MECH-322 Fluid Mechanics Viscous Flow Study Aid

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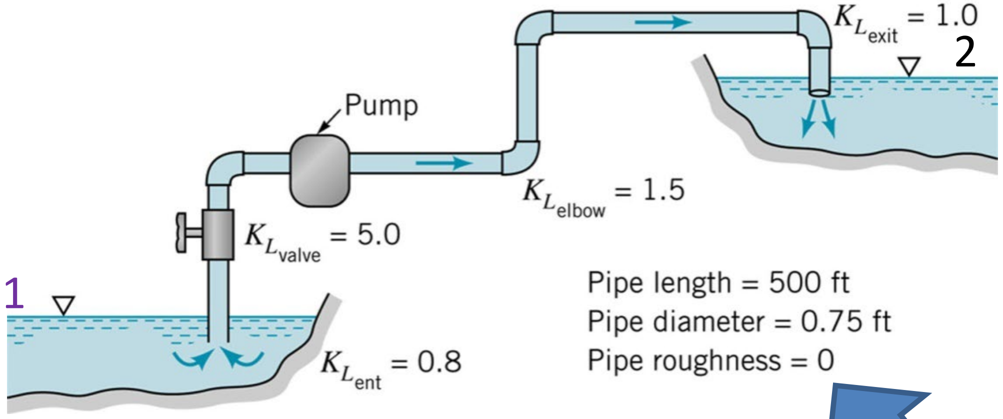
Through this experience, I began to understand the fundamental concepts of the course, thanks to your focus upon the units of each problem. It sounds like a little thing. But is one that is a very important detail to double check the math through each problem. At times I was frustrated and felt as though I could not understand the material, but with persistence and trying to understand the big picture, I began to make much progress. I will remember this class for how it showed me I can learn a great deal in a short amount of time. I appreciate you providing the external pressure to unlock the better engineer in all of us students.

Former Student, Spring 2022

Calculation of head losses

Example – Problem # 8.97: **The pump shown in Figure below delivers a head of 250ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200ft.**



Solution: **The viscous pipe flow equation with losses can be written as**

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L \qquad (1)$$

$$where\ p_1 = p_2 = 0;\ V_1 = V_2 = 0;\ z_1 = 0;\ z_2 = 200\,ft;\ h_p = 250\,ft.$$

Calculation of head losses

CONSTANT PIPE DIAMETER

Thus (1) reduces to:

$$-h_L + h_p = z_2$$

$$\Rightarrow -\left(f_{guess}\frac{L}{D}\frac{V^2}{2g} + \sum_{i=1}^{n}\left(k_{Lminor}\right)_i\left(\frac{V^2}{2g}\right)\right) + h_p = z_2 \qquad (2)$$

Now $\sum_{i=1}^{n}\left(k_{Lminor}\right)_i\left(\frac{V^2}{2g}\right) = \left(0.8+5.0+4\times(1.5)+1.0\right)\left(\frac{V^2}{2g}\right) = 12.8\left(\frac{V^2}{2g}\right)$

Therefo $-\left(\left[f_{GUESS}\frac{(500\,ft)}{(0.75\,ft)} + 12.8\right]\frac{V^2}{2(32.2\,ft/s^2)}\right) + 250\,ft = 200\,ft$

$$\Rightarrow (667 f_{GUESS} + 12.8)V^2 = 3220 \qquad (3):$$

Friction Characteristic Equation $\rightarrow$

$$V(f_{GUESS}) = \left(\frac{3220}{667 f_{GUESS} + 12.8}\right)^{1/2} \rightarrow \text{Guess f: } 0.01 \le f_{GUESS} \le 0.03$$

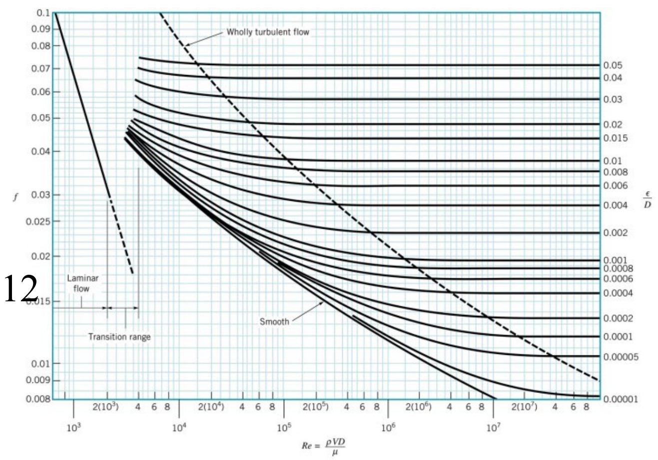
Calculation of head losses

Now $$V = \left(\frac{3220}{667\, f_{GUESS} + 12.8} \right)^{1/2} \quad (3)$$

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{(1.94\, slug/ft^3)\, V\, (0.75\, ft)}{(2.34 \times 10^{-5}\, lb \cdot s/ft^2)} = 6.22 \times 10^4 \bullet V \quad (4)$$

Use Moody's diagram -

Trial & error method:



$Assume\ f_1 = 0.02 \rightarrow V = 11.1\, ft/s \rightarrow \mathrm{Re}_D = 6.9 \times 10^5 \rightarrow f_{moody} = 0.012 \neq 0.02$

$Assume\ f_2 = 0.012 \rightarrow V = 12.4\, ft/s \rightarrow \mathrm{Re}_D = 7.7 \times 10^5 \rightarrow f_{moody} = 0.0121 \simeq 0.012$

$Thus\ V = 12.4\, ft/s$

Hence the power added to the water is

$$\dot{W}_S = Q(\gamma h_p) = (VA)(\gamma h_p) = (12.4\, ft/s)\left(\pi/4 [0.75\, ft]^2\right)(62.4\, lb/ft^3)(250\, ft) = 8.55 \times 10^4 \frac{ft.lb}{s}$$

$$\Rightarrow \dot{W}_S = 8.55 \times 10^4 \frac{ft.lb}{s} \times \frac{1hp}{(550\, ft.lb/s)} = 155hp$$

You may also use HAALAND formula to find *f*.

HALLAND EQUATION
Good for Programming Calculator

$$V = \left(\frac{3220}{667 f_{guess} + 12.8} \right)^{1/2} \quad (3)$$

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{\left(1.94\, slug/ft^3\right) V \left(0.75\, ft\right)}{\left(2.34 \times 10^{-5}\, lb \cdot s/ft^2\right)} = 6.22 \times 10^4 V \quad (4)$$

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left(\left(\frac{\varepsilon / D}{3.7} \right)^{1.11} + \frac{6.9}{\mathrm{Re}} \right)$$

HALLAND EQUATION

	HAALLAND EQUATION													
	DENS	Visc	D			DENS	Visc	D			DENS	Visc	D	
e/D	slugs/ft3	lb-s/ft2	ft	f (guess)	e/D	slugs/ft3	lb-s/ft2	ft	f (guess)	e/D	slugs/ft3	lb-s/ft2	ft	f (guess)
0	1.94	2.34E-05	0.75	0	0	1.94	2.34E-05	0.75	0.01	0	1.94	2.34E-05	0.75	1
f	V	Re	f (FALLAND)		f	V	Re	f (FALLAND)		f	V	Re	f (FALLAND)	
0	15.86	9.862E+05	0.0116		0.01	12.86	7.996E+05	0.0120		1	2.18	1.353E+05	0.0168	
0.0116	12.52	7.784E+05	0.0121		0.0120	12.43	7.731E+05	0.0121		0.0168	11.59	7.206E+05	0.0123	
0.0121	12.42	7.724E+05	0.0121		0.0121	12.42	7.723E+05	0.0121		0.0123	12.39	7.705E+05	0.0121	
0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121	
0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121		0.0121	12.42	7.722E+05	0.0121	

Calculation of head losses/pressure drop

- EXAMPLE: Water at 0.02 m3/s flows through 350m of horizontal cast iron pipe (D=20.27 cm). Determine the head loss and pressure drop?

$$\nu = \frac{\mu}{\rho} = 9.569x10^{-7}\frac{m^2}{s}$$

$$V = \frac{Q}{A} = \frac{0.02\frac{m^3}{s}}{0.032270m^2} = 0.62\frac{m}{s} \to \mathrm{Re} = \frac{VD}{\nu} = 1.31x10^5 \to TURBULENT$$

Table 8.1

ε = Relative Roughness = 0.026 cm

$$\frac{\varepsilon}{D} = \frac{0.026cm}{20.27cm} = 0.0012$$

Moody Diagram $(\mathrm{Re}, \frac{\varepsilon}{D}), f = 0.022$; (or Haaland Equation)

HEAD LOSS

$$\mathrm{h}_L = \frac{fL}{D}\frac{V^2}{2g} = \frac{0.022x350}{0.2027}\frac{(0.62)^2}{2(9.81)} = 0.744 \text{ m of water}$$

PRESSURE DROP

$$\Delta P = \gamma \mathrm{h}_L = 9800\frac{N}{m^3} \bullet 0.744m = 7,280\frac{N}{m^2}$$