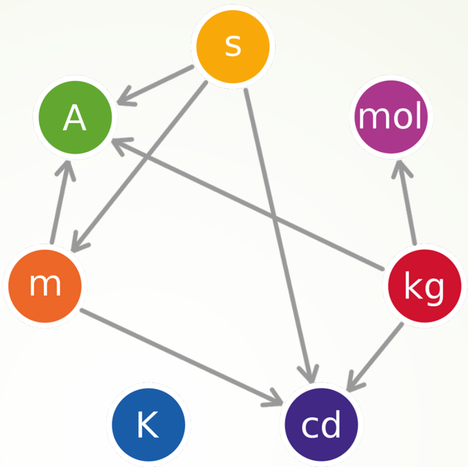
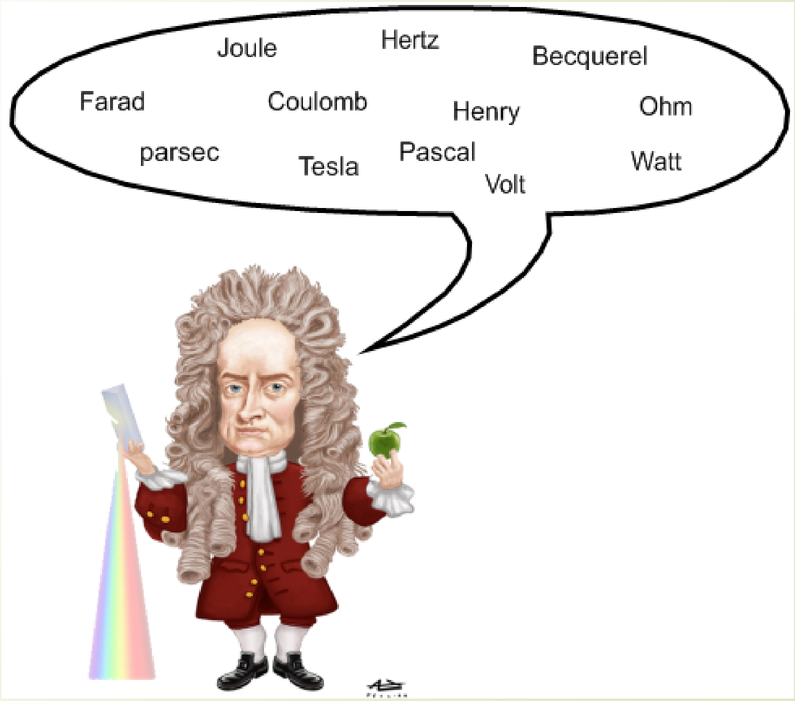
UNIT STUDY AID

MECH-322 Fluid Mechanics





Read the textbook, do the homework, and work with other students constantly to understand the lecture material. MECH-322 ramps up quickly and builds upon itself. Problems take a while to solve, and if you don’t know exactly what you’re doing finishing the exams won’t be possible. Units are a very useful way to double check your work. If you’re not sure if you’re doing it right, units will let you know if you’re mixing up anything.

MECH-322 Student, FALL 2021

FOLLOW THE PATH

As you work to "re-work" the problems from the notes and the homework you should always recall the following basic engineering solution pathways:

1. If you don't understand the definitions used within the problem statement, how can one possibly have any chance of obtaining a valid solution.

2. Never put any number on paper unless one also state the units. (unless it is for certain that is indeed has no units, and can be proven).

3. Always ask? What are the fluid fundamentals driving the solution path?

4. Always form solution is terms of problem "variables" with "units" expressed, and not with numbers.

5. Always check units of final solution to ensure "form" of result is correct with proper units as expected, BEFORE moving to final stage of just plugging a number into a calculator.

UNITS

UNIT CONVERSIONS			
	SI	BKS	Conversion
Force	N	lbf	1N = 0.224809 lbf
Pressure	PA=N/m2	PSF=lbf/ft2	1PA=0.0208855 lbf/ft2
Mass	kg	slug	1kg = 0.0685 slug $\left(\frac{lb_f - s^2}{ft}\right)$
Length	m	ft	1m = 3.28084 ft
Volume	m3	ft3	1m3 = 35.3147 ft3
Velocity	m/s	ft/s	1m/s=3.28084 ft/s
Energy	J	BTU	1BTU = 1,055 J
Power	W (J/s)	ft-lbs/s	1W=0.74 ft-lbf/s=0.00134 hp=3.41BTU/h
Temperature	C (K)	F (R)	C=(F-32)/1.8, K=C+273, R=F+460
Time	s	s	

Engineering Analysis W/O Proper Units Receives 0 Credits

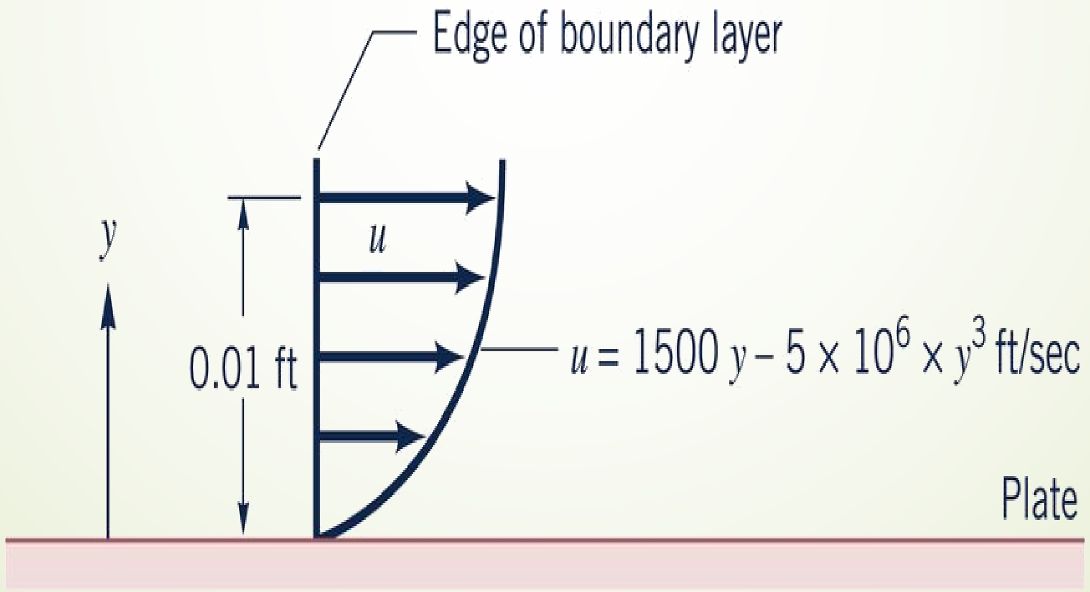
PROPERTIES

FLUID PROPERTIES			
Pressure	N/m2=Pa=F/A	lbf/ft2	1 Pa=0.021 psf
Dynamic Viscosity	N-s/m2=Pa-s	lbf-s/ft2	1Pa-s = 0.02089 lbf-s/ft2
Kinematic Viscosity	m2/s	ft2/s	1m2/s = 10.75381 ft2/s
Density	kg/m3	slugs/ft3	1kg/m3 = 0.00194 slug/ft3
Specific Weight	N/m3=F/V	lbf/ft3	1N/m3 = 0.00637 lbf/ft3
Shear Stress	Pa	lbf/ft2	1 Pa = 0.021 psf

The velocity profile for a fluid flowing in a duct is expressed as:

$$u(y)\left[\frac{m}{s}\right] = A[?]y[m] + B[?]y^2[m^2]$$

where A and B are arbitrary constants, and "y" is the variable distance measured from the bottom wall.



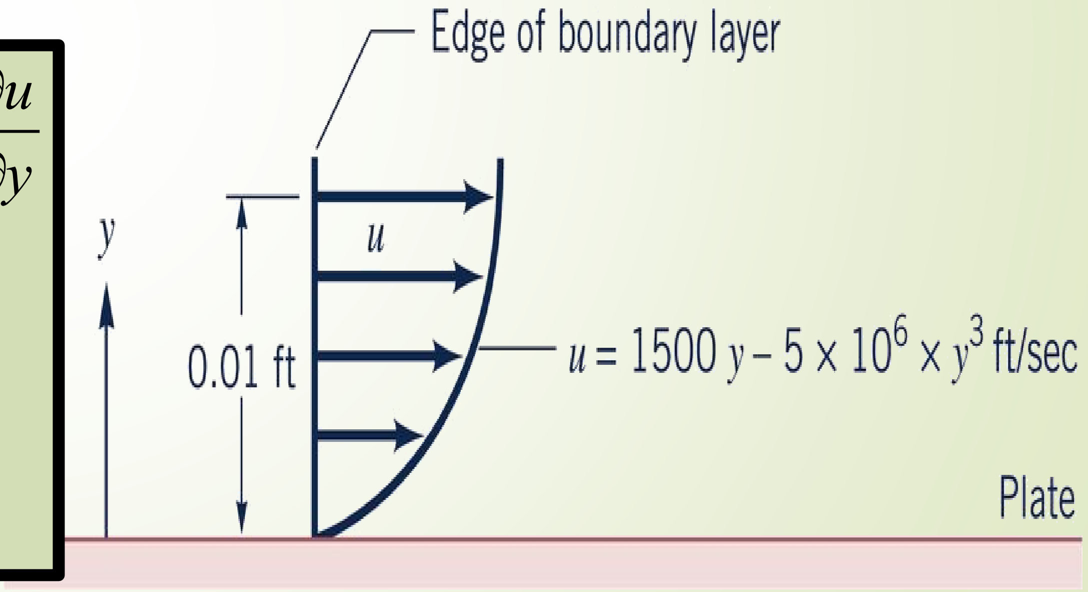
- What must be the correct units for "A" and "B"?
- What is the parametric equation for the **Velocity Gradient**, AND show that units are correct?
- What is the parametric equation for the **Shear Stress**, AND show that units are correct?

$$\dot{\gamma} \equiv \text{rate of shear strain (velocity gradient)} = \frac{\partial u}{\partial y}$$

$$\tau(x,y) \equiv \text{SHEAR STRESS} = \mu\dot{\gamma}$$

$$\tau(x,y) \equiv \text{SHEAR STRESS} = \mu\frac{\partial u}{\partial y}$$

$$\mu \equiv \text{Fluid Viscosity}$$



FUDAMENTAL CONCEPTS

CONSIDER THIS EXPRESSION

$$u(y)\left[\frac{m}{s}\right] = \left[Ay + By^2\right]\left[\frac{m}{s}\right], y[m]$$

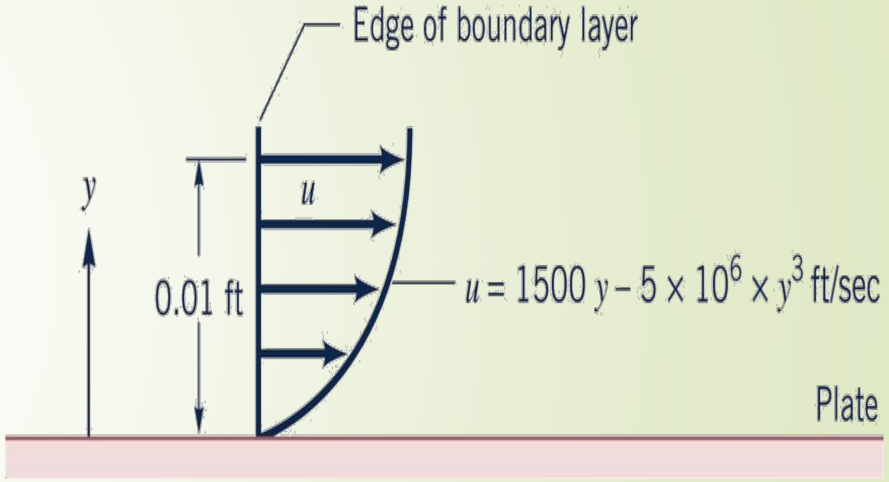
$$\text{LEFT UNITS}[\,] = \{\text{RIGHT UNITS}\}[\,]$$

$$\text{LEFT UNITS}[\,] = \text{"EACH TERM } [\,]\text{"}$$

Can only ADD/SUB LIKE UNITS

$$\left[\frac{m}{s}\right] = \{A[\,]y[m]\}\left[\frac{m}{s}\right] + \{B[\,]y^2[m^2]\}\left[\frac{m}{s}\right]$$

must find [*unit*] and [*unit*]



$$u(y)\left[\frac{m}{s}\right] = \left\{A[\,]y[m] + B[\,]y^2[m^2]\right\}\left[\frac{m}{s}\right]$$

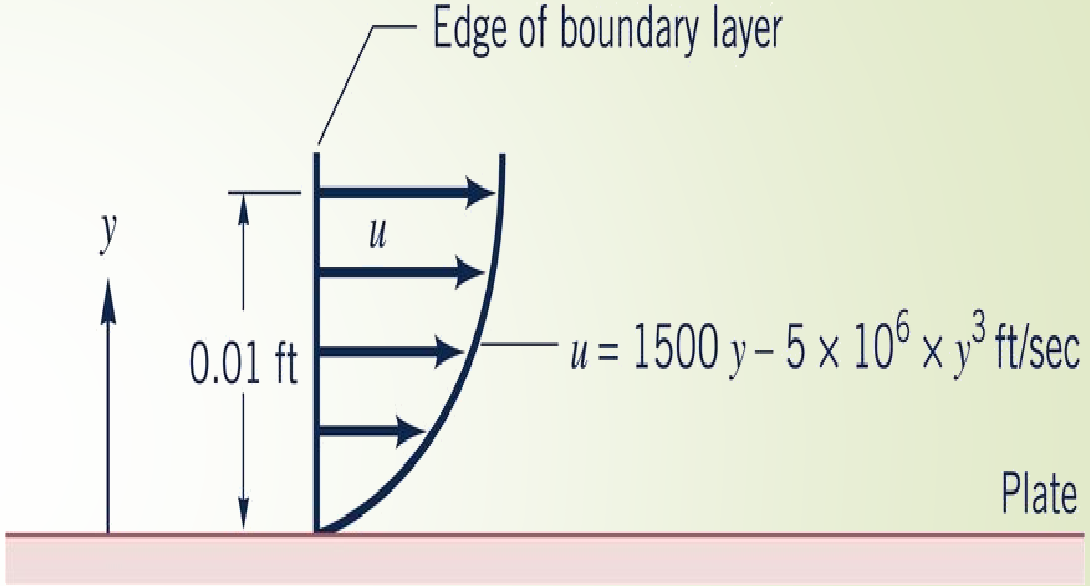
UNITS

$$\left[\frac{m}{s}\right] = A[?][m] + B[?][m^2]$$

$$\left[\frac{m}{s}\right] \leftrightarrow \left[\frac{m}{s}\right] + \left[\frac{m}{s}\right]$$

$$A[\,] = \frac{\frac{m}{s}}{m} = \frac{1}{s}$$

$$B[\,] = \frac{\frac{m}{s}}{m^2} = \frac{1/s}{m}$$



VELOCITY GRADIENT (time rate of strain)

$$\frac{du(y)}{dy}\left[\frac{m/s}{m} = 1/s\right] = A\left[\frac{1}{s}\right] + 2B\left[\frac{1/s}{\cancel{m}}\right]y[\cancel{m}]$$

SHEAR STRESS

$$\tau = \mu\left[\frac{N-s}{m^2}\right]\frac{du}{dy}\left[\frac{1}{s}\right] = \frac{N}{m^2} = \mu\left[\frac{N-\cancel{s}}{m^2}\right]\left\{A\left[\frac{1}{\cancel{s}}\right] + 2B\left[\frac{1/\cancel{s}}{m}\right]y[m]\right\}$$

$$u(y)\left[\frac{ft}{s}\right] = A[]^{3/2}\, y^{1/2} + B[]^{-1/3}\, y^{3} + C[]^{-2/3}\, y^{5/6}$$

Find units for A[], B[], and C[]

$$\frac{ft}{s} = A[]^{3/2}\, ft^{1/2} \rightarrow A[]^{3/2} = \frac{\frac{ft}{s}}{ft^{1/2}} \rightarrow A[] = \left\{\frac{\frac{ft}{s}}{ft^{1/2}}\right\}^{2/3} = A\left[\left\{\frac{ft^{1/2}}{s}\right\}\right]^{2/3}$$

$$\frac{ft}{s} = C[]^{-2/3}\, ft^{5/6} \rightarrow C[]^{-2/3} = \frac{\frac{ft}{s}}{ft^{5/6}} \rightarrow C[] = \left\{\frac{\frac{ft}{s}}{ft^{5/6}}\right\}^{-3/2} = C\left[\left\{\frac{ft^{1/6}}{s}\right\}\right]^{-3/2}$$

$$\frac{ft}{s} = B[]^{-1/3}\, ft^{3} \rightarrow B[]^{-1/3} = \frac{\frac{ft}{s}}{ft^{3}} \rightarrow B[] = \left\{\frac{\frac{ft}{s}}{ft^{3}}\right\}^{-3} = B\left[\left\{\frac{ft^{-2}}{s}\right\}\right]^{-3}$$

Velocity Gradient (time rate of strain)

$$u(y)\left[\frac{ft}{s}\right] = A[\,]^{3/2}\, y^{1/2} + B[\,]^{-1/3}\, y^{3} + C[\,]^{-2/3}\, y^{5/6}$$

$$\frac{du}{dy} = 1/2A\left[\left[\left\{\frac{ft^{1/2}}{s}\right\}\right]^{2/3\ 3/2}\right] y^{-1/2}[ft^{-1/2}] + 3B\left[\left\{\frac{ft^{-2}}{s}\right\}\right]^{-3\ -1/3} y^{2}[ft^{2}]$$

$$+5/6C\left[\left[\left\{\frac{ft^{1/6}}{s}\right\}\right]^{-3/2\ -2/3}\right] y^{-1/6}[ft^{-1/6}]$$

$$= 1/2A\left[\left\{\frac{ft^{1/2}}{s}\right\}\right] y^{-1/2}[ft^{-1/2}] + 3B\left[\left\{\frac{ft^{-2}}{s}\right\}\right] y^{2}[ft^{2}]$$

$$+5/6C\left[\left\{\frac{ft^{1/6}}{s}\right\}\right] y^{-1/6}[ft^{-1/6}]$$

$$\frac{du}{dy}[1/s] = 1/2Ay^{-1/2}[1/s] + 3By^{2}[1/s] + 5/6Cy^{-1/6}[1/s]$$

$$V(y)[in/s] = \left\{ C[] \bullet \left(\omega_0[rad/s]\right)^2 y^3[in^3] \right\} [in/s]$$

$$0 \le y \le \Delta''$$

where C[] is an unknown constant varying with RPM

$$\frac{in}{s} = C[] \left(\frac{rad}{s}\right)^2 in^3$$

$$\frac{\frac{in}{s}}{\left(\frac{rad}{s}\right)^2 in^3} = C[] \rightarrow \left[\frac{s}{rad^2 - in^2}\right]$$

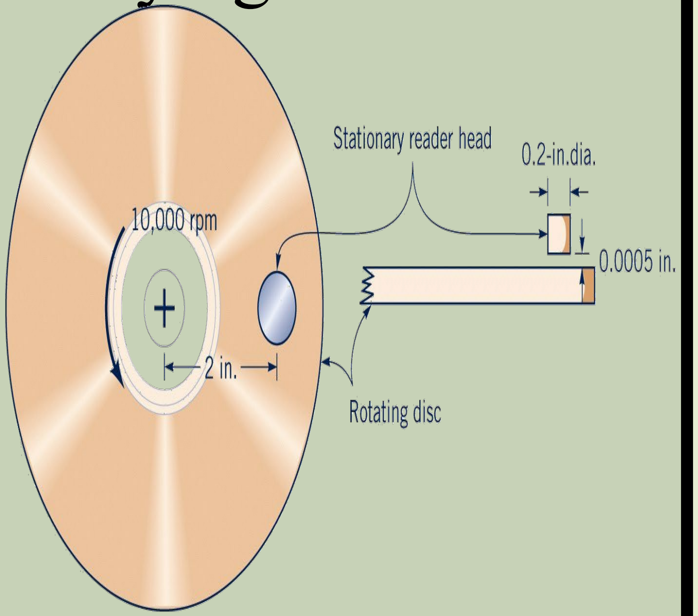


Figure P1.84

$$V(y)[in/s] = C[\frac{s}{\cancel{rad^2}\, in^2}] \bullet \left(\omega_0^2[\cancel{rad^2}/s^2]\right) y^3[in^3] = \frac{in}{s}$$

SHEAR STRESS

$$V(y)[in/s] = \left\{ C[\,] \bullet (\omega_0[rad/s])^2 \; y^3[in^3] \right\}[in/s] \rightarrow \left[\frac{in}{s}\right] \rightarrow \text{Convert to ft/s}$$

$$\tau(y) = \{\mu\} \frac{dV(y)}{dy} = \mu \left(C\left[\frac{s}{\cancel{rad^2}\;\cancel{ft^2}}\right] \bullet \left(\omega_0^2 [\cancel{rad^2} / s^2] \right) \right) \bullet 2y^2[\cancel{ft^2}]$$

$$= \left\{ \frac{lbf-s}{ft^2} \right\} \left[\frac{1}{s} \right] C[\,]\omega_0^2 2y^2[ft^2] = \frac{lbf}{ft^2}$$

$$\tau\left[\frac{lbf}{ft^2}\right](y) = \mu \left[\frac{lbf-s}{ft^2}\right] C[\,]\omega_0^2 2y^2[ft^2]$$

$$0 \le y \le \Delta''$$

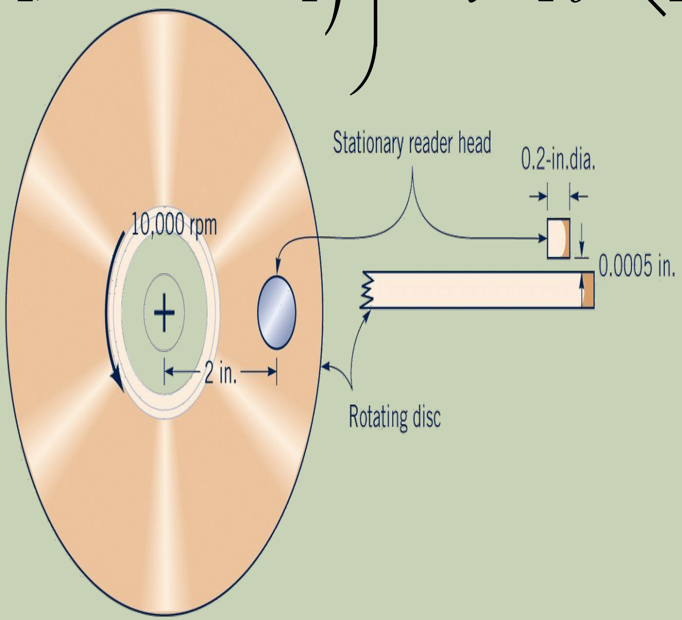


Figure P1.84

Tangential Velocity@Rotating Disk

$$@y=\Delta, V(y)=V_t[in/s]=r_0[in]\omega_0[rad/s]$$

$$V(y)[in/s]=C[]\bullet(\omega_0[rad/s])^2 y^3[in^3]$$

$$V_t[in/s]=r_0[in]\omega_0[rad/s]=C[]\bullet(\omega_0[rad/s])^2\Delta^3[in^3]$$

$$\frac{V_t[in/s]}{(\omega_0^2[rad/s])^2\Delta^3[in^3]}=C[]$$

$$\frac{V_t[in/s]}{(\omega_0^2[rad^2/s^2])\Delta^3[in^3]}=C[\frac{s}{rad^2-in^2}]$$

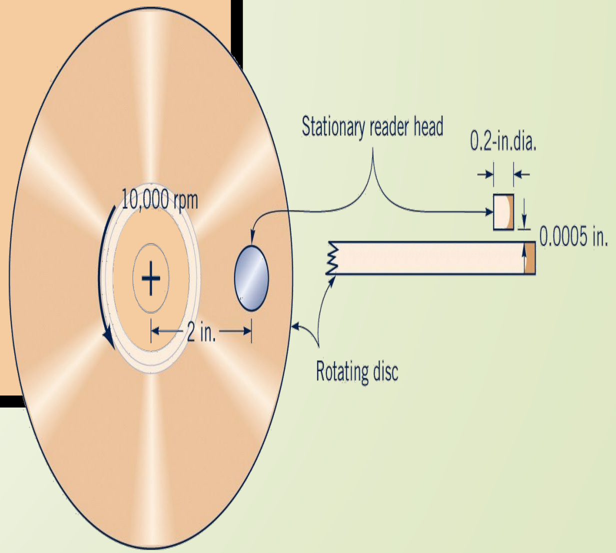


Figure P1.84