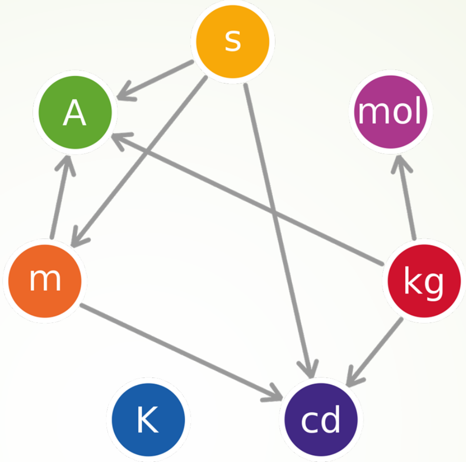
UNIT STUDY AID

MECH-322 Fluid Mechanics



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FOLLOW THE PATH

As you work to "re-work" the problems from the notes and the homework you should always recall the following basic engineering solution pathways:

1. If you don't understand the definitions used within the problem statement, how can one possibly have any chance of obtaining a valid solution.

2. Never put any number on paper unless one also state the units. (unless it is for certain that is indeed has no units, and can be proven).

3. Always ask? What are the fluid fundamentals driving the solution path?

4. Always form solution is terms of problem "variables" with "units" expressed, and not with numbers.

5. Always check units of final solution to ensure "form" of result is correct with proper units as expected, BEFORE moving to final stage of just plugging a number into a calculator.

Read the textbook, do the homework, and work with other students constantly to understand the lecture material. MECH-322 ramps up quickly and builds upon itself. Problems take a while to solve, and if you don’t know exactly what you’re doing finishing the exams won’t be possible. Units are a very useful way to double check your work. If you’re not sure if you’re doing it right, units will let you know if you’re mixing up anything.

MECH-322 Student, FALL 2021

The velocity profile for a fluid flowing in a duct is expressed as:

$$u(y)\left[\frac{m}{s}\right] = A[?]y[m] + B[?]y^2[m^2]$$

where A and B are arbitrary constants,

and "y" is the variable distance measured from the bottom wall.

If the fluid viscosity is μ:

- What must be the correct units for "A" and "B"?
- What is the parametric equation for the velocity gradient, AND show that units are correct?
- What is the parametric equation for the Shear Stress, AND show that units are correct?

$$u(y)\left[\frac{m}{s}\right] = Ay + By^2$$

UNITS

$$\left[\frac{m}{s}\right] = A[?][m] + B[?][m^2]$$

$$\left[\frac{m}{s}\right] \leftrightarrow \left[\frac{m}{s}\right] + \left[\frac{m}{s}\right]$$

$$A[] = \frac{\frac{m}{s}}{m} = \frac{1}{s}$$

$$B[] = \frac{\frac{m}{s}}{m^2} = \frac{1/s}{m}$$

VELOCITY GRADIENT (time rate of strain)

$$\frac{du(y)}{dy}\left[\frac{m/s}{m} = 1/s\right] = A\left[\frac{1}{s}\right] + 2B\left[\frac{1/s}{\not{m}}\right]y[\not{m}]$$

SHEAR STRESS

$$\tau = \mu\left[\frac{N-s}{m^2}\right]\frac{du}{dy}\left[\frac{1}{s}\right] = \frac{N}{m^2} = \mu\left[\frac{N-\not{s}}{m^2}\right]\left\{A\left[\frac{1}{\not{s}}\right] + 2B\left[\frac{1/\not{s}}{m}\right]y[m]\right\}$$

$$u(y)\left[\frac{ft}{s}\right] = A[\,]^{3/2}\,y^{1/2} + B[\,]^{-1/3}\,y^{3} + C[\,]^{-2/3}\,y^{5/6}$$

Find units for A[], B[], and C[]

$$\frac{ft}{s} = A[\,]^{3/2}\,ft^{1/2} \rightarrow A[\,]^{3/2} = \frac{\frac{ft}{s}}{ft^{1/2}} \rightarrow A[\,] = \left\{\frac{\frac{ft}{s}}{ft^{1/2}}\right\}^{2/3} = A\left[\left\{\frac{ft^{1/2}}{s}\right\}\right]^{2/3}$$

$$\frac{ft}{s} = C[\,]^{-2/3}\,ft^{5/6} \rightarrow C[\,]^{-2/3} = \frac{\frac{ft}{s}}{ft^{5/6}} \rightarrow C[\,] = \left\{\frac{\frac{ft}{s}}{ft^{5/6}}\right\}^{-3/2} = C\left[\left\{\frac{ft^{1/6}}{s}\right\}\right]^{-3/2}$$

$$\frac{ft}{s} = B[\,]^{-1/3}\,ft^{3} \rightarrow B[\,]^{-1/3} = \frac{\frac{ft}{s}}{ft^{3}} \rightarrow B[\,] = \left\{\frac{\frac{ft}{s}}{ft^{3}}\right\}^{-3} = B\left[\left\{\frac{ft^{-2}}{s}\right\}\right]^{-3}$$

Velocity Gradient (time rate of strain)

$$u(y)\left[\frac{ft}{s}\right] = A[\,]^{3/2}\, y^{1/2} + B[\,]^{-1/3}\, y^{3} + C[\,]^{-2/3}\, y^{5/6}$$

$$\frac{du}{dy} = 1/2A\left[\left[\left\{\frac{ft^{1/2}}{s}\right\}\right]^{2/3\;3/2}\right] y^{-1/2}\left[ft^{-1/2}\right] + 3B\left[\left\{\frac{ft^{-2}}{s}\right\}\right]^{-3\;-1/3} y^{2}\left[ft^{2}\right]$$

$$+5/6C\left[\left[\left\{\frac{ft^{1/6}}{s}\right\}\right]^{-3/2\;-2/3}\right] y^{-1/6}\left[ft^{-1/6}\right]$$

$$= 1/2A\left[\left\{\frac{ft^{1/2}}{s}\right\}\right] y^{-1/2}\left[ft^{-1/2}\right] + 3B\left[\left\{\frac{ft^{-2}}{s}\right\}\right] y^{2}\left[ft^{2}\right]$$

$$+5/6C\left[\left\{\frac{ft^{1/6}}{s}\right\}\right] y^{-1/6}\left[ft^{-1/6}\right]$$

$$\frac{du}{dy}[1/s] = 1/2Ay^{-1/2}[1/s] + 3By^{2}[1/s] + 5/6Cy^{-1/6}[1/s]$$

$$V[in/s](y) = 0.23 \bullet C[] \bullet \left(\omega_0[rad/s]\right)^2 y^3[in^3]$$

$$0 \le y \le \Delta''$$

where C[] is an unknown constant varying with RPM

$$\frac{in}{s} = C[]\left(\frac{rad}{s}\right)^2 in^3$$

$$\frac{\frac{in}{s}}{\left(\frac{rad}{s}\right)^2 in^3} = C[] = \frac{s}{rad^2 - in^2}$$

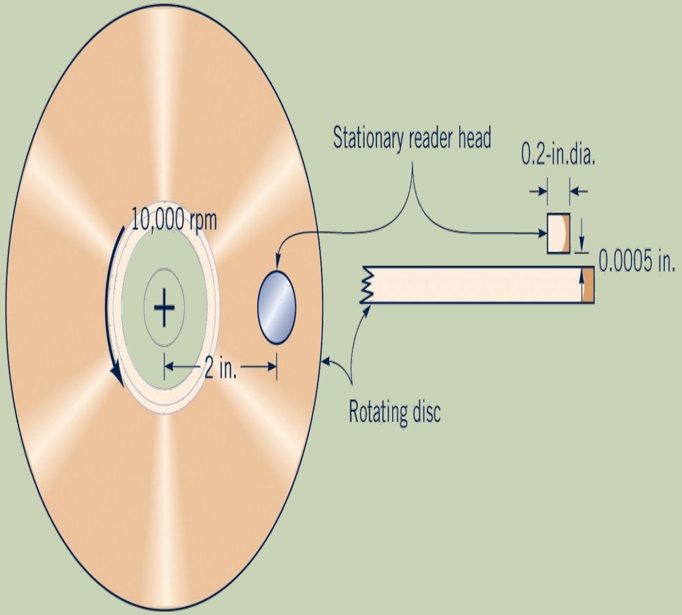

Figure P1.84

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$$V[in/s](y) = 0.23C[\frac{s}{rad^2\, in^2}] \bullet \left(\omega_0^2[rad^2/s^2]\right) y^3[in^3] = \frac{in}{s}$$

SHEAR STRESS

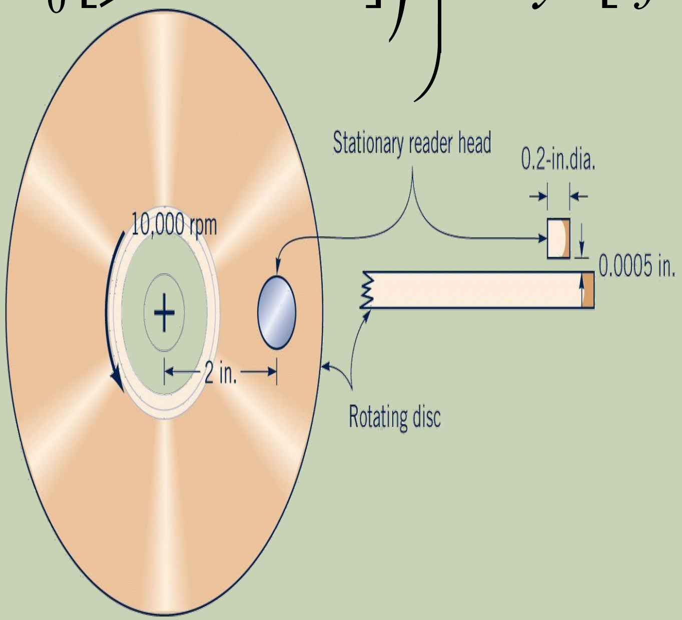
$$V[in/s](y) = 0.23 \bullet C[\,] \bullet \left(\omega_0[rad/s]\right)^2 y^3[in^3] \rightarrow \left[\frac{in}{s}\right] \rightarrow \text{Convert to ft/s}$$

$$\tau(y) = \mu \frac{dV(y)}{dy} = \mu\left(0.23 \bullet C[\frac{s}{\cancel{rad}^2\ \cancel{ft}^2}] \bullet \left(\omega_0^2[\cancel{rad}^2 / s^2]\right)\right) \bullet 2y^2[\cancel{ft}^2]$$

$$= \frac{lbf-s}{ft^2}\left[\frac{1}{s}\right] C[\,]\omega_0^2 2y^2[ft^2] = \frac{lbf}{ft^2}$$

$$\tau\left[\frac{lbf}{ft^2}\right](y) = \mu\left[\frac{lbf-s}{ft^2}\right] C[\,]\omega_0^2 2y^2[ft^2]$$

$$0 \le y \le \Delta''$$



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Tangential Velocity@Rotating Disk

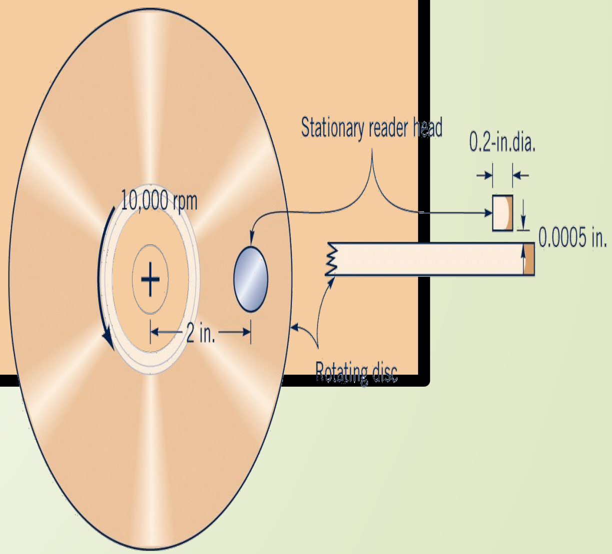
$$@y=\Delta, V(y)=V_t[in/s]=r_0[in]\omega_0[rad/s]$$

$$V[in/s](y)=0.23\bullet C[]\bullet\left(\omega_0[rad/s]\right)^2 y^3[in^3]$$

$$V_t[in/s]=r_0[in]\omega_0[rad/s]=0.23\bullet C[]\bullet\left(\omega_0[rad/s]\right)^2\Delta^3[in^3]$$

$$\frac{V_t[in/s]}{0.23\bullet\left(\omega_0^2[rad/s]\right)^2\Delta^3[in^3]}=C[]$$

$$\frac{V_t[in/s]}{0.23\bullet\left(\omega_0^2[rad^2/s]\right)\Delta^3[in^3]}=C[\frac{s}{rad^2-in^2}](\omega_0)$$



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