TRANSIENT CONDUCTION

LUMPED vs SPATIAL EFFECTS

MECH-420 HEAT TRANSFER

Transient Conduction

- A heat transfer process for which the temperature varies with TIME & SPACE within a solid, T(x,y,z,time).
- It is initiated whenever a system experiences a **SUDDEN** change in operating conditions
  - surface convection conditions (h, T_∞),
  - surface temperature or heat flux
  - internal heat generation
- Solution Techniques
  - Lumped Capacitance
  - Exact Methods
  - Numerical Methods

SPACE SHUTTLE TILE HEATING VIDEO

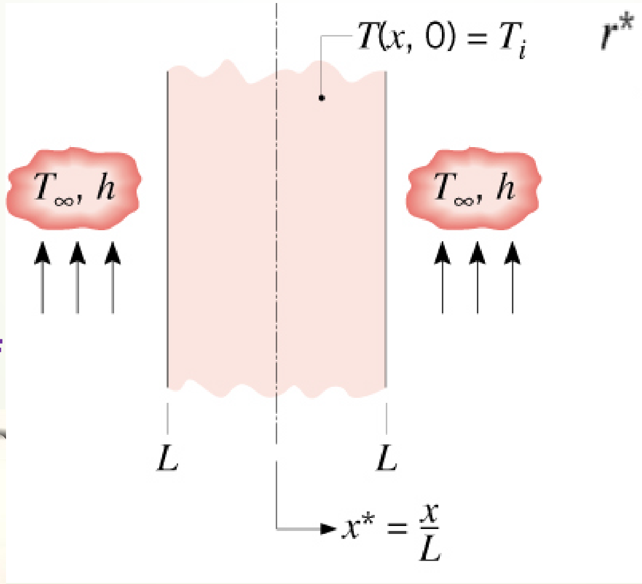
OVERVIEW

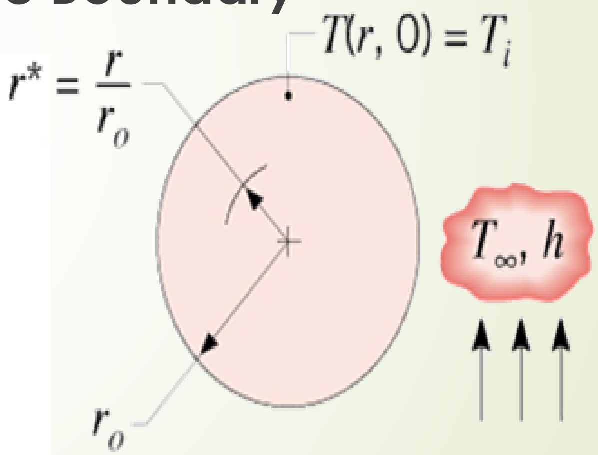
$$Bi = \frac{UL_c}{k_{SOLID}}, \rightarrow BIOT\ \#$$

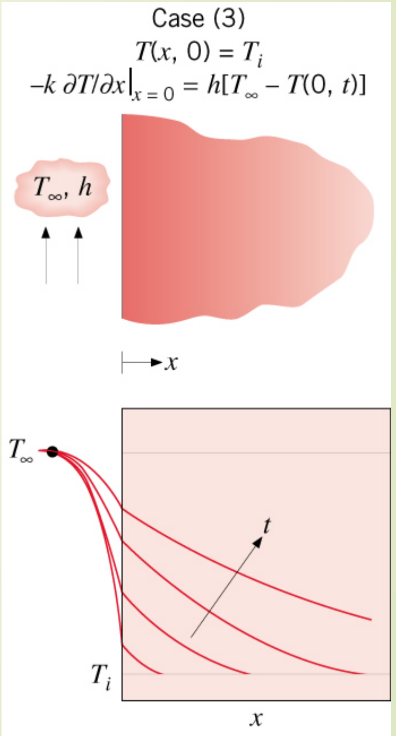
$$WALL \rightarrow L_c = L, CYLINDER\ L_c = \frac{r_0}{2}, SPHERE\ L_c = \frac{r_0}{3}$$

$$Fo = \frac{\alpha t}{L^2}, Fo = \frac{\alpha t}{r_0^2} \rightarrow FOURIER\ \#$$

- **LUMPED ANALYSIS: BIOT # < 0.1**
  - Temp = Function (TIME ONLY)
- **SPATIAL ANALYSIS: BIOT # > 0.1, FO > 0.2**
  - TEMP = FUNCTION (TIME and SPACE): Convective Boundary Conditions ONLY
    - INFINITE WALL
    - INFINITE CYLINDER
    - SPHERE
  - SEMI-INFINITE SOLID
    - Not Influenced by BIOT #

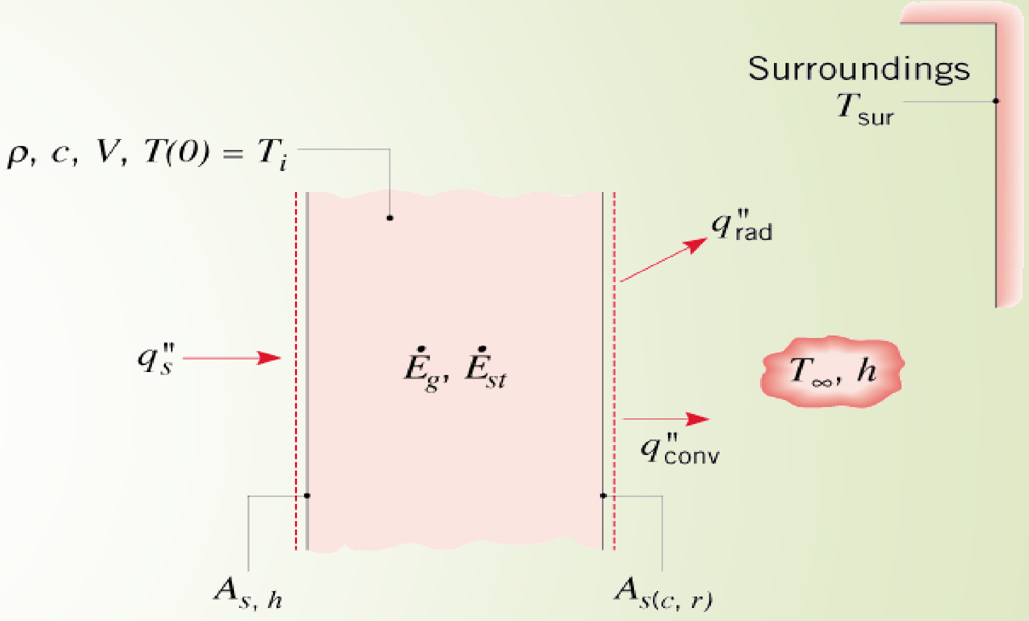






LUMPED CAPACITANCE

- Based on the assumption of a spatially uniform temperature distribution throughout the transient process, $T(x,time) \sim T(time)$
- **Why is this case NEVER actually realized?**



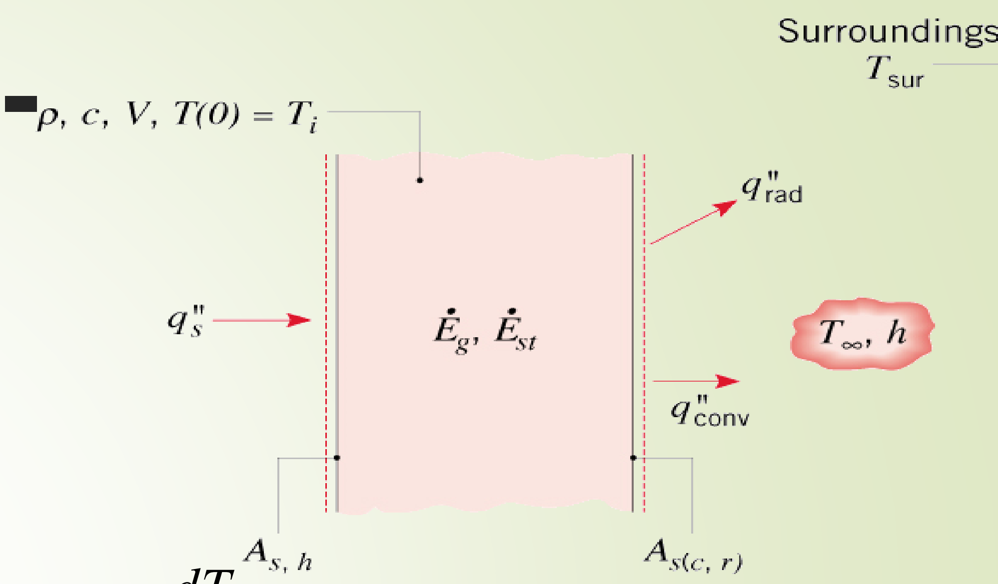
➢Consider a general case, which includes convection, radiation and/or an applied heat flux at specified surfaces as well as internal energy generation

$$\overset{+}{\dot{E}}_{in} - \overset{+}{\dot{E}}_{out} + \overset{\pm}{\dot{E}}_g = \frac{dE_{st}}{dt} \equiv \rho \forall c_p \frac{dT}{dt}$$

Assume:

1. Constant Properties
2. Uniform Heat Transfer Coefficients
3. Small Temperature Wall Gradients

SOLUTION--RECALL-



1^{st} LAW

$$\overset{+}{\dot{E}}_{in} - \overset{+}{\dot{E}}_{out} + \overset{\pm}{\dot{E}}_{g} = \frac{dE_{st}}{dt} \equiv \rho \forall c_p \frac{dT}{dt}$$

$$q''_s(t)A_s - h_c A_c (T_s(t) - T_\infty) - h_r A_r (T_s(t) - T_{surr}) + \dot{S}_{gen}(t)\forall = \rho \forall c_p \frac{dT}{dt}$$

$SOLUTION(neglect$ RADIATION)

let: $a = \frac{h_c A_c}{\rho \forall c_p}, b = \frac{E_{gen}(t)}{\rho \forall c_p}, \theta(t) = T(t) - T_\infty$

$$\frac{d\theta}{dt} + a\theta = b(t) \rightarrow \text{First Order ODE}$$

$$\theta(t) = e^{-at}\int b(t)e^{+at}dt + Ce^{-at}; k > 0$$

$b(t) \equiv$ constant

$$\theta(t) = \frac{b}{a} + Ce^{-at},$$

Initial Condition, t=0, $\theta(t=0) = \theta_i$

$$\theta_i = \frac{b}{a} + C, C = \theta_i - \frac{b}{a}$$

TIME CONSTANT

$b(t) \equiv \text{constant}$

$$\theta(t) = \frac{b}{a} + Ce^{-at}, = \frac{b}{a}(1 - e^{-at}) + \theta_i e^{-at}$$

Initial Condition, t=0, $\theta(t = 0) = \theta_i$

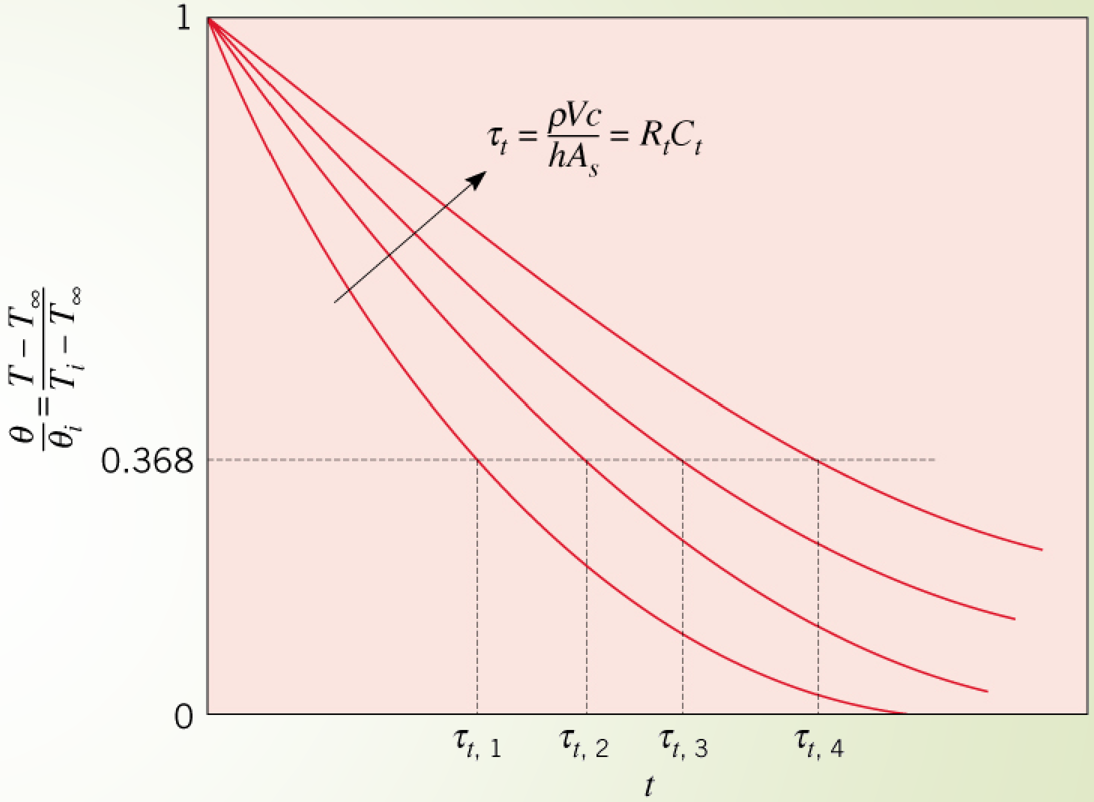
$$\theta_i = \frac{b}{a} + C, C = \theta_i - \frac{b}{a}$$

SPECIAL CASE

$$\frac{b}{a} = 0$$

$$\theta(t) = \theta_i e^{-at}$$

$$\frac{\theta(t)}{\theta_i} = \frac{T(t) - T_\infty}{T - T_\infty} = e^{-at} = e^{\left(\frac{-h_c A_s}{\rho \forall c_p}\right)t} = e^{-\left(\frac{t}{\tau}\right)}$$



$$\frac{1}{\tau} = \left(\frac{h_c A_s}{\rho \forall c_p}\right)$$

$$\tau = \frac{1}{h_c A_s} \bullet \rho \forall c_p \text{[sec]}$$

$$= R_{th} \bullet C_{th}$$

= Thermal Resistance • Lumped Thermal Capacitance

General Solutions (ignore radiation)

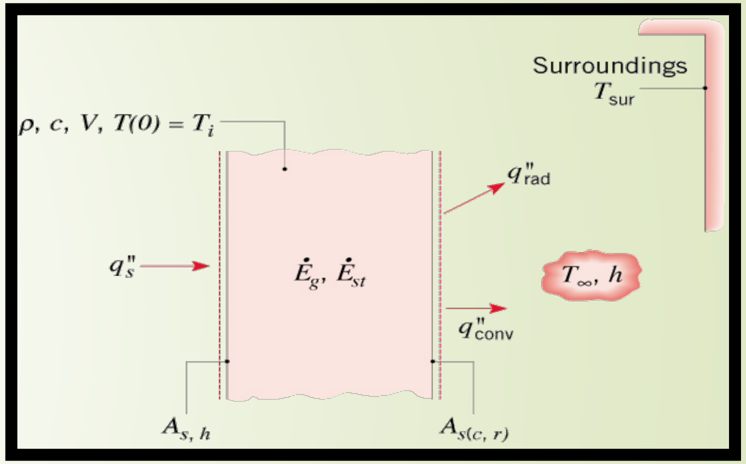
Temperature Given Time

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = \exp\left(-\frac{t}{\tau}\right) + \frac{b/a}{T_i-T_\infty}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]$$

$$a = \frac{1}{\tau};\tau \equiv \frac{\rho\forall c}{hA_{s,c}} \qquad b \equiv \left(q''_s A_s + S_{gen}\forall\right)/\rho\forall c$$

Time Given Temperature

$$t = \ln\left[\frac{T(t)-T_\infty-b/a}{T_i-T_\infty-b/a}\right](-\tau[s])$$



BIOT NUMBER

$$Bi = \frac{R_{th_{CONDUCTION}}}{R_{th_{CONVECTION}}} = \frac{\bar{U}L_c}{k_{SOLID}} \ll 0.1 \rightarrow LUMPED \rightarrow T(t) \text{ ONLY}$$

$$L_c = \frac{\forall}{A_s} \equiv \frac{\text{Volume}}{\text{Surface Area}}$$

$$U = \text{OVERALL HEAT TRANSFER COEF} \frac{W}{m^2 - K}.$$

$$= \left\{ \frac{\left(\sum R_{THERMAL} \frac{K}{W} \right)^{-1}}{(A)} \right\} \frac{W}{m^2 - K}$$

$$= \left\{ \left(\sum R''_{THERMAL} \frac{m^2 - K}{W} \right)^{-1} \right\} \frac{W}{m^2 - K}$$

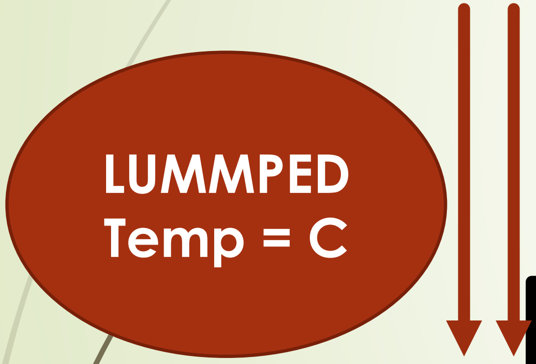
$$L_c = L \rightarrow \text{PLAIN WALL}$$

$$= \frac{r_0}{2} \rightarrow Cylinder$$

$$= \frac{r_0}{3} \rightarrow Sphere$$

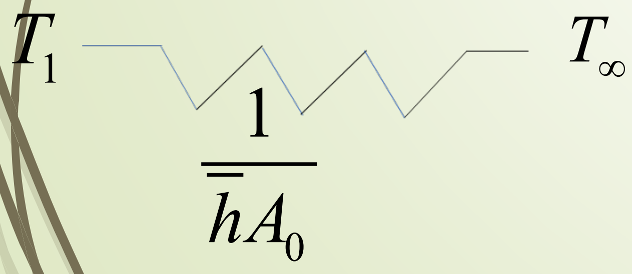
OVERALL THERMAL RESISTANCE

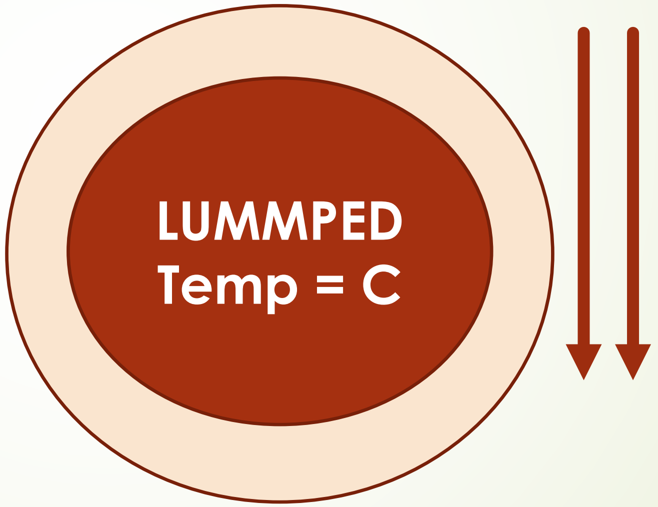
$$UA\left[\frac{W}{K}\right]=\frac{1}{\sum R_{th}\left[\frac{K}{W}\right]}$$ (Overall Resistance: Solid to Fluid)



$$\bar{h}, T_\infty$$

$$R_{th}=\frac{1}{hA_0}$$

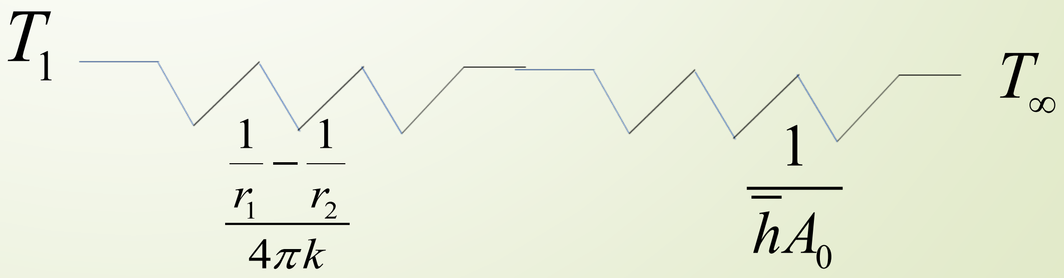




EXTERIOR SHELL

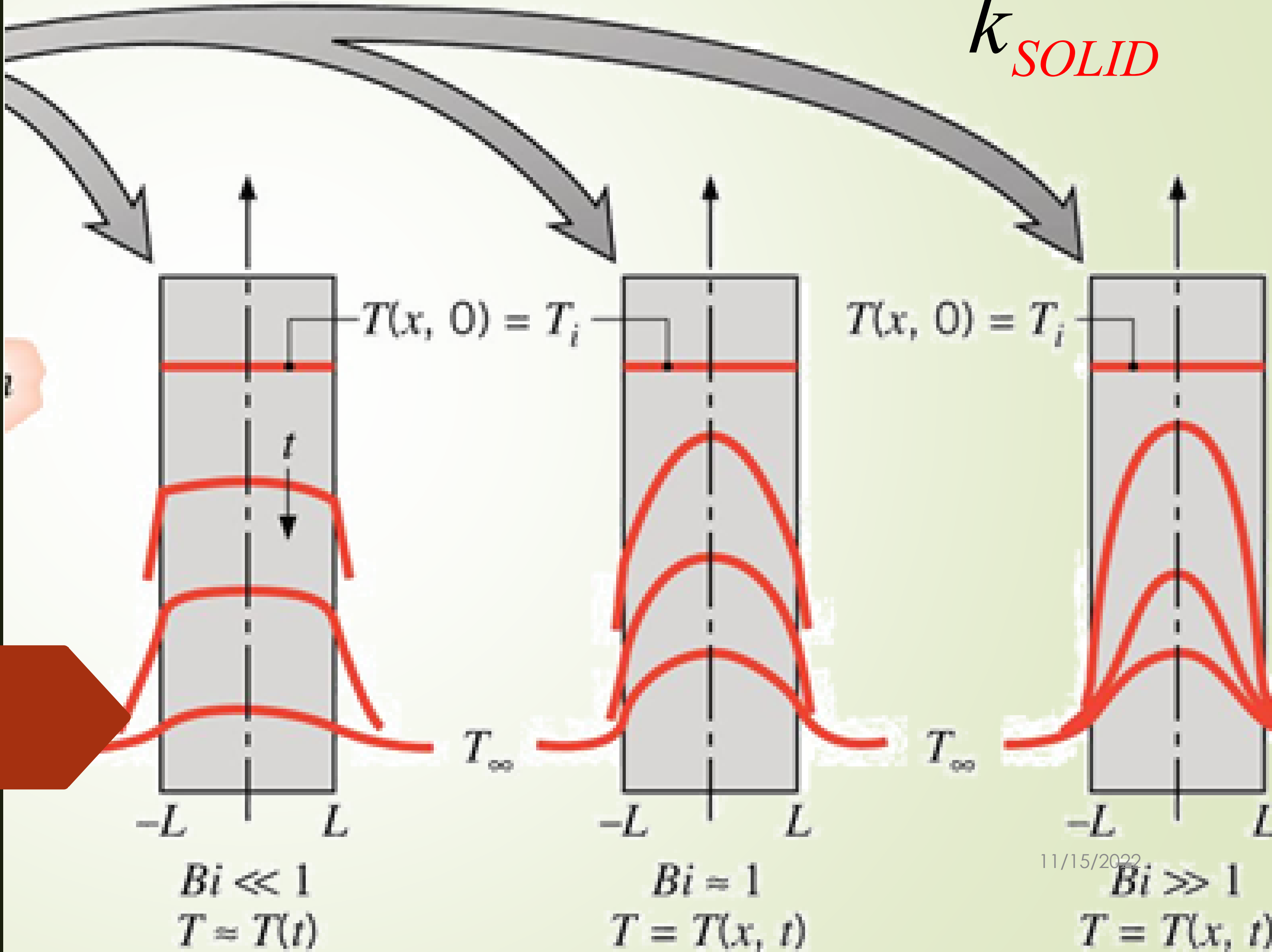
$$\bar{h}, T_\infty$$

$$R_{th}=\frac{\frac{1}{r_1}-\frac{1}{r_2}}{4\pi k}+\frac{1}{hA_0}$$



Biot < 0.1

$$Bi = \frac{UL_c}{k_{SOLID}}$$



Bi < 0.1

Temperature a Function of *TIME* ONLY

$$\frac{d\theta'}{dt} + a\theta' = b$$

$$\theta(t) = T(t) - T_\infty$$

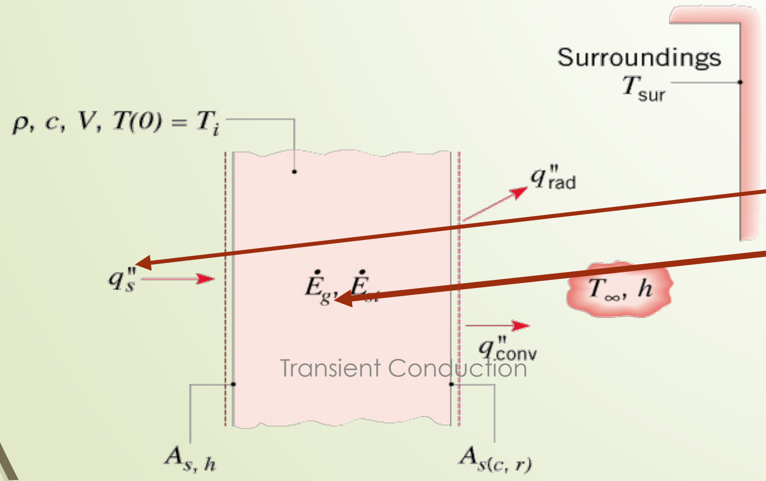
$$\frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau}\right) + \frac{b/a}{T_i - T_\infty}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau}\right)\left(1-\frac{b/a}{T_i - T_\infty}\right) + \frac{b/a}{T_i - T_\infty}$$

$$\ln\left[\frac{T(t)-T_\infty - b/a}{T_i - T_\infty - b/a}\right](-\tau) = t(time)$$

$$a = \frac{1}{\tau};\ \tau \equiv \frac{\rho \forall c}{UA} = R_t C_t$$

$$b = \frac{q_s''[W/m^2]A_s + \dot{E}_g[W]}{\rho \forall c}$$



Total Energy Transfer & HEAT RATE $Bi < 0.1$ (b/a=0)

HEAT TRANSFER RATE

$$q(t)[W]=hA_s(T_s(t)-T_\infty)$$

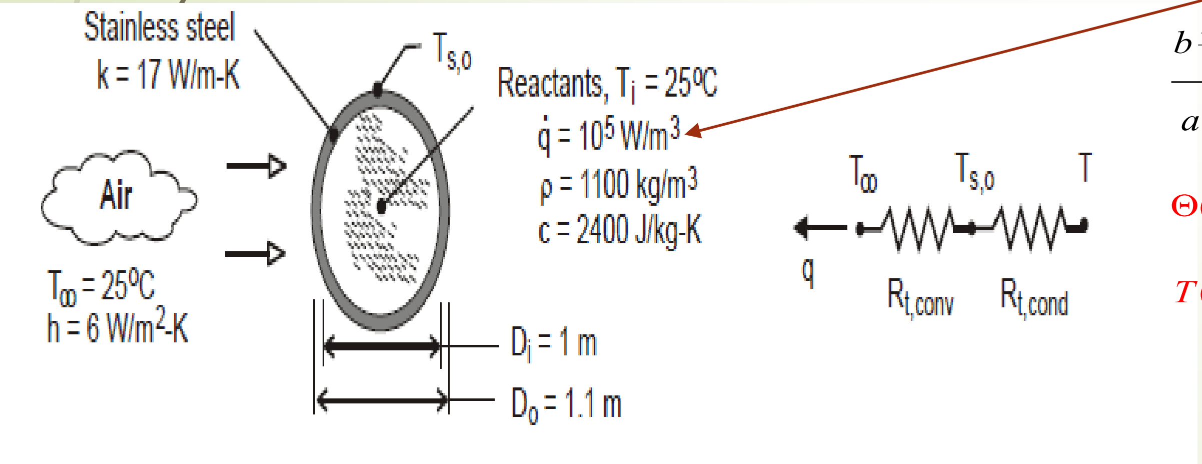
TOTAL ENERGY TRANSFER

$$Q[Joules]=-\Delta E_{st}=\int_0^{t^*} hA_s(T(t)-T_\infty)dt$$

$$=\rho\forall c\Theta_i(1-e^{-\frac{t^*}{\tau}})$$

Problem

- Find temperature of reactants after one hour of reaction time.
- Analysis
  - Transient
  - Internal Heat Generation Rate
  - Assume Lumped
  - Thermal capacitance of steel vessel may be neglected (no storage)



$$\frac{d\Theta}{dt}+\frac{\overline{U}(t)A}{\rho\forall c}\Theta(t)=\frac{\dot{E}_{gen}}{\rho\forall c}$$

$$\frac{d\Theta}{dt}+a\Theta(t)=b$$

$$\Theta(t)=T(t)-T_\infty=\frac{b}{a}+(\Theta_i-\frac{b}{a})e^{-at}$$

$$R_{t_{CONDUCTION}}=\frac{\frac{1}{r_1}-\frac{1}{r_0}}{4\pi k}=8.51x10^{-4}\frac{K}{W},\ R_{t_{CONVECTION}}=\frac{1}{hA_0}=0.0438\frac{K}{W}$$

$$UA=\frac{1}{\sum R_t}=22.4\frac{W}{K}$$

$$a=\frac{\overline{U}(t)A}{\rho\forall c}=\frac{22.4\frac{W}{K}}{1100kg/m3\bullet\frac{4}{3}\pi r_1^3\bullet 2400\frac{J}{kg-K}}=1.620x10^{-5}\frac{1}{s}$$

$$b=\frac{\dot{E}_{gen}}{\rho\forall c}=\frac{\frac{\dot{E}_{gen}}{\forall}}{\rho c}=\frac{10^5W/m3}{1100kg/m3\bullet 2400J/kg-K}=3.788x10^{-3}\frac{K}{s}$$

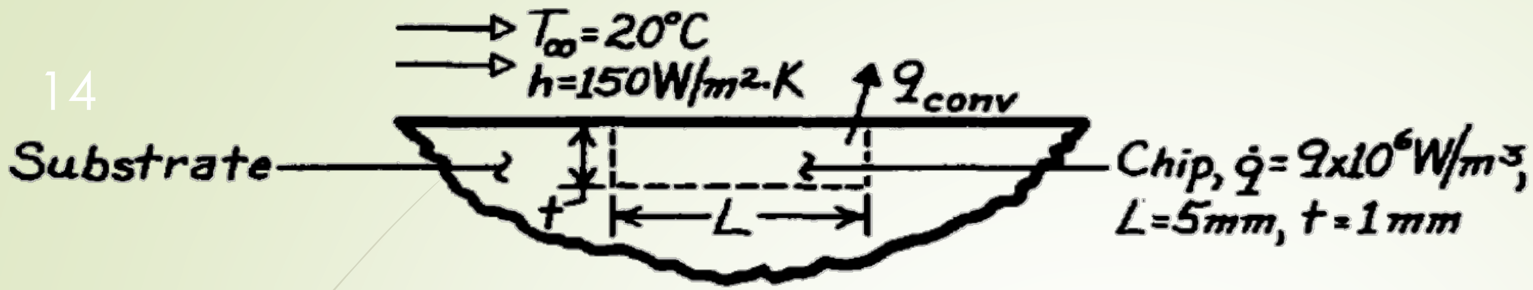
$$\frac{b\frac{K}{s}}{a\frac{1}{s}}=233.8C \text{ and t} = 18{,}000\text{s}$$

$$\Theta(t)=T(t)-T_\infty=\frac{b}{a}+(\Theta_i-\frac{b}{a})e^{-at}$$

$$T(t)=T_\infty+\frac{b}{a}+(\Theta_i-\frac{b}{a})e^{-at},\ \Theta_i=T_i-T_\infty=25.0-25.0=0.0$$

$$=25C+\frac{b}{a}(1-e^{-at})+\Theta_i e^{-at}$$

$$=25C+233.8(1-\exp(-1.62x10^{-5}\frac{1}{s}x18{,}000s))=84.1C$$



INTEGRATED CIRCUIT:

Steady State Temperature and time to come within 1C of steady state.

$$Bi_{wall} = \frac{\bar{U} L_c}{k_{SOLID}} = ?$$

Find Steady State Temperature

1st Law

$$-E_{out} + E_{gen} = 0$$

$$-h(L^2)(T_s - T_\infty) + \dot{S}_{gen}(L^2 \bullet t) = 0$$

$$T_s = T_\infty + \frac{\dot{S}_{gen} t}{h}$$

$$= 80C$$

$$\rightarrow \frac{V}{A_s} = \frac{L^2 \bullet thick}{L^2} = thick$$

$$a = \frac{h}{\rho(thick)c_p} = \frac{150\frac{W}{m^2-K}}{2000\frac{kg}{m^3} \bullet 0.001m \bullet 700\frac{J}{kg-K}} = 0.107s^{-1}$$

$$b = \frac{S_{gen}}{\rho c_p} = 6.429K/s$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau}\right) + \frac{b/a}{T_i - T_\infty}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$

$$a = \frac{1}{\tau}; \tau \equiv \frac{\rho \forall c}{hA_{s,c}} \qquad b \equiv \left(q_s'' A_{s,h} + \dot{E}_g\right)/\rho \forall c$$

Solve for t

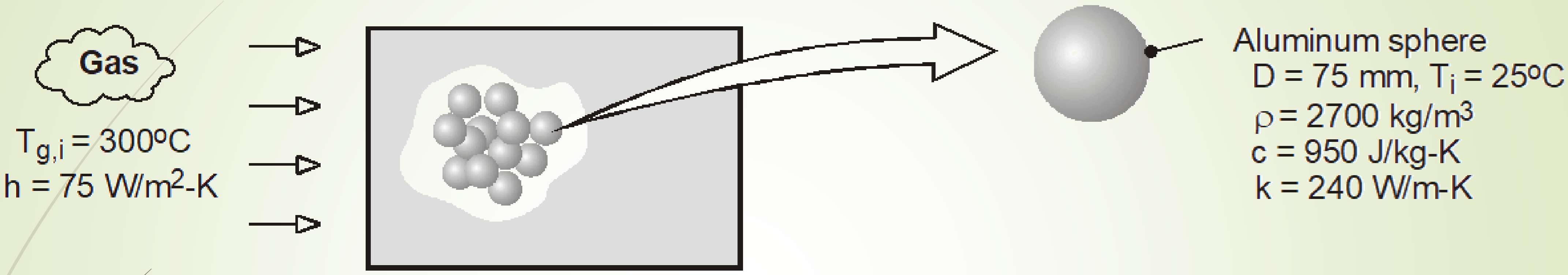
$$\frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau}\right)(1 - \frac{b/a}{T_i - T_\infty}) + \frac{b/a}{T_i - T_\infty}$$

$$\frac{\frac{T(t)-T_\infty}{T_i - T_\infty} - \frac{b/a}{T_i - T_\infty}}{(1 - \frac{b/a}{T_i - T_\infty})} = \exp\left(-\frac{t}{\tau}\right)$$

$$\frac{T(t) - T_\infty - b/a}{T_i - T_\infty - b/a} = \exp\left(-\frac{t}{\tau}\right)$$

$$\ln\left[\frac{T(t) - T_\infty - b/a}{T_i - T_\infty - b/a}\right](-\tau) = t$$

$$\frac{\ln\left[\frac{79 - 20 - 60}{20 - 20 - 60}\right]}{-0.107s^{-1}} = 38.3s$$



Time required for sphere to acquire 90% of MAX possible thermal energy and find center temperature

Analysis

- Neglect heat transfer to/from other spheres or from a sphere by radiation or conduction due to contact with other spheres.
- Constant Properties
- Check ***LUMPED CAPACITANCE*** criteria to see Temperature is function of TIME only, and not a function of SPACE.

$$Bi_{SPHERE} = \frac{h\frac{r_0}{3}}{k_{SOLID}} = \frac{75\frac{W}{m^2-K}\frac{0.0375}{3}}{150\frac{W}{m-K}} = 0.00625 < 0.1 \rightarrow \text{LUMPED}$$

Lumped Capactiance and a uniform temperature throughout at any instant of time.

But all points changing over time together.

$$\text{Q(t)}=\left(\rho\forall c_p\right)\theta_i\left[1-\exp\left(\frac{-t}{\tau_t}\right)\right];\tau_t=\left(\frac{1}{hA_s}\right)\rho\forall c_p \rightarrow \text{Thermal Time Constant}$$

$$\Delta E_{st} \equiv -Q = -\int_0^t \dot{E}_{out}\,dt = -hA_{s,c}\int_0^t \theta dt = -\left(\rho\forall c\right)\theta_i\left[1-\exp\left(-\frac{t}{\tau_t}\right)\right]$$

$$\frac{Q}{Q_i} = \frac{Q}{\left(\rho\forall c\right)\theta_i} = 0.90 = \left[1-\exp\left(-\frac{t}{\tau_t}\right)\right]$$

$$t = -\ln(1-0.90)\bullet\tau_t$$

$$\tau_t = \left(\frac{1}{hA_s}\right)\rho\forall c_p = \frac{\rho c_p}{h}\frac{r_0^3}{3r_0^2} = \frac{kg/m^3}{\frac{W}{m^2-K}}\frac{J-m}{kg-K} = 427\sec$$

$$t = -\ln(0.1)\bullet 427s = 984\sec$$

$$Bi = \frac{UL_c}{k_{SOLID}},$$

Temperature a Function of SPACE and TIME

$$WALL \rightarrow L_c = L, CYLINDER\ L_c = \frac{r_0}{2}, SPHERE\ L_c = \frac{r_0}{3}$$

$$Fo = \frac{\alpha t}{L^2}, Fo = \frac{\alpha t}{r_0^2}$$

Biot# > 0.1

Fourier# > 0.2

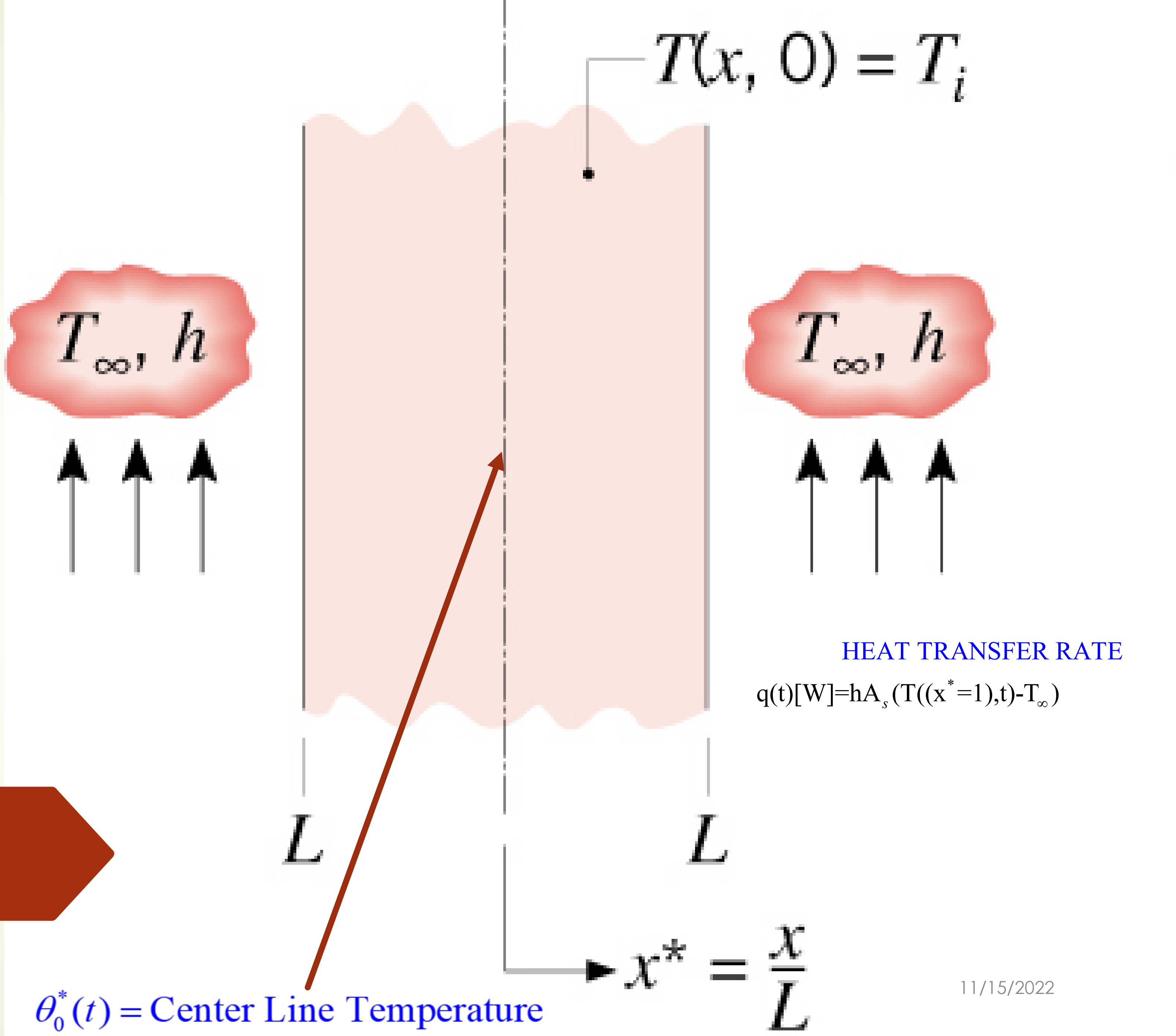
Infinity Wall Validity

$\frac{L}{H} \ll 1.0 \rightarrow$ No Heat Transfer in Y

Long Cylinder Validity

$\frac{r_0}{L} \ll 1.0 \rightarrow$ No Axial Heat Transfer

1D PLANAR ***"FINITE"*** DIMENSION HEAT TRANSFER T(x,t)



HEAT TRANSFER RATE

$q(t)[W]=hA_s(T((x^*=1),t)-T_\infty)$

$\theta_0^*(t) =$ Center Line Temperature

HDE EQUATION SOLUTION

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha}\frac{dT}{dt}, -L \le x \le +L$$

$BC\#1$

$$@x = 0, \frac{dT}{dx} = 0$$

$BC\#2$

$$@x = L, -k\frac{dT}{dx} = h(T(x = L) - T_\infty)$$

Initial Condition

$T(x,t=0)=T_i$

Dimensionless Variables

$$x^* = \frac{x}{L}, t^* = \frac{\alpha t}{L^2} = Fo \rightarrow \text{FOURIER \#}$$

$$\theta^*(x^*, Fo, Bi)) = \frac{\theta(x,t)}{\theta_i(x,t=0)} = \frac{T(x,t) - T_\infty}{T_i - T_\infty}$$

INFINITE SERIES SOLUTION

$$\theta^*(x^*, Fo, Bi) = \sum_{n=1}^{\infty} C_n(Bi)\exp(-\left(\zeta_n^2\right) Fo)\cos(\zeta_n x^*)$$

Approximate Solution, n=1, Fo>0.2

$$\theta^*(x^*, Fo, Bi) = C_1(Bi)\exp(-\left(\zeta_1^2\right) Fo(t))\cos(\zeta_1 x^*)$$

$$\theta^*(x^*, Fo, Bi) = \theta_0^* \cos(\zeta_1 x^*)$$

CENTER LINE($x^* = 0$)

$$\theta_0^*(t) = C_1(Bi)\exp(-\left(\zeta_1^2\right) Fo(t)) \rightarrow C_1(Bi) \leftarrow \text{TABLE 5.1}$$

$$\theta_0^*(t) = \frac{T_0(t) - T_\infty}{T_i - T_\infty}; \theta^*(x,t) = \frac{T(x^*,t^*) - T_\infty}{T_i - T_\infty}$$

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi[a]	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

1D Planar Transient Heat Transfer Convective Boundary Conditions

$$\theta^*(x^*,t^*) = f\left(x^*, Fo, Bi\right)$$

$$\theta^*(t) \equiv \frac{\theta(t)}{\theta_i} = \frac{T(t)-T_\infty}{T_i - T_\infty}$$

$$\theta_o^*(t) \equiv \frac{(T_o(t)-T_\infty)}{(T_i-T_\infty)} \approx C_1 \exp\left(-\zeta_1^2 Fo(t)\right) \rightarrow \text{Time Given}$$

$$Fo = \frac{\alpha t}{L^2} = \frac{-1}{\zeta_1^2} \ln\left[\frac{\theta_o^*(t)}{C_1}\right] \rightarrow \text{Center Temperature Given}$$

ONLY TWO TYPES OF PROBLEMS

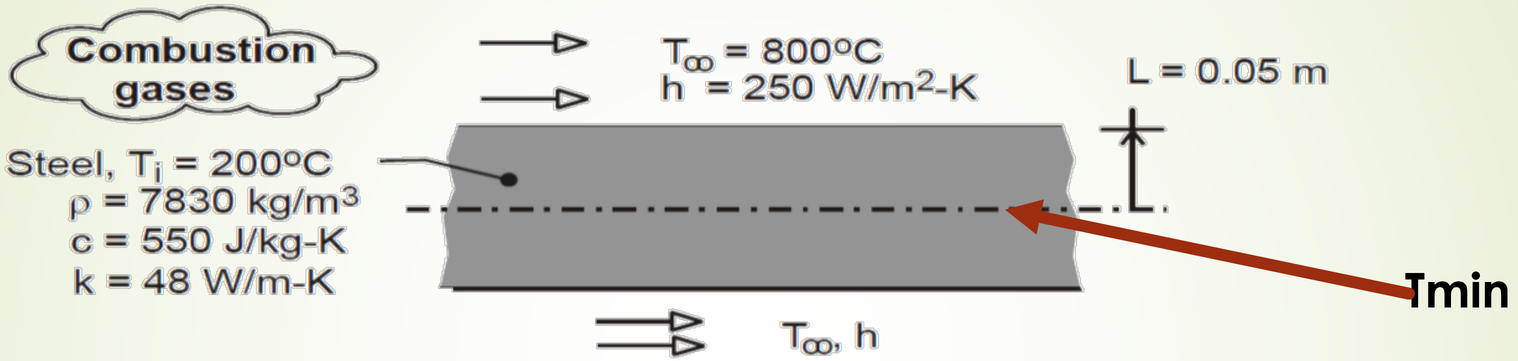
$$\theta^*(x^*,t^*) = \frac{T(t)-T_\infty}{T_i - T_\infty} = \theta_o^*(t^*)\cos\left(\zeta_1 x^*\right)$$

FOR NON-CENTER PLANE LOCATIONS

$$\frac{Q(t)}{Q_o} = \left(1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^*(t)\right)$$

$$Q_o = \rho c V\left(T_i - T_\infty\right)$$

Find time required to achieve a "MINIMUM" temperature of 500C in the slab.



$$Bi = \frac{hL}{k_{solid}} = \frac{250\frac{W}{m-K}0.05m}{48\frac{W-m}{K}} = 0.260 > 0.1 \rightarrow \text{LUMPED IS NOT VALID}$$

$$Bi = \frac{hL}{k} = \frac{250 \frac{W}{m-K} 0.05m}{48 \frac{W-m}{K}} = 0.260 > 0.1 \rightarrow \text{LUMPED IS NOT VALID}$$

$$\theta_0^*(t) = \frac{T_0(t) - T_\infty}{T_i - T_\infty} = \frac{500 - 800}{200 - 800} = 0.50 = C_1(Bi)\exp(-\left(\zeta_1^2\right) Fo(t))$$

TABLE 5.1

Bi=0.260, $\rightarrow \zeta_1 \approx 0.488, C_1 = 1.0396$

$$\alpha = \frac{k}{\rho c} = 1.115x10^{-5} \frac{m^2}{s}$$

SOLVE FOR Fo

$$\frac{\ln\left[\frac{0.50}{C_1(Bi)}\right]}{-\left(\zeta_1^2\right)} = Fo(t) = \frac{\alpha t}{L^2}$$

SOLVE FOR time

$$\frac{\ln\left[\frac{0.50}{C_1(Bi)}\right]}{-\left(\zeta_1^2\right)} \frac{L^2}{\alpha} = t$$

$$\frac{\ln\left[\frac{0.50}{C_1(Bi)}\right]}{-\left(\zeta_1^2\right)} \frac{L^2}{\alpha} = t$$

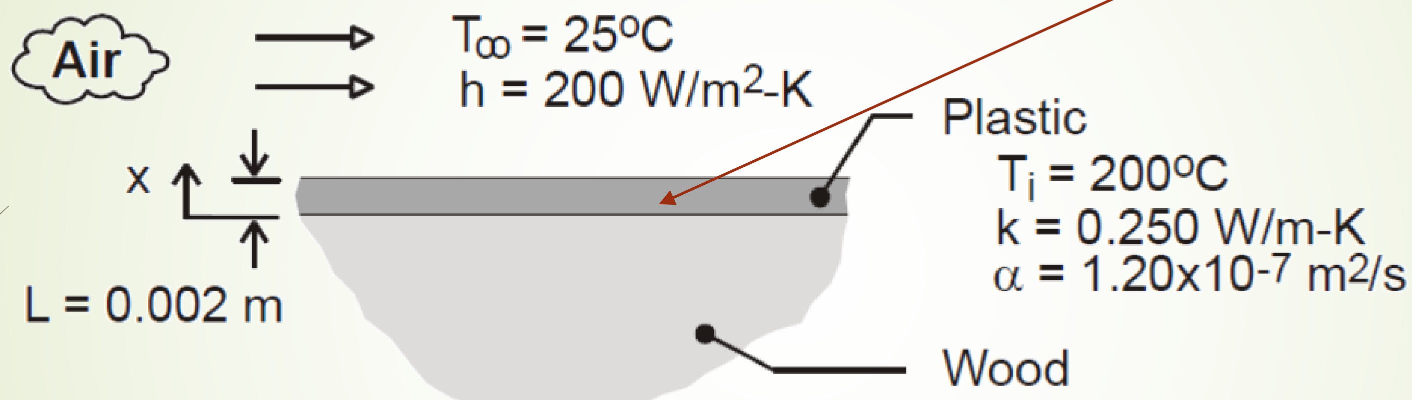
$$\frac{-0.73198L^2}{-\left(\zeta_1^2\right)\alpha} = t = 689s$$

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi[a]	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

Find Time for SURFACE to Reach Safe Temperature (42C) and mid-plane Temperature. (Assume Wood Interface To Be Adiabatic)



$$Bi = \frac{hL}{k_{solid}} = \frac{200\frac{W}{m^2-K}0.002m}{0.25\frac{W}{m-K}} = 1.6 > 0.1$$

LUMPED ANALYSIS NOT VALID

"SPATIAL EFFECTS ARE IMPORTANT"

Surface Safe Temperature = 42C

$$\theta^*(x^*, Fo, Bi) = \theta^*_0(t^*)\cos(\zeta_1 x^*) = C_1(Bi)\exp(-\zeta_1^2 Fo)\cos(\zeta_1 x^*)$$

$$\theta^*_s(t) = \frac{T_s(t) - T_\infty}{T_i - T_\infty} = \frac{42-25}{200-25} = 0.0971 = C_1(Bi)\exp(-\zeta_1^2 Fo)\cos(\zeta_1 x^*)$$

TABLE 5.1

Bi=1.6 $\rightarrow C_1(Bi) = 1.155, \zeta_1 = 0.990$

$$@surface, x^* = \frac{x}{L} = 1.0$$

$$0.0971 = 1.155\exp(-0.990^2 Fo)\cos(0.990)$$

SOLVE FOR Fo

$$\frac{\ln\left[\dfrac{0.0971}{1.155\cos(0.990)}\right]}{-\left(0.990^2\right)} = Fo = 1.914 = \frac{\alpha t}{L^2}\left(t = \frac{1.914L^2}{\alpha \frac{m^2}{s}}\right), or \rightarrow$$

SOLVE FOR "t"

$$\frac{\ln\left[\dfrac{0.0971}{1.155\cos(0.990)}\right]L^2}{-\left(0.990^2\right)\alpha\left[\dfrac{m^2}{s}\right]} = t = 63.8s$$

INTERFACE/CENTER PLANE TEMPERATURE

$$\theta_0^*(t) = C_1(Bi)\exp(-\left(\zeta_1^2\right)Fo(t)) \rightarrow C_1(Bi) \leftarrow \text{TABLE 5.1}$$

$$\theta_0^*(t) = \frac{T_0(t) - T_\infty}{T_i - T_\infty};$$

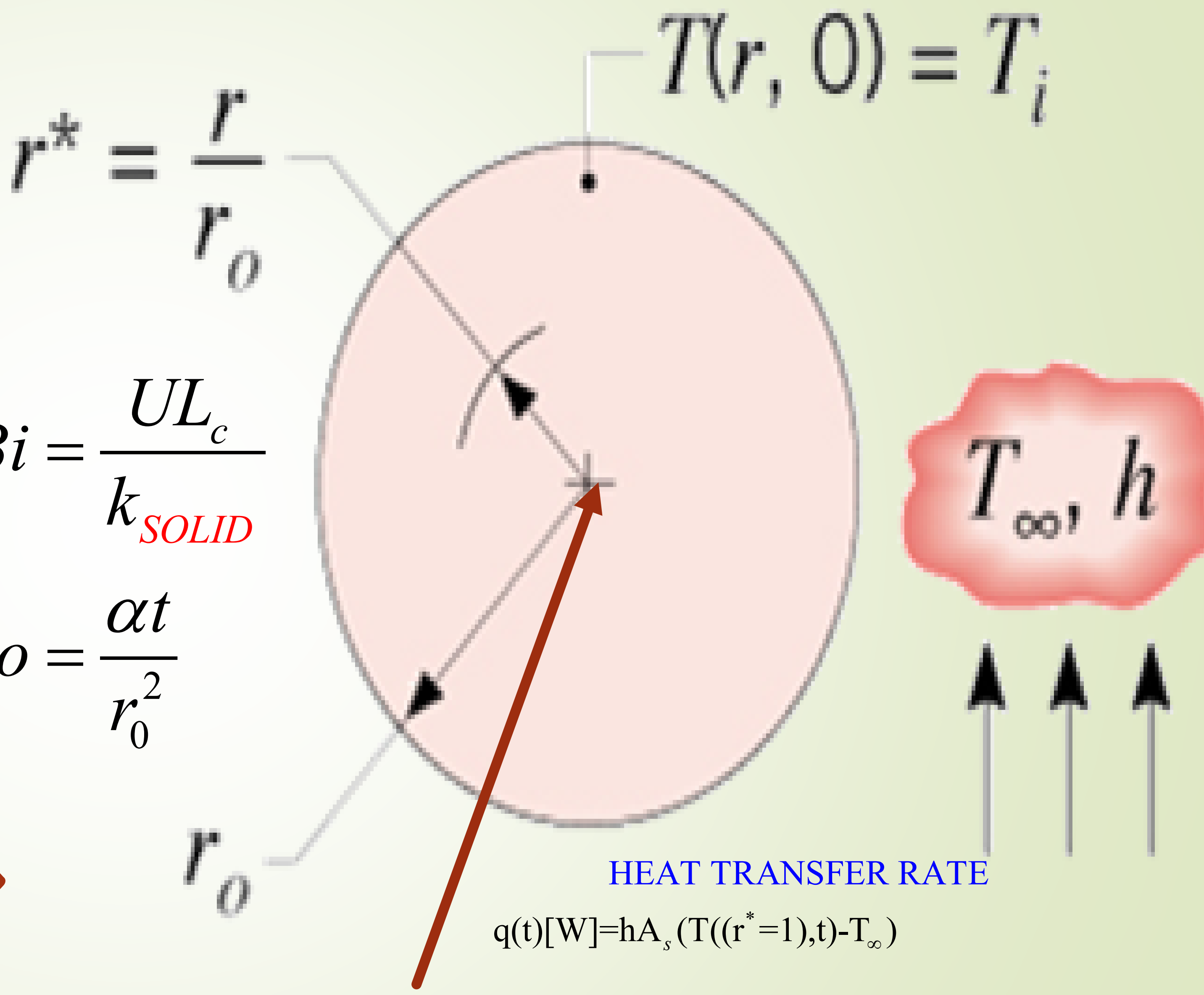
$$T_0(t) = T_\infty + \theta_0^*(t)\left(T_i - T_\infty\right)$$

$$= T_\infty + \left(T_i - T_\infty\right)C_1(Bi)\exp(-\left(\zeta_1^2\right)Fo(t))$$

$$= 25C + (200-25)1.155\exp(-\left(0.990^2\right)1.914)$$

$$= 56C$$

1D RADIAL ***“FINITE”*** DIMENSION HEAT TRANSFER T(r,t)



$$Bi = \frac{UL_c}{k_{SOLID}}$$

$$Fo = \frac{\alpha t}{r_0^2}$$

HEAT TRANSFER RATE

$q(t)[W] = hA_s(T((r^* = 1), t) - T_\infty)$

$\theta_0^*(t) =$ Center Line Temperature

1D RADIAL Transient Heat Transfer Convective Boundary Conditions

$Bi^* = hr_o / k \rightarrow (TABLE 5.1 -- NOTE\ DIFFERENCE)$

$Fo = \alpha t / r_o^2$

INFINITE CYLINDER

$$r^* = \frac{r}{r_0}; Fo = \frac{\alpha t}{r_0^2}$$

$\theta_0^*(t) =$ Center Line Temperature (Normally find this first)

$$\theta_0^*(t) = \frac{\theta_0(t)}{\theta_i} = \frac{T_0(t) - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_o(t))$$

$$\theta^*(r^*, t^*) = \frac{\theta(r,t)}{\theta_i} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = \theta_o^*(t^*) J_0(\xi r^*) = C_1 \exp(-\xi_1^2 Fo(t)) J_0(\xi_1 r^*)$$

$$Q^*(t) = \frac{Q(t)}{Q_0} = 1 - \frac{2\theta_0^*(t)}{\xi_1} J_1(\xi_1) \rightarrow \text{TOTAL ENERGY TRANSFER}$$

$\mathrm{Q}_0 = \rho \forall c_p \theta_i \rightarrow$ Initial Thermal Energy (J)

$\mathrm{C}_1, \xi \rightarrow$ Table 5.1

$\mathrm{J}_0(x), J_1(x) \equiv$ Bessel Functions $\rightarrow$ Appendix B.4

SPHERE

$\theta_0^*(t) =$ Center Line Temperature (Normally find this first)

$$\theta_0^*(t) = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_o)$$

$$\theta^*(r^*, t^*) = \frac{\theta(r,t)}{\theta_i} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = \theta_o^*(t) \frac{\sin(\xi_1 r^*)}{\xi_1 r^*} = C_1 \exp(-\xi_1^2 Fo) \frac{\sin(\xi_1 r^*)}{\xi_1 r^*}$$

$$Q^*(t) = \frac{Q(t)}{Q_0} = 1 - \frac{3\theta_0^*(t)}{\xi_1^3} [\sin(\xi_1) - \xi_1 \cos(\xi_1)] \rightarrow \text{TOTAL ENERGY TRANSFER}$$

$\mathrm{Q}_0 = \rho \forall c_p \theta_i \rightarrow$ Initial Thermal Energy (J)

$\mathrm{C}_1, \xi \rightarrow$ Table 5.1

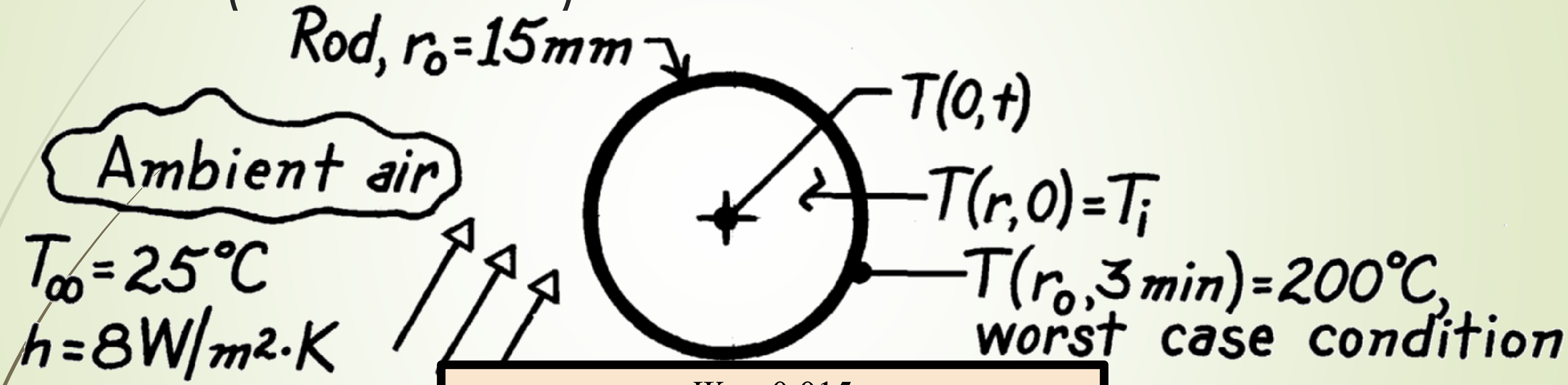
Infinity Wall Validity

$\frac{L}{H} \ll 1.0 \rightarrow$ No Heat Transfer in Y

Long Cylinder Validity

$\frac{r_0}{L} \ll 1.0 \rightarrow$ No Axial Heat Transfer

Long plastic rod heated uniformly in an oven to Ti and cooled to ambient air for 3 minutes. **Minimum INITIAL** temperature of rod should **NOT** be less than 200C (worst case). Find Ti?



$$Bi = \frac{h\frac{r_0}{2}}{k_{solid}} = \frac{8\frac{W}{m^2-K}\frac{0.015}{2}}{0.3\frac{W}{m-K}} = 0.20 > 0.1$$

LUMPED ANALYSIS NOT VALID

"SPATIAL EFFECTS ARE IMPORTANT"

$\rho cp = 1040 kJ/m^3\text{-}K$

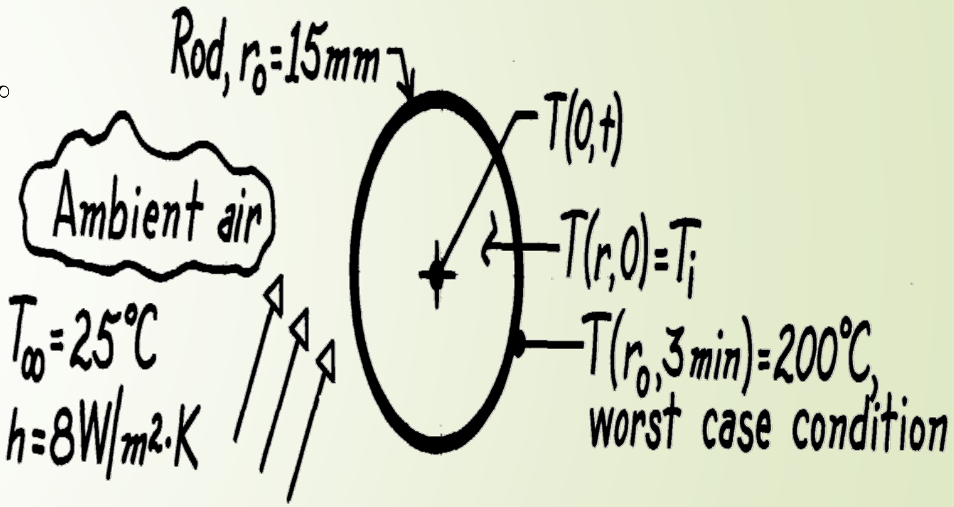
Time is given as "3min"

Temperature is given at outer radius as 200C (minimum starting temperature)

$$r^* = \frac{r}{r_0} = 1.0$$

$$\theta^*(r^*,t^*) = \frac{\theta(r,t)}{\theta_i} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = \theta_0^*(t^*)J_0(\xi r^*) = C_1 \exp(-(\xi_1^2)Fo)J_0(\xi_1 r^*)$$

$$T_i = \frac{T(r,t)-T_\infty}{\theta_0^*(t^*)J_0(\xi r^*)} + T_\infty = \frac{T(r,t)-T_\infty}{C_1 \exp(-(\xi_1^2)Fo)J_0(\xi r^*)} + T_\infty$$



$$Fo = \frac{\alpha t}{r_0^2} = \frac{\frac{0.3W/m-K}{1040x10^3 J//m^3-K}}{0.015^2} 3x60s = 0.2308$$

MODIFIED BI#(TABLE 5.1)

$$Bi = \frac{hr_0}{k_{solid}} = 0.40, \text{TABLE 5.1} \rightarrow \zeta_1 = 0.8516, C_1 = 1.0932$$

$$J_0(\xi r^*) = J_0(0.8516 \bullet 1.0) = 0.7848 (\text{Table B.4})$$

$$T_i = \frac{200-25}{1.0932 \exp(-(0.8516^2)0.2308)0.7848} + 25 = 267C$$

INTERPOLATION EXAMPLE

FIND $J_0(0.8516)$

TABLE B.4

INTERPOLATION

x	$J_0(x)$
0.7	0.8812
?	0.8516
0.8	0.8463

$$x = x_1 + \frac{x_2 - x_1}{J_{02} - J_{01}}(J - J_{01}) = 0.7 + \frac{0.8 - 0.7}{0.8463 - 0.8812}(0.8516 - 0.8812)$$

$x = 0.7848$

What is center temperature at this time (if you KNOW off-center temperature)?

$$\frac{\theta^*}{\theta_0^*} = \frac{\dfrac{T(r^*,t^*)-T_\infty}{T_i - T_\infty}}{\dfrac{T_0(t^*)-T_\infty}{T_i - T_\infty}} = \frac{T(r^*,t^*)-T_\infty}{T_0(t^*)-T_\infty}$$

$$= \frac{C_1 \exp(-(\xi_1^2) Fo(t)) J_0(\xi_1 r^*)}{C_1 \exp(-(\xi_1^2) Fo(t))} = J_0(\xi_1 r^*)$$

$$T_0(t^*) = \frac{T(r^*,t^*)-T_\infty}{J_0(\xi_1 r^*)} + T_\infty = \frac{200-25}{0.7848} + 25 = 248C$$

ANOTHER WAY

$$\theta_0^*(t) = \frac{T_0(t)-T_\infty}{T_i - T_\infty} = C_1 \exp(-(\xi_1^2) Fo(t))$$

$$T_0(t^*) = T_\infty + (T_i - T_\infty) C_1 \exp(-(\xi_1^2) Fo(t))$$

TOTAL ENERGY TRANSFER

$$\frac{Q(t)}{Q_0} = 1 - \frac{2\theta_0^*(t)}{\zeta_1} J_1(\zeta_1)$$

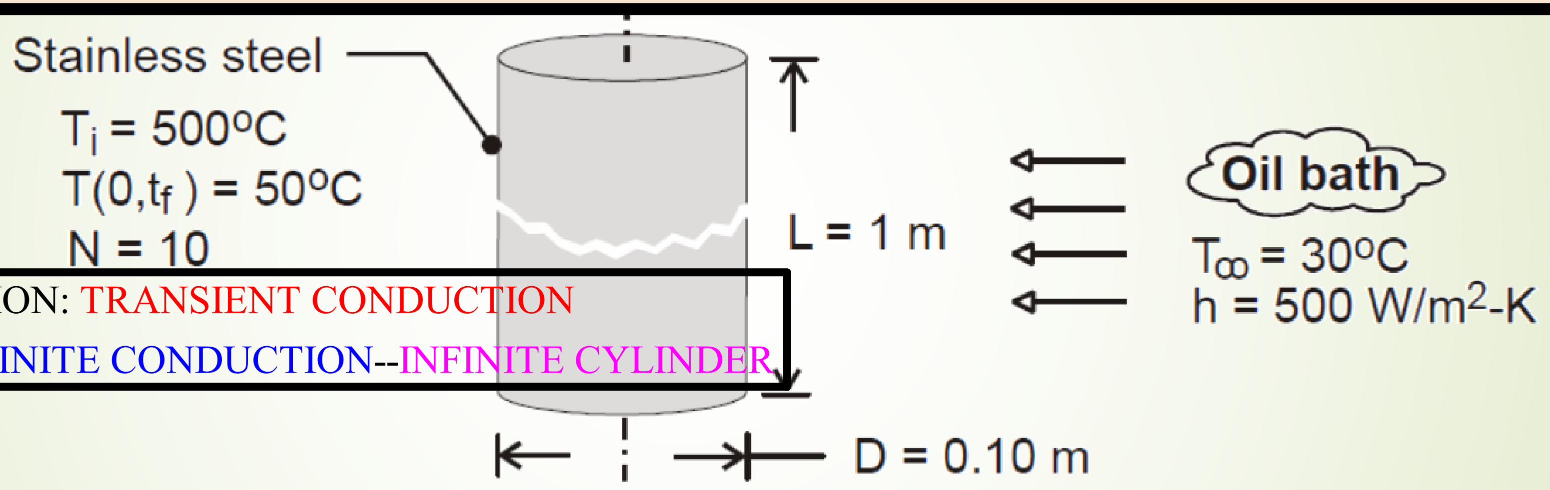
$$Q_0 = \rho \forall c_p (T_i - T_\infty)[J]$$

$$\equiv \text{Max Energy Transfer Possible}$$

Cylindrical 304 SS rods of 100-mm diameter are cooled from 500C in an oil bath of 30C, with convection coefficient of 500 W/m2-K.

How long does it take center to reach 50C?

If rods are L=1m and 10 rods are processed per hour , what is the energy extraction rate from the bath that must be removed by Heat Exchanger.



PROBLEM CLASSIFICATION: TRANSIENT CONDUCTION

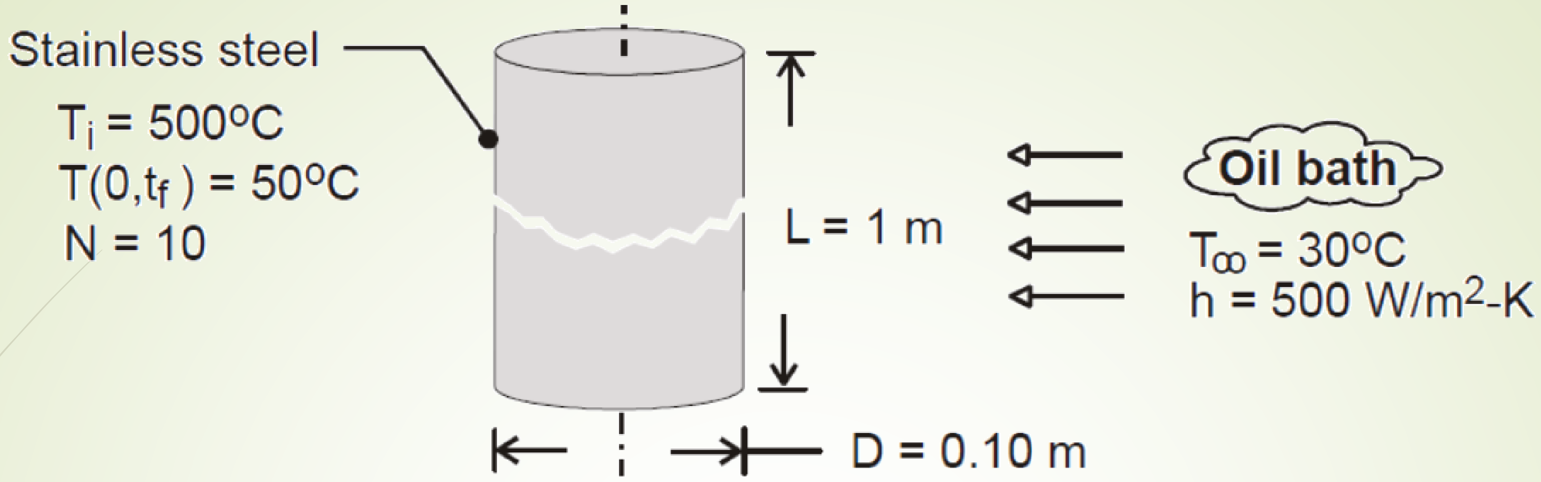
GEOMETRY CLASS: 1D FINITE CONDUCTION--INFINITE CYLINDER

$$T_{film} = \frac{T_i + T_\infty}{2} = 548K, \rho = 7900kg/m3, k = 19W/m-K, cp = 546J/kg-K$$

$$Bi = \frac{h\frac{r_0}{2}}{k_{solid}} = 0.658 > 0.1$$

$\therefore$ LUMPED ANALYSIS NOT VALID

SPATIAL EFFECTS IMPORTANT



$$\theta_0^*(t) = \frac{\theta_0(t)}{\theta_i} = \frac{T_0(t)-T_\infty}{T_i - T_\infty} = C_1 \exp(-(\xi_1^2) F_o(t))$$

$$\frac{T_0(t)-T_\infty}{T_i - T_\infty} = \frac{50-30}{500-30} = 0.0426 = C_1 \exp(-(\xi_1^2) F_o(t))$$

$$\frac{\ln\left(\frac{0.0426}{C_1}\right)}{-(\xi_1^2)} = F_o(t) = \frac{\alpha t}{r_0^2}$$

$$\frac{\ln\left(\frac{0.0426}{C_1}\right)}{-(\xi_1^2)} \frac{r_0^2}{\alpha} = t$$

TABLE 5.1 → MODIFIED BI NUMBER

$$\text{Bi} = \frac{hr_0}{k} = 1.316$$

$C_1 = 1.2486, \zeta_1 = 1.3643 \leftarrow$ INTERPOLATION

$$\frac{\ln\left(\frac{0.0426}{C_1}\right)}{-(\xi_1^2)} \frac{r_0^2}{\alpha} = t$$

$$r_0 = 0.10/2, \alpha = \frac{k}{\rho c_p} = 4.4x10^{-6} m^2/s$$

$$\frac{\ln\left(\frac{0.0426}{1.2486}\right)}{-(1.3643^2)} \frac{(0.10/2)^2}{4.4x10^{-6} m^2/s} = t = 1031s \rightarrow 17\min$$

TOTAL ENERGY TRANSFER

$$Q^*(t) = \frac{Q(t)}{Q_0} = 1 - \frac{2\theta_0^*(t)}{\xi_1} J_1(\xi_1) \rightarrow \text{TOTAL ENERGY TRANSFER}$$

$$\frac{Q(t)}{rod} = Q_0\left(1 - \frac{2\theta_0^*(t)}{\xi_1} J_1(\xi_1)\right)$$

$$Q_0 \rightarrow \text{MAX POSSIBLE ENERGY TRANSFER}$$

$$\rightarrow \rho c_p \forall \theta_i \rightarrow 7900 kg/m^3 \bullet 546 J/kg-K \bullet \left(\pi r_0^2 L\right) \bullet (500-30)K$$

$$= 1.592x10^7 J$$

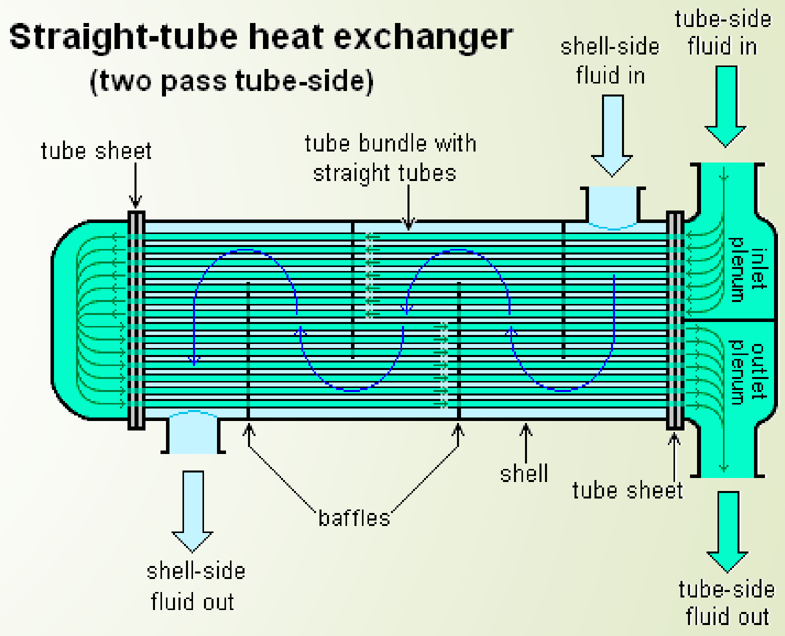
$$J_1(1.3643) = 0.535 \leftarrow \text{TABLE B.4}$$

$$\frac{Q(t)}{rod} = 1.592x10^7 J \bullet (1 - \frac{2x0.0426x0.535}{1.3643})$$

$$= 1.54x10^7 J$$

TOTAL HEAT LOAD FOR 10 RODS

$$\bar{q} = \frac{NQ}{\Delta t} = 10\text{rods} \bullet \frac{\frac{1.54x10^7 J}{1032s}}{rod} = 149kW$$



SURFACE TEMPERATURE

SURFACE TEMPERATURE,

$\rightarrow \mathrm{r}^* = 1.0$

$$\theta^*(r^*,t^*) = \frac{\theta(r,t)}{\theta_i} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = \theta_0^*(t^*)J_0(\xi r^*) = C_1 \exp(-\left(\xi_1^2\right)Fo(t))J_0(\xi_1 r^*)$$

$$T(r,t) = (T_i - T_\infty)\bullet\left[C_1 \exp(-\left(\xi_1^2\right)Fo(t))J_0(\xi_1 r^*)\right] + T_\infty$$

$$= \left(470C\right)\bullet(\theta_0(t))\bullet J_0(\xi_1 r^*) + T_\infty$$

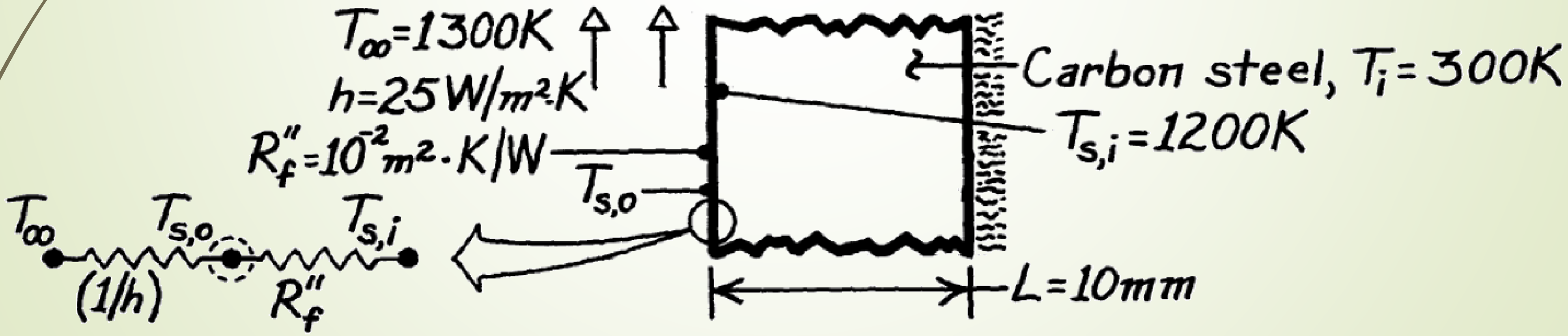
$$= 470C \bullet 0.0426 \bullet 0.586 + 30C$$

$$= 41.7C$$

$J_0(1.3643) = 0.586 \leftarrow$ TABLE B.4

Find time required for inner surface of wall (L=10mm) to reach 1200K. To protect wall from the enviironment a ceramic coating is added with a thermal resistance of $R''_f = 0.01 \frac{m^2-K}{W}$. Opposite wall is well insulated.

ρ=7850kg/m3,c_p=430J/kg-K,k=60W/m-k



ANALYSIS : Heat Transfer to wall is determined by the TOTAL RESISTANCE to Heat Transfer from the gas to the surface of the steel, AND NOT SIMPLY by the convective resistance.

$$UA\left[\frac{W}{m^2-K}\right]=\sum R_t^{-1}=\left\{\frac{1}{h\left[\frac{W}{m^2-K}\right]}+R_f''\frac{m^2-K}{W}\right\}^{-1}$$

$$=\left(\frac{1}{25\left[\frac{W}{m^2-K}\right]}+0.001\frac{m^2-K}{W}\right)^{-1}=20\frac{W}{m^2-K}$$

$$Bi=\frac{UL_c}{k_{solid}}=\frac{20\frac{W}{m^2-K}x0.01m}{60\frac{W}{m-K}}=0.0033<0.1$$

$\therefore$ *LUMPED* OK!!!!

SOLUTION

$$\ln\left[\frac{T(t)-T_\infty-b/a}{T_i-T_\infty-b/a}\right](-\tau)=t(time), b/a=0 \rightarrow \text{NO } S_{gen}$$

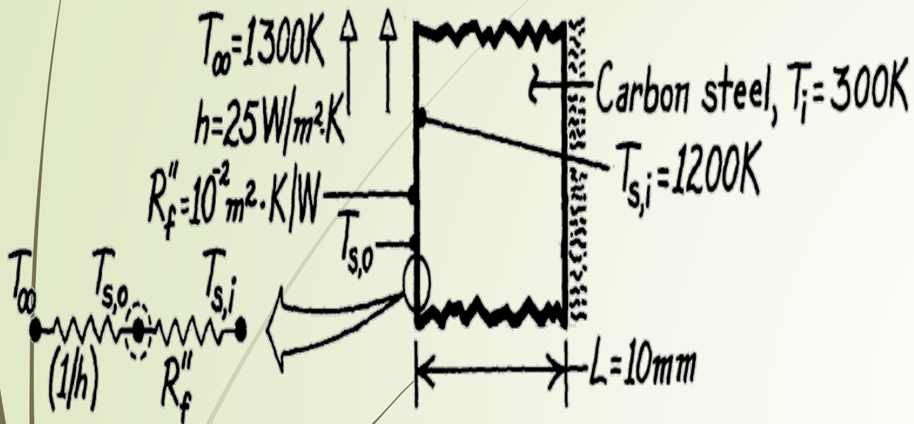
$$\tau \equiv \frac{\rho\forall c}{UA}=\frac{7850\frac{kg}{m^3}\bullet \cancel{A_s} L\bullet 430\frac{J}{kg-K}}{20\frac{W(J/s)}{m^2-K}\bullet \cancel{A_s}}=1688s$$

$$\ln\left[\frac{1200-1300}{300-1300}\right](-\tau)=t(time)$$

$$-2.3\bullet(-\tau)=t$$

$$3887s=t$$

COATING SURFACE TEMPERATURE



APPLYING THERMAL CIRCUITS

$$q[W]=\frac{T_\infty - T_{s,i}}{\sum R_t}=\frac{(1300-1200)K}{\frac{1}{hA\left[\frac{W}{K}\right]}+\frac{R''_f}{A}\left[\frac{K}{W}\right]}$$

$$\div A$$

$$\frac{q[W]}{A}=q''\left[\frac{W}{m^2}\right]=\frac{100}{\cancel{A}\left\{\frac{1}{h\cancel{A}\left[\frac{W}{m^2-K}\right]}+\frac{R''_f}{\cancel{A}}\left[\frac{m^2-K}{W}\right]\right\}}=\frac{100K}{0.05\frac{m^2-K}{W}}=2,000\frac{W}{m^2}$$

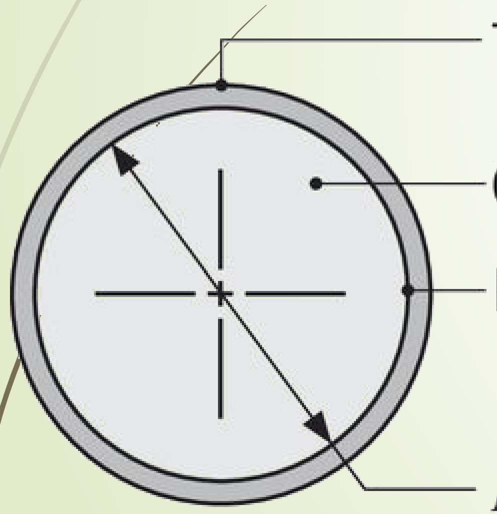
Now use first element in circuit

$$\frac{q}{A}\left[\frac{W}{m^2}\right]=\frac{\left(T_\infty - T_{s,0}\right)K}{\frac{1}{h\frac{W}{m^2-K}}}$$

$$T_{s,o}=T_\infty-\frac{q}{A}\left[\frac{W}{m^2}\right]\bullet\frac{1}{h\frac{W}{m^2-K}}=1300K-2,000\frac{W}{m^2}\bullet 0.04\frac{m^2-K}{W}$$

$$=1220K$$

Long Pyroceran Rod, Initially At A Uniform Temperature Of 900K, and clad with a thin metallic tube giving rise to a thermal contact resistance, is "suddenly" cooled by convection.



$T_\infty = 300K$

$T(r,t=0) = 900K, T(r=0,t_{final}) = 600K$

$h = 100 \dfrac{W}{m^2 - K}$

$R'_{t,c} = 0.12 \dfrac{m-K}{W}$

Find time for center to reach 600K?

PROPERTIES

PYROCERAN

NEED FILM TEMPERATURE

$$T_{film} = \frac{T_i + T_{final}}{2} = 750K \rightarrow \text{use table to obtain properties}$$

$$\rho = 2600 \frac{kg}{m^3}, c = 1100 \frac{J}{kg-K}, k = 3.13 \frac{W}{m-K}$$

CHECK BIOT

$$Bi_{CYLINDER} = \frac{\overline{U}\frac{r_0}{2}}{k_{kolid}}$$

$$UA = \frac{1}{\sum R_t} = \frac{1}{\frac{1}{hA\left[\frac{W}{K}\right]} + R'_{t,c}\left[\frac{m-K}{\frac{W}{L}}\right]}$$

$$\overline{U} = \frac{1}{A}\frac{1}{\sum R_t} = \frac{1}{\pi DL}\frac{1}{\sum R_t}$$

$$= \frac{1}{\pi DL}\frac{1}{\frac{1}{h(A=\pi DL)\left[\frac{W}{K}\right]} + R'_{t,c}\left[\frac{m-K}{\frac{W}{L}}\right]}$$

$$= \frac{1}{\frac{\cancel{\pi DL}}{h(A=\cancel{\pi DL})\left[\frac{W}{K}\right]} + R'_{t,c}\pi D\cancel{L}\left[\frac{m-K}{\frac{W}{\cancel{L}}}\right]}$$

$$= 57.0\frac{W}{m^2-K}$$

$$Bi_{CYLINDER} = \frac{\overline{U}\frac{r_0}{2}}{k_{kolid}}$$

$$= \frac{57\frac{\cancel{W}}{\cancel{m}^2-\cancel{K}}\bullet 10/1000m}{3.13\frac{\cancel{W}}{\cancel{m}-\cancel{K}}}$$

$$= 0.1821 > 0.1 \rightarrow \text{NOT LUMPED}$$

Center Temperature

$$\theta_0^*(t)=\frac{\theta_0(t)}{\theta_i}=\frac{T_0(t)-T_\infty}{T_i-T_\infty}=C_1(B_i)\exp(-\xi_1^2F_o(t))$$

$$\theta_0^*(t)=\frac{\theta_0(t)}{\theta_i}=\frac{T_0(t)-T_\infty}{T_i-T_\infty}=\frac{(600-300)K}{(900-300)K}=0.50$$

MODIFIED BI #

$$\text{Bi}=\frac{\overline{U}r_0}{k_{solid}}=0.182\rightarrow TABLE5.1\rightarrow\xi_1=0.5884,C_1=1.0441$$

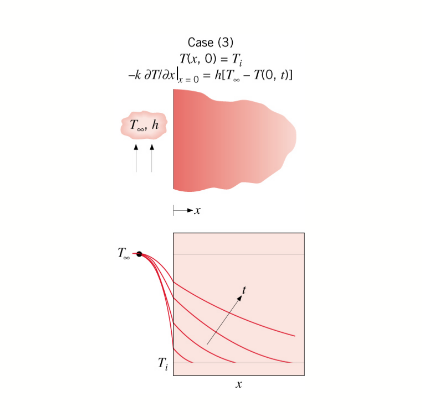
$$F_o(t)=\frac{\alpha t}{r_0^2}=\frac{\ln\left[\frac{\theta_0^*(t)}{C_1(B_i)}\right]}{-(\xi_1^2)}=\frac{-1}{(0.5884)^2}\ln\left[\frac{0.5}{1.0441}\right]=2.127$$

$$\alpha\equiv\text{THERMAL DIFFUSIVITY}=\frac{k}{\rho c}=\frac{W/m-K}{\frac{kg}{m^3}\frac{J}{kg-K}}=\frac{m^2}{s}$$

$$t=F_o(t)\frac{r_0^2[m^2]}{\alpha[\frac{m^2}{s}]}=\frac{2.127(0.010m)^2 2600\frac{kg}{m^3}x1000\frac{J}{kg-K}}{3.13\frac{W}{m-K}}=194s$$

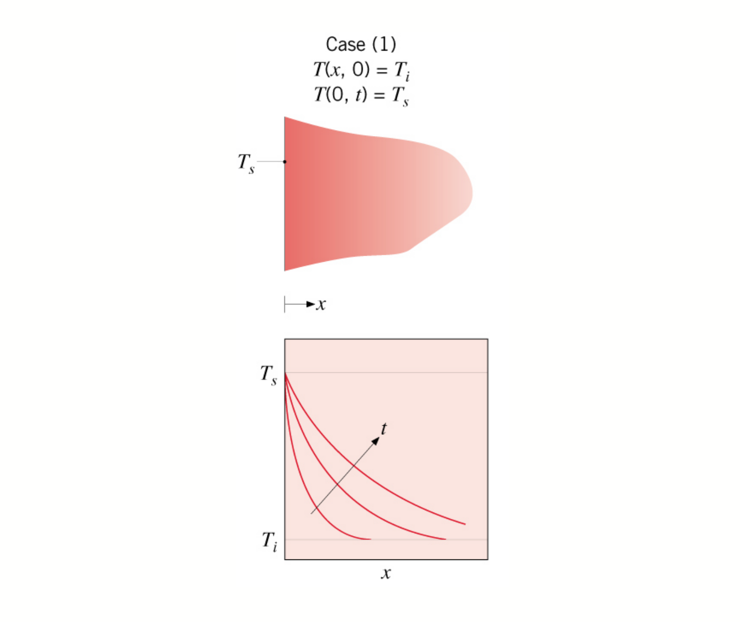
1D Spatial ***"INFNITE"*** DIMENSION HEAT TRANSFER T(x,t)

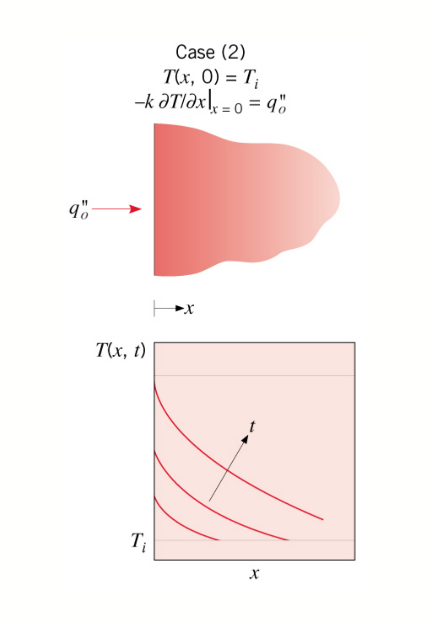
$Bi\#$ Has NO Relevence w/Semi-Infinite SOLIDS



The Semi-Infinite Solid

(Heat Transfer in INFINITE DIRECTION)

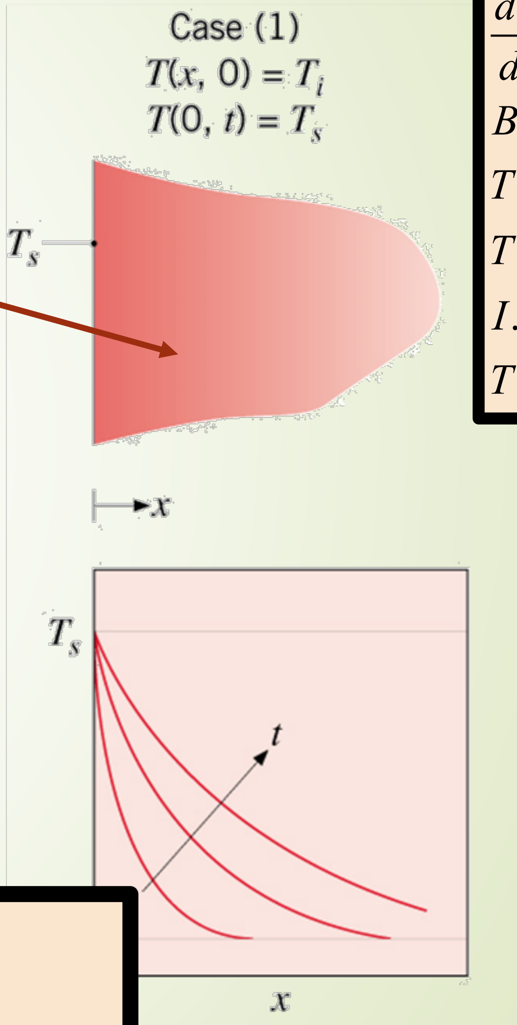




CASE 1: Change in Surface Temperature

$$\frac{T(x,t)-T_s}{T_i-T_s}=\mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$q''_s=\frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}$$



HDE

$$\frac{d^2T}{dx^2}=\frac{1}{\alpha}\frac{dT}{dt}$$

B.C

$$T(x=0,t)=T_s$$

$$T(x\to\infty,t)=T_i$$

I.C.

$$T(x,0)=T_i$$

EXAMPLE

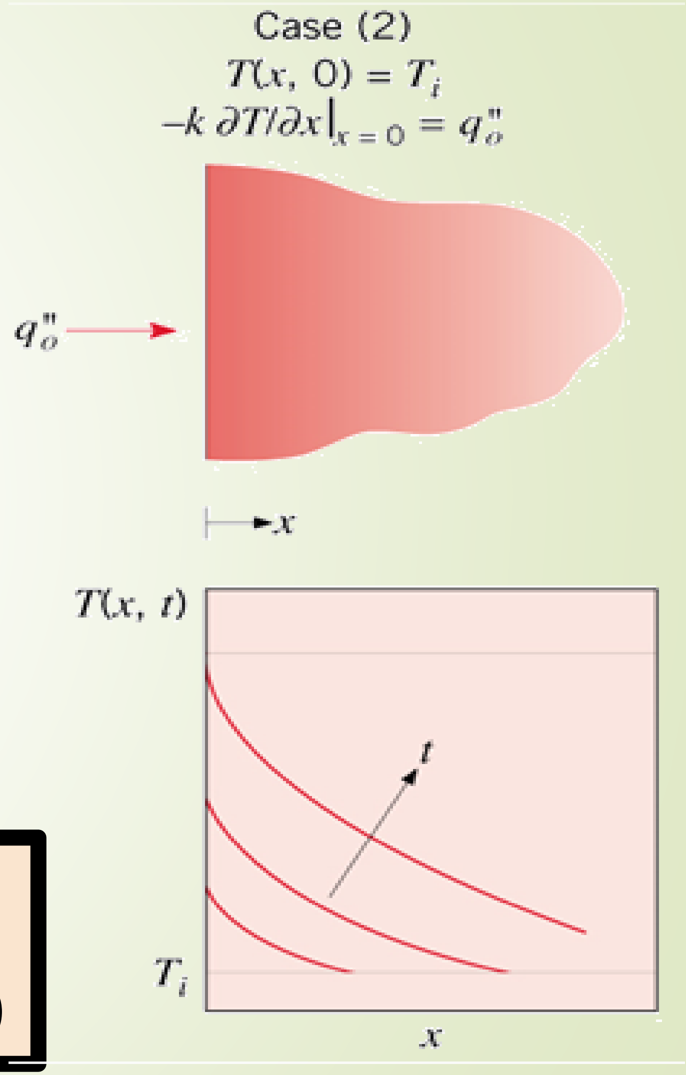
$$erf\left(\frac{x}{2\sqrt{\alpha t}}\right)=erf\left(\frac{0.25m}{2\sqrt{6.92x10^{-7}\frac{m^2}{s}\bullet 1800s}}\right)=erf(3.54)\sim 1.0;(TABLE\ B.2)$$

$$erfc(x)=1.0-erf(x)=0$$

The Semi-Infinite Solid
(Heat Transfer in INFINITE DIRECTION)

CASE 2: Change in Surface **HEAT FLUX** (Table B.2)

$$T(x,t) - T_i = \frac{2q''_o(\alpha t/\pi)^{1/2}}{k}\exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q''_o x}{k}\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

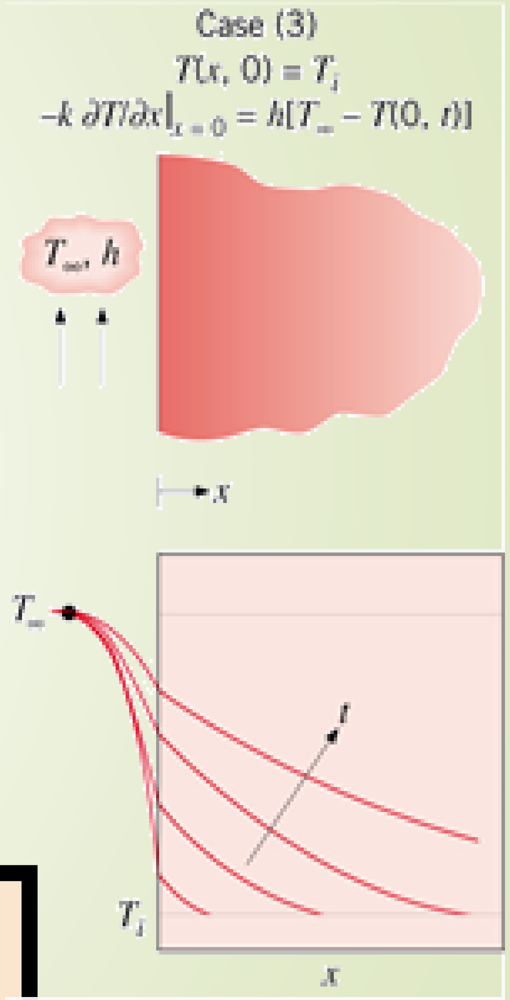


The Semi-Infinite Solid
(Heat Transfer in INFINITE DIRECTION)

CASE 3: Change in Surface **CONVECTION** *(Table B.2)*

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left[T_\infty - T(0,t)\right]$$

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



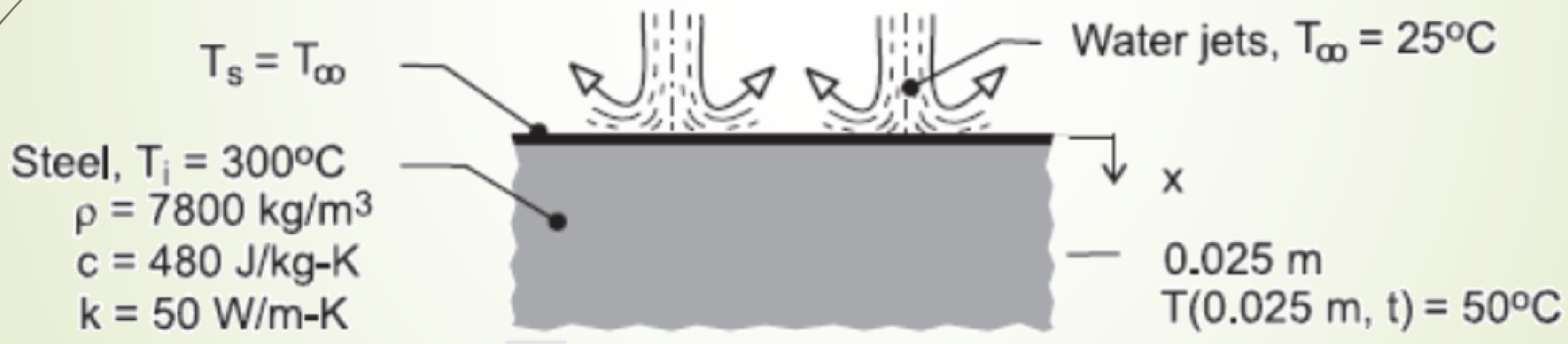
The Semi-Infinite Solid
(Heat Transfer in INFINITE DIRECTION)

For the SUDDEN water quench operation below, find the time required to cool the interior location (x=0.025m) to a temperature of 50C. What is the surface heat flux?

PROBLEM CLASSIFICATION: TRANSIENT CONDUCTION

GEOMETRY CLASS: SEMI-INFINITE SOLID

BI NUMBER is of NO use with this class of problems.!!!!!!!!!!!!!!!!!!!!!!!



The Semi-Infinite Solid
(Heat Transfer in INFINITE DIRECTION)

LARGE CONVECTION; h $\rightarrow \infty \rightarrow (T_s \approx T_\infty)$

$$\frac{T(x,t)-T_s}{T_i - T_s} = \frac{50-25}{300-25} = \frac{25}{275} = 0.09 = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

so

$$0.09 = erf(w)$$

TABLE B.2

0.09=erf(w $\approx$ 0.08)

so

$$0.08 = \frac{x}{2\sqrt{\alpha t}} \rightarrow t = \frac{x^2}{4w^2\alpha} = \frac{0.025^2}{4x0.08^2\,x\,\frac{k}{\rho c_p}} = 1.828x10^3 \text{ sec} = 0.5hr$$

$$q_s''(t) = \frac{k(T_s - T)_i}{\sqrt{\pi\alpha t}}$$

$$= -49.6kW/m^2$$