

MECH-322 FLUID MECHANICS

VISCOUS FLOW LOSS AND PUMPS

Water is pumped from the tank and exits as a free jet, and the pump manufacture has provided the pump head vs volume flow rate curve as shown.

Determine the flow rate through the 100 meter straight smooth pipe section.

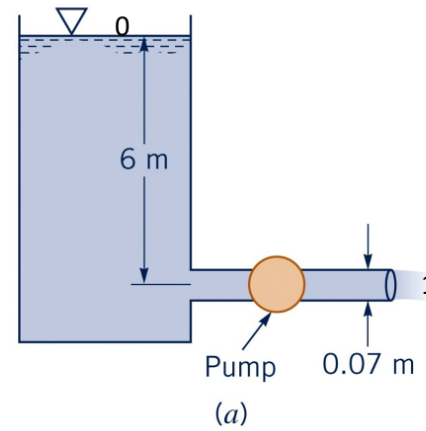


Figure P5.118a
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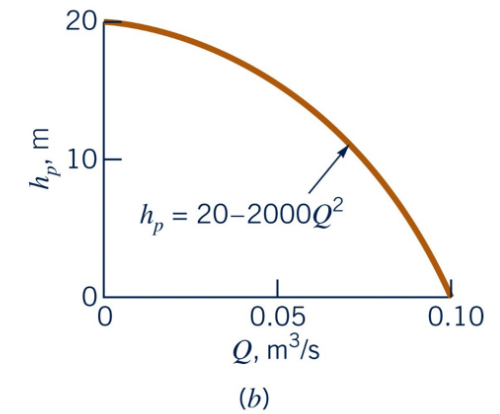


Figure P5.118b
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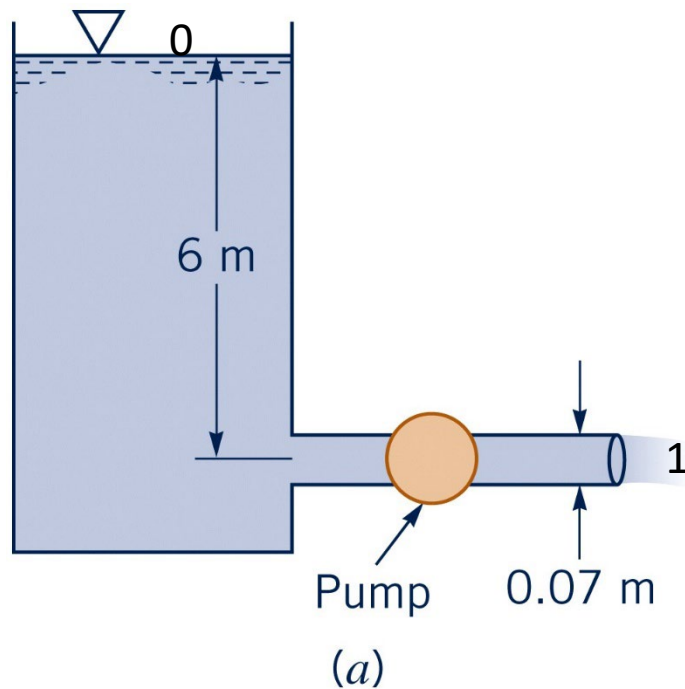


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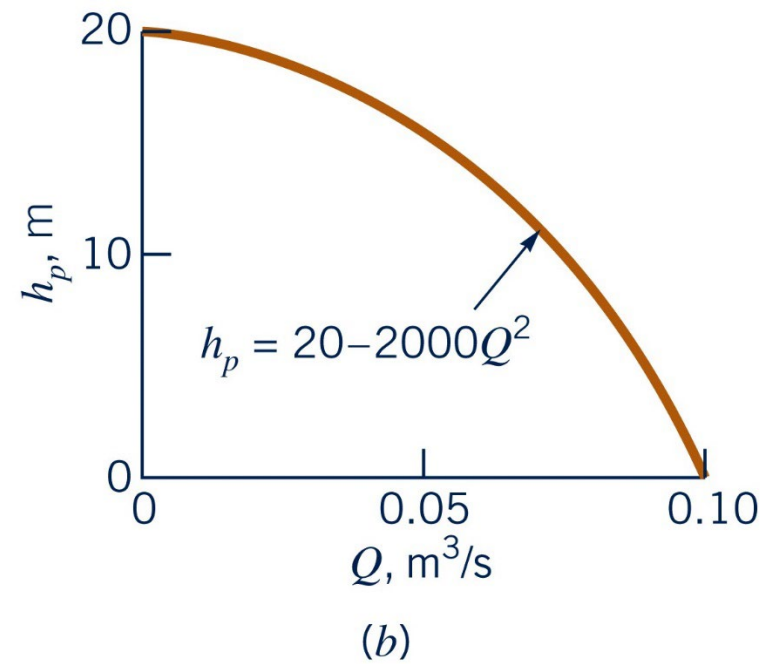


Figure P5.118b
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ENERGY CONSERVATION (Single Inlet/Exit)

$$\frac{p_0}{\gamma} + z_0 + \frac{V_0^2}{2g} + h_p = \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}} \quad (1)$$

$$p_0 = p_1 = V_0 = z_1 = h_T = 0$$

$$z_0 = 6m$$

$$6 + h_p = \frac{V_1^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}}$$

MASS CONSERVATION

$$V_1 = \frac{Q}{A_1} \quad (2)$$

Combine MASS with Energy & Pump Head

$$6 + h_p = \frac{\left[\frac{Q}{A_1} \right]^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}}$$

$$6 + (20 - 2000Q^2) = \frac{\left[\frac{Q}{A_1} \right]^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}} \quad (3)$$

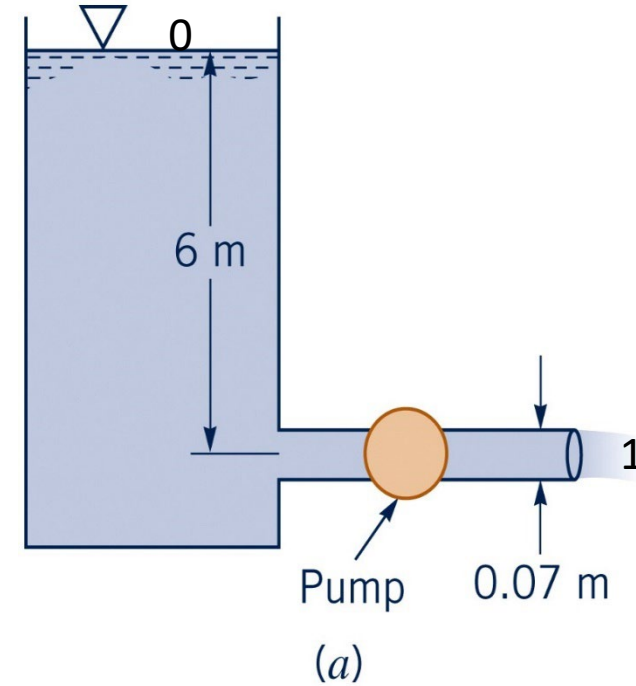


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PUMP HEAD UNITS

$$\begin{aligned}hp[m] &= 20 - 2000Q^2 \\ &= 20[m] - 2000[]Q^2 \left[\frac{m^3}{s} \right]^2\end{aligned}$$

UNIT ANALYSIS

$$m = 20[m] - 2000[]Q^2 \frac{m^6}{s^2}$$

$$m \frac{s^2}{m^6} = []$$

$$\frac{s^2}{m^5} = []$$

$$hp[m] = 20[m] - 2000 \left[\frac{s^2}{m^5} \right] Q^2 \left[\frac{m^3}{s} \right]^2$$

$$6 + (20 - 2000Q^2) = \frac{\left[\frac{Q}{A_1}\right]^2}{2g} + h_{L_{MINOR}} + h_{L_{MAJOR}} \quad (3)$$

MINOR and MAJOR LOSSES

$$h_{L_{MINOR}} = (k_{inlet} + k_{exit}) \frac{V^2}{2g} = (0.5 + 1.0) \frac{\left[\frac{Q}{A_1}\right]^2}{2g}$$

$$h_{L_{MAJOR}} = f \frac{L V^2}{D 2g} = f \frac{L}{D} \frac{\left[\frac{Q}{A_1}\right]^2}{2g}$$

COMBINE WITH ENERGY EQUATION

$$6 + (20 - 2000Q^2) = \frac{\left[\frac{Q}{A_1}\right]^2}{2g} + (0.5 + 1.0) \frac{\left[\frac{Q}{A_1}\right]^2}{2g} + f \frac{L}{D} \frac{\left[\frac{Q}{A_1}\right]^2}{2g}$$

$$6 + (20 - 2000Q^2) = Q^2 \frac{(2.5 + f \frac{L}{D})}{A_1^2 2g}$$

$$26 = Q^2 \left(\frac{(2.5 + f \frac{L}{D})}{A_1^2 2g} + 2000 \right)$$

$$\sqrt{\frac{26[m]}{\left(\frac{(2.5 + f \frac{L}{D})}{A_1^2 2g [m^4 \frac{m}{s^2}]} + 2000 \left[\frac{s^2}{m^5} \right] \right)}} = Q \rightarrow \text{FRICTION CHARACTERISTIC EQUATION}$$

$$\sqrt{\frac{m}{\frac{s^2}{m^5}}} = \sqrt{\frac{m^6}{s^2}} = \frac{m^3}{s} = Q$$



Trial-n-Error Solution

$$\sqrt{\frac{26[m]}{\left(\frac{1.5 + f \frac{L}{D}}{A_1^2 2g \left[\frac{m^4 m}{s^2} \right] + 2000 \left[\frac{s^2}{m^5} \right] \right)}} = Q \rightarrow \text{FRICTION CHARACTERISTIC EQUATION}$$

1. guess " f_{guess}^1 "
2. Solve for Q
3. Solve for $V=Q/A$
4. Solve for $Re_D = \frac{\rho V D}{\mu}$
5. Check " f_{guess}^1 " guess with HAALAND eqn.

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left(\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right)$$

$$f_{halland}^1 = \left[\frac{1}{-1.8 \log_{10} \left(\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right)} \right]^2$$

6. Compare $f_{halland}$ to f_{guess}
7. If not close(x.xxx), re-iterate
8. NEW $f_{guess}^2 = f_{halland}^1 \rightarrow \text{GO TO 2}$

HAALLAND EQUATION				
	DENS	Visc	D	
e/D	kg/m3	Pa-s	m	f (guess)
0	999	1.12E-03	0.07	0
		L	A	
		m	m2	
	m3/s	100	0.00384845	m/s
f	Q	Re	f (FALLAND)	V
0	0.060252	9.775E+05	0.0116	15.65607
0.0116	0.020102	3.261E+05	0.0141	5.223338
0.0141	0.018423	2.989E+05	0.0144	4.787201
0.0144	0.018288	2.967E+05	0.0144	4.752007
0.0144	0.018276	2.965E+05	0.0144	4.749025

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0	999	1.12E-03	0.07	1
		L	A	
		m	m2	
	m3/s	100	0.003848451	m/s
f	Q	Re	f (FALLAND)	V
1	0.002298	3.728E+04	0.0222	0.59713
0.0222	0.014967	2.428E+05	0.0149	3.889074
0.0149	0.017964	2.914E+05	0.0144	4.667822
0.0144	0.018249	2.961E+05	0.0144	4.741799
0.0144	0.018273	2.965E+05	0.0144	4.748156

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