MECH-420 SPECIAL TOPICS STUDY AID

TYPED INDIVIDUAL WORK
SUBMIT TO BLACKBOARD W/
SOLUTION AND SPREADSHEET/MATHLAB
DUE FRIDAY WEEK #2, 10:00 AM

Submit typed solution and spreadsheet/MATLAB

MECH-420 with Dr. Berry has pushed me to limits I did not know I had. But I can say with 100% certainty I am a better student and will be a better engineer as a direct result of it. Taking this course with Dr. Berry was a blessing. You will work hard in this course, but you will MASTER the concept as they apply to the real world.

MECH-420
Heat Transfer

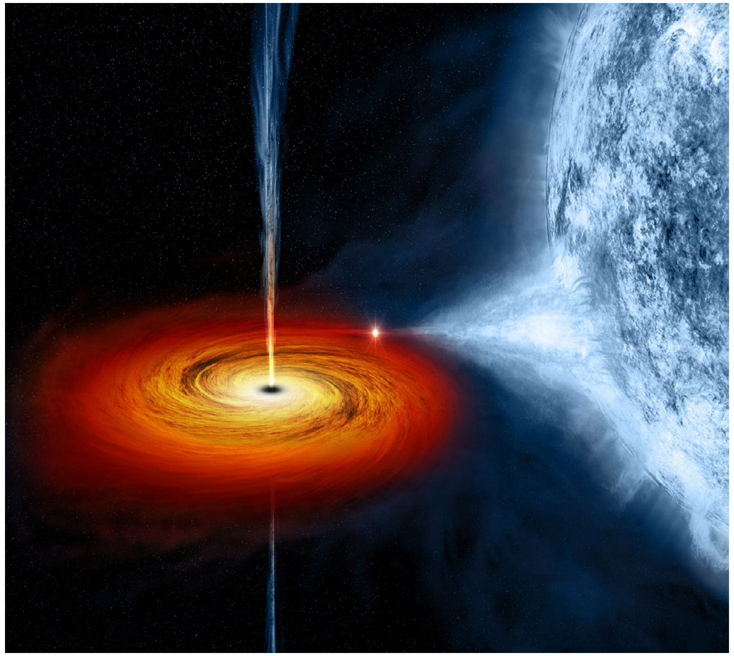
KNOWLEDGE YOU SEEK, DO YOU?

DO OR DO NOT. THERE IS NO TRY

HOMEWORK

FOLLOW THE PATH

1.The comet Centaur is headed straight for a BLACK HOLE emitting thermal radiation causing the temperature of comet to be measured as:



$$T(x,y,z,t) = (2xt + 12y^2x^2zt^{-1/2} - 34y^3xzt^3)\ K$$

Where “x,y,”z are in meters, and “t” is in seconds.

a. What are the units on the constants “2”, “12”, and “34”.

b. Determine the expression/relationship for the *Time Rate of Change of Temperature* $\frac{DT}{Dt}(K/s)$ for the doomed comet Centaur and **VERIFY** the units on each term.

UNIT ANALYSIS

$$T(x,y,z,t) = (2xt + 12y^2x^2zt^{-1/2} - 34y^3xzt^3)[K]$$

$$[K] = 2[][m][s]$$

$$2[] = \frac{[K]}{[m][s]} = \frac{K}{m-s}$$

$$[K] = 12[][m^5][s]^{-1/2}$$

$$12[] = \frac{[K]}{[m^5][s]^{-1/2}} = \frac{K}{m^5 - s^{-1/2}}$$

$$[K] = 34[][m^5][s^3]$$

$$34[] = \frac{[K]}{[m^5][s^3]} = \frac{K}{m^5 - s^3}$$

TIME RATE OF CHANGE

$$T(x,y,z,t) = (2xt + 12y^2x^2zt^{-1/2} - 34y^3xzt^3)[K]$$

$$\frac{DT}{Dt}\left[\frac{K}{s}\right] = 2x - 1/2 * 12 * y^2x^2zt^{-3/2} - 3 * 34 * y^3xzt^2$$

$$= \left[2x - 6y^2x^2t^{-3/2} - 102y^3xzt^2\right][?]$$

ENTER UNITS

$$=2\left[\frac{K}{\cancel{m} - s}\right]x[\cancel{m}] - 6\left[\frac{K}{\cancel{m^5} - s^{-1/2}}\right]y^2x^2z[\cancel{m^5}]t^{-3/2}[s^{-3/2}]$$

$$-102\left[\frac{K}{\cancel{m^5} - s^3}\right]y^3xz[\cancel{m^5}]t^2[s^2]$$

HOMEWORK

2. FIND MOST GENERAL SOLUTION

$$\frac{dS}{dx} + 2S = \sin\left(\frac{3\pi x}{L}\right) + \cos\left(\frac{\pi x}{2L}\right), L = \text{last 2 digits of ID} + 10$$

$$S(x) = ?$$

INITIAL CONDITION

S(x=0)=20

FIND EXACT SOLUTION

$$\text{Plot } S(x); 0 \le \frac{x}{L} \le 1$$

Submit typed solution and spreadsheet/MATLAB

$$\frac{dS}{dx} + 2S = \sin(\frac{3\pi x}{L}) + \cos(\frac{\pi x}{2L})$$

GENERAL SOLUTION

$$\frac{dy}{dx} + p(x)y = f(x)$$

$$y(x) = Ce^{-\int p(x)dx} + e^{-\int p(x)dx} \int e^{\int + p(x)dx} f(x)dx$$

$C \equiv$ Arbitrary Constant of Integration Obtained

$$y = S, p(x) = 2, f(x) = \sin(\frac{3\pi x}{L}) + \cos(\frac{\pi x}{2L})$$

$$S(x) = Ce^{-2x} + e^{-2x} \int e^{2x} \left[\sin(\frac{3\pi x}{L}) + \cos(\frac{\pi x}{2L}) \right] dx$$

$$S(x) = Ce^{-2x} + e^{-2x}\int e^{2x}\left[\sin\left(\frac{3\pi x}{L}\right) + \cos\left(\frac{\pi x}{2L}\right)\right]dx$$

STANDARD INTEGRATION FORMULAS

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx) - b\cos(bx)\right]$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx) + b\sin(bx)\right]$$

$$S(x) = Ce^{-2x} + e^{-2x}\int e^{2x}\left[\sin\left(\frac{3\pi x}{L}\right) + \cos\left(\frac{\pi x}{2L}\right)\right]dx$$

MOST GENERAL SOLUTION

$$S(x) = Ce^{-2x} + e^{-2x}\left[\frac{e^{2x}}{4+\left(\frac{3\pi}{L}\right)^2} 2\sin\left(\frac{3\pi}{L}x\right) - \frac{3\pi}{L}\cos\left(\frac{3\pi}{L}x\right)\right]$$

$$+e^{-2x}\left[\frac{e^{2x}}{4+\left(\frac{\pi}{2L}\right)^2} 2\cos\left(\frac{\pi}{2L}x\right) + \frac{\pi}{2L}\sin\left(\frac{\pi}{2L}x\right)\right]$$

EXACT SOLUTION

INITIAL CONDITION:@x=0,S(x=0)=S_0

$$S(x)=Ce^{-2x}+e^{-2x}\left[\frac{e^{2x}}{4+\left(\frac{3\pi}{L}\right)^2}2\sin\left(\frac{3\pi}{L}x\right)-\frac{3\pi}{L}\cos\left(\frac{3\pi}{L}x\right)\right]$$

$$+e^{-2x}\left[\frac{e^{2x}}{4+\left(\frac{\pi}{2L}\right)^2}2\cos\left(\frac{\pi}{2L}x\right)+\frac{\pi}{2L}\sin\left(\frac{\pi}{2L}x\right)\right]$$

$$S(x=0)=S_0=C+\left[\frac{-\frac{3\pi}{L}}{4+\left(\frac{3\pi}{L}\right)^2}\right]+\left[\frac{2}{4+\left(\frac{\pi}{2L}\right)^2}\right]$$

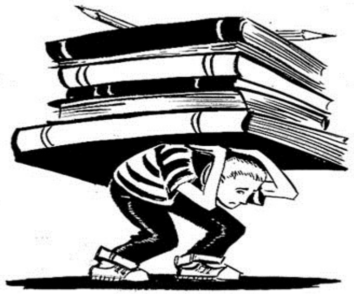
$$C=S_0+\left[\frac{\frac{3\pi}{L}}{4+\left(\frac{3\pi}{L}\right)^2}\right]-\left[\frac{2}{4+\left(\frac{\pi}{2L}\right)^2}\right]$$

TRUST THE PATH

3. A spherical shell of inner radius "r_1" and outer radius "r_2" serves as radiation containment vessel is exposed to a convective fluid, $T_\infty(r_2) = 300K$, and convective heat transfer coeff. "h=25W/m^2-K". The internal material has a volumetric heat generation rate defined as:

$$\dot{q}(r) = \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0\left[\frac{W}{m^3}\right](2.0 + \frac{r^4}{r_1^4});\ 0 \le r \le r_1,$$

$$r_1 = 2.5m, r_2 = 5m, S_0 = 20kW/m^3.$$



If $V_{sphere} = \frac{4}{3}\pi r^3$, find:

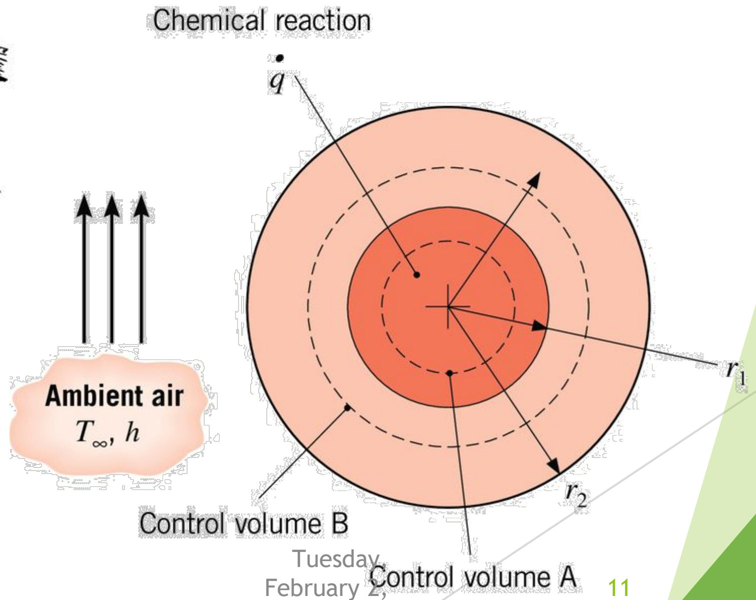
a) Total heat generated [W],

b) steady state surface temperature $T_s(S_0, r_1, r_2)$

c) Plot $T_s(S_0, r_1, r_2)$ vs r_1 as $0 \le r_1 \le 4.5m$

(submit typed PDF solution and spreadsheet/MATLAB)

Blackboard



$$\dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] = S_0\left[\frac{W}{m^3}\right](2.0+\frac{r^4}{r_1^4});\ 0 \le r \le r_1,$$

$$r_1 = 2.5m, r_2 = 5m, S_0 = 20kW / m^3.$$

If $V_{sphere} = \frac{4}{3}\pi r^3$, find: $\dot{E}_{gen} = ?$

$$\dot{E}_{gen} = \int_0^{r_1} \dot{S}_{gen}(r)\left[\frac{W}{m^3}\right] dV, dV_{sphere} = 4\pi r^2 dr$$

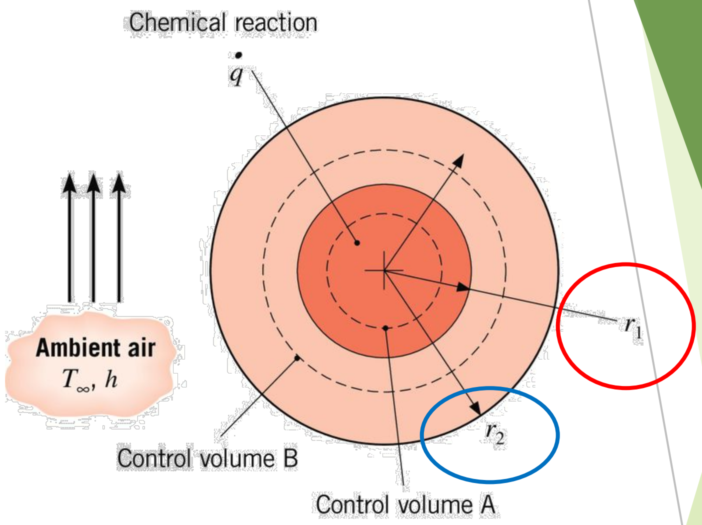
$$\dot{E}_{gen} = 4\pi S_0\left[\frac{W}{m^3}\right]\int_0^{r_1}(2.0+\frac{r^4}{r_1^4})r^2 dr = 4\pi S_0\left[\frac{W}{m^3}\right]\int_0^{r_1}(2.0r^2+\frac{r^6}{r_1^4})dr$$

$$\dot{E}_{gen} = 4\pi S_0\left[\frac{W}{m^3}\right][2\frac{r^3}{3}+\frac{r^7}{7r_1^4}]_{0-r_1}$$

$$\dot{E}_{gen} = 4\pi S_0\left[\frac{W}{m^3}\right]\left[\frac{2}{3}(r_1^3)+\frac{1}{7}(\frac{r_1^7}{r_1^4})\right][m^3]$$

$$\dot{E}_{gen} = 4\pi S_0\left[\frac{W}{m^3}\right]r_1^3(\frac{2}{3}+\frac{1}{7}) = 4\pi S_0\left[\frac{W}{m^3}\right]r_1^3(\frac{14}{21}+\frac{3}{21})$$

$$\dot{E}_{gen}(S_0, r_1) = 4\pi S_0\left[\frac{W}{m^3}\right]r_1^3\frac{17}{21} = \frac{68}{21}\pi S_0\left[\frac{W}{m^3}\right]r_1^3[m^3]$$



EXTERNAL STATE SURFACE TEMPERATURE

$$\dot{E}_{gen}(S_0, r_1) = 4\pi S_0 \left[\frac{W}{m^3}\right] r_1^3 \frac{17}{21} = \frac{68}{21}\pi S_0 \left[\frac{W}{m^3}\right] r_1^3 [m^3]$$

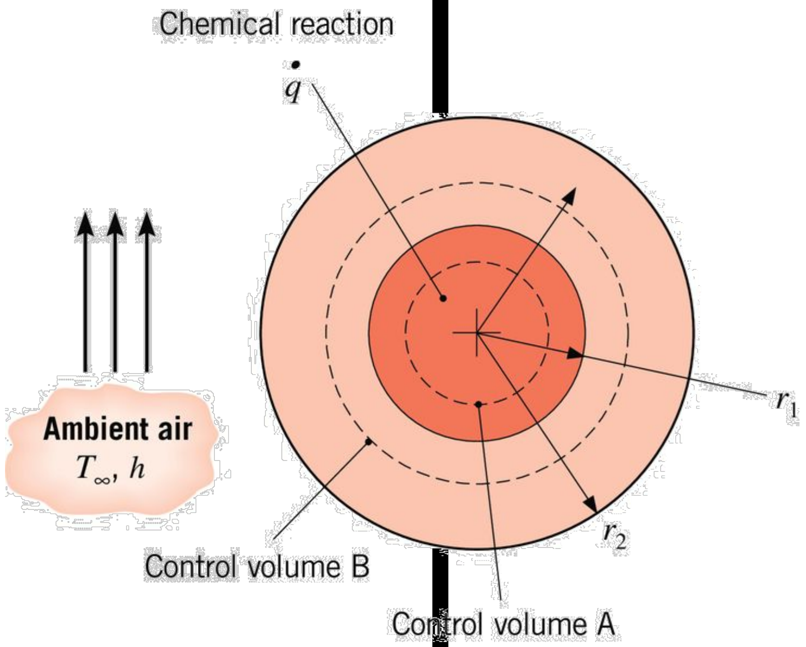
External Surface Temperature ($T(r=r_2)$)

$$\dot{E}_{gen}(S_0, r_1) = \dot{E}_{out} = \bar{h} A_0 (r = r_2)(T_s - T_\infty)$$

$$\frac{\dot{E}_{gen}(S_0, r_1)}{\bar{h} A_0 (r = r_2)} + T_\infty = T_s(S_0, r_1, r_2)$$

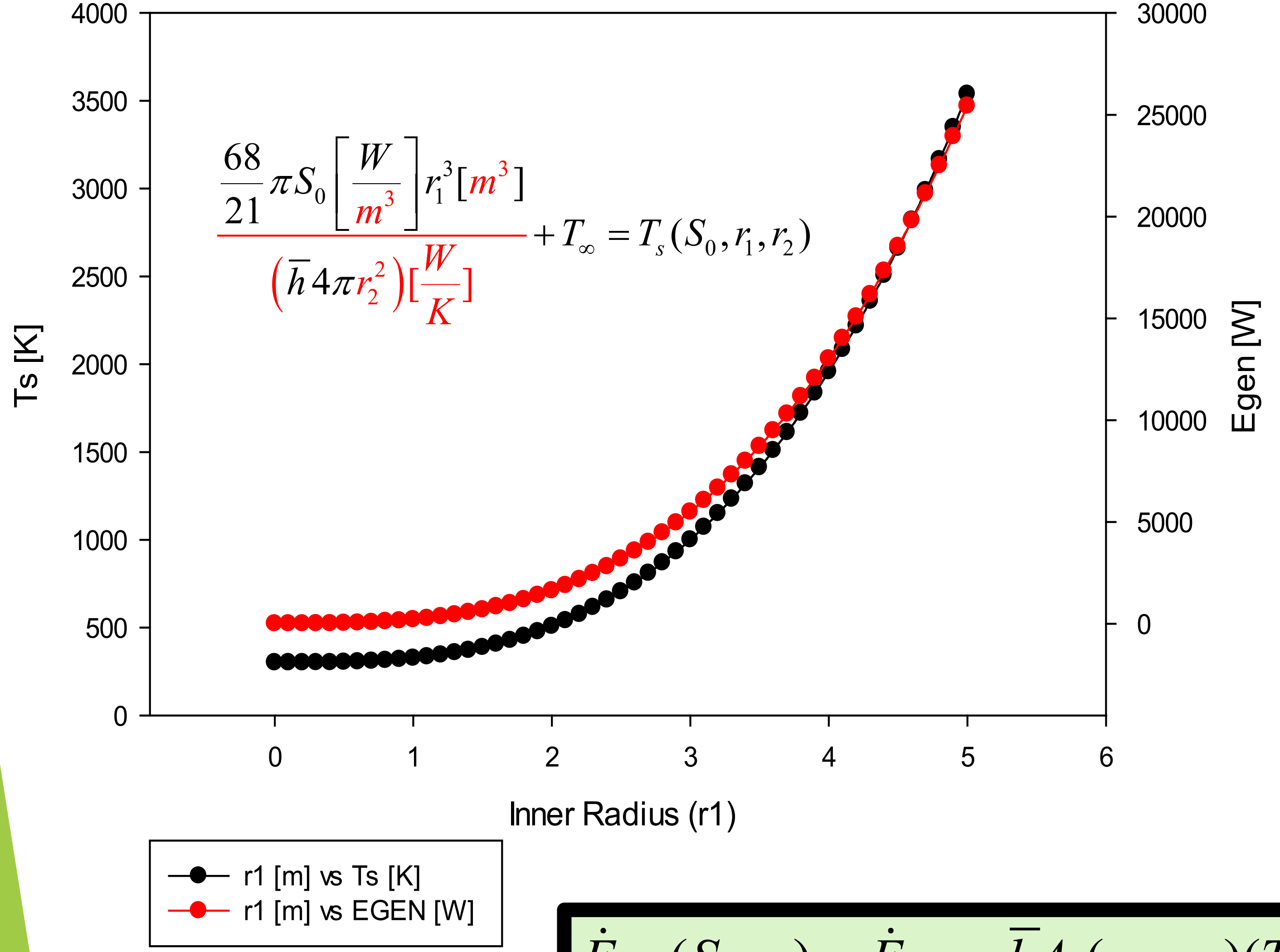
$$A_0 = 4\pi r_2^2$$

$$\frac{\frac{68}{21}\pi S_0 \left[\frac{W}{m^3}\right] r_1^3 [m^3]}{\left(\bar{h} 4\pi r_2^2\right)\left[\frac{W}{K}\right]} + T_\infty = T_s(S_0, r_1, r_2)$$



$$r_1 = 0 - 4.5m, r_2 = 5m, S_0 = 20kW/m^3, h = 25\frac{W}{m^2 - K}, T_\infty = 300K.$$

External Surface Temperature vs Inner Radius



m	W/m3	W/m2-K	K
r2	S0	h	Tf
5	20000	25	300

m	kW	K	kW
r1	EGEN	Ts	q-conv
0	0.00	300	0.00
0.5	25.43	303.2381	25.43
1	203.46	325.9048	203.46
1.5	686.66	387.4286	686.66
2	1,627.64	507.2381	1,627.64
2.5	3,178.99	704.7619	3,178.99
3	5,493.30	999.4286	5,493.30
3.5	8,723.16	1410.667	8,723.16
4	13,021.15	1957.905	13,021.15
4.5	18,539.88	2660.571	18,539.88
5	25,431.94	3538.095	25,431.94

$$\dot{E}_{gen}(S_0, r_1) = \dot{E}_{out} = \bar{h}A_0(r = r_2)(T_s - T_\infty)$$