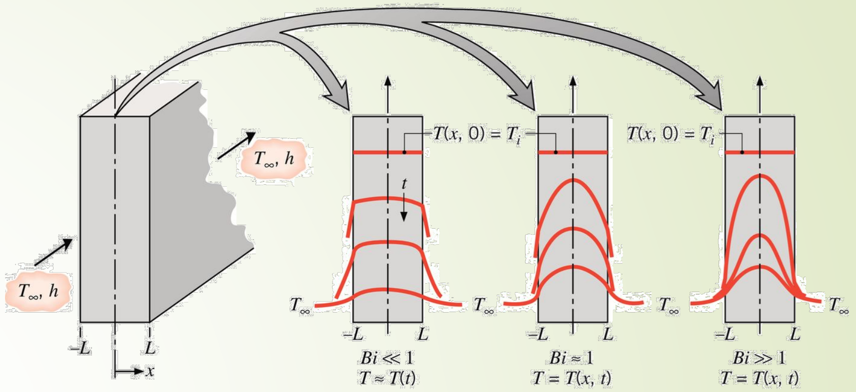
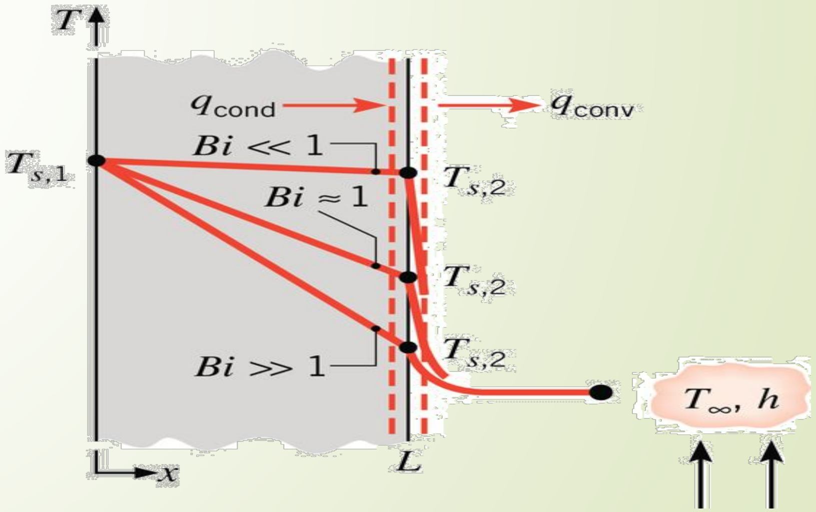
Transient Conduction

MECH-420 Heat Transfer





Transient Conduction

- A heat transfer process for which the temperature varies with time, as well as location within a solid.

- It is initiated whenever a system experiences a change in operating conditions.

- It can be induced by changes in:
  - surface convection conditions (h, T_∞),
  - surface radiation conditions (h_r, T_{sur}),
  - a surface temperature or heat flux, and/or
  - internal energy generation.

- Solution Techniques
  - The Lumped Capacitance Method
  - Exact Solutions
  - The Finite-Difference Method (numerical)

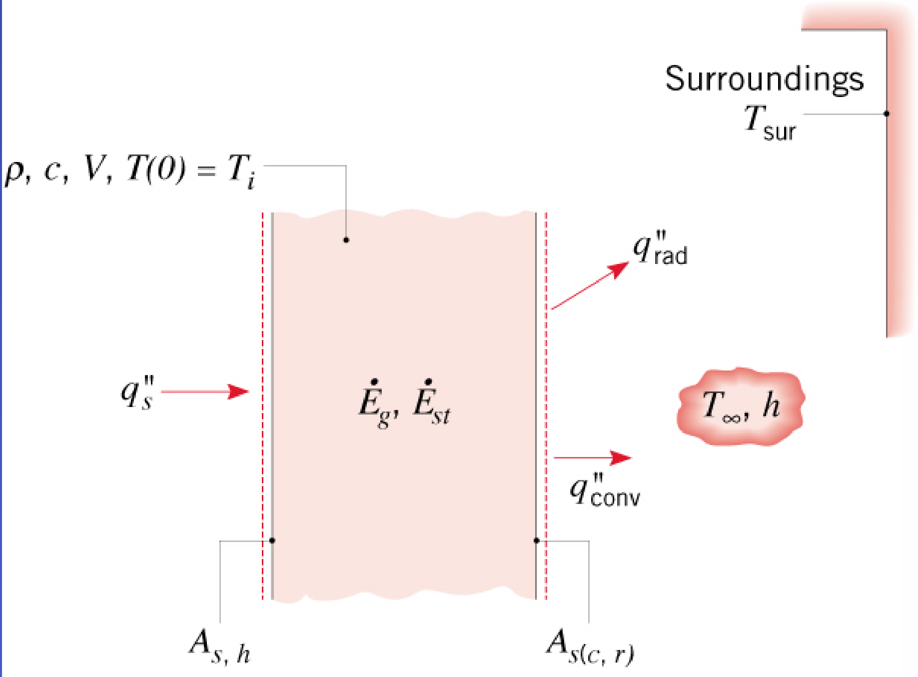
The Lumped Capacitance Method

- Based on the assumption of a spatially uniform temperature distribution throughout the transient process. Hence, $T(\vec{r},t) \approx T(t)$.

- Why is the assumption never fully realized in practice?

- General Lumped Capacitance Analysis:

  - Consider a general case, which includes convection, radiation and/or an applied heat flux at specified surfaces $(A_{s,c}, A_{s,r}, A_{s,h})$, as well as internal energy generation



- First Law:

$$\frac{dE_{st}}{dt} = \rho \forall c \frac{dT}{dt} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g$$

- Assuming energy outflow due to convection and radiation and inflow due to an applied heat flux q_s'',

$$\rho \forall c \frac{dT}{dt} = q_s'' A_{s,h} - h A_{s,c}\left(T - T_\infty\right) - h_r A_{s,r}\left(T - T_{\text{sur}}\right) + \dot{E}_g \qquad (5.15)$$

- May h and h_r be assumed to be constant throughout the transient process?

- How must such an equation be solved?

- Special Cases (Exact Solutions, $T(0) \equiv T_i$)
  - Negligible Radiation $\left(\theta \equiv T - T_\infty,\ \theta' \equiv \theta - b/a\right)$:

$$a \equiv hA_{s,c} / \rho\forall c \qquad b \equiv \left(q_s'' A_{s,h} + \dot{E}_g\right) / \rho\forall c$$

The non-homogeneous differential equation is transformed into a homogeneous equation of the form:

$$\frac{d\theta'}{dt} + a\theta' = b$$

Integrating from $t = 0$ to any t and rearranging,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}\left[1 - \exp(-at)\right] \qquad (5.25)$$

To what does the foregoing equation reduce as steady state is approached?

How else may the steady-state solution be obtained?

Special Case (Convection)

- Negligible Radiation and Source Terms $\left(h >> h_r,\ \dot{E}_g = 0,\ q_s'' = 0\right)$:

$$\rho \forall c \frac{dT}{dt} = -hA_{s,c}\left(T - T_\infty\right) \qquad (5.2)$$

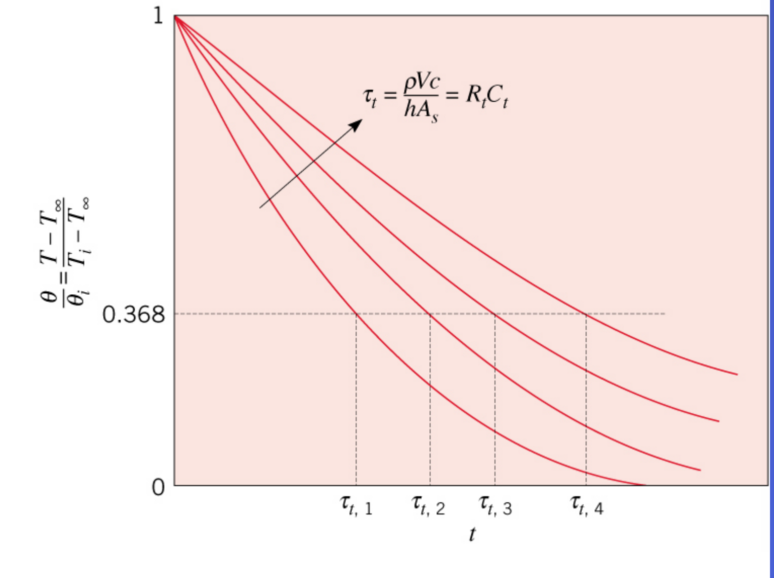
$$\frac{\rho \forall c}{hA_{s,c}} \int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t dt$$

Note: $\theta \equiv T - T_\infty$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_{s,c}}{\rho \forall c}\right)t\right] = \exp\left[-\frac{t}{\tau_t}\right] \qquad (5.6)$$

The thermal time constant is defined as

$$\tau_t \equiv \underbrace{\left(\frac{1}{hA_{s,c}}\right)}_{\text{Thermal Resistance, } R_t} \underbrace{\left(\rho \forall c\right)}_{\text{Lumped Thermal Capacitance, } C_t} \qquad (5.7)$$



The change in thermal energy storage due to the transient process is

$$\Delta E_{st} \equiv -Q = -\int_0^t \dot{E}_{out}\, dt = -hA_{s,c}\int_0^t \theta\, dt = -\left(\rho \forall c\right)\theta_i\left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right] \qquad (5.8)$$

- Negligible Convection and Source Terms $\left(h_r >> h,\ \dot{E}_g = 0,\ q_s'' = 0\right)$:

Assuming radiation exchange with large surroundings,

$$\rho \forall c \frac{dT}{dt} = -\varepsilon A_{s,r} \sigma \left(T^4 - T_{\text{sur}}^4\right)$$

$$\frac{\varepsilon A_{s,r} \sigma}{\rho \forall c} \int_0^t dt = \int_{T_i}^{T} \frac{dT}{T_{\text{sur}}^4 - T^4}$$

$$t = \frac{\rho \forall c}{4 \varepsilon A_{s,r} \sigma T_{\text{sur}}^3} \left\{ \ln \left| \frac{T_{\text{sur}} + T}{T_{\text{sur}} - T} \right| - \ln \left| \frac{T_{\text{sur}} + T_i}{T_{\text{sur}} - T_i} \right| + 2 \left[\tan^{-1} \left(\frac{T}{T_{\text{sur}}} \right) - \tan^{-1} \left(\frac{T_i}{T_{\text{sur}}} \right) \right] \right\} \quad (5.18)$$

This result necessitates implicit evaluation of $T(t)$.

BIOT NUMBER

$$Bi = \frac{R_{th_{CONDUCTION}}}{R_{th_{CONVECTION}}} = \frac{UL_c}{k_{SOLID}} \ll 0.1 \rightarrow LUMPED \rightarrow T(t) \text{ ONLY}$$

$$L_c = \frac{\forall}{A_s} \equiv \frac{\text{Volume}}{\text{Surface Area}}$$

$$U = \text{OVERALL HEAT TRANSFER COEF} \frac{W}{m^2 - K}.$$

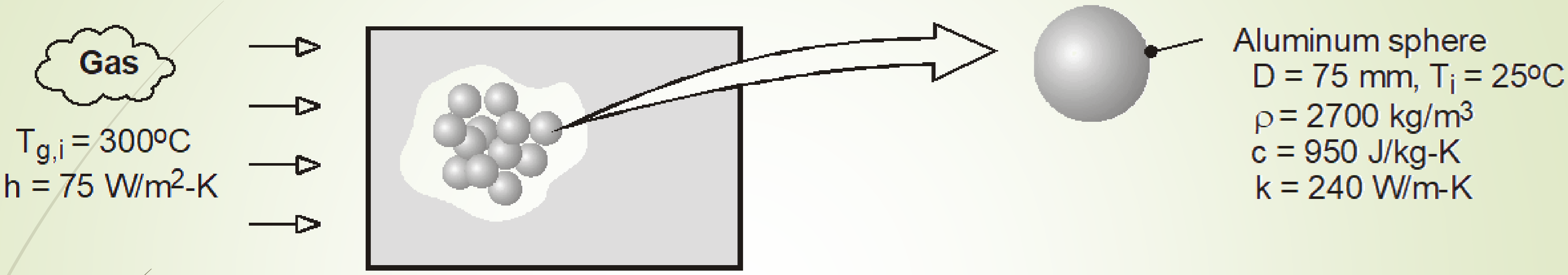
$$= \left\{ \frac{\left(\sum R_{THERMAL} \frac{K}{W} \right)^{-1}}{(A)} \right\} \frac{W}{m^2 - K}$$

$$= \left\{ \left(\sum R''_{THERMAL} \frac{m^2 - K}{W} \right)^{-1} \right\} \frac{W}{m^2 - K}$$

$$L_c = L \rightarrow \text{PLAIN WALL}$$

$$= \frac{r_0}{2} \rightarrow Cylinder$$

$$= \frac{r_0}{3} \rightarrow Sphere$$



Time required for sphere to acquire 90% of MAX possible thermal energy and find center temperature

Analysis

- Neglect heat transfer to/from other spheres or from a sphere by radiation or conduction due to contact with other spheres.
- Constant Properties
- Check ***LUMPED CAPACITANCE*** criteria to see Temperature is function of TIME only, and not a function of SPACE.

$$Bi_{SPHERE} = \frac{h\frac{r_0}{3}}{k_{SOLID}} = \frac{75\frac{W}{m^2-K}\frac{0.0375}{3}}{150\frac{W}{m-K}} = 0.00625 < 0.1 \rightarrow \text{LUMPED}$$

Lumped Capactiance and a uniform temperature throughout at any instant of time.

But all points changing over time together.

$$\text{Q(t)}=\left(\rho\forall c_p\right)\theta_i\left[1-\exp\left(\frac{-t}{\tau_t}\right)\right];\tau_t=\left(\frac{1}{hA_s}\right)\rho\forall c_p \rightarrow \text{Thermal Time Constant}$$

$$\Delta E_{st} \equiv -Q = -\int_0^t \dot{E}_{out}dt = -hA_{s,c}\int_0^t \theta dt = -\left(\rho\forall c\right)\theta_i\left[1-\exp\left(-\frac{t}{\tau_t}\right)\right]$$

$$\frac{Q}{Q_i} = \frac{Q}{\left(\rho\forall c\right)\theta_i} = 0.90 = \left[1-\exp\left(-\frac{t}{\tau_t}\right)\right]$$

$$t = -\ln(1-0.90)\bullet\tau_t$$

$$\tau_t = \left(\frac{1}{hA_s}\right)\rho\forall c_p = \frac{\rho c_p}{h}\frac{r_0^3}{3r_0^2} = \frac{kg/m^3}{\frac{W}{m^2-K}}\frac{J-m}{kg-K} = 427\sec$$

$$t = -\ln(0.1)\bullet 427s = 984\sec$$

TEMPERATURE IF Given TIME

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=\exp\left(-\frac{t}{\tau}\right)+\frac{b/a}{T_i-T_\infty}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]$$

$$a=\frac{1}{\tau};\tau\equiv\frac{\rho\forall c}{hA_{s,c}} \qquad b\equiv\left(\cancel{q_s''A_{s,h}}+\cancel{\dot{E}_g}\right)/\rho\forall c$$

$$T(t)=T_\infty+\left(T_i-T_\infty\right)\exp\left(-\frac{t}{\tau}\right)$$

$$=300+(25-300)\exp(-984/427)$$

$$=272.5C$$

TIME IF Given TEMPERATURE

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=\exp\left(-\frac{t}{\tau}\right)+\frac{b/a}{T_i-T_\infty}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]$$

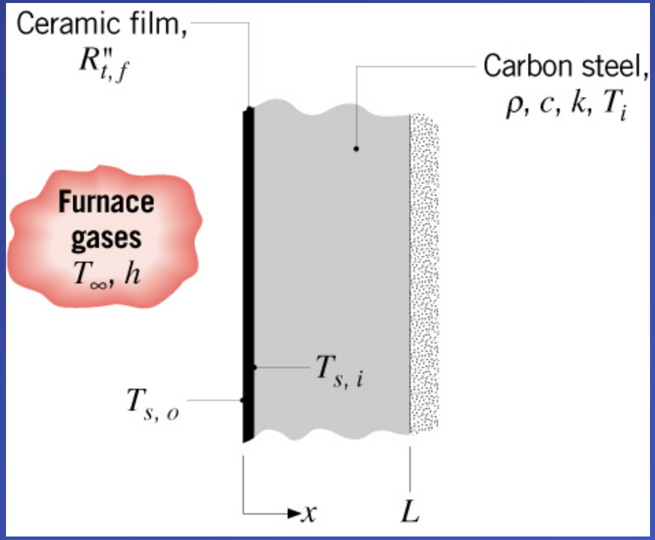
$$\frac{T(t)-T_\infty}{T_i-T_\infty}=\exp\left(-\frac{t}{\tau}\right)\left(1-\frac{b/a}{T_i-T_\infty}\right)+\frac{b/a}{T_i-T_\infty}$$

$$\ln\left[\frac{T(t)-T_\infty-b/a}{T_i-T_\infty-b/a}\right](-\tau)=t$$

$$a=\frac{1}{\tau};\tau\equiv\frac{\rho\forall c}{\mathrm{h}A_{s,c}}=\mathrm{R}_t\mathrm{C}_t$$

$$\mathrm{b}=\frac{q_s''A_s+\dot{E}_g}{\rho\forall c}$$

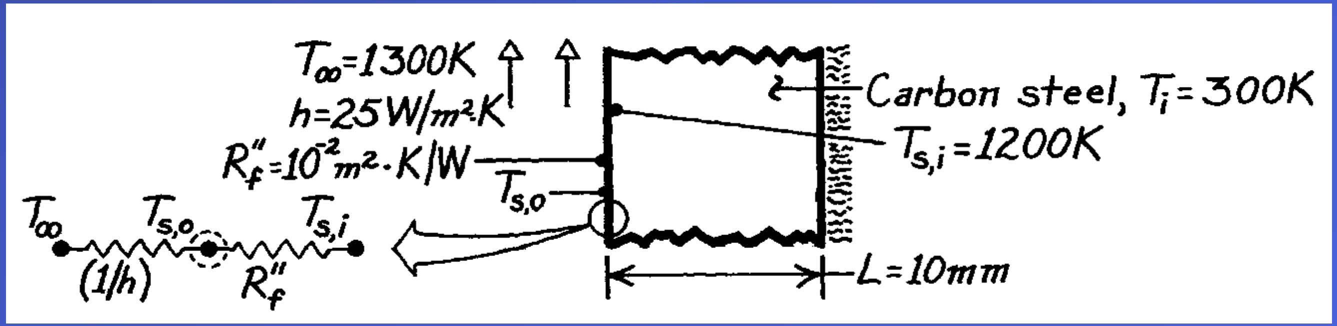
Problem 5.22: Heating of coated furnace wall during start-up.



KNOWN: Thickness and properties of furnace wall. Thermal resistance of ceramic coating on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of coating surface temperature.

Schematic:



ASSUMPTIONS: (1) Constant properties, (2) Negligible coating thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel: $\rho = 7850$ kg/m^3, $c = 430$ J/kg·K, $k = 60$ W/m·K.

ANALYSIS: HEAT TRANSFER TO THE WALL IS DETERMINED BY THE TOTAL RESISTANCE TO HEAT TRANSFER FROM THE GAS TO THE SURFACE OF THE STEEL, AND NOT SIMPLY BY THE CONVECTION RESISTANCE.

Hence, with

$$U = \left(R''_{tot}\right)^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ \text{W/m}^2\cdot\text{K}} + 10^{-2}\ \text{m}^2\cdot\text{K/W}\right)^{-1} = 20\ \text{W/m}^2\cdot\text{K}.$$

$$Bi = \frac{UL}{k} = \frac{20\ \text{W/m}^2\cdot\text{K} \times 0.01\ \text{m}}{60\ \text{W/m}\cdot\text{K}} = 0.0033 << 0.1$$

and the lumped capacitance method can be used.

(a) From Eqs. (5.6) and (5.7),

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-t/\tau_t\right) = \exp\left(-t/R_t C_t\right) = \exp\left(-Ut/\rho Lc\right)$$

$$\tau_t \equiv \left(\frac{1}{hA_{s,c}}\right)\left(\rho \forall c\right) = \frac{C_t}{R_t} = \frac{C_t}{UA}$$

$$t = -\frac{\rho Lc}{U} \ln \frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ \text{kg/m}^3\,(0.01\ \text{m})\,430\ \text{J/kg}\cdot\text{K}}{20\ \text{W/m}^2\cdot\text{K}} \ln \frac{1200 - 1300}{300 - 1300}$$

$$t = 3886\ \text{s} = 1.08\ \text{h}.$$

<

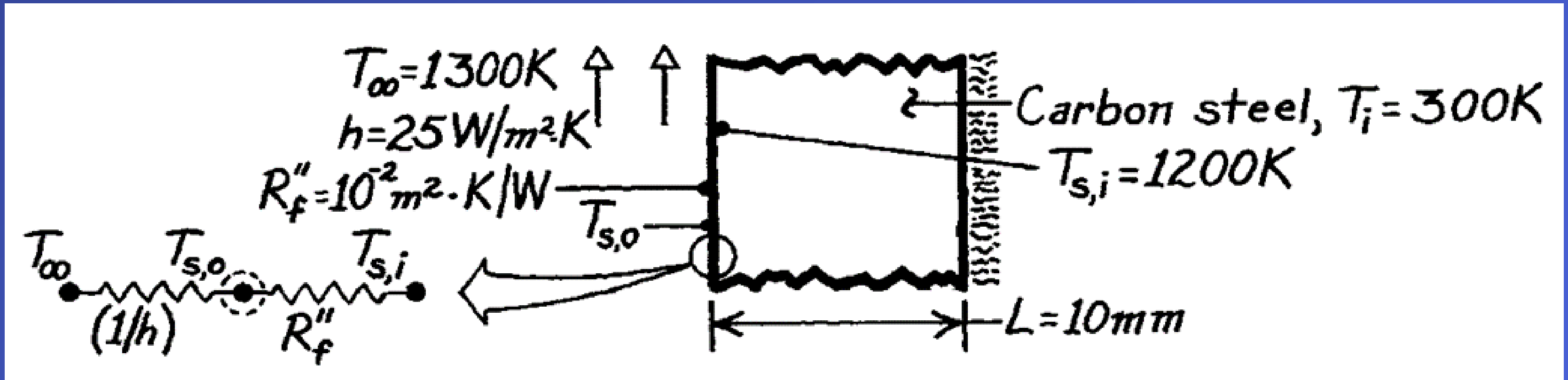
(b) Performing an energy balance at the outer surface (s,o),

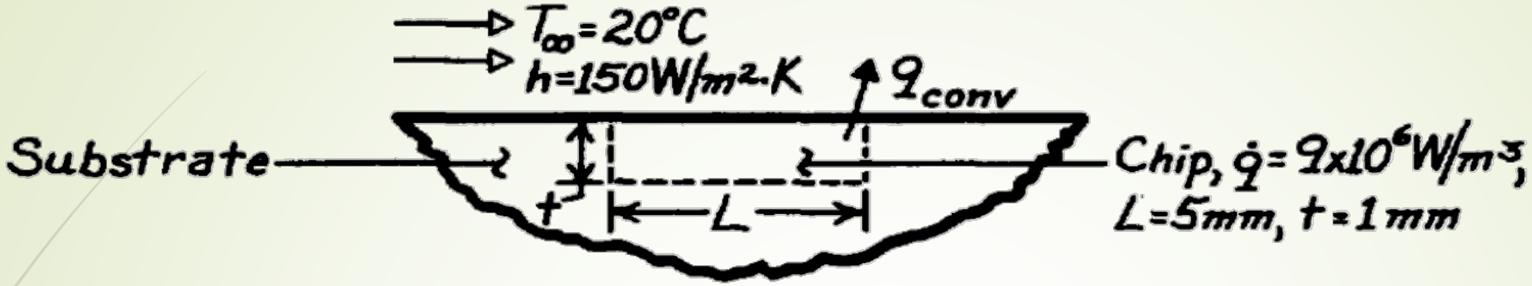
$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R''_f$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + \left(1/R''_f\right)} = \frac{25\ \mathrm{W/m^2 \cdot K} \times 1300\ \mathrm{K} + 1200\ \mathrm{K}/10^{-2}\mathrm{m^2 \cdot K/W}}{\left(25+100\right)\mathrm{W/m^2 \cdot K}}$$

$$T_{s,o} = 1220\ \mathrm{K}. \qquad <$$

How does the coating affect the thermal time constant?





INTEGRATED CIRCUIT:

Steady State Temperature and time to come within 1C of steady state.

Find Steady State Temperature

1^{st} Law

$$-E_{out} + E_{gen} = 0$$

$$-h(L^2)(T_s - T_\infty) + \dot{S}_{gen}(L^2 \bullet t) = 0$$

$$T_s = T_\infty + \frac{\dot{S}_{gen}}{h}$$

$$= 80C$$

$$\rightarrow \frac{V}{A_s} = \frac{L^2 \bullet thick}{L^2} = thick$$

$$a = \frac{h}{\rho(thick)c_p} = \frac{150\frac{W}{m^2-K}}{2000\frac{kg}{m^3} \bullet 0.001m \bullet 700\frac{J}{kg-K}} = 0.107s^{-1}$$

$$b = \frac{S_{gen}}{\rho c_p} = 6.429K/s$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau}\right) + \frac{b/a}{T_i - T_\infty}\left[1-\exp\left(-\frac{t}{\tau}\right)\right]$$

$$a = \frac{1}{\tau}; \tau \equiv \frac{\rho \forall c}{hA_{s,c}} \qquad b \equiv \left(q_s'' A_{s,h} + \dot{E}_g\right)/\rho\forall c$$

Solve for t

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau}\right)\left(1-\frac{b/a}{T_i - T_\infty}\right) + \frac{b/a}{T_i - T_\infty}$$

$$\frac{\frac{T(t)-T_\infty}{T_i - T_\infty} - \frac{b/a}{T_i - T_\infty}}{\left(1-\frac{b/a}{T_i - T_\infty}\right)} = \exp\left(-\frac{t}{\tau}\right)$$

$$\frac{T(t)-T_\infty - b/a}{T_i - T_\infty - b/a} = \exp\left(-\frac{t}{\tau}\right)$$

$$\ln\left[\frac{T(t)-T_\infty - b/a}{T_i - T_\infty - b/a}\right](-\tau) = t$$

$$\frac{\ln\left[\frac{79-20-60}{20-20-60}\right]}{-0.107s^{-1}} = 38.3s$$