



## UNIT STUDY AID & PARAMETRIC THOUGHT

**Understanding units and unit conversion is the first primary skill for parametric engineering modeling!!!**

- ✓ Be able to determine correct dimensional units for constants within SI and BKS measurement systems if given any arbitrary function.
- ✓ Considering the velocity stream of  $\vec{V}(x, y) \left[ \frac{m}{s} \right] = \left\{ 10.2 \left[ \right] x^{\frac{1}{3}} - 20.4 \left[ \right] e^{-\frac{y[m]}{3 \left[ \right]}} \right\} \left[ \frac{m}{s} \right]$ , the chemical molar dispersion [moles/sec] is measured as:

$$\psi(t, x, y) \left[ \frac{\text{moles}}{s} \right] = 3.5 \left[ \right] e^{-5 \left[ \right] t \left[ s \right]} + 20.2 \left[ \right] x^2 \left[ m^2 \right] - 0.3 \left[ \right] y^3 \left[ m^3 \right], \text{ where } x \text{ and } y \text{ are in meters.}$$

What are the units [ ] in brackets for both

$$\vec{V} \left[ \frac{m}{s} \right], \psi \left[ \frac{\text{moles}}{s} \right], \frac{\partial \psi}{\partial t} \left[ \frac{\frac{\text{moles}}{s}}{s} = \frac{\text{moles}}{s^2} \right], \text{ and, } \frac{\partial \psi}{\partial x} \left[ \frac{\frac{\text{moles}}{s}}{m} = \frac{\text{moles}}{s \cdot m} \right] ?$$

**SHOW THAT UNITS ARE CORRECT FOR EACH TERM**

$$\vec{V}(x, y) \left[ \frac{m}{s} \right] = \left\{ 10.2 \left[ \right] x^{\frac{1}{3}} - 20.4 \left[ \right] e^{-\frac{y[m]}{3 \left[ \right]}} \right\} \left[ \frac{m}{s} \right]$$

"10.2"

$$\left[ \frac{m}{s} \right] = \left[ \right] m^{\frac{1}{3}} \rightarrow \left[ \right] = \frac{\left[ \frac{m}{s} \right]}{m^{\frac{1}{3}}} = \frac{m^{\frac{2}{3}}}{s}$$

"20.4"

$$\left[ \frac{m}{s} \right] = \left[ \right] [\text{unitless}] \rightarrow \left[ \right] = \frac{m}{s}$$

"3"

$$\frac{y[m]}{3 \left[ \right]} = [\text{unitless}] \rightarrow \left[ \right] = m$$

$$\psi(t, x, y) \left[ \frac{\text{moles}}{s} \right] = 3.5 \left[ \right] e^{-5 \left[ \right] t[s]} + 20.2 \left[ \right] x^2 [m^2] - 0.3 \left[ \right] y^3 [m^3]$$

"3.5"

$$\frac{\text{moles}}{s} = \left[ \right] [\text{unitless}] \rightarrow \left[ \right] = \frac{\text{moles}}{s}$$

"5"

$$[\text{unitless}] = \left[ \right] s \rightarrow \left[ \right] = \frac{1}{s}$$

"20.2"

$$\frac{\text{moles}}{s} = \left[ \right] m^2 \rightarrow \left[ \right] = \frac{\frac{\text{moles}}{s}}{m^2} = \frac{\text{moles}}{s - m^2}$$

"0.3"

$$\frac{\text{moles}}{s} = \left[ \right] m^3 \rightarrow \left[ \right] = \frac{\frac{\text{moles}}{s}}{m^3} = \frac{\text{moles}}{s - m^3}$$

$$\psi(t, x, y) \left[ \frac{\text{moles}}{s} \right] = 3.5 \left[ e^{-5[t/s]} \right] + 20.2 \left[ x^2 [m^2] \right] - 0.3 \left[ y^3 [m^3] \right]$$

$$\frac{\partial \psi(t, x, y)}{\partial t} \left[ \frac{\text{moles}}{s} \right] = \frac{\text{moles}}{s^2} = \overbrace{\left( -3.5 \left[ \frac{\text{moles}}{s} \right] \cdot 5 \left[ \frac{1}{s} \right] \right)}^{\text{IT IS IMPORTANT TO CARRY UNITS WITH THESE NUMBERS}} e^{-5[t/s]} \quad \text{NEVER WRITE ANY NUMBER WITHOUT PROPER UNITS}$$

$$= -(3.5 \cdot 5) \left[ \frac{\text{moles}}{s^2} \right] e^{-5[t/s]}$$

$$\frac{\partial \psi(t, x, y)}{\partial x} \left[ \frac{\text{moles}}{s} \right] = \frac{\text{moles}}{s \cdot m} = 2 \cdot 20.2 \left[ \frac{\text{moles}}{s \cdot m^2} \right] x [m]$$

$$= \left\{ 2 \cdot 20.2 \left[ \frac{\text{moles}}{s \cdot m^2} \right] x [\cancel{m}] \right\} \left[ \frac{\text{moles}}{s \cdot m} \right]$$



As staff scientist for Cosmic Physics Inc. on the deep space research vessel **PROTIUS ONE**, your team after months of analysis has measured data and have determined the following equation for the Neutron Power Flux ( $\Psi$ ) emitted within a Black Hole where "c" is the speed of light (m/s), "t" is time, and "J" is Joules:

$$\Psi(t) \left[ \frac{J}{s} \right] = A [ ]^{3/2} t^{1/2} + B [ ]^{-1/3} t^3 + C [ ]^{-2/3} t^{5/6} + D [ ] c [m / s]^2 t^{-12}$$

- Determine the units of time varying constants A[], B[], C[], and D[].
- The time rate of change of the neutron power flux ( $\frac{d\Psi}{dt}$ ) provides the power absorption rate of radiant energy. Determine ( $\frac{d\Psi}{dt} \left[ \frac{J / s}{s} \right]$ ) and show that units are **CORRECT**.

$$\left[ \frac{J}{s} \right] = A[]^{3/2} [s]^{1/2}$$

$$A[]^{3/2} = \frac{J}{s^{1/2}} = \frac{J}{s^{3/2}}$$

$$A[] = \left[ \frac{J}{s^{3/2}} \right]^{2/3} = \frac{J^{2/3}}{s}$$

$$\left[ \frac{J}{s} \right] = B[]^{-1/3} [s]^3$$

$$B[]^{-1/3} = \frac{J}{s^3} = \frac{J}{s^4}$$

$$\frac{J}{s} = C[]^{-2/3} [s]^{5/6}$$

$$C[]^{-2/3} = \frac{J}{s^{5/6}} = \frac{J}{s^{11/6}}$$

$$C[] = \left[ \frac{J}{s^{11/6}} \right]^{-3/2} = J^{-3/2} s^{\frac{33}{12}}$$

$$\left[ \frac{J}{s} \right] = D[] [m/s]^2 [s]^{-12}$$

$$D[] = \frac{\frac{J}{s}}{[m/s]^2 s^{-12}} = \frac{J/m^2}{s^{-13}}$$

## Gradient

$$\frac{d\Psi}{dt} \left[ \frac{J/s}{s} \right] = 1/2 A^{3/2} t^{-1/2} - 3B^{-1/3} t^2 + 5/6 C^{-2/3} t^{-1/6} - 12Dc[m/s]^2 t^{-13}$$

UNITS

$$\left[ \frac{J/s}{s} \right] = A^{3/2} [s]^{-1/2} = \left[ \frac{J^{2/3}}{s} \right]^{3/2} s^{-1/2} = \left[ \frac{J/s}{s} \right]$$

$$A[] = \frac{J^{2/3}}{s}$$

$$\left[ \frac{J/s}{s} \right] = B^{-1/3} t^2 = \left[ \frac{J^{-3}}{s^{12}} \right]^{-1/3} s^2 = \left[ \frac{J}{s^4} \right] s^2 = \left[ \frac{J/s}{s} \right]$$

$$B[] = \frac{J^{-3}}{s^{12}}$$

$$\frac{J/s}{s} = C^{-2/3} t^{-1/6} = \left[ J^{-3/2} s^{\frac{33}{12}} \right]^{-2/3} s^{-1/6} = J s^{-72/36} = \left[ \frac{J/s}{s} \right]$$

$$C[] = J^{-3/2} s^{\frac{33}{12}}$$

$$\left[ \frac{J/s}{s} \right] = Dc[m/s]^2 t^{-13} = \left[ \frac{J/m^2}{s^{-13}} \right] [m/s]^2 s^{-13} = \frac{J}{s^{-13} s^{13} s^2} = \left[ \frac{J/s}{s} \right]$$

$$D[] = \frac{J/m^2}{s^{-9}}$$