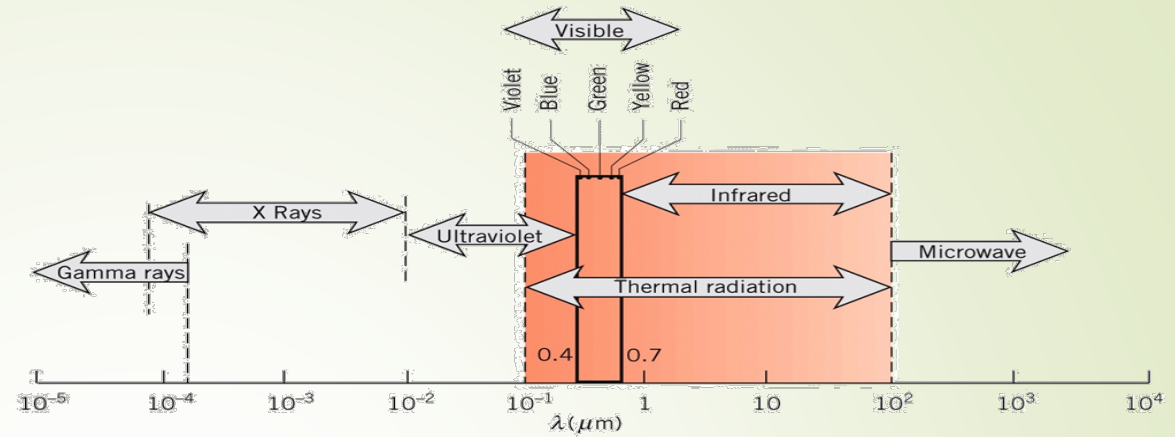
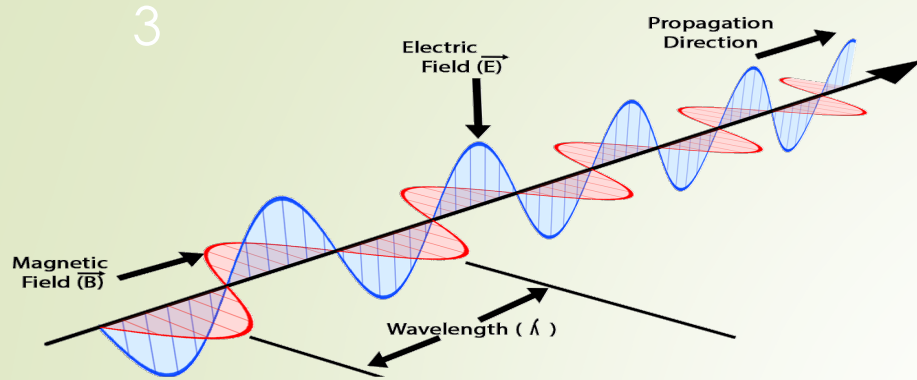


Figure 12.3
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Radiation may be viewed as the propagation of “electromagnetic waves” with a frequency of ν and a wavelength of λ , where the properties are related by: $\lambda = \frac{c}{\nu}$, where “c” is the speed of light in the medium. For example, for a vacuum, $c = 2.998 \times 10^8 \text{ m/s}$. The complete electromagnetic spectrum is delineated below.

WAVE PROPAGATION



BLACKBODY RADIATION

- ▶ A “BLACKBODY” is an ideal surface with the following attributes:
- ▶ A blackbody **absorbs all incident radiation**, regardless of wavelength and direction.
- ▶ For a prescribed temperature and wavelength, **no surface can emit more energy** than a blackbody.
- ✓ Although the radiation emitted by a blackbody is a function of wavelength and temperature, it's **independent of direction**.



TOTAL EMISSIVE POWER

The total emissive power (radiation heat flux) of a blackbody over all wavelengths can be determined as:

$$E_b \left[\frac{W}{m^2} \right] = \int_0^{\infty} E_{\lambda}(\lambda) d\lambda = \sigma T^4 [K^4]$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 - K^4}; \text{ Stefan-Boltzmann Constant}$$

Therefore, the total radiation heat transfer rate for a blackbody with area surface, A_s , and Temperature, T_s is:

$$q[W] = A_s \sigma T_s^4$$

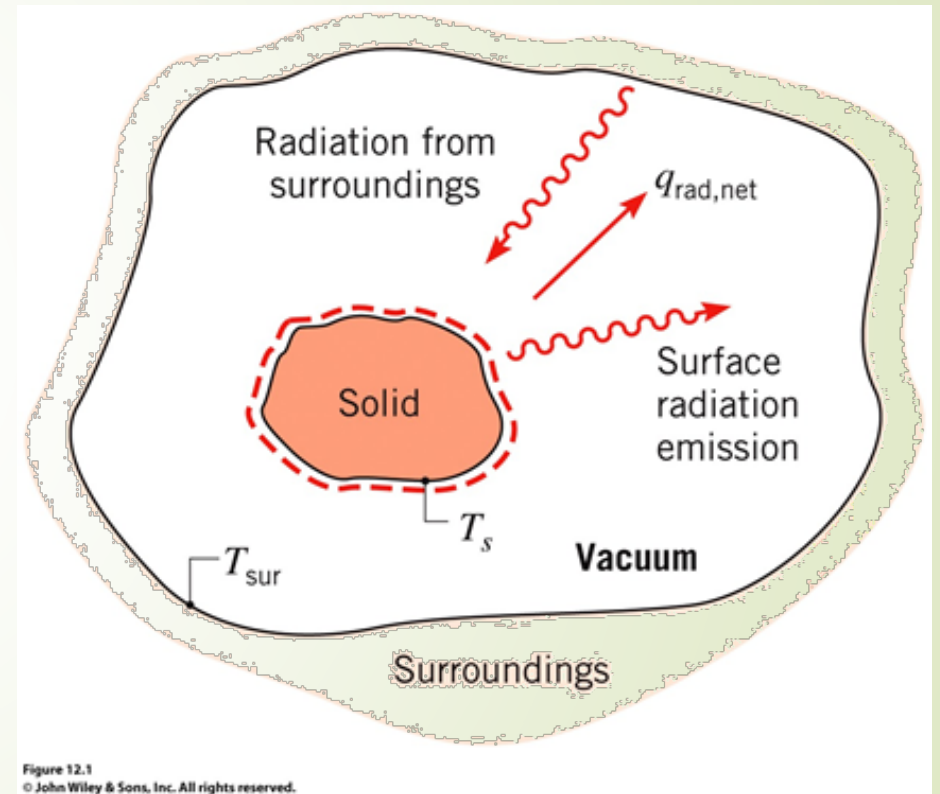


NET RADIATION EXCHANGE

► Consider a small blackbody object at Temperature T_s and completely enclosed and exchanging radiation with the surroundings at Temperature $T_{sur} < T_s$ as shown below.

The 'net' radiation exchange between the blackbody and the surrounding enclosure is:

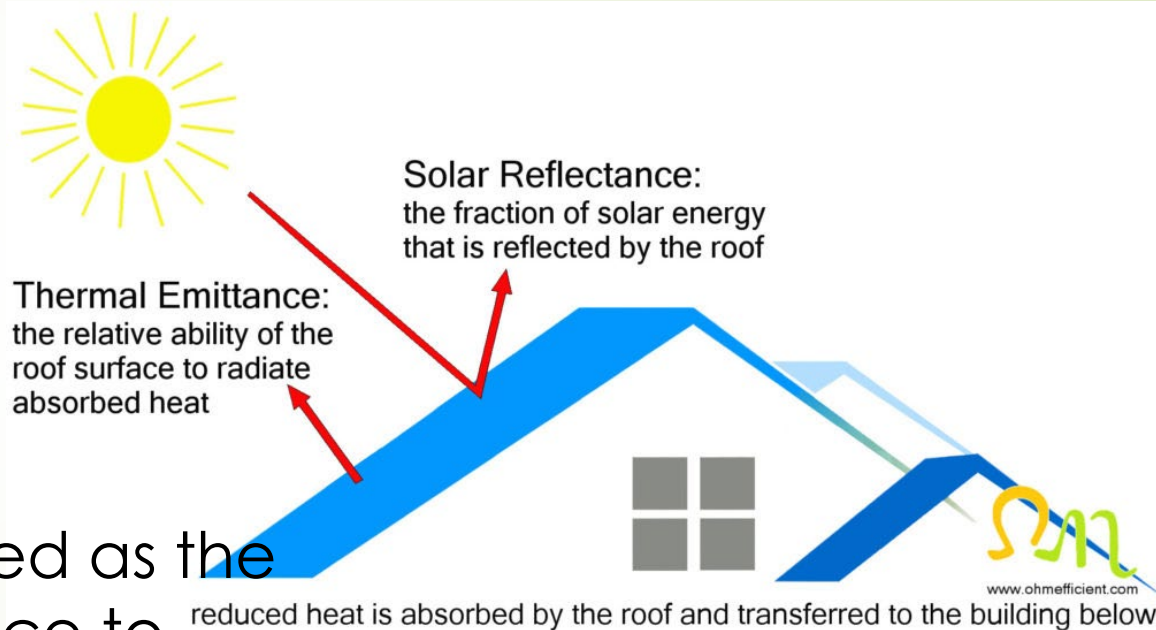
$$q''_{rad} \left[\frac{W}{m^2} \right] = \sigma \frac{W}{m^2 - K^4} (T_s^4 - T_{surr}^4)$$



EMISSION FROM REAL SURFACES

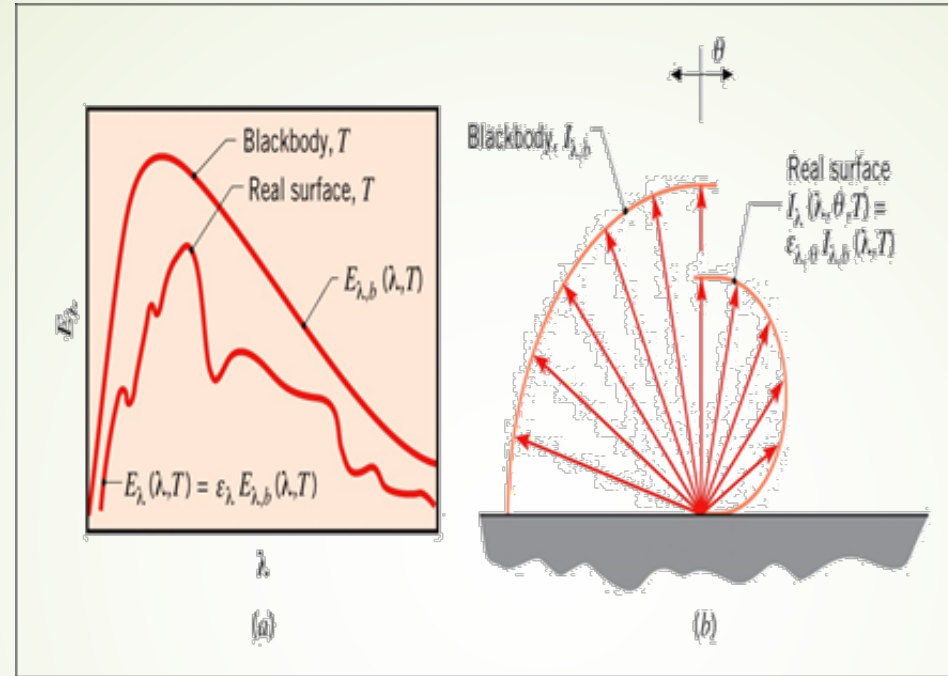
A blackbody is an **ideal emitter** in the sense that no surface can emit more radiation than a blackbody at the same temperature. A **“real” surface has a resistance** therefore will have a radiative emissive power less than a blackbody and is designated as a surface property known as the surface “emissivity”, ϵ

The **surface emissivity** may then be defined as the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature.



The figure shows the comparison between a real surface and a blackbody or ideal surface (a), and the directional distribution comparison (b). Therefore, the emissive power of a ‘real’ surface is with area A_s and Temperature T_s is:

$$E[W] = \varepsilon A_s \sigma T_s^4$$



8 Blackbody vs Real Surfaces

REAL SURFACE RESISTANCE

Every “**real**” surface has a resistance to thermal radiation emission. This resistance and net radiation heat transfer exchange can be expressed by:

$$q_{net} = \frac{E_b - J}{\frac{1 - \varepsilon}{\varepsilon A}}$$

Where $E_b - J$ is the driving surface potential and where J is known as the surface Radiosity (W/m²) and $\frac{1 - \varepsilon}{\varepsilon A}$ is the surface resistance to radiation emission. Note for a blackbody $\varepsilon = 1$, and the resistance goes to zero.

Radiation Balance – 2 Surface Problem

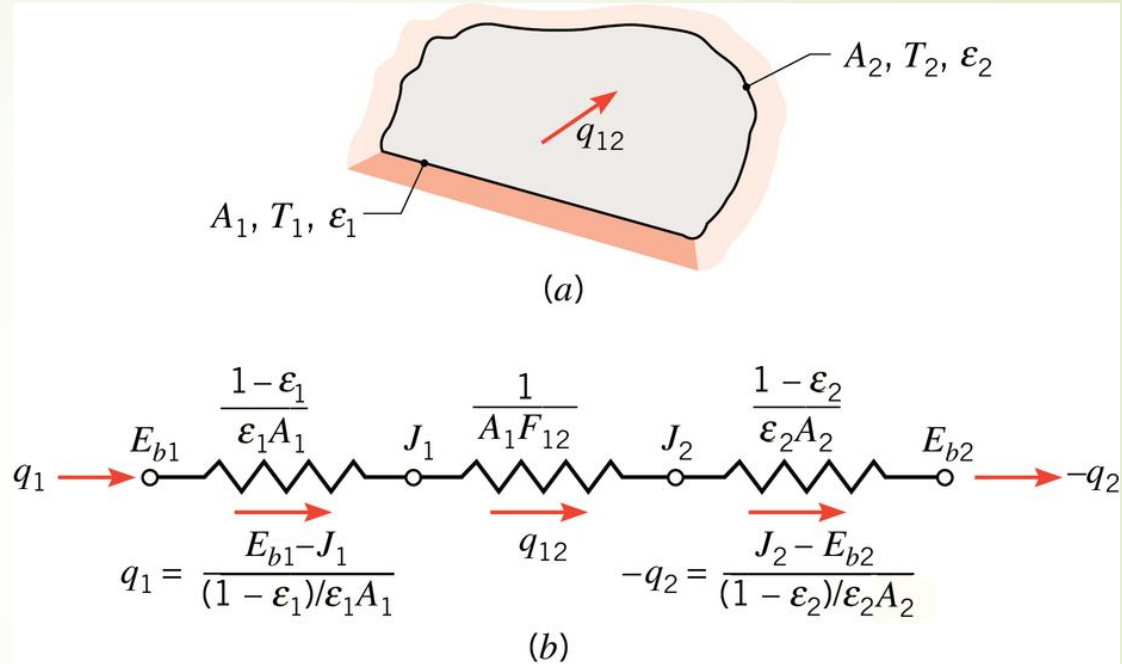
10

$$1: q_1 = \frac{E_{b1} - J_1}{\frac{1 - \epsilon_1}{\epsilon_1 A_1}} = \frac{J_1 - J_2}{A_1 F_{12}}$$

$$2: q_2 = \frac{E_{b2} - J_2}{\frac{1 - \epsilon_2}{\epsilon_2 A_2}} = \frac{J_2 - J_1}{A_2 F_{21}}$$

Two equations and two unknowns for J_1 and J_2 .

Assuming T_1 and T_2 are known.



"n" Surfaces Exchange

To complete the exchange analysis we need to consider a radiation energy balance for each surface shown above to the right. Due to the distance between surface and the **RELATIVE SHAPE** of each surface, not all the energy that is emitted by surface "1", say will reach surface "2".

This distance and geometry differences result in a "surface" resistance for surface "i" of the form: $\frac{1}{A_i F_{ij}}$

. Where F_{ij} (shape/view factor) is the fraction of energy that leaves surface "i", and strikes surface "j" directly.

So a radiation balance of an arbitrary surface "i" and exchanging radiation with "n" other surfaces (including itself) becomes:

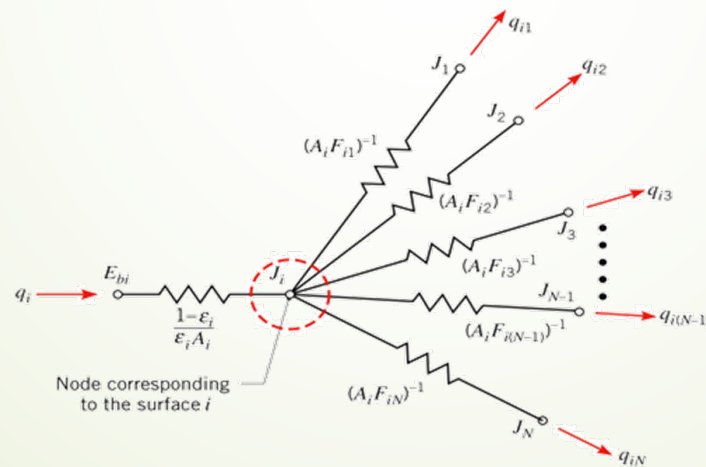


Figure 13.10
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$$\frac{E_{bi} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}} = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}}$$

applied to every surface if "T" is known and once J's are known of if "q" is known:

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{bi} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

SHAPE FACTORS

SHAPE FACTORS--RECIPROCALITY

The shape factor $A_i F_{ij}$ is the fraction of energy that leaves surface "i" and strikes surface "j". Of course this must be a reciprocal relationship, i.e.:

In general for any two arbitrary surfaces

$$A_i F_{ij} = A_j F_{ji}$$

Example, for surfaces 1 and 2

$$A_1 F_{12} = A_2 F_{21}$$

Also, since the "F" represents a fraction of the total energy leaving a surface and since energy is conserved the following summation rule applies as:

$$\sum_{j=1}^n F_{ij} = 1; \text{ for every surface}$$

for example for surface 1:

$$F_{11} + F_{12} = 1; \text{ and}$$

for surface 2, etc.

$$F_{21} + F_{22} = 1$$

INSULATED SURFACES AND SURFACES WITH LARGE AREAS

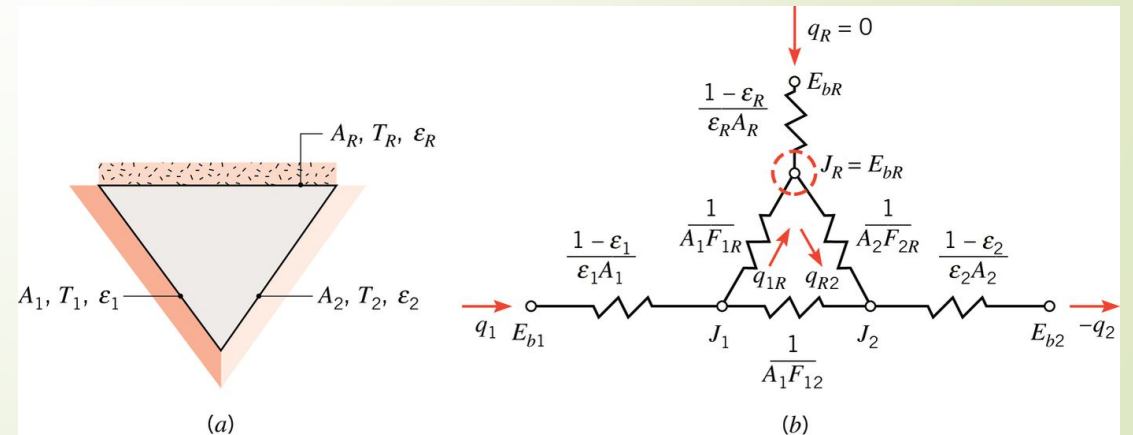
► For **insulated** surfaces (re-re-radiating), this behaves “like” a **blackbody** and as such the surface resistance goes to zero and $E_b = J$ ($q = 0$).

► Likewise for “large” areas (i.e. **LARGE ROOM**), the surface resistance approaches zero and once again the reduction in the thermal circuit becomes $E_b = J$ ($q = 0$). For example, consider the thermal circuit for the following 3-surface problem with one insulated surface.



Surface "r" Balance

$$\frac{J_r - J_1}{\frac{1}{A_r F_{r1}}} + \frac{J_r - J_2}{\frac{1}{A_r F_{r2}}} = 0$$



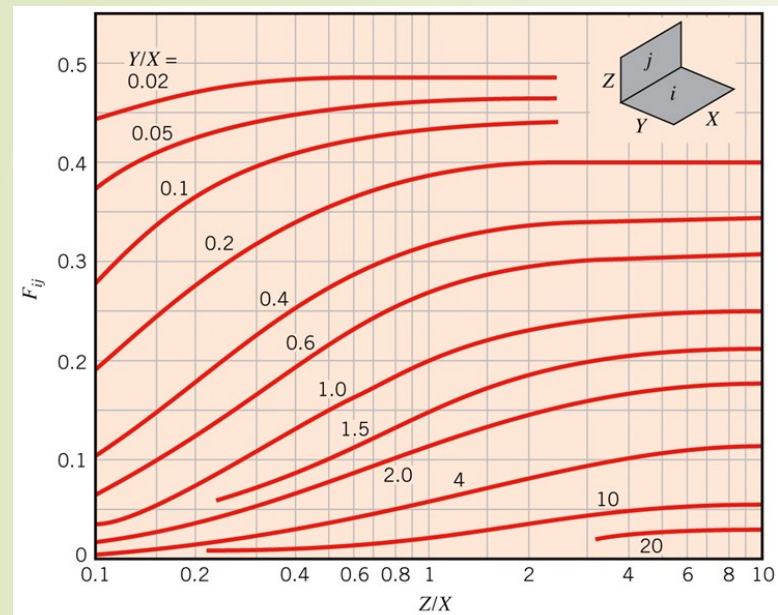


Figure 13.6
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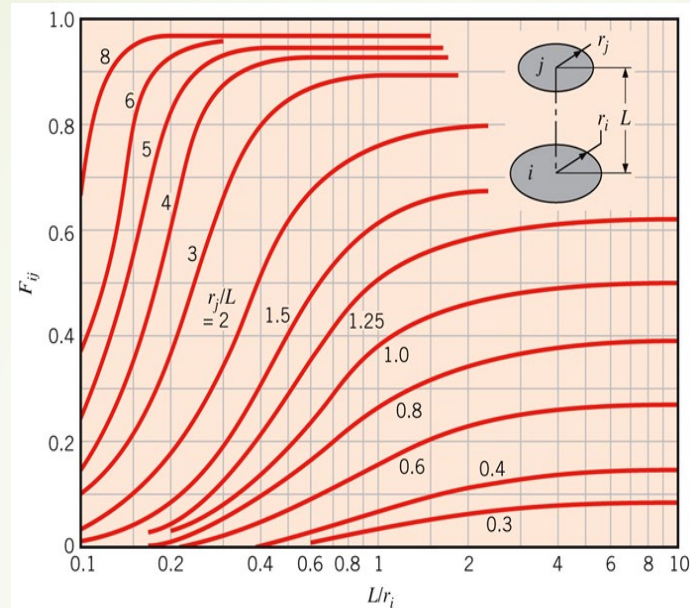


Figure 13.5
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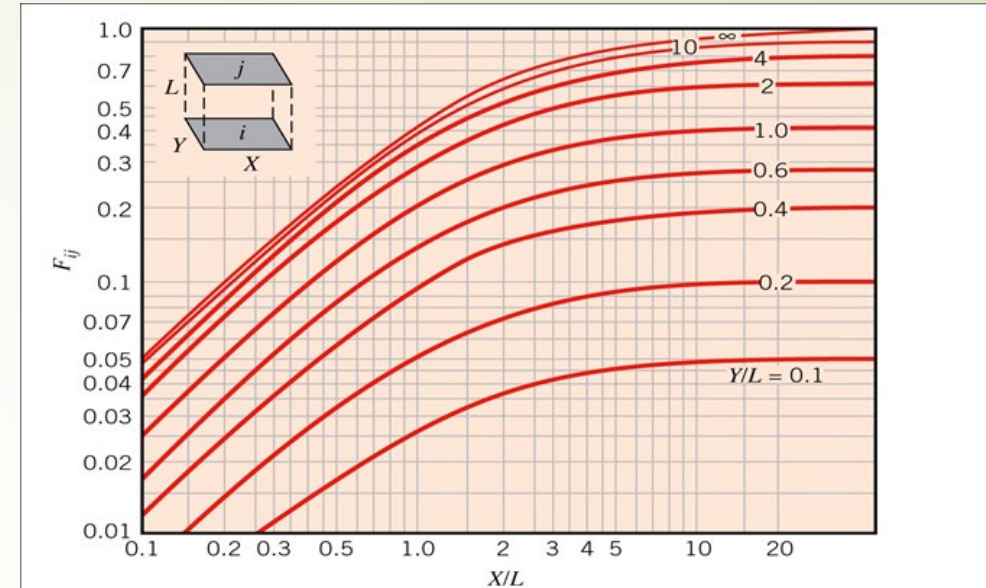


Figure 13.4
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14 View Factors Standard Surfaces

A small oven consists of a "cubical" box, with power supplied from the floor.

The walls loose heat to the outside surroundings with wall Temp = 400K.

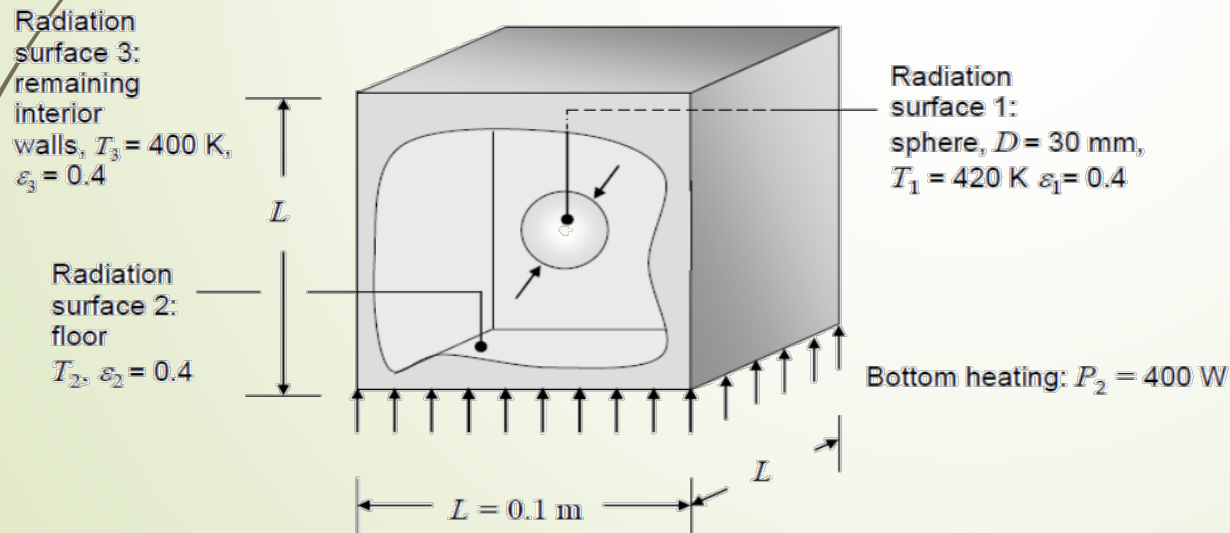
A spherical object is placed in the center, $D = 30\text{mm}$.

Sometime after the sphere is placed in the oven, the sphere Temp (T_1) = 420K.

Find:

- View Factors, F_{12} , F_{13} , F_{21} , F_{31} , F_{23} , F_{32}
- floor temperature,
- Is it steady state?

SCHEMATIC:



Assumptions:

- Surfaces are grey and diffuse;
- Uniform irradiation and radiosity,
- All floor power goes to heater, no storage,
- No Convection

ANALYSIS

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Due to symmetry, view factor from the sphere to each of the SIX walls must be equal, thus:

Surface: 1 = Sphere, 2 = Floor, 3 = Other Walls

$F_{12} = 1/6$, → fraction leaving "1" and strikes "2"

$F_{13} = 5/6$, → fraction leaving "1" and strikes "3"

APPLY RECIPROCALITY

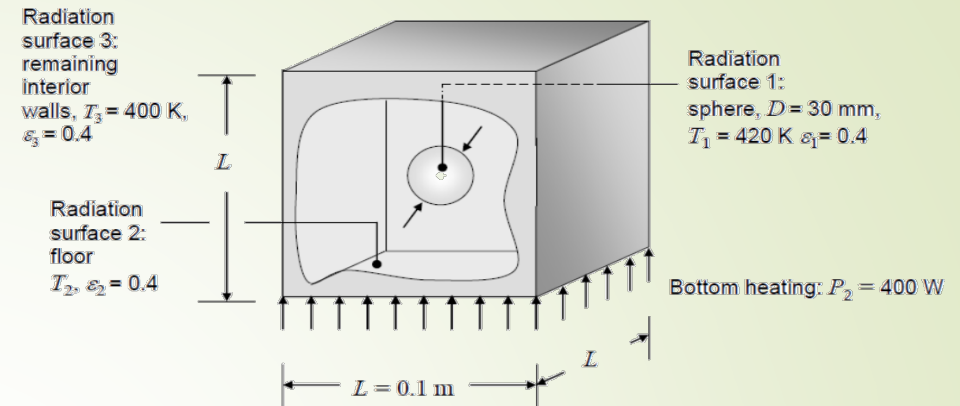
$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{4\pi r^2 \cdot F_{12}}{L^2} = \frac{4\pi 0.015^2 \cdot \frac{1}{6}}{0.1^2} = 0.0471$$

$$A_3 F_{31} = A_1 F_{13}$$

$$F_{31} = \frac{A_1 F_{13}}{A_3} = \frac{4\pi r^2 \cdot F_{13}}{5L^2} = \frac{4\pi 0.015^2 \cdot \frac{5}{6}}{5 \cdot 0.1^2} = 0.0471$$

SCHEMATIC:



CONSERVATION

$$\sum_i F_{ij} = 1$$

surface 2 does not see itself, $i=2$

$$F_{21} + F_{22} + F_{23} = 1$$

$F_{23} = 1 - F_{21} = 0.9529$, and from RECIPROCALITY

$$A_3 F_{32} = A_2 F_{23}$$

$$F_{32} = \frac{A_2 F_{23}}{A_3} = \frac{L^2 \cdot 0.9529}{5 \cdot L^2} = 0.19058$$

Finally from Conservation

$i=3$

$$F_{31} + F_{32} + F_{33} = 1$$

$$F_{33} = 1 - F_{31} - F_{32} = 1 - 0.0471 - 0.19058 = 0.76232$$

(fraction of walls that see itself)

SURFACE ENERGY BALANCE

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$$\frac{E_{bi} - J_i}{1 - \varepsilon_i} = \sum_{j=1}^n \frac{J_i - J_j}{A_i F_{ij}}$$

applied to every surface if "T" is known
and once J's are known or if "q" is known:

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{A_i F_{ij}} = \frac{E_{bi} - J_i}{\varepsilon_i A_i}$$

Surface 1 and 3 → T is KNOWN

"1"

$$\frac{E_{b1} - J_1}{1 - \varepsilon_1} = \frac{J_1 - J_2}{A_1 F_{12}} + \frac{J_1 - J_3}{A_1 F_{13}}$$

"3"

$$\frac{E_{b3} - J_3}{1 - \varepsilon_3} = \frac{J_3 - J_1}{A_3 F_{31}} + \frac{J_3 - J_2}{A_3 F_{32}}$$

Surface "2" → q is known

$$q_2 = \frac{J_2 - J_1}{A_2 F_{21}} + \frac{J_2 - J_3}{A_2 F_{23}} = 400W$$

UNKNOWN

3 Equations, 3 Unknowns

J_1, J_2, J_3

EQUATION SETUP

$$0.00189 \left[m^2 \right] (E_{b1} - J_1) \left[\frac{W}{m^2} \right] = 0.000471 \left[m^2 \right] (J_1 - J_2) \left[\frac{W}{m^2} \right] + 0.00236 m^2 (J_1 - J_3) \left[\frac{W}{m^2} \right]$$

$$0.0333 \left[m^2 \right] (E_{b3} - J_3) \left[\frac{W}{m^2} \right] = 0.00236 \left[m^2 \right] (J_3 - J_1) \left[\frac{W}{m^2} \right] + 0.00953 m^2 (J_3 - J_2) \left[\frac{W}{m^2} \right]$$

$$400W = 0.000471 \left[m^2 \right] (J_2 - J_1) \left[\frac{W}{m^2} \right] + 0.00953 m^2 (J_2 - J_3) \left[\frac{W}{m^2} \right]$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.4$$

$$E_{b1} = \sigma T_1^4 = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \cdot (420)^4 K^4 = 1764 \frac{W}{m^2}$$

$$E_{b3} = \sigma T_3^4 = 1452 \frac{W}{m^2}$$

SOLVING

$$J_1 = 1.24 \times 10^4 \frac{W}{m^2}$$

$$J_2 = 5.28 \times 10^4 \frac{W}{m^2}$$

$$J_3 = 1.29 \times 10^4 \frac{W}{m^2}$$

MATRIX SOLUTION

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$$0.000471 \left[m^2 \right] (J_1 - J_2) \left[\frac{W}{m^2} \right] + 0.00236 m^2 (J_1 - J_3) \left[\frac{W}{m^2} \right] = 0.00189 \left[m^2 \right] (E_{b1} - J_1) \left[\frac{W}{m^2} \right]$$

$$0.00236 \left[m^2 \right] (J_3 - J_1) \left[\frac{W}{m^2} \right] + 0.00953 m^2 (J_3 - J_2) \left[\frac{W}{m^2} \right] = 0.0333 \left[m^2 \right] (E_{b3} - J_3) \left[\frac{W}{m^2} \right]$$

$$0.000471 \left[m^2 \right] (J_2 - J_1) \left[\frac{W}{m^2} \right] + 0.00953 m^2 (J_2 - J_3) \left[\frac{W}{m^2} \right] = 400W$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{Bmatrix} J_1 \\ J_2 \\ J_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\begin{aligned} c_1 &= -0.000471 \\ c_2 &= 0.000471 + 0.00953 \\ c_3 &= -0.00953 \\ F_2 &= 400 \end{aligned}$$

$$\begin{aligned} a_1 &= 0.000471 + 0.000236 + 0.00189 \\ a_2 &= -0.000471 \\ a_3 &= -0.00236 \\ F_1 &= 0.00189 \end{aligned}$$

$$\begin{aligned} b_1 &= -0.00236 \\ b_2 &= -0.00953 \\ b_3 &= 0.00236 + 0.00953 + 0.0333 \\ F_2 &= 0.0333 \end{aligned}$$

SOLVING

$$J_1 = 1.24 \times 10^4 \frac{W}{m^2}$$

$$J_2 = 5.28 \times 10^4 \frac{W}{m^2}$$

$$J_3 = 1.29 \times 10^4 \frac{W}{m^2}$$

FLOOR TEMPERATURE

$$i = 2$$

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_i} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

$$E_{b2} = J_2 + q_2 \frac{1 - \epsilon_2}{\epsilon_2 A_2} = \sigma T_2^4$$

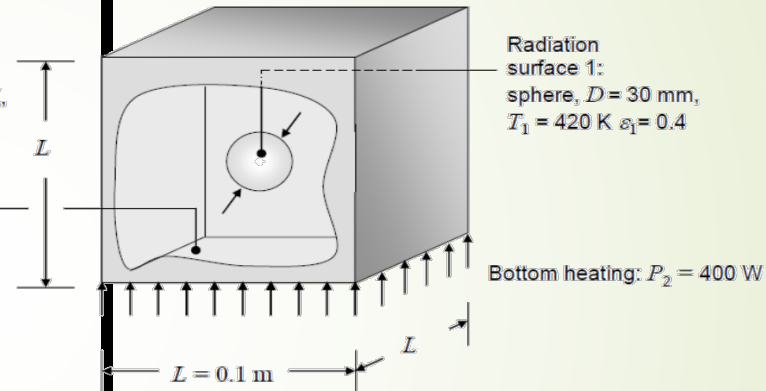
$$= 5.28 \times 10^4 \frac{W}{m^2} + 400W \frac{1 - 0.4}{0.4 \cdot 0.1^2} = 1.128 \times 10^5 \frac{W}{m^2}$$

$$T_2 = \left[\frac{E_{b2}}{\sigma} \right]^{1/4} = 1188K$$

SCHEMATIC:

Radiation surface 3:
remaining interior walls, $T_3 = 400\text{ K}$, $\epsilon_3 = 0.4$

Radiation surface 2:
floor
 $T_2, \epsilon_2 = 0.4$



NET RADIATION LEAVING SPHERE

SCHMATIC:

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_i} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

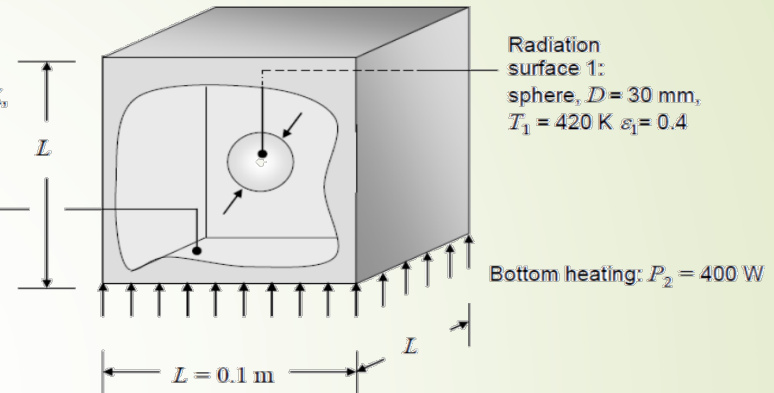
$i = 1$

$$q_1 = \frac{E_{b_1} - J_1}{\frac{1 - \epsilon_1}{\epsilon_1 A_1}} = 0.00189 \text{ m}^2 \left(1764 \left[\frac{\text{W}}{\text{m}^2} \right] - 1.24 \times 10^4 \left[\frac{\text{W}}{\text{m}^2} \right] \right)$$

$$= -20 \text{ W}$$

Radiation surface 3: remaining interior walls, $T_3 = 400 \text{ K}$, $\epsilon_3 = 0.4$

Radiation surface 2: floor $T_2, \epsilon_2 = 0.4$



“q” is negative; thus heat is “INCOMING” to the sphere at the rate of 20W.

RADIATION HEAT TRANSFER BETWEEN FLOOR-to-WALLS

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$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_i} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

$$q_{23} = \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}} = \frac{(5.28 \times 10^4 - 1.29 \times 10^4) \frac{W}{m^2}}{\frac{1}{(0.1^2 \cdot 0.9529) [m^2]}}$$

$$= 0.009529 [m^2] \cdot (5.28 \times 10^4 - 1.29 \times 10^4) \left[\frac{W}{m^2} \right]$$

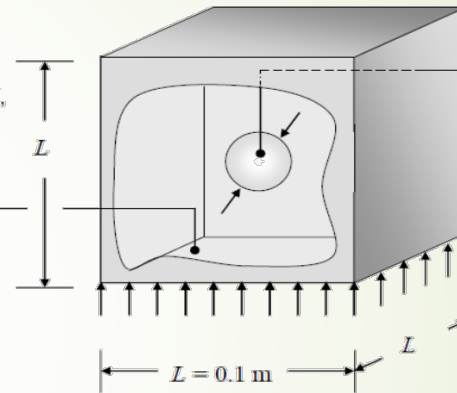
$$= 0.0380207 \times 10^4 W$$

$$= 380.2071 W = -q_{32} \text{ (WALLS to FLOOR)}$$

SCHEMATIC:

Radiation surface 3:
remaining interior walls, $T_3 = 400 \text{ K}$, $\epsilon_3 = 0.4$

Radiation surface 2:
floor, $T_2, \epsilon_2 = 0.4$



Radiation surface 1:
sphere, $D = 30 \text{ mm}$,
 $T_1 = 420 \text{ K}$, $\epsilon_1 = 0.4$

Bottom heating: $P_2 = 400 \text{ W}$

RADIATION HEAT TRANSFER BETWEEN FLOOR-to-SPHERE

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$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_i} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

$$q_{21} = \frac{J_2 - J_1}{\frac{1}{A_2 F_{21}}} = \frac{(5.28 - 1.24) \times 10^4}{\frac{1}{0.1^2 \cdot 0.0471}} = 19.0284W$$

RADIATION HEAT TRANSFER BETWEEN WALL-to-SPHERE

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_i} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

$$q_{31} = \frac{J_3 - J_1}{\frac{1}{A_3 F_{31}}} = \frac{(1.29 - 1.24) \times 10^4}{\frac{1}{5(0.1^2) \cdot 0.0471}} = 1.1175W = -q_{13}$$

NET RADIATION LEAVING WALL

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{\frac{1}{A_i F_{ij}}} = \frac{E_{b_i} - J_i}{\frac{1 - \epsilon_i}{\epsilon_i A_i}}$$

$$i = 3$$

$$q_3 = \frac{E_{b_3} - J_3}{\frac{1 - \epsilon_3}{\epsilon_3 A_3}} = \frac{(1452 - 1.29 \times 10^4) \frac{W}{m^2}}{\frac{1 - 0.4}{0.4 \cdot 5 \cdot 0.1^2 [m^2]}} = -381.6W$$

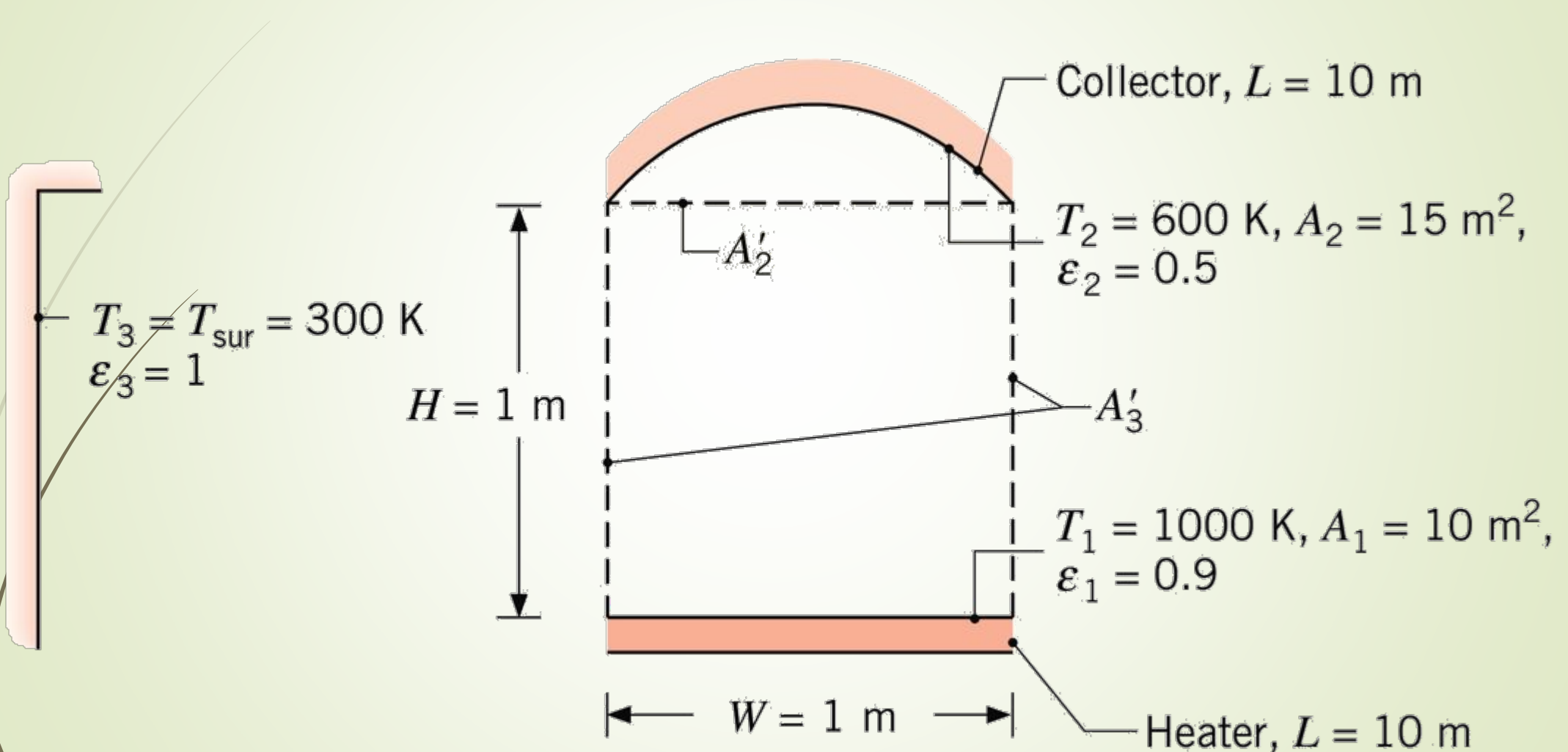
“q” is negative: Radiation is STRIKING WALL

TOTAL RADIATION TO/FROM FLOOR

Total Radiation TO/FROM FLOOR

$$\begin{aligned}q_{floor} &= q_{floor \rightarrow walls} + q_{floor \rightarrow sphere} \\ &= 380.2071W \text{ (page 21)} + 19.0284W \text{ (page 22)} \\ &= 399.2355W \rightarrow \text{SHOULD BE 400W}\end{aligned}$$

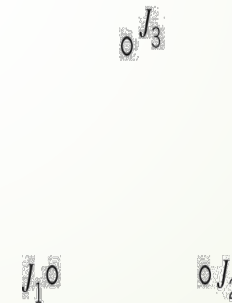
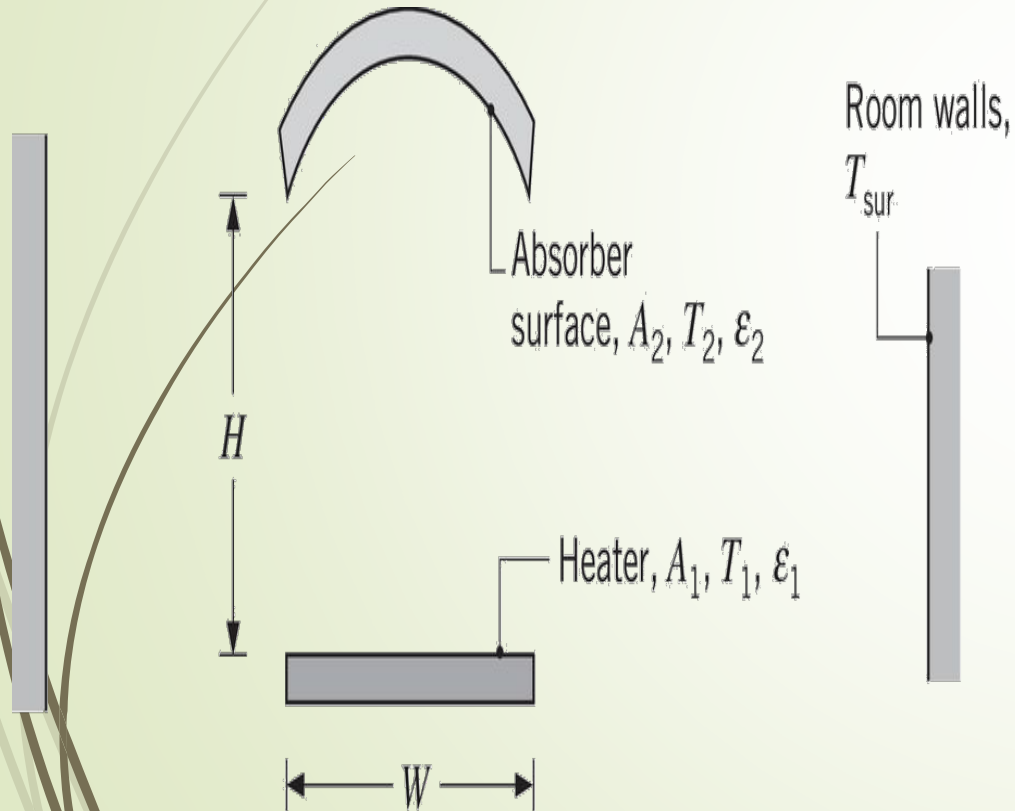
Curved, solar absorber surface cured by use of infrared heater in a large room.



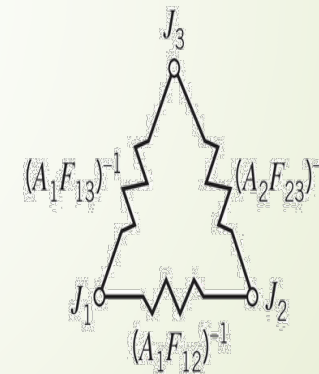
ANALYSIS

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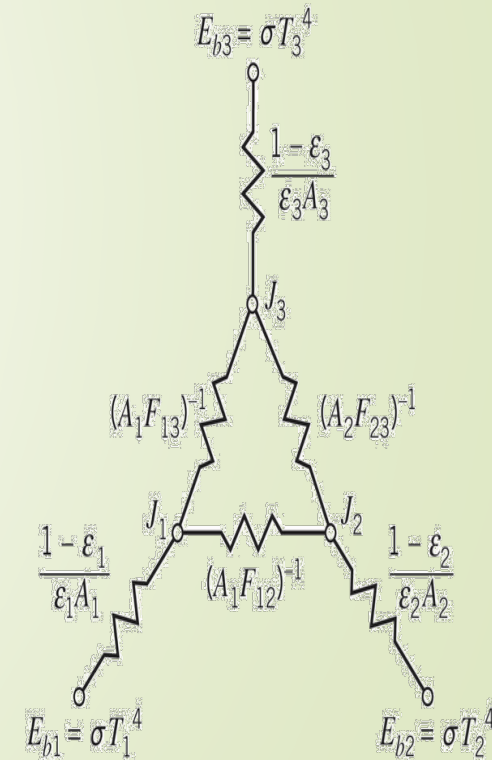
The system is viewed as 3 surface enclosure, with the **third surface being the surrounding room**, which behaves as a **BLACKBODY**. What is the net rate of heat transfer to the absorber surface.



Step 1



Step 2



Step 3 / 2022

Surface 3 - Large Room

$$J_3 = E_{b3} = \sigma T_3^4 = 459 \text{ W / m}^2$$

NODAL BALANCE - 1

$$q_1 = \frac{E_{b1} - J_1}{1 - \varepsilon_1} = \frac{J_1 - J_2}{A_1 F_{12}} + \frac{J_1 - J_3}{A_1 F_{13}}$$

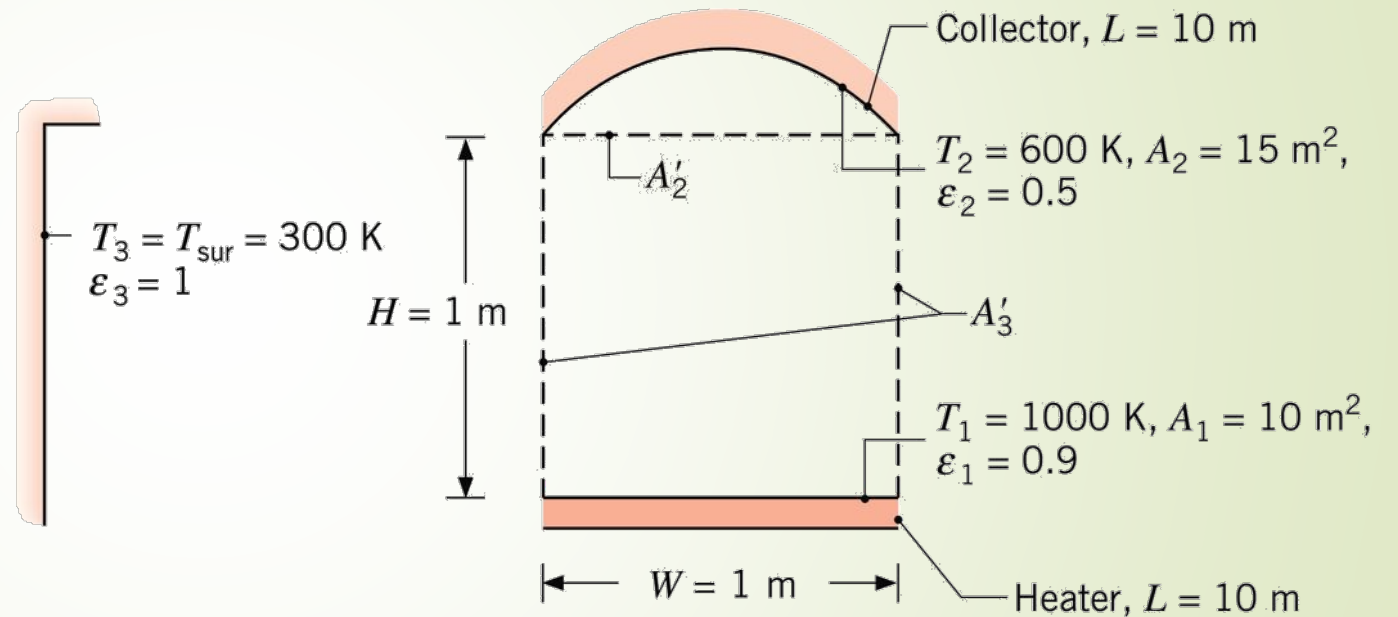
NODAL BALANCE - 2

$$q_2 = \frac{E_{b2} - J_2}{1 - \varepsilon_2} = \frac{J_2 - J_1}{A_2 F_{21}} + \frac{J_2 - J_3}{A_2 F_{23}}$$

2 Equations, 2 Unknowns

$$E_{b1} = \sigma T_1^4 = 56,700 \text{ W / m}^2$$

$$E_{b2} = \sigma T_2^4 = 7348 \text{ W / m}^2$$



SHAPE FACTORS

$F_{12} = F_{12'} \rightarrow A_2'$ is shown as the rectangular base of the absorber surface

FIGURE 13.2, 13.4

$$Y/L = 10/1 = 10, X/L = 1/1 = 1$$

$$F_{12} = 0.39$$

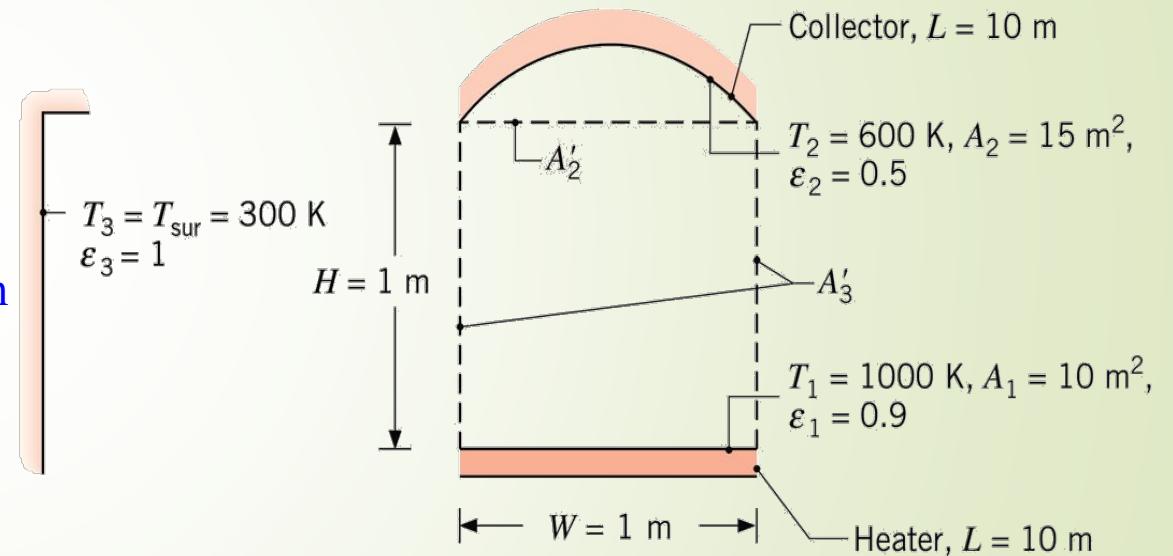
$$\cancel{F_{11}} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{12} = 0.61$$

Radiation propagating from surface 2 to 3 must pass through hypothetical surface A_2' .

$$A_2 F_{23} = A_2' F_{2'3}$$

By Symmetry: $F_{2'3} = F_{13}$

$$F_{23} = \frac{A_2'}{A_2} F_{2'3} = \frac{A_2'}{A_2} F_{13} = \frac{10}{15} 0.61 = 0.41$$



NODAL MATRIX SOLUTION

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NODAL BALANCE - 1

$$q_1 = \frac{E_{b1}}{1 - \varepsilon_1} + \frac{J_3}{1} = \frac{J_1 - J_2}{1} + \frac{J_1}{1} + \frac{J_1}{1 - \varepsilon_1} = J_1 \left[\frac{1}{1} + \frac{1}{1} + \frac{1}{1 - \varepsilon_1} \right] + J_2 \left[\frac{-1}{1} \right]$$

$$\frac{E_{b1}}{\varepsilon_1 A_1} + \frac{J_3}{A_1 F_{13}} = \frac{J_1 - J_2}{A_1 F_{12}} + \frac{J_1}{A_1 F_{13}} + \frac{J_1}{\varepsilon_1 A_1} = J_1 \left[\frac{1}{A_1 F_{12}} + \frac{1}{A_1 F_{13}} + \frac{1}{\varepsilon_1 A_1} \right] + J_2 \left[\frac{-1}{A_1 F_{12}} \right]$$

NODAL BALANCE - 2

$$q_2 = \frac{E_{b2}}{1 - \varepsilon_2} + \frac{J_3}{1} = \frac{J_2 - J_1}{1} + \frac{J_2}{1} + \frac{J_2}{1 - \varepsilon_2} = J_1 \left[\frac{-1}{1} \right] + J_2 \left[\frac{1}{1} + \frac{1}{1} + \frac{1}{1 - \varepsilon_2} \right]$$

$$\frac{E_{b2}}{\varepsilon_2 A_2} + \frac{J_3}{A_2 F_{23}} = \frac{J_2 - J_1}{A_2 F_{21}} + \frac{J_2}{A_2 F_{23}} + \frac{J_2}{\varepsilon_2 A_2} = J_1 \left[\frac{-1}{A_2 F_{21}} \right] + J_2 \left[\frac{1}{A_2 F_{23}} + \frac{1}{A_2 F_{23}} + \frac{1}{\varepsilon_2 A_2} \right]$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} J_1 \\ J_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} \frac{E_{b1}}{1 - \varepsilon_1} + \frac{J_3}{1} \\ \frac{E_{b2}}{1 - \varepsilon_2} + \frac{J_3}{1} \end{Bmatrix}$$

$$\begin{bmatrix} \frac{E_{b1}}{\varepsilon_1 A_1} + \frac{J_3}{A_1 F_{13}} \\ \frac{E_{b2}}{\varepsilon_2 A_2} + \frac{J_3}{A_2 F_{23}} \end{bmatrix}$$

NODAL MATRIX SOLUTION

NODAL BALANCE - 1

$$q_1 = J_1 \left[\begin{array}{c} \frac{1}{A_1 F_{12}} + \frac{1}{A_1 F_{13}} + \frac{1}{\epsilon_1 A_1} \\ \frac{1}{A_1 F_{12}} \\ \frac{1}{A_1 F_{13}} \\ \frac{1}{\epsilon_1 A_1} \end{array} \right] + J_2 \left[\begin{array}{c} -1 \\ 1 \end{array} \right] = J_1 a + J_2 b$$

NODAL BALANCE - 2

$$q_2 = J_1 \left[\begin{array}{c} -1 \\ 1 \end{array} \right] + J_2 \left[\begin{array}{c} \frac{1}{A_2 F_{23}} + \frac{1}{A_2 F_{23}} + \frac{1}{\epsilon_2 A_2} \\ \frac{1}{A_2 F_{23}} \\ \frac{1}{A_2 F_{23}} \\ \frac{1}{\epsilon_2 A_2} \end{array} \right] = J_1 c + J_2 d$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} J_1 \\ J_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} \frac{E_{b1}}{1-\epsilon_1} + \frac{J_3}{1} \\ \frac{E_{b1}}{\epsilon_1 A_1} + \frac{A_1 F_{13}}{1} \\ \frac{E_{b2}}{1-\epsilon_2} + \frac{J_3}{1} \\ \frac{E_{b2}}{\epsilon_2 A_2} + \frac{A_2 F_{23}}{1} \end{Bmatrix}$$

$$J_3 = 459W / m^2$$

$$J_1 = 51,541W / m^2, J_2 = 12,487W / m^2$$

$$E_{b1} = \sigma T_1^4 = 56,700W / m^2$$

$$E_{b2} = \sigma T_2^4 = 7348W / m^2$$